Input Price Discrimination with Advertising

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Input Price Discrimination with Advertising*

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Abstract

Input price discrimination reallocates production from efficient to inefficient firms and is therefore known to reduce welfare relative to uniform input pricing. We challenge this conventional result by introducing downstream informative advertising. When the downstream firm engaging in informative advertising is productively inefficient, input price discrimination lowers its input price. In this case, the downstream firm engages more aggressively in informative advertising; consequently, input price discrimination improves welfare. We further show that a similar result arises under persuasive advertising.

JEL codes: D43, L10, L13

Keywords: input price discrimination, uniform input price, informative advertising, persuasive advertising, vertical relationship.

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1 Introduction

It is well known that input price discrimination harms welfare relative to uniform input pricing because it reallocates production from efficient to inefficient downstream firms: the upstream firm sets a higher input price for the efficient firm and a lower input price for the inefficient one (DeGraba, 1990; Valletti, 2003; Yoshida, 2000).[1] We challenge this conventional wisdom by introducing downstream advertising.

Specifically, we consider a three-stage game played by an upstream firm and two downstream firms that differ in production efficiency. One downstream firm can engage in informative advertising. In the first stage, the advertising firm chooses its advertising level. In the second stage, the upstream firm sets either discriminatory input prices or a uniform input price. In the third stage, the downstream firms compete in quantities.

Our model yields three main findings. First, the informative advertising level is higher under input price discrimination than under uniform pricing only when the advertising firm is the inefficient one. The intuition is as follows. Under input price discrimination, the upstream firm charges the inefficient downstream firm a lower input price than the efficient one. When the advertising firm is inefficient, it therefore faces a lower input price and has a stronger incentive to advertise. If the advertising firm is efficient, however, it faces a higher input price and advertises less.

Second, when the advertising firm is inefficient, consumer surplus and total surplus are both higher under input price discrimination than under uniform pricing. This result contrasts with the conventional wisdom that input price discrimination always harms welfare compared to uniform pricing. The higher advertising level under discrimination, as explained above, expands the fraction of informed consumers and thus enlarges the market, which benefits consumers and raises total surplus. When the advertising firm is efficient, however, the reverse holds: discrimination raises that firm's input price, reduces

[1] Katz (1987) also shows that input price discrimination can reduce welfare even without differences in production efficiency among downstream firms.

its advertising, and lowers both consumer and total surplus relative to uniform pricing.

Third, we examine persuasive advertising and obtain qualitatively similar results. As with informative advertising, input price discrimination encourages advertising and raises consumer surplus only when the advertising firm is inefficient. In that case, however, input price discrimination raises total surplus only when persuasive advertising is sufficiently efficient. The reason is that discrimination simultaneously reallocates output toward the inefficient firm, which reduces surplus, and strengthens the advertising incentive, which raises surplus. As advertising becomes more efficient, the positive advertising effect eventually dominates the negative reallocation effect. Notably, this result differs from the informative advertising case: there, as long as the advertising firm is inefficient, discrimination always improves total surplus. Under informative advertising, the advertising level does not affect input prices; under persuasive advertising, a higher advertising level raises input prices and worsens double marginalization, so the total surplus ranking depends on advertising efficiency.

Our work relates to two strands of literature. The first strand identifies conditions under which input price discrimination can be welfare-enhancing (Arya and Mittendorf, 2010; Chen, 2022; Gaudin and Lestage, 2022; Hu et al., 2022; Lestage and Li, 2022; Li and Shuai, 2022; Lømo, 2023; Li and Zhang, 2024). None of these studies considers downstream advertising. The closest study is Lestage and Li (2022), which allows a downstream firm to undertake cost-reducing investment. They show that input price discrimination impedes investment by a technological leader, fosters investment by a laggard, and may improve welfare when the laggard can overtake the leader. Our mechanism differs fundamentally. In Lestage and Li (2022), the investment affects only the investing firm's own cost; in our model, advertising affects demand for both downstream firms and the entire downstream market. Hence, the welfare-enhancing channel in our model is distinct from theirs. In addition, whereas Lestage and Li (2022) obtain welfare improvement only when the laggard can overtake the leader, we show that, conditional

only on the inefficient firm being the advertiser, input price discrimination always improves welfare under informative advertising.

The second strand studies informative and persuasive advertising in vertical structures. Zhang et al. (2020) examine upstream encroachment when the upstream or downstream firm engages in advertising. Building on Zhang et al. (2020), Hu and Mizuno (2021) analyze endogenous timing in a vertical market with upstream advertising. Hu et al. (2024) study the endogenous choice between price and quantity contracts when the upstream firm advertises. Hu and Mizuno (2026) examine profitable entry with upstream and downstream advertising and extend the downstream market to n firms. These papers assume differentiated products, and their results typically require sufficient product differentiation. Different from these paper, we introduce downstream cost asymmetry and consider a homogeneous final good, comparing input price discrimination with uniform input pricing with one downstream firm's advertising.

The remainder of the paper is organized as follows. Section 2 describes the basic model. Section 3 analyzes informative advertising. Section 4 examines persuasive advertising. Section 5 concludes.

2 Model

We consider a market with one upstream firm, firm U, and two downstream firms, firms Di $(i = 1, 2)$. Firm U produces an input at a constant marginal cost normalized to zero and sells it to firm Di at input price w_i. We consider two input pricing schemes: input price discrimination and uniform input pricing. Under uniform input pricing, $w_1 = w_2 = w$.

Each downstream firm requires one unit of the input to produce one unit of the final good. In addition to the input price paid to firm U, firm $D1$ may incur an extra marginal cost of $(1 - x)c$, whereas firm $D2$ may incur an extra marginal cost of xc, where c represents the inefficient marginal cost and $x \in \{0, 1\}$ is a dummy variable.

To guarantee that both downstream firms produce positive quantities, we assume that $c \in [0, 2/7)$. When $x = 1$, firm $D1$ is efficient and firm $D2$ is inefficient, whereas when $x = 0$, firm $D1$ is inefficient and firm $D2$ is efficient. We refer to the former as Case (i) and the latter as Case (ii). The downstream firms produce a homogeneous final good.[2] We denote its price as p and the output of firm Di as Q_i.

Furthermore, firm $D2$ can engage in informative advertising. We denote the advertising level by $\theta \in [0, 1]$, which represents the proportion of consumers who receive the advertisement. Consumers who receive the advertisement are aware of the final good and can purchase it, whereas those who do not are unaware of the final good and therefore cannot purchase it. Firm $D2$ incurs an advertising cost of $k\theta^2$, where k represents the inefficiency of advertising. To guarantee an interior solution, we assume that $k > (2 + 5c)^2/288$.

We define the profits of downstream firms as follows:

$$\pi_{D1} = [p - w_1 - (1 - x)c]Q_1, \quad \pi_{D2} = (p - w_2 - xc)Q_2 - k\theta^2.$$

Additionally, the profit of firm U is as follows:

$$\pi_U = w_1 Q_1 + w_2 Q_2.$$

Each consumer has the following utility function: $u(q) = q - q^2/2 - pq$, where q denotes consumption of the final good. If a consumer does not receive the advertisement, the consumer's consumption is zero, that is, $q = 0$. For a consumer who receives the advertisement, utility maximization yields the individual demand function $q = 1 - p$. Since a fraction θ of consumers receive the advertisement, the condition equating total supply and total demand is $Q_1 + Q_2 = \theta q$. Accordingly, the inverse demand function in the downstream market is as follows:

$$p = 1 - \frac{Q_1 + Q_2}{\theta}.$$

[2]We also find the similar results with final product differentiation.

Therefore, consumer and total surpluses are as follows.

$$CS = \frac{(Q_1 + Q_2)^2}{2\theta}, \quad TS = CS + \pi_{D1} + \pi_{D2} + \pi_U.$$

Finally, the timing of this game is as follows. In stage 1, firm $D2$ chooses the level of informative advertising. In stage 2, under input price discrimination, firm U chooses discriminatory input prices; under uniform input pricing, firm U chooses a uniform input price. In stage 3, both downstream firms engage in quantity competition. We solve this game using backward induction.

3 Results

In Subsection 3.1, we consider Case (i), and in Subsection 3.2, we consider Case (ii). For each case, we analyze whether input price discrimination or uniform input pricing leads to larger consumer and total surpluses.

3.1 Case (i): Inefficient Advertising Firm ($x = 1$)

In this subsection, we consider the case in which firm $D2$, which engages in informative advertising, is inefficient (i.e., $x = 1$).

3.1.1 Input Price Discrimination

We suppose that firm U employs input price discrimination. In stage 3, from the first-order conditions, we obtain the following outputs.

$$Q_1 = \frac{\theta(1 - 2w_1 + w_2 + c)}{3}, \quad Q_2 = \frac{\theta(1 + w_1 - 2w_2 - 2c)}{3}.$$

In stage 2, substituting the above outcomes into π_U and solving the first-order conditions, we obtain the discriminatory input prices as follows.

$$w_1 = \frac{1}{2}, \quad w_2 = \frac{1 - c}{2}.$$

Therefore, we obtain Lemma 1.

Lemma 1 *We consider the case where firm $D2$ is more inefficient than firm $D1$. Under input price discrimination, firm $D2$, which engages in informative advertising, faces a lower input price than firm $D1$.*

The intuition behind Lemma 1 is as follows. Firm $D2$ is more inefficient than firm $D1$ and therefore faces a cost disadvantage. Accordingly, firm U offers firm $D2$ a lower input price to offset this cost disadvantage.

We consider stage 1. Substituting the above outcomes into π_{D2}, from the first-order condition, we obtain the equilibrium level of informative advertising.

$$\theta^d = \frac{(1-2c)^2}{72k}.$$

The superscript d denotes input price discrimination. Based on the above, the consumer and total surpluses in equilibrium are as follows.

$$CS^d = \frac{(2-c)^2(1-2c)^2}{5184k}, \quad TS^d = \frac{(1-2c)^2(19-16c+19c^2)}{5184k}.$$

3.1.2 Uniform Input Pricing

We suppose that firm U employs uniform input pricing. The outcomes in stage 3 are the same as those under input price discrimination.

In stage 2, substituting these outcomes into π_U and imposing $w_1 = w_2 = w$, the first-order condition yields the following uniform input price.

$$w = \frac{2-c}{4}.$$

We consider stage 1. Substituting the above outcomes into π_{D2}, we obtain the equilibrium level of informative advertising from the first-order condition.

$$\theta^u = \frac{(2-7c)^2}{288k}.$$

The superscript u denotes uniform input pricing. Furthermore, the consumer and total surpluses in equilibrium are as follows.

$$CS^u = \frac{(2-c)^2(2-7c)^2}{20736k}, \quad TS^u = \frac{(2-7c)^2(76-52c+115c^2)}{82944k}.$$

3.1.3 Comparison: Desirability of Input Price Discrimination

We analyze which input pricing scheme yields larger consumer and total surpluses. We first compare the levels of informative advertising under the two pricing schemes, leading to Lemma 2.

Lemma 2 *We consider the case where firm $D2$ is more inefficient than firm $D1$. The level of informative advertising under input price discrimination is higher than that under uniform input pricing.*

Proof. Calculating $\theta^d - \theta^u$ yields the following result.

$$\theta^d - \theta^u = \frac{(4-11c)c}{96k}.$$

Since $c \in (0, 2/7)$, we obtain $\theta^d > \theta^u$. □

The intuition behind Lemma 2 is as follows. As shown in Lemma 1, when firm $D2$ is more inefficient than firm $D1$, firm $D2$ faces a lower input price under input price discrimination. The lower input price makes firm $D2$ more aggressive in informative advertising.

We next compare consumer and total surpluses between the two pricing schemes, which leads to Proposition 1.

Proposition 1 *We consider the case where firm $D2$ is more inefficient than firm $D1$. (i) The consumer surplus under input price discrimination is always larger than that under uniform input pricing. (ii) The total surplus under input price discrimination is always larger than that under uniform input pricing.*

Proof. Calculating $CS^d - CS^u$ and $TS^d - TS^u$ yields the following results.

$$CS^d - CS^u = \frac{(2-c)^2(4-11c)c}{6912k}, \quad TS^d - TS^u = \frac{(96-344c+392c^2-491c^3)c}{9216k}.$$

Since $c \in (0, 2/7)$, we have $CS^d > CS^u$ and $TS^d > TS^u$.□

The intuition behind Proposition 1 is as follows. As shown in Lemma 2, the level of informative advertising is higher under input price discrimination than under uniform input pricing. A higher level of informative advertising increases the fraction of consumers who receive the advertisement and, consequently, can purchase the final good. Therefore, the consumer surplus is larger under input price discrimination, as stated in Proposition 1 (i). Similarly, the total surplus is larger under input price discrimination, as indicated in Proposition 1 (ii). This result contrasts with the conventional finding in the literature on input price discrimination.

3.2 Case (ii): Efficient Advertising Firm ($x = 0$)

In this subsection, we consider the case in which firm $D2$ is more efficient than firm $D1$ (i.e., $x = 0$). Under input price discrimination and uniform input pricing, the outputs in stage 3 are as follows.

$$Q_1 = \frac{\theta(1 - 2w_1 + w_2 - 2c)}{3}, \quad Q_2 = \frac{\theta(1 + w_1 - 2w_2 + c)}{3}.$$

In stage 2, under input price discrimination, we obtain the discriminatory input prices as follows.

$$w_1 = \frac{1 - c}{2}, \quad w_2 = \frac{1}{2}.$$

Therefore, we find that firm $D2$ faces a higher input price than firm $D1$ because it is more efficient. Under uniform input pricing, we obtain the uniform input price as follows.

$$w = \frac{2 - c}{4}.$$

In stage 1, we obtain the equilibrium level of informative advertising under each input pricing scheme.

$$\widetilde{\theta}^d = \frac{(1 + c)^2}{72k}, \quad \widetilde{\theta}^u = \frac{(2 + 5c)^2}{288k}.$$

We thus find $\widetilde{\theta}^d < \widetilde{\theta}^u$. As mentioned earlier, under input price discrimination, firm $D2$ faces a higher input price. Therefore, firm $D2$ has a weaker incentive to invest in informative advertising under input price discrimination than under uniform input pricing. Based on the above, we obtain the consumer and total surpluses in equilibrium under each input pricing scheme as follows.

$$\widetilde{CS}^d = \frac{(2-c)^2(1+c)^2}{5184k}, \quad \widetilde{CS}^u = \frac{(2-c)^2(2+5c)^2}{20736k},$$
$$\widetilde{TS}^d = \frac{(1+c)^2(19-22c+22c^2)}{5184k}, \quad \widetilde{TS}^u = \frac{(2+5c)^2(76-100c+139c^2)}{82944k}.$$

Therefore, we find that $\widetilde{CS}^d < \widetilde{CS}^u$. This is because, under input price discrimination, firm $D2$ engages less in informative advertising. For the same reason, we obtain $\widetilde{TS}^d < \widetilde{TS}^u$. This result is consistent with the well-known result in the literature on input price discrimination.

4 Robustness: Persuasive Advertising

In addition to informative advertising, the advertising literature examines persuasive advertising, which increases consumers' willingness to pay. In this section, we examine whether our main results are robust under persuasive advertising.

We define the utility function of a representative consumer as $u = (1+\theta)q - q^2/2 - pq$, where θ denotes the level of persuasive advertising chosen by firm $D2$. The inverse demand function is then $p = 1 + \theta - q$. Using the condition that equates total supply and total demand, $Q_1 + Q_2 = q$, we obtain $p = 1 + \theta - Q_1 - Q_2$.

Firm $D2$ incurs a persuasive advertising cost of $k\theta^2$. We assume $k > 1/36$ so that firm $D2$'s profit is concave. The profit functions of all firms are the same as those in Section 2. Consumer surplus is redefined as follows: $CS = (Q_1 + Q_2)^2/2$, while total surplus is defined as in Section 2. The game proceeds as in Section 2, except that firm $D2$ engages in persuasive advertising instead of informative advertising in stage 1.

Following Section 3, Subsection 4.1 first considers Case (i), in which firm $D2$ is more inefficient than firm $D1$. Subsection 4.2 then considers Case (ii), in which firm $D2$ is more efficient than firm $D1$.

4.1 Case (i): Inefficient Advertising Firm ($x = 1$)

In this subsection, we consider the case in which firm $D2$ is more inefficient than firm $D1$ (i.e., $x = 1$).

4.1.1 Input Price Discrimination

We suppose that firm U employs input price discrimination. In stage 3, based on the first-order conditions, we obtain the following outputs.

$$Q_1 = \frac{1 + \theta - 2w_1 + w_2 + c}{3}, \quad Q_2 = \frac{1 + \theta + w_1 - 2w_2 - 2c}{3}.$$

In stage 2, substituting these outcomes into π_U and solving the first-order conditions, we obtain the discriminatory input prices.

$$w_1 = \frac{1 + \theta}{2}, \quad w_2 = \frac{1 + \theta - c}{2}.$$

From these outcomes, we obtain Lemma 3.

Lemma 3 *We consider the case where firm $D2$ is more inefficient than firm $D1$. Under input price discrimination, firm $D2$, which engages in persuasive advertising, faces a lower input price than firm $D1$.*

The intuition behind Lemma 3 is similar to that behind Lemma 1. Since firm $D2$ is more inefficient than firm $D1$, it faces a cost disadvantage. Firm U therefore offers firm $D2$ a lower input price to compensate for this disadvantage.

In stage 1, substituting the above outcomes into π_{D2} and solving the first-order condition, we obtain the equilibrium level of persuasive advertising.

$$\theta^{Pd} = \frac{1 - 2c}{-1 + 36k},$$

where the superscript P denotes persuasive advertising. Therefore, we obtain the consumer and total surpluses in equilibrium as follows:

$$CS^{Pd} = \frac{[-c + (24 - 12c)k]^2}{8(-1 + 36k)^2},$$
$$TS^{Pd} = \frac{7c^2 - (8 + 208c + 56c^2)k + (2880 - 2880c + 3312c^2)k^2}{8(-1 + 36k)^2}.$$

4.1.2 Uniform Input Pricing

We suppose that firm U adopts uniform input pricing. The outcomes in stage 3 are identical to those under input price discrimination. In stage 2, using $w_1 = w_2 = w$, we obtain the uniform input price from the first-order condition.

$$w = \frac{2 + 2\theta - c}{4}.$$

In stage 1, substituting the above outcomes into π_{D2}, we obtain the equilibrium level of persuasive advertising from the first-order condition.

$$\theta^{Pu} = \frac{2 - 7c}{2(-1 + 36k)}.$$

Therefore, the consumer and total surpluses in equilibrium are as follows.

$$CS^{Pu} = \frac{[-c + (12 - 6c)k]^2}{2(-1 + 36k)^2},$$
$$TS^{Pu} = \frac{12c^2 - (4 + 212c + 73c^2)k + (1440 - 1440c + 2952c^2)k^2}{4(-1 + 36k)^2}.$$

4.1.3 Comparison: Desirability of Input Price Discrimination

We compare the outcomes under input price discrimination and uniform input pricing. First, we examine the levels of persuasive advertising under the two pricing schemes and obtain the following lemma.

Lemma 4 *We consider the case where firm D2 is more inefficient than firm D1. (i) The level of persuasive advertising under input price discrimination is higher than that*

under uniform input pricing. (ii) The difference between persuasive advertising levels under the two input pricing schemes increases as persuasive advertising becomes more efficient.

Proof. Calculating $\theta^{Pd} - \theta^{Pu}$ and $\partial(\theta^{Pd} - \theta^{Pu})/\partial k$ yields the following results.

$$\theta^{Pd} - \theta^{Pu} = \frac{3c}{2(-1+36k)}, \quad \frac{\partial(\theta^{Pd} - \theta^{Pu})}{\partial k} = -\frac{54c}{(-1+36k)^2}.$$

Therefore, we have $\theta^{Pd} > \theta^{Pu}$ and $\partial(\theta^{Pd} - \theta^{Pu})/\partial k < 0$.□

The intuition behind Lemma 4 is as follows. As shown in Lemma 3, under input price discrimination, firm $D2$ faces a lower input price than firm $D1$. As a result, firm $D2$ has a stronger incentive to invest in persuasive advertising under input price discrimination, yielding Lemma 4 (i). Furthermore, this incentive increases as persuasive advertising becomes more efficient, yielding Lemma 4 (ii).

Next, we obtain Proposition 2 on consumer and total surpluses.

Proposition 2 *We consider the case where firm $D2$ is more inefficient than firm $D1$. (i) The consumer surplus under input price discrimination is always larger than that under uniform input pricing. (ii) The total surplus under input price discrimination is larger than that under uniform input pricing if persuasive advertising is sufficiently efficient: $k < k_{TS}^P = (12 + 5c + \sqrt{3(48 + 40c - 173c^2)})/(288c)$. Otherwise, the total surplus under input price discrimination is smaller than that under uniform input pricing.*

Proof. First, calculating $CS^{Pd} - CS^{Pu}$ yields the following result.

$$CS^{Pd} - CS^{Pu} = \frac{3[-c + (16 - 8c)k]c}{8(-1+36k)^2} > 0.$$

Next, we obtain the following result by calculating $TS^{Pd} - TS^{Pu}$.

$$TS^{Pd} - TS^{Pu} = \frac{[-17c + (216 + 90c)k - 2592ck^2]c}{8(-1+36k)^2}.$$

The sign of $TS^{Pd} - TS^{Pu}$ corresponds to that of the numerator. Therefore, if $k < k_{TS}^{P}$, we obtain $TS^{Pd} - TS^{Pu} > 0$. From the above, we obtain Proposition 2.□

The intuition behind Proposition 2 is as follows. First, we consider Proposition 2 (i). As shown in Lemma 4 (i), the level of persuasive advertising is higher under input price discrimination than under uniform input pricing. Because persuasive advertising increases consumers' willingness to pay, the consumer surplus is larger under input price discrimination. Next, we consider Proposition 2 (ii). Input price discrimination encourages inefficient firm $D2$ to increase its output, thereby reducing the total surplus. However, it gives firm $D2$ a stronger incentive to engage in persuasive advertising. As shown in Lemma 4 (ii), this positive effect becomes more pronounced as persuasive advertising becomes more efficient. Therefore, when persuasive advertising is sufficiently efficient, the positive effect of higher advertising dominates the negative effect of reallocating output toward the inefficient firm, and input price discrimination improves the total surplus.

Thus, we find that our main results under informative advertising remain valid to some extent under persuasive advertising. However, while input price discrimination always improves total surplus under informative advertising, it improves total surplus under persuasive advertising only when persuasive advertising is sufficiently efficient. Under informative advertising, the advertising level does not affect input prices. In contrast, under persuasive advertising, a higher advertising level raises input prices, thereby worsening the double marginalization problem. Consequently, whether input price discrimination improves total surplus under persuasive advertising depends on the efficiency of advertising, unlike under informative advertising.

4.2 Case (ii): Efficient Advertising Firm ($x = 0$)

In this subsection, we analyze the case in which firm $D2$ is more efficient than firm $D1$ (i.e., $x = 0$). In stage 3, we obtain the following outputs.

$$Q_1 = \frac{1 + \theta - 2w_1 + w_2 - 2c}{3}, \quad Q_2 = \frac{1 + \theta + w_1 - 2w_2 + c}{3}.$$

In stage 2, we obtain the discriminatory input prices under input price discrimination.

$$w_1 = \frac{1 + \theta - c}{2}, \quad w_2 = \frac{1 + \theta}{2}.$$

Thus, firm $D2$ faces a higher input price than firm $D1$. Under uniform input pricing, we obtain the uniform input price.

$$w = \frac{2 + 2\theta - c}{4}.$$

In stage 1, we obtain the equilibrium level of persuasive advertising under each input pricing scheme.

$$\widetilde{\theta}^{Pd} = \frac{1 + c}{-1 + 36k}, \quad \widetilde{\theta}^{Pu} = \frac{2 + 5c}{2(-1 + 36k)}.$$

Therefore, we find that $\widetilde{\theta}^{Pd} < \widetilde{\theta}^{Pu}$. This is because, under input price discrimination, firm $D2$ faces a higher input price and thus has a weaker incentive to invest in persuasive advertising.

Finally, we obtain the consumer and total surpluses in equilibrium under each input pricing scheme as follows.

$$\begin{aligned}
\widetilde{CS}^{Pd} &= \frac{[c + (24 - 12c)k]^2}{8(-1 + 36k)^2}, \quad \widetilde{CS}^{Pu} = \frac{[c + (12 - 6c)k]^2}{2(-1 + 36k)^2}, \\
\widetilde{TS}^{Pd} &= \frac{7c^2 - (8 - 224c + 272c^2)k + (2880 - 2880c + 3312c^2)k^2}{8(-1 + 36k)^2}, \\
\widetilde{TS}^{Pu} &= \frac{12c^2 - (4 - 220c + 289c^2)k + (1440 - 1440c + 2952c^2)k^2}{4(-1 + 36k)^2}.
\end{aligned}$$

Using these outcomes, we obtain $\widetilde{CS}^{Pd} < \widetilde{CS}^{Pu}$ and $\widetilde{TS}^{Pd} < \widetilde{TS}^{Pu}$. This is because firm $D2$ invests less in persuasive advertising under input price discrimination than under

uniform input pricing. Therefore, when the downstream firm engaging in advertising is more efficient in production, input price discrimination reduces consumer and total surpluses under informative advertising and also under persuasive advertising.

5 Conclusions

We have revisited the welfare effect of input price discrimination in a vertical market with downstream advertising. In our model, one of two downstream firms that differ in production efficiency can engage in advertising, and the upstream firm either discriminates in input prices or charges a uniform price.

We obtain three main results. First, input price discrimination raises informative advertising only when the advertising firm is the inefficient one, because discrimination gives that firm a lower input price and strengthens its advertising incentive. Second, in that case, both consumer surplus and total surplus are higher under discrimination than under uniform pricing: the higher advertising level expands the informed share of consumers and enlarges the market, reversing the conventional result that discrimination harms welfare by reallocating output from efficient to inefficient firms. When the efficient firm is the advertiser, the conventional result is restored. Third, our main results under informative advertising extend to persuasive advertising: discrimination again raises advertising and consumer surplus only when the inefficient firm advertises, but in that case it raises total surplus only when persuasive advertising is sufficiently efficient.

Competition authorities typically presume that input price discrimination is harmful because it reallocates output from efficient to inefficient firms. Our analysis shows that when the inefficient firm is also the advertiser, discrimination strengthens the market-expanding advertising incentive and may raise welfare. The welfare ranking therefore depends not only on cost asymmetry, but also on which downstream firm undertakes advertising. Under persuasive advertising, the efficiency of advertising matters as well.

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