Free Entry and Multiple Downstream Markets

Qing Hu
Ryo Masuyama
Tomomichi Mizuno

November 2026

Discussion Paper No. 2616

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

Free Entry and Multiple Downstream Markets*

Qing Hu[†ac], Ryo Masuyama[‡bc], Tomomichi Mizuno[§c]

[a] Faculty of Business and Commerce, Kansai University

[b] Faculty of Economics, Kushiro Public University of Economics

[c] Graduate School of Economics, Kobe University

November 11, 2026

Abstract

This study analyzes how intermarket spillovers through a common upstream firm affect the efficiency of free entry. We consider a model in which an upstream firm supplies a common input to two independent downstream markets, one with free entry and the other with an incumbent. We show that the effect of entry on the input price depends on the relative size of the entry market. When the entry market is relatively small, entry lowers the input price and generates positive spillovers, which tend to result in insufficient entry. In contrast, when it is relatively large, entry raises the input price and generates negative spillovers, which tend to result in excess entry. We further show that this mechanism relies on uniform input pricing but not on monopoly in either downstream or upstream markets. These findings highlight the importance of intermarket spillovers for evaluating the efficiency of free entry.

JEL codes: D43, L13, L40

Keywords: free entry; vertical market; multiple markets

*The authors gratefully acknowledge the financial support of the Japan Society for the Promotion of Science (JSPS), KAKENHI Grant Numbers JP22H00043, JP23H00764, 24K04863, and 25K21888. The usual disclaimer applies.

†Qing Hu, Faculty of Business and Commerce, Kansai University, 3-3-35 Yamate-cho, Suita-shi, Osaka, 564-8680, Japan. E-mail: huqing549@gmail.com

‡Ryo Masuyama, Faculty of Economics, Kushiro Public University of Economics, 4-1-1 Ashino, Kushiro, Hokkaido, 085-8585, Japan. E-mail: ry.masuyama@gmail.com

§Tomomichi Mizuno. Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada, Kobe, Hyogo, 657-8501, Japan. E-mail: mizuno@econ.kobe-u.ac.jp

1 Introduction

Free entry promotes market competition, thereby lowering prices and increasing consumer surplus. However, the equilibrium level of entry is not necessarily socially optimal. This is because potential entrants consider only their own profits when deciding whether to enter and do not fully internalize the competitive effects of entry or its impact on welfare. The welfare implications of free entry have therefore been extensively studied in the industrial organization literature, beginning with the well-known excess-entry theorem (Mankiw and Whinston, 1986).

However, in a vertically related market, additional effects beyond the welfare effects discussed above may arise. When entry changes the demand for an upstream input, it may also affect the input price, thereby influencing welfare in downstream markets other than the entry market. Bhattacharyya and Nain (2011) provide empirical evidence that horizontal mergers among downstream firms affect input prices by changing input demand. Their findings suggest that changes in the number of firms in one downstream market can influence not only that market itself but also other downstream markets connected through a common upstream supplier.

Such a vertically related market structure is observed, for example, in the polyvinyl chloride (PVC) industry. PVC resin is an upstream input in the production of various downstream products, including PVC pipes and medical PVC products. The PVC pipe market has many downstream producers, and barriers to entry are relatively low compared with more strictly regulated applications. In contrast, the production of medical PVC products is subject to stringent requirements concerning product standards, certification, and quality control, which can constitute significant barriers to entry. Thus, the PVC industry provides an example in which a common upstream input is supplied to downstream markets with different barriers to entry: relatively free entry in the PVC

pipe market and more limited entry in the medical PVC products market.

The PVC industry also provides a motivation for our assumption of uniform input pricing across downstream markets. Transactions for a given grade of PVC resin within a particular region are commonly based on market reference prices. Although transaction prices may differ depending on the grade, region, contract terms, and other conditions, market reference prices for standardized grades provide a common benchmark for transactions. Under such a pricing structure, changes in input demand in one downstream market can affect the input price faced by firms in other downstream markets. For example, the entry of PVC pipe producers may increase aggregate demand for PVC resin, thereby affecting the input price and the production costs of medical PVC products. Firms deciding whether to enter the PVC pipe market do not internalize this effect on the medical PVC market.

Motivated by this industry structure, this study examines how intermarket spillovers arising from upstream input pricing affect the efficiency of free entry. We consider a vertically related market structure in which a common upstream firm supplies an input to two independent downstream markets. Free entry is allowed in one downstream market, whereas the other downstream market is served by an incumbent firm.

The main findings of this study are as follows. First, when the entry market is larger than the other downstream market, additional entry raises the input price. In contrast, when the entry market is relatively small, additional entry lowers the input price. This result arises from differences in demand elasticities between the two downstream markets. When the entry market is larger (smaller) than the other downstream market, demand in the entry market is less (more) elastic than that in the other market. Consequently, additional entry makes aggregate input demand less (more) elastic, inducing the upstream firm to raise (lower) the input price.

Second, the efficiency of free entry depends on the size of the entry market relative to that of the other downstream market, rather than on the size of the entry market alone. When the entry market is relatively small, additional entry generates a positive spillover to the other market through a reduction in the input price, so insufficient entry tends to arise. In contrast, when the entry market is relatively large, additional entry raises the input price and generates a negative spillover to the other market, which may lead to excess entry. In either case, entrants fail to internalize the effects of their entry on the other downstream market through the upstream input price. Consequently, the equilibrium number of firms under free entry may differ from the socially optimal level.

We further examine the robustness of the mechanism under alternative market structures. First, we consider input price discrimination across downstream firms. In this case, the intermarket spillover through the input price disappears, and the entry market always exhibits insufficient entry. This result highlights that uniform input pricing is a crucial assumption for the main mechanism. In contrast, monopoly in either the other downstream market or the upstream market is not essential. When the non-entry downstream market is served by multiple firms or when multiple upstream firms compete in the input market, the same qualitative relationship between the relative size of the entry market and the efficiency of free entry continues to hold. These results suggest that the key mechanism arises from the linkage between downstream markets through a common input price and the degree of market power in the upstream market, rather than from the monopoly structures assumed in the main model.

These findings suggest that entry policy should take into account conditions in other markets connected through vertical relationships, in addition to those in the entry market itself. In particular, when a common upstream supplier serves multiple downstream markets, the welfare effects of entry should be evaluated by considering spillover effects

on other downstream markets through the input price.

This study is closely related to two strands of the literature: one on free entry in vertically related markets and the other on how downstream market structure affects input prices. The efficiency of free entry has long been a central topic in industrial organization. Early studies established that free entry generally results in excess entry (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). This literature has subsequently been extended to various market environments, including vertically related markets.[1] Ghosh and Morita (2007a) show that insufficient entry may arise in a bilateral oligopoly, while Ghosh and Morita (2007b) demonstrate that insufficient entry may also occur in the upstream market. Basak and Mukherjee (2016) show that when a monopolistic upstream firm supplies inputs to downstream firms operating under constant returns to scale, the downstream market always exhibits insufficient entry. Extending their framework to allow for technological asymmetry among downstream firms, Cao and Wang (2020) demonstrate that excess entry may arise when entrants possess sufficiently efficient production technologies. However, these studies consider only a single downstream market and therefore do not examine how intermarket spillovers through input pricing affect the efficiency of free entry.

The other strand of the literature examines the relationship between downstream competition and upstream input prices. Tyagi (1999) and Koulamas and Kyparisis (2010) analyze how changes in the number of downstream firms affect the input price. Arya and Mittendorf (2010) and Yenipazarli (2023) further investigate how downstream market structure influences input pricing when a monopolistic upstream firm supplies multiple downstream markets. More recently, Monden and Mizuno (2026) and Monden et al. (2025) study a market structure similar to ours and show how the nature of

[1]In addition to the studies discussed below, Milliou and Petrakis (2024) analyze downstream free entry in a vertically related market with secret contracts.

downstream competition affects the upstream input price. However, these studies assume that the number of firms is exogenous and therefore do not examine the efficiency of free entry.

By integrating the literature on free entry in vertically related markets with the literature on input pricing, this study is the first to demonstrate how intermarket spillovers through a common upstream firm affect the efficiency of free entry. Our analysis identifies a new mechanism through which free entry may result in either excess entry or insufficient entry.

The remainder of the paper is organized as follows. Section 2 presents the main model. Section 3 characterizes the equilibrium and analyzes the efficiency of the free-entry equilibrium. Section 4 examines the robustness of the main mechanism under alternative market structures. Section 5 presents the conclusion.

2 Main model

A monopolistic upstream firm, denoted by U, supplies an input to two downstream markets (market X and market Y) at the input price w. The upstream firm's production cost is assumed to be zero. In market Y, a monopolistic downstream firm, denoted by Y, supplies the final good. In market X, firms can enter by paying a fixed entry cost f. We suppose that n downstream firms enter market X, where $n \geq 1$. Each entrant i transforms one unit of the input into one unit of the final good. Downstream firms incur no production cost other than the payment for the input. We assume that downstream firms in market X produce a homogeneous final good and that the goods sold in markets X and Y are independent. We denote the output of downstream firm i in market X by q_i and that of downstream firm Y by q_Y. The inverse demand functions in markets X

and Y are $p_X = a_X - \sum_{j=1}^{n} q_j$ and $p_Y = a_Y - q_Y$, respectively, where a_X and a_Y represent the sizes of markets X and Y. We define $r = a_X/a_Y$ as the size of market X relative to that of market Y. To guarantee an interior equilibrium, we assume that $1/2 < r < 2$.

The profit functions of the firms are as follows:

$$\pi_Y = (p_Y - w)q_Y, \quad \pi_i = \begin{cases} (p_X - w)q_i - f & \text{if downstream firm } i \text{ enters,} \\ 0 & \text{otherwise,} \end{cases}$$

$$\pi_U = w\left(q_Y + \sum_{j=1}^{n} q_j\right).$$

Consumer surplus in market X is $CS_X = (\sum_{j=1}^{n} q_j)^2/2$, while consumer surplus in market Y is $CS_Y = q_Y^2/2$. Total surplus is therefore $TS = CS_X + CS_Y + \pi_Y + \sum_{j=1}^{n} \pi_j + \pi_U$.

The timing of the game is as follows. In stage 1, potential entrants decide whether to enter market X. In stage 2, the upstream firm chooses the input price. In stage 3, downstream firms simultaneously choose their outputs. We solve this game using backward induction.

3 Equilibrium analysis

The first-order conditions in stage 3 yield $q_i = (a_X - w)/(1 + n)$ and $q_Y = (a_Y - w)/2$. Substituting these outputs into the profit function of upstream firm U, solving the first-order condition, and using $r = a_X/a_Y$, we obtain the input price as follows:

$$w(n) = \frac{a_Y(1 + n + 2nr)}{2(1 + 3n)}.$$

Accordingly, we obtain the following outcomes in stage 2.

$$\pi_i(n) = \frac{a_Y^2[1+n-2(1+2n)r]^2}{4(1+n)^2(1+3n)^2} - f, \quad \pi_Y(n) = \frac{a_Y^2(1+5n-2nr)^2}{16(1+3n)^2},$$
$$\pi_U(n) = \frac{a_Y^2(1+n+2nr)^2}{8(1+n)(1+3n)},$$
$$CS_X(n) = \frac{a_Y^2 n^2[1+n-2(1+2n)r]^2}{8(1+n)^2(1+3n)^2}, \quad CS_Y(n) = \frac{a_Y^2(1+5n-2nr)^2}{32(1+3n)^2},$$
$$TS(n) = CS_X(n) + CS_Y(n) + \pi_Y(n) + \sum_{j=1}^{n} \pi_j(n) + \pi_U(n).$$

Differentiating $w(n)$ with respect to n yields the following proposition.

Proposition 1 *(i) As the number of downstream firms n in market X increases, the input price increases when $r > 1$. That is, $\partial w(n)/\partial n > 0$ if $r > 1$. (ii) The effect of the number of downstream firms in market X on the input price becomes stronger as the size difference between markets X and Y increases. That is, $\partial^2 w(n)/\partial r \partial n > 0$.*

Proof. See Appendix.

The intuition behind Proposition 1 is as follows. First, we consider Proposition 1 (i). Under a linear inverse demand function, the demand for the final good becomes less price elastic as the intercept of the inverse demand function, which represents market size, increases. This property also holds for the price elasticity of input demand. Hence, when market X is larger than market Y, input demand from market X is less elastic. As the number of firms in market X increases, the share of relatively inelastic input demand rises. Consequently, the upstream firm raises the input price as the number of downstream firms increases. Moreover, as r becomes larger, the proportion of input demand from market X increases, strengthening the effect of downstream entry on the input price. When market X is smaller than market Y, the same mechanism

applies in the opposite direction. Thus, we obtain the result stated in Proposition 1 (ii), $\partial^2 w(n)/\partial n \partial r > 0$.

Next, differentiating the firms' profits and consumer surplus with respect to n yields the following proposition.

Proposition 2 *As the number of downstream firms n in market X increases, (i) the profit of downstream firm i decreases, whereas the profit of the upstream firm U increases. Moreover, the profit of downstream firm Y increases if $r < 1$. That is, $\partial \pi_i(n)/\partial n < 0$; $\partial \pi_U(n)/\partial n > 0$; $\partial \pi_Y(n)/\partial n > 0$ if $r < 1$. (ii) Consumer surplus in market X increases. Furthermore, consumer surplus in market Y also increases if $r < 1$. That is, $\partial CS_X(n)/\partial n > 0$; $\partial CS_Y(n)/\partial n > 0$ if $r < 1$.*

Proof. See Appendix.

The intuition behind Proposition 2 is straightforward. For Proposition 2 (i), as the number of downstream firms in market X increases, competition in market X intensifies, reducing the profit of downstream firm i. At the same time, the increase in aggregate input demand raises the profit of the upstream firm U. Although downstream firm Y does not compete directly with firm i, changes in n affect the input price. Proposition 1 shows that when $r < 1$, additional entry lowers the input price, thereby increasing the profit of downstream firm Y.

The intuition behind Proposition 2 (ii) is as follows. As the number of firms in market X increases, total output in market X rises, thereby increasing consumer surplus in that market. Consumer surplus in market Y depends on the marginal cost of downstream firm Y. As shown in Proposition 1, when $r < 1$, additional entry lowers the input price, thereby reducing the firm's marginal cost and increasing consumer surplus in market Y.

To simplify the analysis of stage 1, we first show that n, the equilibrium number of

downstream firms in market X, decreases with the entry cost f. This allows us to treat n as an exogenous parameter instead of f without affecting the analysis, provided that the corresponding value of f is chosen accordingly. Proposition 2 (i) shows that the profit of downstream firm i is decreasing in n. Therefore, it remains only to show that the gross profit, $\pi_i(n) + f$, converges to zero as n approaches infinity. This follows immediately because the degree of n in the denominator of the first term of $\pi_i(n)$ exceeds that in the numerator.

Therefore, the equilibrium number of firms n is a decreasing function of the entry cost f, and any number of firms $n \geq 1$ can be induced by a unique entry cost f. We denote the equilibrium number of firms by $n = N(f)$. The inverse function $f = N^{-1}(n)$ then exists. Hence, in the remainder of the paper, we treat n as a parameter and assume that the corresponding value of f is chosen, without loss of generality.

Under free entry, the equilibrium condition is $\pi_i(n) = 0$. Solving this condition for f, we obtain the entry cost as follows:

$$f^* = \frac{a_Y^2[1 + n - 2(1 + 2n)r]^2}{4(1 + n)^2(1 + 3n)^2}.$$

Differentiating $TS(n)$ with respect to n and evaluating it at $f = f^*$, we obtain the following result:

$$\left.\frac{\partial TS(n)}{\partial n}\right|_{f=f^*} = \frac{a_Y^2(\Phi_0 + \Phi_1 r - \Phi_2 r^2)}{8(1 + n)^3(1 + 3n)^3},$$

where $\Phi_0 = (1 - n)(1 + n)^3$, $\Phi_1 = 1 + 26n + 100n^2 + 134n^3 + 59n^4$, and $\Phi_2 = 2n(5 + 23n + 39n^2 + 29n^3)$. Since the sign of this derivative is determined by the numerator, $\Phi_0 + \Phi_1 r - \Phi_2 r^2$, we obtain the following proposition for $1/2 < r < 2$.

Proposition 3 *Market X exhibits excess entry if $r > r^{TS}$ and insufficient entry if*

$r < r^{TS}$, *where*

$$r^{TS} = \frac{\Phi_1 + \sqrt{\Phi_1^2 + 4\Phi_0\Phi_2}}{2\Phi_2}.$$

Proof. See Appendix.

Figure 1 illustrates the threshold characterized in Proposition 3. The horizontal axis represents the number of downstream firms in market X, n, while the vertical axis represents the relative size of market X, r. The curve represents the threshold r^{TS}. Insufficient entry occurs in the region below the curve, whereas excess entry occurs in the region above it. Figure 1 also shows that insufficient entry tends to arise when the relative size of market X is small.

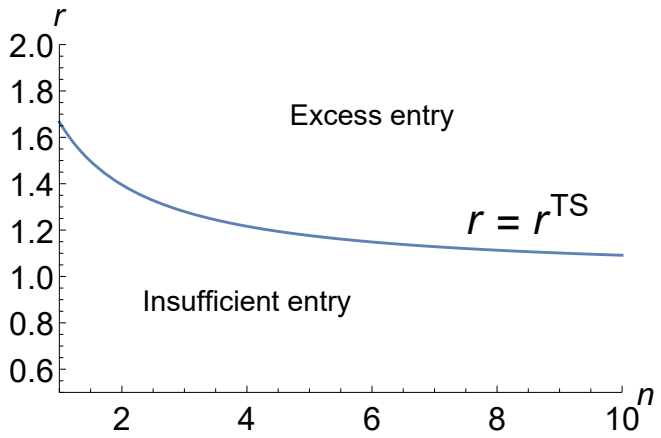


Figure 1: Condition for insufficient entry.

To provide intuition for Proposition 3, we decompose the effect of the number of downstream firms n on total surplus. Using the results of Proposition 2, the sign of each component can be written as follows:

$$\frac{\partial TS(n)}{\partial n} = \underbrace{\frac{\partial CS_X(n)}{\partial n}}_{+} + \underbrace{\frac{\partial CS_Y(n)}{\partial n}}_{+ \text{ if } r<1} + n\underbrace{\frac{\partial \pi_i(n)}{\partial n}}_{-} + \underbrace{\pi_i(n)}_{+} + \underbrace{\frac{\partial \pi_Y(n)}{\partial n}}_{+ \text{ if } r<1} + \underbrace{\frac{\partial \pi_U(n)}{\partial n}}_{+} \qquad (1)$$

If $\partial TS(n)/\partial n|_{f=f^*} > 0$ holds at the free-entry equilibrium, the market exhibits insufficient entry. Equation (1) shows that when r is small, an additional welfare gain arises from market Y. Moreover, this positive effect becomes stronger as r decreases.

The intuition is as follows. Proposition 1 shows that when $r < 1$, additional entry lowers the input price, thereby increasing both consumer surplus and downstream firms' profits in market Y. However, potential entrants do not internalize these welfare gains in market Y when making their entry decisions. Consequently, when $r < 1$, as the relative size of market X decreases, additional entry causes a larger reduction in the input price, thereby generating a stronger positive spillover to market Y. This tends to result in insufficient entry.

When $r > 1$, additional entry raises the input price, thereby worsening the double marginalization problem. As r increases, the increase in the input price caused by additional entry becomes larger. However, because entering firms do not internalize the double marginalization problem, free entry tends to result in excess entry when r is large.

Proposition 3 also yields the following policy implication. Previous studies have shown that free entry tends to result in excess entry in non-vertical markets (Mankiw and Whinston, 1986), whereas in vertically related markets insufficient entry may arise depending on the characteristics of the market (Basak and Mukherjee, 2016; Cao and Wang, 2020; Ghosh and Morita, 2007a; Ghosh and Morita, 2007b). In contrast, this study shows that entry affects not only welfare in market X but also welfare in market Y through changes in the upstream input price. These intermarket spillovers determine the efficiency of the free-entry equilibrium, and they arise through changes in the input price, as reflected by the terms $\partial CS_Y(n)/\partial n$ and $\partial \pi_Y(n)/\partial n$ in equation (1). Therefore, designing optimal entry policy requires considering spillover effects on related markets in addition to the effects of entry within the entry market. For example, when the entry

market is relatively small, additional entry lowers the input price and thereby increases welfare in the neighboring market. If this positive spillover is sufficiently large, entry subsidies may improve social welfare. Conversely, when the entry market is relatively large, additional entry raises the input price and reduces welfare in the neighboring market. In such a case, policies that discourage entry may be socially desirable.

4 Robustness

To clarify the mechanisms underlying the main model, this section extends it in three directions. First, we consider the case in which input price discrimination is allowed. Second, we consider the case in which downstream market Y is oligopolistic. Third, we consider the case in which the upstream market is oligopolistic. We show that the assumption of uniform input pricing is crucial to our results, whereas the assumptions of monopoly in market Y and the upstream market are not.

4.1 Input price discrimination

In the main model, the upstream firm adopts uniform input pricing. This assumption is reasonable when the upstream market supplies inputs such as PVC resin, as transactions are commonly based on a common reference price for the same grade and region. This feature of input prices has also been considered in previous studies, and the assumption of uniform input pricing has been accepted to some extent. In addition, regulations, such as the Robinson-Patman Act, prohibit input price discrimination.

However, some industries practice input price discrimination. In this study, downstream markets affect each other through a uniform input price, and this effect underlies the central mechanism of the analysis. Input price discrimination changes this key mech-

anism. The purpose of this subsection is to clarify that the results of this study do not apply to industries in which input price discrimination is practiced.

We suppose that upstream firm U can offer different input prices to each downstream firm. We denote the input prices faced by downstream firm Y and downstream firm i in market X by w_Y and w_i, respectively. Then, the profits of the firms are defined as follows.

$$\pi_Y = (p_Y - w_Y)q_Y, \quad \pi_i = \begin{cases} (p_X - w_i)q_i - f & \text{if downstream firm } i \text{ enters,} \\ 0 & \text{otherwise,} \end{cases}$$
$$\pi_U = w_Y q_Y + \sum_{j=1}^{n} w_j q_j.$$

The other assumptions are the same as those in the main model.

The first-order conditions in the final stage and the sum of the first-order conditions for market X yield the following equations:

$$\frac{\partial \pi_Y}{\partial q_Y} = a_Y - 2q_Y - w_Y = 0, \quad \frac{\partial \pi_i}{\partial q_i} = a_X - q_i - \sum_{j=1}^{n} q_j - w_i = 0,$$
$$\sum_{k=1}^{n} \frac{\partial \pi_k}{\partial q_k} = n a_X - (n+1)\sum_{j=1}^{n} q_j - \sum_{j=1}^{n} w_j = 0.$$

Therefore, the outputs in the final stage are as follows.

$$\bar{q}_i = \frac{a_X - (1+n)w_i + \sum_{j=1}^{n} w_j}{1+n}, \quad \sum_{j=1}^{n} \bar{q}_j = \frac{n a_X - \sum_{j=1}^{n} w_j}{1+n}, \quad \bar{q}_Y = \frac{a_Y - w_Y}{2}.$$

Substituting these outcomes into the profit function of the upstream firm and using

the first-order conditions, we obtain the following equations:

$$\frac{\partial \pi_U}{\partial w_i} = \frac{1}{1+n}\left[a_X - 2nw_i + \sum_{j\neq i} w_j + (1+n)\sum_{j\neq i} w_j \frac{\partial \bar{q}_j}{\partial w_i}\right] = 0, \quad \frac{\partial \pi_U}{\partial w_Y} = \frac{a_Y - 2w_Y}{2} = 0.$$

Using the symmetry of the downstream firms in market X and $\partial \bar{q}_j/\partial w_i = 1/(1+n)$, the input prices are as follows.

$$\bar{w_i} = \frac{a_X}{2}, \quad \bar{w_Y} = \frac{a_Y}{2}.$$

Using these outcomes, the profit of a downstream firm in market X and total surplus are as follows.

$$\bar{\pi}_i(n) = \frac{a_X^2}{4(1+n)^2} - f, \quad \overline{TS}(n) = \frac{a_X^2 n(4+3n)}{8(1+n)^2} + \frac{7a_Y^2}{32} - nf.$$

Differentiating total surplus with respect to n and substituting the fixed cost that makes the downstream firms' profit equal to zero, $\bar{f} = a_X^2/[4(1+n)^2]$, leads to the following result:

$$\left.\frac{\partial \overline{TS}(n)}{\partial n}\right|_{f=\bar{f}} = \frac{a_X^2}{4(1+n)^3} > 0.$$

Therefore, we obtain the following proposition.

Proposition 4 *When the upstream firm practices input price discrimination, market X exhibits insufficient entry.*

Basak and Mukherjee (2016) show that when firms in a downstream market with free entry purchase an input from a monopolistic upstream firm and produce under constant returns to scale, insufficient entry arises. Proposition 4 is consistent with their result and shows that the property of insufficient entry is preserved even when there are multiple downstream markets.

The intuition behind this result rests on the following two properties. First, in the main model, an increase in the input price caused by additional entry in market X is necessary for excess entry in market X to arise. Indeed, one of the necessary conditions for excess entry in Proposition 3 is $r > 1$, which corresponds to the region in which the input price increases as the number of downstream firms in market X increases.

The second property is as follows. When the upstream firm practices input price discrimination, input demand in market X is separated from that in market Y. The input demand in each downstream market is linear in the input price, and the choke price in each downstream market is independent of the number of downstream firms. Therefore, the intercept of the inverse demand function for input is also independent of the number of downstream firms. Hence, the number of downstream firms affects only the slope of the linear input demand function, implying that the input price chosen by the upstream firm is independent of the number of downstream firms.

These two properties explain why market X exhibits insufficient entry. Even if an additional downstream firm enters market X, the input price does not change. Thus, as long as additional entry does not increase the input price, market X exhibits insufficient entry.

4.2 Oligopoly in market Y

In the main model, downstream market Y is monopolistic. In this subsection, we consider the case in which market Y has $m \geq 1$ firms. In market Y, q_{Yk} denotes the output of downstream firm k, and the inverse demand function is given by $p_Y = a_Y - \sum_{j=1}^{m} q_{Yj}$. The profit of downstream firm k in market Y is $\pi_{Yk} = (p_Y - w)q_{Yk}$, while the profit of upstream firm U is $\pi_U = w(\sum_{j=1}^{n} q_j + \sum_{j=1}^{m} q_{Yj})$. Consumer surplus in market Y is $CS_Y = (\sum_{j=1}^{m} q_{Yj})^2/2$, and total surplus is $TS = CS_X + CS_Y + \sum_{j=1}^{m} \pi_{Yj} + \sum_{j=1}^{n} \pi_j + \pi_U$.

The other assumptions are the same as those in the main model.

The outputs in the final stage are $\hat{q}_i = (a_X - w)/(1+n)$ and $\hat{q}_{Yj} = (a_Y - w)/(1+m)$. Substituting these outcomes into the upstream firm's profit function and solving the first-order condition, we obtain the following input price:

$$\hat{w} = \frac{a_X(1+m)n + a_Y m(1+n)}{2(m+n+2mn)}.$$

Substituting the outcome into the profit of a downstream firm in market X yields the following condition determining the number of entrants.

$$\hat{f}^* = \frac{a_Y^2[m(1+n) - (2m+n+3mn)r]^2}{4(1+n)^2(m+n+2mn)^2}.$$

Substituting the outcomes from stages 2 and 3 into total surplus, differentiating it with respect to n, and evaluating the resulting derivative at $f = \hat{f}^*$, we obtain the following result.

$$\left.\frac{\partial TS}{\partial n}\right|_{f=\hat{f}^*} = \frac{a_Y^2(\hat{\Phi}_0 + \hat{\Phi}_1 r - \hat{\Phi}_2 r^2)}{4(1+m)(1+n)^3(m+n+2mn)^3}. \tag{2}$$

where $\hat{\Phi}_0 = m^2(1+n)^3(m+n-2m^2n)$, $\hat{\Phi}_1 = m(1+n)[m^3(1+12n+26n^2+16n^3) + m^2n(11+34n+26n^2) + mn(2+13n+14n^2) + n^2(2+3n)]$, and $\hat{\Phi}_2 = (1+m)n[m^3(5+20n+28n^2+14n^3) + m^2n(4+13n+12n^2) - mn(1+n-3n^2) - n^2]$. Therefore, we obtain the following proposition.

Proposition 5 *When the downstream market Y has m firms, market X exhibits excess*

entry if $r > \hat{r}^{TS}(m)$*, where*

$$\hat{r}^{TS}(m) = \frac{\hat{\Phi}_1 + \sqrt{\hat{\Phi}_1^2 + 4\hat{\Phi}_0\hat{\Phi}_2}}{2\hat{\Phi}_2}.$$

Proof. See Appendix.

The result of this proposition is similar to that of Proposition 3 and therefore shares the same intuition.

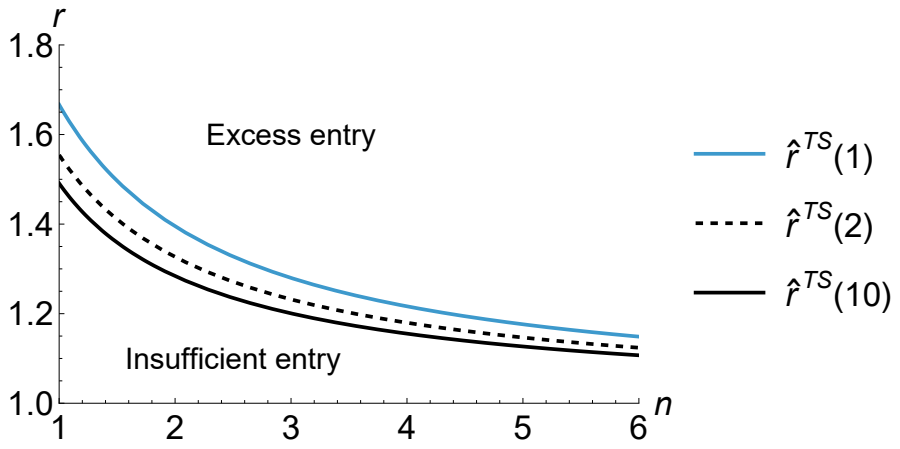


Figure 2: Condition for insufficient entry with multiple firms in market Y.

Figure 2 plots the threshold characterized in Proposition 5 for $m = 1, 2, 10$. The horizontal axis represents the number of downstream firms in market X, n, while the vertical axis represents the relative size of market X, $r = a_X/a_Y$. The blue curve represents the case with $m = 1$, which corresponds to the condition in Proposition 3. The black dashed curve represents the case with $m = 2$, while the black solid curve represents the case with $m = 10$. The figure shows that the threshold $\hat{r}^{TS}(m)$ shifts downward as m increases. Therefore, an increase in the number of firms in market Y expands the region in which excess entry arises in market X. The intuition behind this property is

as follows. As the number of firms m in downstream market Y increases, competition in that market becomes more intense, increasing input demand. As a result, in determining the input price, the upstream firm places more weight on the characteristics of market Y. When $r > 1$, input demand in market X is inelastic, while that in market Y is elastic. As the number of firms in market Y increases, input demand in market Y becomes more elastic, widening the difference in input demand elasticities between the two markets. Consequently, an increase in the number of firms in market X causes a larger rise in the input price when market Y has more firms. Therefore, an increase in the number of firms in market Y tends to promote excess entry in market X.

4.3 Upstream oligopoly

In the main model, we consider the case in which the upstream market is monopolistic. In this subsection, we consider a setting in which the upstream market is oligopolistic. This analysis shows that the assumption of upstream monopoly is not essential to our results; rather, the degree of market power in the upstream market plays a key role.

We suppose that the upstream market has $u \geq 1$ firms that produce a homogeneous input and compete in quantities.[2] We denote the output of upstream firm $k \in \{1, \ldots, u\}$ by x_k. Each upstream firm produces the input at zero marginal cost and sells it at input price w. The profit of upstream firm k is $\pi_{Uk} = wx_k$. Therefore, total surplus is $TS = CS_X + CS_Y + \pi_Y + \sum_{j=1}^{n} \pi_j + \sum_{j=1}^{u} \pi_{Uj}$.

The determination of the input price follows the successive Cournot model (Salinger, 1988; Greenhut and Ohta, 1979).[3] Specifically, downstream firms choose their outputs

[2]If upstream firms compete in prices, the input price is always equal to marginal cost, which is zero, as a result of price competition with a homogeneous good. The analysis then reduces to free entry in a market consisting only of downstream firms, and the result of Mankiw and Whinston (1986) applies, implying that downstream market X exhibits excess entry.

[3]The successive Cournot model has also been used in recent studies. See, for example, Avenel (2026),

taking the input price as given. The derived inverse demand function for the input is obtained from the condition that input demand, $q_Y(w) + \sum_{j=1}^{n} q_j(w)$, equals the total supply of the input by upstream firms, $\sum_{j=1}^{u} x_j$. Upstream firms then engage in quantity competition facing this derived inverse demand function.[4]

The timing of the game is as follows. In stage 1, downstream firms decide whether to enter market X. In stage 2, each upstream firm chooses its output, and the input price is determined by the derived inverse demand function. In stage 3, each downstream firm chooses its output, taking the input price as given. The other settings are the same as those in the main model.

The outputs in the final stage are the same as those in the main model. Total output in the downstream markets is as follows.

$$q_Y(w) + \sum_{j=1}^{n} q_j(w) = \frac{n(a_X - w)}{1+n} + \frac{a_Y - w}{2}.$$

Using the market clearing condition for the input, $q_Y(w) + \sum_{j=1}^{n} q_j(w) = \sum_{j=1}^{u} x_j$, and solving for w, we obtain the following inverse demand function.

$$w\left(\sum_{j=1}^{u} x_j\right) = \frac{2a_X n + a_Y(1+n) - 2(1+n)\sum_{j=1}^{u} x_j}{1+3n}.$$

Using this inverse demand function and solving each upstream firm's first-order condi-

Matsumura et al. (2025), and Tsuritani (2025).

[4]In off-equilibrium subgames, there may be a problem in which input supply and demand do not coincide. This issue can be addressed by the competitive allocation rule used in Avenel (2026). Even if the competitive allocation rule is explicitly incorporated into the model, the equilibrium outcome is the same as that in Salinger (1988). Therefore, as in much of the previous literature, we follow the approach of Salinger (1988).

tion, we obtain the following output for each upstream firm.

$$x_k = \frac{2a_X n + a_Y(1+n)}{2(1+n)(1+u)}.$$

Substituting the outcome into the profit of a downstream firm in market X and equating the resulting profit to zero, we obtain the following condition determining the number of entrants.

$$\tilde{f} = \frac{a_Y^2[1+n-(1+n+u+3nu)r]^2}{(1+n)^2(1+3n)^2(1+u)^2}$$

We denote total surplus after substituting the outcomes from stages 2 and 3 by $\widetilde{TS}(n)$. Differentiating it with respect to n and evaluating the derivative under the condition that determines the number of entrants yields the following result.

$$\left.\frac{\partial \widetilde{TS}(n)}{\partial n}\right|_{f=\tilde{f}} = \frac{a_Y^2(\tilde{\Phi}_0 + \tilde{\Phi}_1 r - \tilde{\Phi}_2 r^2)}{2(1+n)^3(1+3n)^3(1+u)^2},$$

where $\tilde{\Phi}_0 = (1+n)^3[-4n+(1+3n)u]$, $\tilde{\Phi}_1 = (1+n)[8n(1+n)^2+(1+3n)(1+14n+17n^2)u]$, $\tilde{\Phi}_2 = 2n[2(1+n)^3+2(1+n)(1+3n)u+(1+3n)^3u^2]$. Therefore, we obtain the following proposition.

Proposition 6 *When the upstream market has u firms, market X exhibits insufficient entry if $u \leq 4$ and $r < \tilde{r}^{TS}(u)$, and excess entry otherwise, where*

$$\tilde{r}^{TS}(u) = \frac{\tilde{\Phi}_1 + \sqrt{\tilde{\Phi}_1^2 + 4\tilde{\Phi}_0\tilde{\Phi}_2}}{2\tilde{\Phi}_2}.$$

Proof. See Appendix.

The result of this proposition is similar to that of Proposition 3 and therefore shares

the same intuition.

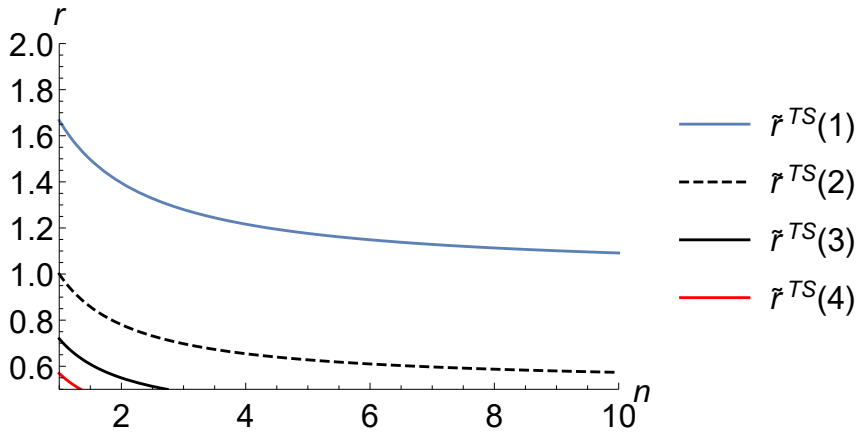


Figure 3: Condition for insufficient entry with multiple upstream firms.

Figure 3 plots the threshold characterized in Proposition 6 for $u = 1, 2, 3, 4$. The horizontal axis represents the number of downstream firms in market X, n, while the vertical axis represents the relative size of market X, $r = a_X/a_Y$. The blue curve represents the case with $u = 1$, which corresponds to the condition in Proposition 3. The black dashed curve represents the case with $u = 2$, the black solid curve represents the case with $u = 3$, and the red solid curve represents the case with $u = 4$. Figure 3 shows that the threshold $\tilde{r}^{TS}(u)$ shifts downward as u increases. Therefore, an increase in the number of upstream firms expands the region in which excess entry arises in market X. The intuition behind this property is as follows. An increase in the number of upstream firms intensifies competition in the upstream market, resulting in a lower input price. In the main model, changes in the input price play an important role in causing insufficient entry. As the number of upstream firms increases, changes in the input price become smaller, thereby weakening the mechanism through which a reduction in the input price leads to insufficient entry. Therefore, the region in which excess entry occurs expands as the number of upstream firms increases.

5 Conclusion

This study examines how intermarket spillovers affect the efficiency of free entry in vertically related markets. We consider two independent downstream markets supplied by a common upstream firm, with free entry in one and an incumbent firm in the other. In this setting, changes in the number of firms in the entry market affect the input price chosen by the upstream firm, thereby generating spillover effects on the other downstream market.

The analysis yields two main findings. First, the effect of additional entry on the input price depends on the relative size of the entry market. When the entry market is larger than the other downstream market, additional entry raises the input price. In contrast, when the entry market is relatively small, additional entry lowers the input price. Second, because changes in the input price affect surplus not only in the entry market but also in the other downstream market, these intermarket spillovers play a key role in determining the efficiency of the free-entry equilibrium. Specifically, when the entry market is relatively small, additional entry generates positive spillovers in the other downstream market, so insufficient entry tends to arise. Conversely, when the entry market is relatively large, negative spillovers reduce surplus in the other downstream market and may lead to excess entry.

These findings imply that the efficiency of free entry cannot be evaluated solely by considering competition within the entry market. It is also necessary to account for spillover effects on the other downstream market through changes in the common input price. By integrating the literature on free entry in vertically related markets with the literature on input pricing, this study identifies a new mechanism through which intermarket spillovers influence the efficiency of the free-entry equilibrium.

We also examine the robustness of this mechanism under alternative market struc-

tures. When the upstream firm can charge different input prices to downstream firms, intermarket spillovers disappear, and the entry market always exhibits insufficient entry. This result highlights the importance of uniform input pricing for the main mechanism. By contrast, monopoly in either the non-entry downstream market or the upstream market is not essential. When the non-entry downstream market is served by multiple firms, or when multiple upstream firms compete in the input market, the qualitative relationship between the relative size of the entry market and the efficiency of free entry continues to hold. These results indicate that the central mechanism arises from the linkage between downstream markets through a common input price rather than from the monopoly structures assumed in the main model.

From a policy perspective, the design of entry regulation or entry promotion policies should take into account spillover effects on related downstream markets through a common upstream supplier, in addition to welfare effects within the entry market. In industries where a common upstream supplier serves multiple downstream markets, entry policy should therefore be designed with a focus on overall social welfare rather than the welfare of an individual market.

The analysis can be extended in several directions. One possible extension is to allow for free entry in multiple downstream markets, which would enable us to examine how entry decisions interact across markets. Another direction is to consider more general forms of input pricing, including endogenous input price discrimination. Such extensions could provide further insights into how the structure of upstream pricing shapes intermarket spillovers and the efficiency of free entry.

Appendix

A1. Proof of Proposition 1

First, we consider Proposition 1 (i). Differentiating $w(n)$ with respect to n, we obtain the following result.

$$\frac{\partial w(n)}{\partial n} = -\frac{a_Y(1-r)}{(1+3n)^2}.$$

Therefore, we find that $\partial w(n)/\partial n > 0$ when $r > 1$.

Next, we consider Proposition 1 (ii). Differentiating $\partial w(n)/\partial n$ with respect to r, we obtain the following result.

$$\frac{\partial^2 w(n)}{\partial r \partial n} = \frac{a_Y}{(1+3n)^2} > 0.$$

Thus, we obtain Proposition 1.□

A2. Proof of Proposition 2

First, we consider Proposition 2 (i). Differentiating $\pi_i(n)$, $\pi_U(n)$, and $\pi_Y(n)$ with respect to n, we obtain the following results.

$$\frac{\partial \pi_i(n)}{\partial n} = -\frac{a_Y^2[-1-n+(2+4n)r][-3-6n-3n^2+(4+12n+12n^2)r]}{2(1+n)^3(1+3n)^3} < 0,$$
$$\frac{\partial \pi_U(n)}{\partial n} = \frac{a_Y^2(1+n+2nr)[-1-n+(2+4n)r]}{4(1+n)^2(1+3n)^2} > 0,$$
$$\frac{\partial \pi_Y(n)}{\partial n} = \frac{a_Y^2(1-r)(1+5n-2nr)}{4(1+3n)^3} > 0 \text{ if } r < 1.$$

Next, we consider Proposition 2 (ii). Differentiating $CS_X(n)$ and $CS_Y(n)$ with respect

to n, we obtain the following results.

$$\frac{\partial CS_X(n)}{\partial n} = \frac{a_Y^2 n[-1-n+(2+4n)r][-1-2n-n^2+(2+8n+10n^2)r]}{4(1+n)^3(1+3n)^3} > 0,$$
$$\frac{\partial CS_Y(n)}{\partial n} = \frac{a_Y^2(1-r)(1+5n-2nr)}{8(1+3n)^3} > 0 \text{ if } r < 1.$$

Therefore, we obtain Proposition 2.□

A3. Proof of Proposition 3

As shown earlier, by differentiating total surplus with respect to n and evaluating it under the free-entry equilibrium condition, we obtain the following result:

$$\left.\frac{\partial TS(n)}{\partial n}\right|_{f=f^*} = \frac{a_Y^2(\Phi_0+\Phi_1 r-\Phi_2 r^2)}{8(1+n)^3(1+3n)^3},$$

This sign coincides with that of $\Phi_0+\Phi_1 r-\Phi_2 r^2$ in the numerator. As this is quadratic in r, we denote its two roots as follows.

$$r_1 = \frac{\Phi_1-\sqrt{\Phi_1^2+4\Phi_0\Phi_2}}{2\Phi_2}, \quad r^{TS} = \frac{\Phi_1+\sqrt{\Phi_1^2+4\Phi_0\Phi_2}}{2\Phi_2}.$$

We now show that $r_1 \le 1/2$ and $1/2 < r^{TS} < 2$ for any $n \ge 1$.

First, we show that $r_1 \le 1/2$.

$$\frac{1}{2} - r_1 = \frac{-(1+16n+54n^2+56n^3+n^4)+(1+n)(1+3n)\sqrt{\psi}}{4n(5+23n+39n^2+29n^3)},$$

where $\psi = 1+84n+446n^2+708n^3+361n^4$. We find that the numerator is positive for

$n \geq 1$, as shown by the following calculation.

$$(1+n)^2(1+3n)^2\psi - (1+16n+54n^2+56n^3+n^4)^2$$
$$= 4n(15+194n+1077n^2+3288n^3+5961n^4+6426n^5+3731n^6+812n^7) > 0.$$

Next, we have $1/2 < r^{TS}$, as shown by the following equation.

$$r^{TS} - \frac{1}{2} = \frac{1+16n+54n^2+56n^3+n^4+(1+n)(1+3n)\sqrt{\psi}}{4n(5+23n+39n^2+29n^3)} > 0.$$

Finally, we show that $r^{TS} < 2$.

$$2 - r^{TS} = \frac{-1+14n+84n^2+178n^3+173n^4-(1+n)(1+3n)\sqrt{\psi}}{4n(5+23n+39n^2+29n^3)}$$

We find that the numerator is positive from the following calculation and $n \geq 1$.

$$(-1+14n+84n^2+178n^3+173n^4)^2 - (1+n)^2(1+3n)^2\psi$$
$$= 8n(-15-139n-519n^2-771n^3+603n^4+3975n^5+5819n^6+3335n^7) > 0.$$

Therefore, we obtain this proposition.□

A4. Proof of Proposition 5

As shown earlier, differentiating total surplus with respect to n and evaluating it under the free-entry equilibrium condition yields the following result:

$$\left.\frac{\partial TS(n)}{\partial n}\right|_{f=\hat{f}^*} = \frac{a_Y^2(\hat{\Phi}_0+\hat{\Phi}_1 r-\hat{\Phi}_2 r^2)}{4(1+m)(1+n)^3(m+n+2mn)^3},$$

This sign coincides with that of $\hat{\Phi}_0 + \hat{\Phi}_1 r - \hat{\Phi}_2 r^2$ in the numerator. This quadratic expression has two roots, which we denote as follows.

$$\hat{r}_1 = \frac{\hat{\Phi}_1 - \sqrt{\hat{\Phi}_1^2 + 4\hat{\Phi}_0\hat{\Phi}_2}}{2\hat{\Phi}_2}, \quad \hat{r}^{TS} = \frac{\hat{\Phi}_1 + \sqrt{\hat{\Phi}_1^2 + 4\hat{\Phi}_0\hat{\Phi}_2}}{2\hat{\Phi}_2}.$$

We show that $\hat{r}_1 \leq 1/2$ and $1/2 < \hat{r}^{TS} < 2$ for any $n \geq 1$ and $m \geq 1$.

First, we show that $\hat{r}_1 \leq 1/2$.

$$\frac{1}{2} - \hat{r}_1 = \frac{\left[\begin{array}{l} -m^4(1+8n+18n^2+14n^3+2n^4) - m^3n(6+21n+19n^2) \\ -m^2n(2+12n+15n^2-n^3) - mn^2(3+7n) - n^3 \\ +m(1+n)(m+n+2mn)\sqrt{\hat{\psi}} \end{array}\right]}{\left[\begin{array}{l} 2(1+m)n(m^3(5+20n+28n^2+14n^3) + m^2n(4+13n+12n^2) \\ +mn(-1-n+3n^2) - n^2) \end{array}\right]},$$

where $\hat{\psi} = m^4(1+20n+72n^2+88n^3+36n^4) + 4m^3n(10+47n+66n^2+30n^3) + 2m^2n(12+77n+130n^2+68n^3) + 4mn^2(8+22n+15n^2) + n^3(8+9n)$. We find that the numerator is positive from the following equation, $n \geq 1$, and $m \geq 1$.

$$\begin{aligned} & m^2(1+n)^2(m+n+2mn)^2\hat{\psi} \\ & \quad - \left[\begin{array}{l} m^4(1+8n+18n^2+14n^3+2n^4) + m^3n(6+21n+19n^2) \\ +m^2n(2+12n+15n^2-n^3) + mn^2(3+7n) + n^3 \end{array}\right]^2 \\ & = (1+m)n \left[\begin{array}{l} m^3(5+20n+28n^2+14n^3) + m^2n(4+13n+12n^2) \\ +mn(-1-n+3n^2) - n^2 \end{array}\right] \\ & \quad \times \left[\begin{array}{l} m^4(2+13n+32n^2+32n^3+10n^4) + m^3(4+29n+78n^2+83n^3+26n^4) \\ +m^2n(8+39n+54n^2+17n^3) + mn^2(5+12n+3n^2) + n^3 \end{array}\right] > 0. \end{aligned}$$

Next, we obtain that $1/2 < \hat{r}^{TS}$ from the following equation.

$$\hat{r}^{TS} - \frac{1}{2} = \frac{\left[\begin{array}{l} m^4(1+8n+18n^2+14n^3) + m^3n(6+21n+19n^2) \\ +m^2n(2+12n+15n^2) + mn^2(3+7n) + (n^3 - m^2n^4 + 2m^4n^4) \\ +m(1+n)(m+n+2mn)\sqrt{\hat{\psi}} \end{array}\right]}{\left[\begin{array}{l} 2(1+m)n(m^3(5+20n+28n^2+14n^3) + m^2n(4+13n+12n^2) \\ +mn(-1-n+3n^2) - n^2) \end{array}\right]} > 0.$$

Finally, we show that $\hat{r}^{TS} < 2$.

$$2 - \hat{r}^{TS} = \frac{\left[\begin{array}{l} m^4(-1+7n+42n^2+70n^3+40n^4) + m^3n(9+51n+104n^2+78n^3) \\ +m^2n(-2-3n+21n^2+46n^3) + mn^2(-6-13n+9n^2) - 4n^3 \\ -m(1+n)(m+n+2mn)\sqrt{\hat{\psi}} \end{array}\right]}{\left[\begin{array}{l} 2(1+m)n(m^3(5+20n+28n^2+14n^3) + m^2n(4+13n+12n^2) \\ +mn(-1-n+3n^2) - n^2) \end{array}\right]}$$

We find that the numerator is positive from the following calculation, $n \geq 1$, and $m \geq 1$.

$$\left[\begin{array}{l} m^4(-1+7n+42n^2+70n^3+40n^4) \\ +m^3n(9+51n+104n^2+78n^3) \\ +m^2n(-2-3n+21n^2+46n^3) \\ +mn^2(-6-13n+9n^2)-4n^3 \end{array}\right]^2 - m^2(1+n)^2(m+n+2mn)^2\hat{\psi}$$

$$= 4(1+m)n\left[\begin{array}{l} m^3(5+20n+28n^2+14n^3)+m^2n(4+13n+12n^2) \\ +mn(-1-n+3n^2)-n^2 \end{array}\right]$$

$$\times\left[\begin{array}{l} (-4m^4n-21m^2n^2+26m^4n^4)+(-8mn^2-4n^3-18mn^3+31m^2n^4) \\ +(-2m^4-5m^2n-9m^2n^3+34m^4n^3)+10m^4n^2+6mn^4 \\ +m^3(-1-5n+3n^2+43n^3+52n^4) \end{array}\right] > 0.$$

Thus, we have this proposition.□

A5. Proof of Proposition 6

As shown earlier, we obtain the following result by differentiating total surplus with respect to n and evaluating it under the free-entry equilibrium condition:

$$\left.\frac{\partial \widetilde{TS}(n)}{\partial n}\right|_{f=\tilde{f}} = \frac{a_Y^2(\tilde{\Phi}_0+\tilde{\Phi}_1 r-\tilde{\Phi}_2 r^2)}{2(1+n)^3(1+3n)^3(1+u)^2},$$

This sign coincides with that of $\tilde{\Phi}_0+\tilde{\Phi}_1 r-\tilde{\Phi}_2 r^2$ in the numerator. This quadratic function of r has two roots, denoted as follows.

$$\tilde{r}_1 = \frac{\tilde{\Phi}_1-\sqrt{\tilde{\Phi}_1^2+4\tilde{\Phi}_0\tilde{\Phi}_2}}{2\tilde{\Phi}_2}, \quad \tilde{r}^{TS} = \frac{\tilde{\Phi}_1+\sqrt{\tilde{\Phi}_1^2+4\tilde{\Phi}_0\tilde{\Phi}_2}}{2\tilde{\Phi}_2}.$$

We show that $\tilde{r}_1 \leq 1/2$ and $\tilde{r}^{TS} < 2$ for any $n \geq 1$ and $u \geq 1$. In addition, we show that $\tilde{r}^{TS} < 1/2$ if $u \geq 5$.

First, we show that $\tilde{r}_1 \leq 1/2$.

$$\frac{1}{2} - \tilde{r}_1 = \frac{\left[\begin{array}{l} -u + 2n(-2 - 7u + u^2) + 6n^2(-2 - 10u + 3u^2) + 2n^3(-6 - 49u + 27u^2) \\ +n^4(-4 - 51u + 54u^2) + (1+n)(1+3n)\sqrt{\tilde{\psi}} \end{array} \right]}{4n[2 + 2u + u^2 + n(6 + 8u + 9u^2) + 3n^2(2 + 2u + 9u^2) + n^3(2 + 27u^2)]}$$

where $\tilde{\psi} = u[u + 4n(8 + 11u + 2u^2) + 2n^2(80 + 115u + 28u^2) + 4n^3(56 + 91u + 30u^2) + n^4(96+193u+72u^2)]$. As $(1+n)(1+3n)\sqrt{\tilde{\psi}}$ in the numerator is positive, we only need to show that the sum of the remaining terms is positive. $-u+2n(-2-7u+u^2)+6n^2(-2-10u+3u^2)+2n^3(-6-49u+27u^2)+n^4(-4-51u+54u^2)$ is clearly positive for $u \geq 2$. In addition, when $u = 1$, the numerator is $-(1+16n+54n^2+56n^3+n^4)+(1+n)(1+3n)\sqrt{\tilde{\psi}}$. From $(1+n)^2(1+3n)^2\tilde{\psi} - (1+16n+54n^2+56n^3+n^4)^2 = 4n(15+194n+1077n^2+3288n^3+5961n^4+6426n^5+3731n^6+812n^7) > 0$, the numerator is positive. Therefore, we find that $\tilde{r}_1 \leq 1/2$.

Next, we show that $\tilde{r}^{TS} < 2$.

$$2 - \tilde{r}^{TS} = \frac{\left[\begin{array}{l} -u + n(8 - 2u + 8u^2) + 12n^2(2 - u + 6u^2) + n^3(24 - 62u + 216u^2) \\ +n^4(8 - 51u + 216u^2) - (1+n)(1+3n)\sqrt{\tilde{\psi}} \end{array} \right]}{4n[2 + 2u + u^2 + n(6 + 8u + 9u^2) + 3n^2(2 + 2u + 9u^2) + n^3(2 + 27u^2)]}$$

The numerator is positive from the following calculation, $n \geq 1$, and $u \geq 1$.

$$\left[\begin{array}{l} -u + n(8 - 2u + 8u^2) + 12n^2(2 - u + 6u^2) \\ +n^3(24 - 62u + 216u^2) + n^4(8 - 51u + 216u^2) \end{array} \right]^2 - (1+n)^2(1+3n)^2\tilde{\psi}$$
$$= 8n \left[\begin{array}{l} 2 + 2u + u^2 + n(6 + 8u + 9u^2) \\ +3n^2(2 + 2u + 9u^2) + n^3(2 + 27u^2) \end{array} \right]$$
$$\times \left[\begin{array}{l} (-3u - 100n^2u - 105n^4u + 216n^4u^2) + (-26nu - 182n^3u + 216n^3u^2) \\ +4n(1 + 2u^2) + 12n^2(1 + 6u^2) + 12n^3 + 4n^4 \end{array} \right] > 0.$$

Thus, we have $\tilde{r}^{TS} < 2$.

We now show that $\tilde{r}^{TS} \leq 1/2$ if $u \geq 5$.

$$\frac{1}{2} - \tilde{r}^{TS} = \frac{\left[\begin{array}{l} -u + 2n(-2 - 7u + u^2) + 6n^2(-2 - 10u + 3u^2) + 2n^3(-6 - 49u + 27u^2) \\ +n^4(-4 - 51u + 54u^2) - (1+n)(1+3n)\sqrt{\tilde{\psi}} \end{array} \right]}{4n[2 + 2u + u^2 + n(6 + 8u + 9u^2) + 3n^2(2 + 2u + 9u^2) + n^3(2 + 27u^2)]}$$

We examine the sign of the numerator using the following equation.

$$\left[\begin{array}{l} -u + 2n(-2 - 7u + u^2) + 6n^2(-2 - 10u + 3u^2) \\ +2n^3(-6 - 49u + 27u^2) + n^4(-4 - 51u + 54u^2) \end{array} \right]^2 - (1+n)^2(1+3n)^2\tilde{\psi}$$
$$= 4n[2 + 2u + u^2 + n(6 + 8u + 9u^2) + 3n^2(2 + 2u + 9u^2) + n^3(2 + 27u^2)]\tilde{\Psi}(u).$$

where $\tilde{\Psi}(u) = -3u + n(2 - 28u + u^2) + n^2(6 - 92u + 9u^2) + n^3(6 - 124u + 27u^2) + n^4(2 - 57u + 27u^2)$. The sign of the above expression is determined by the sign of $\tilde{\Psi}(u)$. We have $\partial\tilde{\Psi}(u)/\partial u > 0$ for $u \geq 5$ and $\tilde{\Psi}(5) = -15 - 113n - 229n^2 + 61n^3 + 392n^4 > 0$. Therefore, we obtain $\tilde{r}^{TS} \leq 1/2$ for any n when $u \geq 5$. Finally, for $u \in \{1, 2, 3, 4\}$, evaluating $\tilde{r}^{TS}(u)$

at $n = 1$ yields $5/3$, 1, $(832 + 64\sqrt{249})/2560$, and $(1088 + 256\sqrt{31})/4416$, respectively, all of which are greater than $1/2$. Therefore, we obtain this proposition.□

References

Arya, A. and Mittendorf, B. (2010) Input price discrimination when buyers operate in multiple markets. *Journal of Industrial Economics*, 58(4), 846–867.

Avenel, E. (2026) Tidying up the successive Cournot oligopoly model. *Research in Economics*, 101169.

Basak, D. and Mukherjee, A. (2016) Social efficiency of entry in a vertically related industry. *Economics Letters*, 139, 8–10.

Bhattacharyya, S. and Nain, A. (2011). Horizontal acquisitions and buying power: A product market analysis. *Journal of Financial Economics*, 99(1), 97–115.

Cao, H. and Wang, L. F. (2020) Social efficiency of entry in a vertically related industry revisited. *Economics Letters*, 192, 109200.

Ghosh, A. and Morita, H. (2007a) Social desirability of free entry: a bilateral oligopoly analysis. *International Journal of Industrial Organization*, 25(5), 925–934.

Ghosh, A. and Morita, H. (2007b) Free entry and social efficiency under vertical oligopoly. *RAND Journal of Economics*, 38(2), 541–554.

Greenhut, M. L. and Ohta, H. (1979) Vertical integration of successive oligopolists. *American Economic Review*, 69(1), 137–141.

Koulamas, C. and Kyparisis, G.J. (2010) A note on the effects of downstream efficiency on upstream pricing. *European Journal of Operational Research*, 200(3), 926–928.

Mankiw, N. G. and Whinston, M. D. (1986) Free entry and social inefficiency. *RAND Journal of Economics*, 17(1), 48–58.

Matsumura, T., Henry Wang, X., and Zeng, C. (2025) Welfare improving common ownership in successive oligopolies: The role of the input market. *Canadian Journal of Economics*, 58(1), 169–192.

Milliou, C. and Petrakis, E. (2024). Vertical contracts and downstream entry. *Journal of Industrial Economics*, 72(1), 598–629.

Monden, A. and Mizuno, T. (2026). Input pricing and common ownership between asymmetric firms. *Economic Modelling*, 107735.

Monden, A., Takezawa, S., and Mizuno, T. (2025). Quantity versus price competitions in a vertical relationship with separate downstream markets. *Economics Bulletin*, 45(3), 1620-1629.

Salinger, M. A. (1988) Vertical mergers and market foreclosure. *Quarterly Journal of Economics*, 103(2), 345–356.

Suzumura, K. and Kiyono, K. (1987) Entry barriers and economic welfare. *Review of Economic Studies*, 54(1), 157–167.

Tsuritani, R. (2025) Endogenous timing with upstream entry. *Applied Economics Letters*, 32(3), 445–448.

Tyagi, R.K. (1999) On the effects of downstream entry. *Management Science*, 45(1), 59–73.

Yenipazarli, A. (2021) Downstream entry revisited: Economic effects of entry in vertically-related markets. *Omega*, 103, 102428.