Personalization, Disclosure, and Investment Distortions

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Personalization, Disclosure, and Investment Distortions

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Abstract

This paper studies how pricing structures shape AI investment incentives in data-driven markets. We develop a Hotelling model in which effective mismatch costs are jointly determined by firm investment and consumer disclosure. Firms compete either under symmetric personalized pricing or under an asymmetric structure in which one firm personalizes while its rival sets a uniform price.

Under symmetric personalized pricing, allocation follows effective costs, and investment affects welfare only through technological improvements. Private and socially second-best incentives therefore coincide. Under asymmetric pricing, however, allocation is governed by total prices. Investment shifts the market boundary and induces strategic price responses by the uniform-price rival, generating a distortion wedge between private and social incentives. The direction of this wedge determines whether investment is socially insufficient or excessive.

Our findings show that the welfare consequences of AI investment depend fundamentally on the pricing structure through which competition operates.

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1 Introduction

1.1 Motivation

Recent advances in artificial intelligence (AI) and the expansion of digital markets have reshaped how firms interact with consumers. Personalization—ranging from targeted advertising and customized product recommendations to individualized pricing—has become central to many digital business models. Large platforms such as Google and Meta rely on machine-learning systems trained on vast user data to improve advertiser–audience matching, while firms in subscription-based streaming services and online retail environments increasingly use AI-driven recommendation systems to reduce consumers'search and mismatch costs.[*1]

At the same time, regulatory scrutiny of data use and AI deployment has intensified. Major initiatives such as the GDPR, the Digital Markets Act, and the AI Act govern data access, transparency, and digital competition.[*2] Similar debates are ongoing in the United States and Asia. These developments coexist with expanding public and private investment in AI capabilities.[*3]

Digital markets also differ in the extent to which firms can personalize prices. In some settings—such as online advertising—competing platforms rely extensively on data-driven and highly granular pricing mechanisms. In other markets, technologically advanced firms may experiment with dynamic or individualized offers while rivals continue to rely on largely uniform pricing schemes. These observable differences motivate a comparison between symmetric and asymmetric personalization environments.

The competitive environments analyzed in this paper are stylized yet consistent with observed patterns in differentiated digital industries. Across subscription-based digital services and online retail markets, firms deploy AI technologies that reduce consumers'search and mismatch costs, while pricing practices exhibit heterogeneity across firms and over time. Differences in data access, technological capacity, and regulatory exposure plausibly contribute to variation in personalization capabilities across competitors.[*4]

From this perspective, our theoretical comparison between symmetric and asymmetric personalization environments provides a structured way to interpret how AI-driven efficiency

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[*1] Examples include data-driven advertising systems (OECD, 2018), personalized discounts in retail (Anderson et al., 2023), and algorithmic matching in platform markets.

[*2] See (European Union, 2016), (European Union, 2022), and (European Union, 2024).

[*3] See OECD (2018), Wallace and Castro (2018), and recent national AI strategies (e.g., OECD (2023); European Commission (2020); Government of Japan (2022)).

[*4] Related discussions of data-driven innovation and its competitive implications include Acquisti et al. (2016), Goldfarb and Tucker (2012), and de Cornière and Taylor (2025).

gains translate into market outcomes through competitive interaction. Improvements in matching quality can intensify competition by increasing consumers'responsiveness to price and quality differences. In such settings, private incentives to invest in AI may diverge from socially desirable levels, and the direction of this divergence can depend on the competitive environment. The analysis therefore clarifies why investment outcomes may differ across otherwise similar digital industries. This interpretation is consistent with a broader literature showing that the relationship between competition and innovation depends on market conditions, and with recent work emphasizing that symmetric and asymmetric personalization environments can yield sharply different competitive and welfare outcomes.[*5]

1.2 Main Results and Mechanism

To study how pricing structures shape investment incentives, we develop a Hotelling-type duopoly with horizontally differentiated consumers. The central object is the effective cost function $\psi(s, x)$, which captures how investment s and consumer disclosure y jointly determine location-dependent reductions in mismatch costs.

We compare two pricing environments. In the symmetric structure, both firms adopt personalized pricing. In the asymmetric structure, one firm personalizes while the rival sets a uniform price. Under symmetric personalized pricing, allocation responds directly to effective costs, so investment affects welfare only through inframarginal reductions in mismatch costs; private and socially optimal (second-best) incentives coincide. Under asymmetric pricing, allocation depends on total prices, so investment both reduces mismatch costs and triggers strategic price responses by the uniform-price rival. This interaction creates a distortion wedge between technological efficiency and price-based allocation, making welfare outcomes structure-dependent.

The divergence between private and social incentives depends on how investment affects this distortion wedge: mitigating the wedge yields underinvestment, while amplifying it may generate overinvestment. These mechanisms hold under general functional assumptions and are illustrated using a seeded exponential specification. More broadly, our findings connect to the classic literature on strategic investment in oligopoly, where incentives depend on the mode of competition (Bulow et al., 1985).[*6]

Our framework extends this logic to data-driven personalization. Here, investment does not merely reduce marginal production costs; instead, it shapes a location-dependent effective mismatch technology jointly determined with consumer disclosure. Moreover, the effective

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[*5] See Aghion et al. (2005) and Rhodes and Zhou (2024).

[*6] See also Brander and Spencer (1983), Fudenberg and Tirole (1984), and Matsumura and Matsushima (2012).

mode of competition is mediated by the pricing structure. Under symmetric personalized pricing, allocation responds directly to effective costs, dampening strategic distortions. Under asymmetric pricing, however, investment triggers strategic price responses by the uniform-price rival, creating a distortion wedge analogous to a price-competition environment.

In this sense, our model embeds classic strategic investment theory within a data-driven, structure-dependent setting. Taken together, the results establish a central message: AI investment is neither inherently welfare-enhancing nor welfare-reducing; its social consequences depend on the pricing structure governing market allocation.

1.3 Related Literature

A central distinction in the literature is whether mismatch or data-analytics quality is exogenous or endogenous. We jointly endogenize disclosure, investment, and pricing structures in a horizontally differentiated market, and study how their interaction governs the alignment between private and social investment incentives.

■**Personalized pricing with exogenous mismatch.** A large body of work studies competitive personalized pricing in Hotelling-type environments where mismatch costs or matching quality are taken as given.[*7] Classic analyses such as Thisse and Vives (1988) show how spatial price policies shape equilibrium outcomes. More recent contributions, including Chen et al. (2020) and Rhodes and Zhou (2024), analyze competitive personalized pricing and characterize how price discrimination affects market coverage and welfare. Similarly, Matsushima et al. (2023) examine personalized pricing when consumers exhibit heterogeneous mismatch costs, treating the distribution of mismatch as exogenous. Earlier work on customer recognition and one-to-one promotions (Villas-Boas, 1999; Shaffer and Zhang, 2002) also studies firm behavior under personalized pricing without modeling the endogenous formation of matching technology. In these models, personalization affects prices and allocation, but the determinants of mismatch are not strategic variables. For example, Chen et al. (2022) analyze data-driven improvements in matching within a Hotelling framework while treating analytics effectiveness as exogenous.[*8]

■**Privacy, disclosure, and data-driven competition.** Another strand focuses on consumers'privacy and disclosure decisions in competitive settings. Rhodes and Zhou (2026) provide a general framework analyzing personalization and privacy choice in competitive markets, treating

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[*7] For a comprehensive overview of price discrimination theory, see Armstrong (2006).

[*8] Related analyses of information quality and personalized pricing include Bloch and Demange (2018), which study how improved information or targeting affects pricing and welfare under exogenous technological environments.

personalization technology as exogenous while endogenizing consumers'disclosure decisions.[*9] Ali et al. (2023) analyze voluntary disclosure and personalized pricing under opt-in regimes, while Anderson et al. (2023) study personalized targeted discounts and the interaction between poaching and privacy. Ichihashi (2020) study how consumers optimally disclose data when firms monetize information.[*10] Related work examines privacy regulation and competition (Taylor and Wagman, 2014; Casadesus-Masanell and Hervas-Drane, 2015; Conti and Reverberi, 2021; Lefouili et al., 2024). These studies endogenize disclosure or privacy protection but treat technological effectiveness as given or in reduced form, so disclosure affects pricing without generating a location-dependent matching technology through investment. Our analysis is also related to recent frameworks linking data advantages to competitive outcomes (de Cornière and Taylor, 2025). [*11] Related work also studies how data-enabled learning generates persistent competitive advantages in digital markets (Hagiu and Wright, 2023). While these models capture how data asymmetries affect market power, they treat data advantages in reduced form and abstract from the joint determination of disclosure and investment that characterizes our framework.

■Strategic investment and endogenous technology. A third strand analyzes strategic investment and endogenous cost or quality choices in oligopoly. A central insight in this literature is that the sign of investment distortions depends on the mode of competition: under quantity competition, cost-reducing investment may lead to overinvestment, whereas under price competition it may generate underinvestment (Brander and Spencer, 1983; Fudenberg and Tirole, 1984; Bulow et al., 1985). In spatial settings, Matsumura and Matsushima (2012) extend this logic to Hotelling competition.

Our framework builds on this tradition by introducing two features: investment shapes a location-dependent mismatch technology jointly determined with disclosure, and the effective mode of competition is governed by the pricing structure. By endogenizing both disclosure and pricing, we show how classic strategic investment distortions arise in a data-driven environment.

Our framework integrates these strands by modeling disclosure as a continuous choice and allowing investment to determine, jointly with disclosure, a location-dependent effective cost function. This structure makes pricing central and generates systematic divergences between

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[*9] While Rhodes and Zhou (2026) analyze consumers'disclosure decisions under exogenous personalization technology, our framework makes the effectiveness of personalization endogenous through firm investment, generating structure-dependent distortions between private and social incentives.

[*10] See also Hidir and Vellodi (2021) for an analysis of privacy and price discrimination under consumer heterogeneity.

[*11] Foundational contributions on data targeting and information design include Bergemann and Bonatti (2011) and Bergemann et al. (2018).

private and socially optimal investment incentives.

The remainder of the paper is organized as follows. Section 2 introduces the model and defines the effective cost structure. Section 3 analyzes investment incentives under symmetric personalized pricing. Section 4 studies the asymmetric pricing environment and characterizes the resulting investment distortions. Section 5 presents simulations based on a seeded exponential specification of AI technology. Section 6 discusses broader implications and policy considerations. Section 7 concludes.

2 Model

This section formalizes the economic environment in which AI investment, consumer disclosure, and pricing structures interact, focusing on how alternative pricing rules translate technological improvements into market allocation. Our spatial competition framework builds on the canonical Hotelling model with stability conditions established by d'Aspremont et al. (1979). We adopt the standard linear transportation cost environment as a baseline and introduce endogenous data-driven reductions in effective mismatch costs.

2.1 Consumers and Market Structure

We consider a standard Hotelling duopoly with a unit mass of consumers uniformly distributed on $[0,1]$, a framework commonly used to study personalized pricing and data-driven competition (Chen et al., 2020, 2022; Rhodes and Zhou, 2026). Firm A is located at 0 and firm B at 1. A consumer located at $x \in [0,1]$ incurs mismatch (transportation) cost equal to the distance $z_i(x)$ from firm $i \in \{A, B\}$, where $z_A(x) = x$ and $z_B(x) = 1 - x$. Each consumer has a sufficiently large baseline valuation $v > 0$ so that the market is fully covered. Firms have zero marginal cost of production.

2.2 Pricing structures and Consumer Utility

We distinguish between personalized and uniform pricing structures.

■**Personalized Pricing.** Under personalized pricing, firm i sets a consumer-specific price $p_i(x)$. Under personalized pricing, the utility of a consumer located at x who purchases from firm i is given by

$$u_i^P(x) = v - p_i(x) - (1 - \phi(s_i, y))\, z_i - d(y). \tag{1}$$

Here, y denotes the amount of information disclosed by the consumer to firm i, and s_i represents firm i's investment in AI technology. Because each consumer purchases from a single firm, we omit firm subscripts on y. The function $\phi(s_i, y)$ captures the extent to which AI technology reduces the mismatch cost. Consumer-specific pricing is standard in the literature on personalized competition (e.g., Fudenberg and Tirole, 1984; Chen et al., 2020).

■**Uniform Pricing.** Under uniform pricing, firm i sets a common price p_i for all consumers, and consumers do not disclose any information to firms. The utility of a consumer located at x who purchases from firm i is then given by

$$u_i^U(x) = v - p_i - z_i. \tag{2}$$

Under uniform pricing, AI and disclosure do not enter utility, as prices cannot depend on consumer-specific information. Uniform pricing corresponds to list-price competition and can coexist with personalized pricing environments (e.g., Chen et al., 2022).

2.3 Technology and Cost Functions

■**AI Technology.** Firm i's AI technology is represented by $\phi(s_i, y) \in [0, 1)$, which depends on the firm's investment level s_i and the consumer's disclosure level y. We assume the following:

$$\phi_s > 0, \quad \phi_y > 0, \quad \phi_{sy} > 0, \quad \phi_{ss} \leq 0, \quad \phi_{yy} \leq 0. \tag{3}$$

These conditions ensure complementarity between investment and disclosure and diminishing marginal returns. Since $\phi(s_i, y) < 1$, AI technology cannot fully eliminate mismatch costs. These assumptions are consistent with models in which data improves product matching or reduces mismatch costs (e.g., Chen et al., 2022; Ichihashi, 2020; Hagiu and Wright, 2023). Unlike those studies, we endogenize the effectiveness of data-processing technology through investment and disclosure.

■**Information Disclosure Costs.** The cost of information disclosure is given by a function $d(y)$ satisfying

$$d'(y) > 0, \quad d''(y) \geq 0, \quad d(0) = 0. \tag{4}$$

■**Investment Cost.** Firm i incurs an investment cost $C(s_i)$, which satisfies

$$C'(s_i) > 0, \quad C''(s_i) \geq 0, \quad C(0) = 0. \tag{5}$$

These assumptions ensure well-behaved optimization and are consistent with the functional specification adopted in Section 5.

2.4 Optimal Information Disclosure and Effective Cost

Consumers optimally choose disclosure in the final stage, which yields an effective cost representation. This subsection characterizes optimal disclosure and introduces the notion of effective cost. To simplify exposition, we focus on a consumer located at x who purchases from firm A. An analogous argument applies to firm B.

Under personalized pricing, a consumer at distance x chooses $y \geq 0$ to minimize $(1 - \phi(s_i, y))x + d(y)$. For an interior solution, the first-order condition is

$$\phi_y(s_i, y)\, x = d'(y). \tag{FOC}$$

Given $\phi_{yy} \leq 0$ and $d'' \geq 0$, the objective is convex in y, yielding a unique solution $y^*(s_i, x)$. The following lemma summarizes the existence, uniqueness, and comparative statics of the optimal disclosure choice.

Lemma 1. *For any given investment level $s \geq 0$ and consumer location $x \in [0,1]$, the consumer's optimal disclosure choice*

$$y^*(s,x) \;\in\; \arg\min_{y\geq 0} \{(1 - \phi(s,y))x + d(y)\}$$

exists and is unique. Moreover, under the maintained assumptions on the AI technology ϕ and the disclosure cost function d, the optimal disclosure satisfies

$$\frac{\partial y^*(s,x)}{\partial s} > 0, \qquad \frac{\partial y^*(s,x)}{\partial x} > 0.$$

Proof. The proof is provided in Appendix A.1. □

This lemma implies that higher investment and greater distance increase disclosure incentives (e.g., Ali et al., 2023; Lefouili et al., 2024).

■Effective Cost. Define the effective cost as the minimized mismatch cost after optimal disclosure:

$$\psi(s,x) \;\equiv\; \min_{y\geq 0} \{(1 - \phi(s,y))\, x + d(y)\}\,. \tag{6}$$

Using this definition, the utility of a consumer purchasing from firm i under personalized pricing can be written compactly as

$$u^P(x) = v - p(x) - \psi(s,x). \tag{7}$$

■Properties of the Effective Cost. The maintained assumptions on ϕ and d imply the following structural properties.

Lemma 2. *Define*

$$\psi(s,x) = (1 - \phi(s, y^*(s,x)))x + d(y^*(s,x)).$$

where $y^*(s,x)$ is given by Lemma 1. Then, for all (s,x) with $x \geq 0$,

$$\psi_x > 0, \qquad \psi_s \leq 0, \qquad \psi_{sx} \leq 0, \qquad \psi_{xx} \leq 0.$$

Proof. The proof is provided in Appendix A.2. ☐

Thus, investment lowers effective mismatch costs while preserving monotonicity and concavity in distance. The curvature of ψ with respect to s is left unrestricted and depends on the functional form (Appendix D). In the asymmetric model, $s_B = 0$, implying $y_B^* = 0$ and $\psi(0, 1-x) = 1-x$.

2.5 Functional Forms

We adopt tractable functional forms consistent with the maintained assumptions.

■**AI Technology (Seeded Exponential).** For the remainder of the analysis, we adopt the seeded exponential specification

$$\phi(s_i, y) = 1 - \exp\bigl(-a(\sigma + s_i)y\bigr), \qquad a > 0,\ \sigma \geq 0. \tag{8}$$

The parameter σ captures baseline AI capability. This specification satisfies the maintained assumptions and preserves complementarity with diminishing returns.[*12]

■**Costs.** The cost of information disclosure and AI investment are assumed to be quadratic:

$$d(y) = \frac{b}{2}y^2, \qquad C(s_i) = \frac{c}{2}s_i^2, \qquad b > 0,\ c > 0. \tag{9}$$

■**Inverse-Function Representation of Optimal Disclosure.** Under these functional forms, the first-order condition for optimal disclosure becomes

$$by = a(\sigma + s_i)z_i \exp\bigl(-a(\sigma + s_i)y\bigr) \tag{10}$$

The left-hand side is strictly increasing in y, while the right-hand side is strictly decreasing, implying a unique solution that defines the optimal disclosure as an inverse function. This solution can be written in closed form using the Lambert W function, though the main analysis relies only on existence, uniqueness, and monotonicity.

While the main analysis proceeds under general assumptions, these functional forms provide a tractable illustration. Appendix D.1–D.2 detail the optimal disclosure decision, the resulting effective cost, and its curvature properties under the seeded exponential specification.

[*12] Exponential learning or matching functions are widely used to capture diminishing returns in data processing and digital learning (e.g., Goldfarb and Tucker, 2012; Hagiu and Wright, 2023. The seeded structure allows baseline capability and investment to enter symmetrically while preserving complementarity.

2.6 Consumer Surplus, Profits, and Social Welfare

■**Consumer Surplus.** Consumer surplus is the sum of utilities integrated over the corresponding demand sets. Let D_i denote the demand set of firm $i \in \{A, B\}$, with $D_A \cup D_B = [0,1]$ and $D_A \cap D_B = \varnothing$. Consumer surplus is then given by

$$U = \int_{D_A} u_A(x)\,dx + \int_{D_B} u_B(x)\,dx. \tag{11}$$

■**Firm Profits.** With zero marginal production cost, firm i's profit equals total price revenue net of investment cost:

$$\pi_i = \int_{D_i} p_i(x)\,dx - C(s_i). \tag{12}$$

Under personalized pricing, prices may vary across consumers, while under uniform pricing we have $p_i(x) \equiv p_i$. Uniform pricing is treated as a special case of personalized pricing.

■**Social Welfare.** Social welfare is defined as the sum of consumer surplus and the profits of both firms:

$$W = U + \pi_A + \pi_B. \tag{13}$$

Under this definition, prices are pure transfers between consumers and firms. Welfare comparisons across pricing structures therefore reflect differences in effective costs and investment incentives rather than redistributive price effects.

3 Benchmark: Symmetric Personalized Pricing with Investment

We begin with a benchmark in which both firms perfectly personalize prices and invest in AI technology. Because allocation is governed directly by effective costs, technological improvements translate one-for-one into competitive outcomes without additional distortions.

The game consists of two stages. In Stage 1, firms simultaneously choose their investment levels. In Stage 2, given investments, firms engage in personalized price competition for each consumer. We solve the game by backward induction and characterize the subgame perfect equilibrium.

3.1 Personalized Price Competition

Consider a consumer located at $x \in [0,1]$. Let the effective cost of purchasing from firm $i \in \{A, B\}$ be $\psi(s_i, z_i(x))$, where $z_A(x) = x$ and $z_B(x) = 1-x$. Given (s_A, s_B), Stage 2 reduces to independent Bertrand competition at each location due to perfect price discrimination, as in (Thisse and Vives, 1988; Chen et al., 2020).

To simplify notation and clarify the structure of competition, we use the relative effective cost advantage from firm A's perspective, $\Delta(x; s_A, s_B) \equiv \psi(s_B, 1-x) - \psi(s_A, x)$. A positive $\Delta(x)$ indicates a cost advantage for firm A, and a negative value an advantage for firm B. Because effective costs depend on investment, $\Delta(x)$ captures how investment asymmetries translate into competitive advantages.

Because marginal production costs are normalized to zero and firms can set prices individually for each consumer, price competition at each x yields equilibrium personalized prices

$$p_A^*(x) = \max\{0, \Delta(x; s_A, s_B)\}, \qquad p_B^*(x) = \max\{0, -\Delta(x; s_A, s_B)\}. \tag{14}$$

Thus, prices equal effective cost differences, and the high-cost firm's price is driven to zero.

3.2 Marginal Consumer and Market Allocation

In an interior equilibrium, the market is divided by a marginal consumer x^* who is indifferent between firms. The boundary is characterized by

$$\Delta(x^*; s_A, s_B) = 0 \quad \Longleftrightarrow \quad \psi(s_A, x^*) = \psi(s_B, 1-x^*). \tag{15}$$

This condition uniquely determines the boundary where effective costs coincide. Consumers with $x < x^*$ purchase from A, and those with $x > x^*$ from B. The boundary depends on both firms'investment levels through effective costs.

3.3 Investment Decisions

Anticipating Stage 2, firm A chooses s_A to maximize

$$\begin{aligned} \pi_A(s_A, s_B) &= \int_0^{x^*(s_A,s_B)} p_A^*(x)\,dx - C(s_A) \\ &= \int_0^{x^*(s_A,s_B)} \Delta(x; s_A, s_B)\,dx - C(s_A). \end{aligned} \tag{16}$$

A symmetric expression applies to firm B. Differentiating π_A with respect to s_A yields

$$\frac{\partial \pi_A}{\partial s_A} = \int_0^{x^*(s_A,s_B)} \frac{\partial \Delta}{\partial s_A}\,dx + \Delta(x^*; s_A, s_B)\frac{\partial x^*}{\partial s_A} - C'(s_A) \tag{17}$$

The second term captures the boundary effect but vanishes because $\Delta(x^*) = 0$. Since $\Delta_{s_A}(x) = -\psi_s(s_A, x) > 0$, the first-order condition for firm A's optimal investment simplifies to

$$\int_0^{x^*(s_A,s_B)} \{-\psi_s(s_A, x)\}dx = C'(s_A). \tag{FOC-A}$$

The term $-\psi_s(s, x)$ represents the **location-based technological effect**, i.e., the marginal reduction in effective cost at location x. Aggregating over firm A's market yields the total technological effect governing investment incentives.

To characterize equilibrium investment, we also impose the second-order condition (SOC). Differentiating once more and applying Leibniz's rule yields

$$\frac{d^2\pi_A}{ds_A^2} = \int_0^{x^*(s_A,s_B)} \left\{-\psi_{ss}(s_A,x)\right\} dx + \left\{-\psi_s(s_A,x^*)\right\} \frac{\partial x^*}{\partial s_A} - C''(s_A). \tag{18}$$

From Section 2, $\psi_s(s,x) < 0$ holds for all $x > 0$, and the market boundary satisfies $\partial x^*/\partial s_A \geq 0$. Hence, the boundary term enters weakly positively and weakens concavity. Since ψ_{ss} is unrestricted in general, the SOC cannot be signed globally. A sufficient condition for strict concavity is therefore

$$\int_0^{x^*(s_A,s_B)} \left\{-\psi_{ss}(s_A,x)\right\} dx + \left\{-\psi_s(s_A,x^*)\right\} \frac{\partial x^*}{\partial s_A} < C''(s_A), \tag{19}$$

which ensures $d^2\pi_A/ds_A^2 < 0$ and hence uniqueness of the optimal investment. In Section 5 we verify concavity and the existence of an interior solution under the seeded exponential specification [*13].

By symmetry, the equilibrium investment levels satisfy $s_A = s_B = s^P$. In this case, the boundary condition simplifies to $\psi(s^P, x) = \psi(s^P, 1-x)$, which immediately yields

$$x^* = \tfrac{1}{2}. \tag{20}$$

Thus, in the symmetric benchmark equilibrium, $x^* = \frac{1}{2}$ and does not depend on investment. An interior equilibrium exists when the FOC holds and the SOC is satisfied, implying

$$s^* > 0. \tag{21}$$

Thus, investment incentives in the benchmark model are governed solely by the aggregate technological effect. We now evaluate whether private incentives coincide with the socially optimal level.

3.4 Social Evaluation

Consider a social planner who chooses investment levels. Social welfare is

$$\begin{aligned} W(s_A,s_B) = &\int_0^{x^*(s_A,s_B)} \left[v - \psi(s_A,x)\right] dx \\ &+ \int_{x^*(s_A,s_B)}^1 \left[v - \psi(s_B,1-x)\right] dx - C(s_A) - C(s_B) \end{aligned} \tag{22}$$

Under complete personalized pricing, prices are pure transfers and allocation is determined by effective cost minimization. Welfare therefore depends only on effective and investment costs.

[*13] Since we assume $C'(0) = 0$, it holds $\frac{d\pi_A}{ds_A}\Big|_{s_A=0} = \int_0^{x^*(0,s_B)} \{-\psi_s(0,x)\} dx > 0$. Thus, firm A strictly benefits from marginally increasing investment above zero.

The marginal effect of firm A's investment from the social planner's perspective is

$$\frac{dW}{ds_A} = \int_0^{x^*(s_A, s_B)} \{-\psi_s(s_A, x)\} dx - C'(s_A)$$

The boundary term vanishes, so this coincides with firm A's private FOC. An analogous argument applies to firm B. We therefore obtain the following benchmark result.

Proposition 1. *In the symmetric benchmark model with complete personalized pricing, the privately optimal investment coincides with the socially optimal investment:*

$$s_A^* = s_B^* = s^W.$$

This benchmark property reflects the logic of competitive price discrimination in Hotelling environments, where complete personalization implies effective-cost-based allocation and prices act as transfers (Thisse and Vives, 1988; Chen et al., 2020).[*14] Because investment affects welfare only through inframarginal cost reductions, private and social incentives coincide. In the next section, we show that this alignment breaks down under asymmetric pricing. The existence of an interior solution is examined numerically in Section 5.

4 The Asymmetric Model

4.1 Analytical roadmap

We analyze a three-stage asymmetric model in which firm A personalizes and invests, while firm B sets a uniform price and does not invest. The equilibrium is derived by backward induction.

In Stage 3, given s and p_B, consumers choose where to purchase. Those buying from A optimally disclose information, determining effective cost $\psi(s, x)$. The marginal consumer x^* satisfies

$$\psi(s, x^*) = p_B + (1 - x^*),$$

and therefore depends on both s and p_B, that is, $x^* \equiv x^*(s, p_B)$. Firm A's personalized price schedule is then given by $p_A(x) = p_B + (1 - x) - \psi(s, x)$ for all $x \leq x^*$. Unlike the benchmark, allocation is not determined solely by effective costs: because firm B sets a uniform price, the boundary reflects total cost comparisons.

In Stage 2, firm B chooses its uniform price to maximize profit, taking into account how the marginal consumer responds to price changes. Firm B's optimal pricing condition must

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[*14] This coincidence is not accidental. The social planner chooses only investment levels, while prices are determined by firms; hence the welfare benchmark is second-best.

therefore be satisfied jointly with the boundary condition. Solving Stage 2 yields $p_B^*(s)$ and the corresponding personalized schedule $p_A^*(x)$.

Given these optimized pricing decisions, in Stage 1 firm A chooses its investment level s to maximize profit, anticipating the induced responses of prices and the marginal consumer.

Unlike the benchmark, the marginal consumer responds to investment, and A's incentives interact with B's price response. Investment therefore affects effective costs, the boundary, and strategic pricing, creating a distortion channel absent under symmetric personalization.

4.2 Equilibrium analysis

4.2.1 Personalized pricing and consumer behavior

In Stage 3, consumers purchasing from A optimally disclose information, determining effective cost $\psi(s, x)$. Firm B does not invest and induces no disclosure, so $\psi_B(0, 1-x) = 1-x$. We write $\psi_A(s, x) = \psi(s, x)$ when no confusion arises. As shown in Section 2, optimal disclosure is uniquely summarized by $\psi(s, x)$, which we take as given in Stage 3.

A consumer at $x \in [0, 1]$ purchases from A if $u_A(x) \geq u_B(x) \Leftrightarrow p_A(x)+\psi(s, x) \leq p_B+(1-x)$. Firm A sets $p_A(x) \leq p_B + (1-x) - \psi(s, x)$ whenever non-negative. In equilibrium, consumers with $x \leq x^*$ purchase from firm A, where the marginal consumer x^* is determined by the boundary condition

$$\psi(s, x^*) = (1 - x^*) + p_B, \tag{BC}$$

because firm A offers $p_A(x^*) = 0$. Unlike the benchmark, where $\psi_A(s, x^*) = \psi_B(s, 1 - x^*)$ and $x^* = \frac{1}{2}$ under symmetry, the asymmetric boundary condition becomes

$$\psi_A(s, x^*) = \psi_B(0, 1 - x^*) + p_B.$$

Since firm A invests while firm B does not, and the boundary condition in the asymmetric model is determined by total cost rather than effective cost alone, the equality can be restored only at a larger value of x^*. Hence, the marginal consumer satisfies the following [*15]:

$$x^* > \tfrac{1}{2}. \tag{23}$$

At the margin, $\psi_A(s, x^*) > \psi_B(0, 1-x^*)$, even though consumer x^* purchases from A. Thus, some consumers who would minimize effective cost instead purchase from A due to firm B' s uniform pricing. This price-induced distortion renders the asymmetric allocation inefficient relative to the benchmark.

[*15] Since $s_A = s > 0$ and $s_B = 0$, at $x = \frac{1}{2}$ we have $\psi_A(s, \frac{1}{2}) < \psi_B(0, \frac{1}{2})$. Because $\psi_x > 0$, restoring the boundary condition requires increasing x, implying $x^* > \frac{1}{2}$.

4.2.2 firm B's pricing decision

In Stage 2, given s, firm B chooses p_B anticipating the boundary response. Its problem is

$$\max_{p_B \geq 0} \pi_B(p_B; s) = p_B\bigl(1 - x^*(p_B; s)\bigr), \tag{24}$$

where $x^*(p_B; s)$ denotes the marginal consumer implicitly determined by p_B and s. The boundary adjusts endogenously to satisfy $\psi(s, x^*) = p_B + (1 - x^*)$.

Differentiating the boundary condition with respect to p_B and applying the implicit function theorem yields

$$x_p^* \equiv \frac{\partial x^*}{\partial p_B} = \frac{1}{1 + \psi_x(s, x^*)} > 0, \tag{25}$$

since $\psi_x > 0$ by construction. Thus, a higher p_B shifts the boundary rightward and reduces B's demand.

Firm B's first-order condition is therefore

$$\frac{\partial \pi_B}{\partial p_B} = (1 - x^*(p_B; s)) - p_B x_p^* = 0, \tag{26}$$

which implicitly defines the optimal uniform price, $p_B^*(s)$. The positivity of x_p^* captures competitive pressure to firm B: a higher price reduces B's demand.

Rewriting this condition yields

$$p_B^*(s) = \frac{1 - x^*(s)}{x_p^*(p_B^*(s), s)} = (1 - x^*(s))\bigl(1 + \psi_x(s, x^*(s))\bigr). \tag{FOC-B}$$

This first-order condition characterizes firm B's optimal pricing rule for a given investment level.[*16] The first-order condition for firm B (FOC-B) and the boundary condition (BC) derived in Stage 3 must be satisfied simultaneously. Substituting the former into the latter, the boundary condition reduces to

$$\psi(s, x^*(s)) = (1 - x^*(s))\bigl(2 + \psi_x(s, x^*(s))\bigr). \tag{BC-red}$$

Equation (BC-red) marks the key departure from the benchmark: the boundary depends not only on effective cost levels but also on the slope ψ_x, which enters through firm B's pricing condition. This interaction generates the distortion channel emphasized in Section 1.2.

After solving the Stage-2 subgame, we examine how the equilibrium boundary responds to firm A's investment decision. To this end, we first characterize the response of the marginal consumer. The term $-\psi_s(s, x)$, as we mentioned in Section 3, represents the location-based technological effect. By contrast, the cross-partial term $\psi_{xs}(s, x)$ captures how investment alters the slope of effective cost with respect to distance. This generates a **distance-sensitivity**

[*16] This corresponds to the standard unit-elastic residual demand condition under zero marginal cost.

effect, $(1-x^*(s))\psi_{xs}(s,x^*(s))$, which reflects how investment changes the relative competitive position at the boundary.

We refer to $x^*_s \equiv \frac{dx^*}{ds}$ as the **boundary-response effect**. The inequality

$$\psi_s(s,x^*(s)) \le (1-x^*(s))\psi_{xs}(s,x^*(s))$$

as the **dominance condition**. The following lemma formalizes this observation.

Lemma 3. *In the asymmetric equilibrium, the boundary-response effect is weakly positive whenever the dominance condition holds. Formally,*

$$x^*_s \ge 0 \quad \textit{if} \quad \psi_s(s,x^*(s)) \le (1-x^*(s))\psi_{xs}(s,x^*(s)).$$

The proof in Appendix B.1 applies the implicit function theorem to the reduced boundary condition.

We next characterize firm B's equilibrium price response, $p^*_s \equiv \frac{dp^*_B}{ds}$, referred to as the **price-response effect**.

Lemma 4. *In the asymmetric equilibrium, firm B's price responds negatively to firm A's investment under the same condition as in Lemma 3. Formally,*

$$p^*_s \le 0$$

Proof. The proof is provided in Appendix B.2. Here we give a brief sketch for completeness. Differentiating p^*_B with respect to s,

$$\frac{dp^*_B}{ds} = \Big[(1-x^*(s))\psi_{xx}(s,x^*(s)) - \big(1+\psi_x(s,x^*(s))\big)\Big]x^*_s + (1-x^*(s))\psi_{xs} \tag{27}$$

From the model assumption, $\psi_x > 0$, $\psi_{xx} \le 0$, and $\psi_{xs} \le 0$. Moreover, when Lemma 3 is applied, $x^*_s \ge 0$. Together these imply $p^*_s \le 0$. □

Taking this price reaction as given, we now turn to firm A's investment decision in Stage 1.

4.2.3 firm A's investment decision

In Stage 1, firm A chooses s, anticipating $p^*_B(s)$, the marginal consumer $x^*(s)$, and the induced pricing schedule $p^*_A(x) = p^*_B(s) + (1-x) - \psi(s,x)$ for $x \in [0, x^*(s)]$. Firm A's objective function is

$$\begin{aligned} \pi_A(s) &= \pi_A\big(s,\ p^*_B(s),\ x^*(s)\big) \\ &= \int_0^{x^*(s)} \Big[p^*_B(s) + (1-x) - \psi(s,x)\Big]\,dx - C(s). \end{aligned} \tag{28}$$

Differentiating $\pi_A(s)$ with respect to s yields the first-order condition

$$\frac{d\pi_A}{ds} = \int_0^{x^*(s)} \big(- \psi_s(s,x)\big)\, dx \ + \ x^* p_s^*(s) \ + \ \big(p_B^*(s) + (1-x^*) - \psi(s,x^*)\big)\, x_s^* \ - \ C_s(s) = 0$$

$$\Rightarrow \int_0^{x^*(s)} \big(- \psi_s(s,x)\big)\, dx \ + x^*(s) p_s^*(s) - C_s(s) = 0. \tag{FOC-A}$$

The first term is the **technological effect**, the aggregate reduction in effective costs. The second, $x^*(s)p_s^*(s)$, is the **competition effect**, reflecting firm B's price response. The last term is marginal investment cost. By the envelope theorem and the boundary condition, terms involving x_s^* vanish since $p_B^*(s) + (1-x^*) - \psi(s,x^*) = 0$. [*17] Under Lemma 4, higher investment intensifies effective competition on firm B's side, implying $p_s^*(s) \leq 0$. Hence the competition effect weakens firm A's investment incentives. Unlike the benchmark, investment alters firm B's pricing rule through the boundary condition, introducing a strategic distortion.

In the specific functional form we consider, the first-order condition can be positive at $s = 0$, so that a strictly positive interior investment level may arise. As investment increases, however, the competition effect may become stronger, potentially offsetting the technological effect. [*18]

Taking the second derivative with respect to s,

$$\begin{aligned} \frac{d^2\pi_A}{ds^2} &= \int_0^{x^*(s)} (-\psi_{ss})\, dx + x^*(s) p_{ss}^* + (p_s^* - \psi_s)\, x_s^* - C_{ss} \\ &= \int_0^{x^*(s)} (-\psi_{ss})\, dx + x^*(s) p_{ss}^* + (1+\psi_x)(x_s^*)^2 - C_{ss} \end{aligned} \tag{29}$$

The second derivative decomposes into curvature of effective costs, curvature of firm B's price response, endogenous boundary adjustments, and investment-cost curvature. Because the signs of higher-order price and boundary responses are not restricted in general, the SOC cannot be signed without additional assumptions. A detailed decomposition is provided in Appendix B.3. Despite this ambiguity, welfare comparisons remain possible by evaluating the social marginal benefit at the private optimum. The analytical indeterminacy motivates the functional specification and numerical analysis in Section 5.

4.3 Social Evaluation of Investment

We evaluate investment from a second-best social perspective, taking Stage 2–3 behavior as given and choosing s to maximize welfare.

[*17] This reflects that the marginal consumer earns zero surplus at the optimum.

[*18] At $s = 0$, the marginal investment cost vanishes and a marginal increase in investment reduces effective costs at first order. By contrast, price and boundary responses do not react immediately to an infinitesimal change in investment. Hence, in the vicinity of $s = 0$, the technological effect tends to dominate the competitive effect.

■Social welfare. Since prices are pure transfers, social welfare equals gross valuation minus effective costs and investment costs:

$$W(s) = \int_0^{x^*(s)} \big[v - \psi(s,x)\big]dx + \int_{x^*(s)}^1 \big[v - (1-x)\big]dx - C(s). \tag{30}$$

Differentiating with respect to s yields

$$\frac{dW}{ds} = \int_0^{x^*(s)} \big(-\psi_s(s,x)\big)dx - p_B^*(s)x_s^*(s) - C_s(s), \tag{31}$$

using the boundary condition $\psi(s,x^*) = p_B^*(s) + (1-x^*)$.

■Private marginal incentive. Firm A's profit is

$$\pi_A(s) = \int_0^{x^*(s)} \big[p_B^*(s) + (1-x) - \psi(s,x)\big]dx - C(s).$$

Differentiation yields

$$\frac{d\pi_A}{ds} = \int_0^{x^*(s)} (-\psi_s(s,x))dx + x^*(s)p_s^*(s) - C_s(s). \tag{32}$$

■Wedge between social and private incentives. Subtracting (32) from (31) yields

$$\frac{dW}{ds} - \frac{d\pi_A}{ds} = -\frac{d}{ds}\big(x^*(s)p_B^*(s)\big). \tag{33}$$

We refer to $x^*p_B^*$ as the **uniform-pricing distortion**. Equation (33) shows that the wedge between social and private incentives is entirely governed by the responsiveness of this distortion to investment. Expanding the derivative,

$$\frac{d}{ds}\big(x^*p_B^*\big) = p_B^*x_s^* + x^*p_s^*. \tag{34}$$

The term decomposes into two channels: $p_B^*x_s^*$ (boundary effect) and $x^*p_s^*$ (competition effect).

Evaluating at the privately optimal investment level s^P, defined by $d\pi_A/ds = 0$, we obtain

$$\left.\frac{dW}{ds}\right|_{s=s^P} = -\frac{d}{ds}\big(x^*p_B^*\big)\Big|_{s=s^P}. \tag{35}$$

Hence, private investment is socially insufficient if and only if investment reduces the distortion, that is, $\frac{d}{ds}(x^*p_B^*) < 0$. Conversely, private investment is socially excessive if investment enlarges the distortion.

The following proposition summarizes this result.

Proposition 2. *Suppose that Lemma 3 holds ($x_s^* > 0$) and that firm B's price responds to investment according to Lemma 4. Then, at the privately optimal investment level s^P:*

1. *Investment is socially insufficient if*
$$p_B^*(s^P)x_s^*(s^P) + x^*(s^P)p_s^*(s^P) < 0.$$
2. *Investment is socially excessive if*
$$p_B^*(s^P)x_s^*(s^P) + x^*(s^P)p_s^*(s^P) > 0.$$

Investment affects allocative efficiency through two channels: it reduces effective costs and shifts the distorted boundary. If firm B's price response is sufficiently strong, the distortion shrinks and investment is socially insufficient. If boundary expansion dominates, distortion increases and investment is socially excessive. The wedge arises solely from asymmetric pricing; under symmetric personalization, $x^*p_B^* = 0$ and incentives coincide.

5 Simulation

This section evaluates the model numerically under the seeded exponential specification introduced in Subsection 2.5, which satisfies all maintained assumptions (Appendix D.1–D.2). We compute equilibrium prices, market boundaries, and investment incentives that cannot be characterized in closed form.

5.1 Numerical Setup

■**Market boundary.** In the benchmark model (BM) with symmetric personalized pricing, the market boundary is fixed at $x^* = 1/2$ and does not respond to investment. Investment therefore affects only effective costs. In the asymmetric model (AM), the boundary is endogenous and satisfies
$$\psi(s, x^*) = p_B^*(s) + (1 - x^*).$$

Investment thus affects both effective costs and market shares through this condition.

■**Investment problem.** Firm A chooses s anticipating the pricing equilibrium. In both models, the problem reduces to a one-dimensional maximization in s (Sections 3 and 4).

■**Computational procedure.** For each parameter configuration, we compute[*19]:

1. the market boundary $x^*(s)$ (AM),
2. firm B's equilibrium price $p_B^*(s)$ (AM),
3. firm A's profit $\pi_A(s)$,
4. the privately optimal investment s^*,
5. the socially optimal investment s^W.

[*19] The parameter roles are structurally interpreted in Appendix D.3.2

Table1: Baseline equilibrium outcomes under the seeded exponential specification.

	Benchmark model (BM)	Asymmetric model (AM)
Baseline parameters	$a = b = c = 1,\ \sigma = 0.2$	$a = b = c = 1,\ \sigma = 0.2$
Private investment	$s^* \approx 0.0084$	$s^P = 0$
Social investment	$s^W = s^*$	$s^W \approx 0.0503$
Market boundary	$x^* = \frac{1}{2}$	$x^* \approx 0.7510$
Firm A average price	$\bar{p}_A \approx 0.490$	$\bar{p}_A \approx 0.7434^\dagger$
Firm B price	—	$p_B^* \approx 0.4901^\dagger$
Firm A profit	$\pi_A \approx 0.245$	$\pi_A \approx 0.5583^\dagger$
Firm B profit	—	$\pi_B \approx 0.1220^\dagger$

Notes: In AM, outcomes marked by † are evaluated at $s = 0.01$ for illustration, since the private optimum is $s^P = 0$.

5.2 Numerical Results

This subsection evaluates the mechanisms identified in Sections 3 and 4 under the seeded exponential specification. While the general analysis leaves the curvature of effective cost with respect to investment ambiguous, the specific functional form allows us to verify the second-order condition and compute equilibrium outcomes numerically.

Baseline equilibrium values for both the benchmark model (BM) and the asymmetric model (AM) are reported in Table 1. Comparative statics are illustrated in Figures A.1–A.4 (BM) and Figures B.1–B.4 (AM). Each figure varies one parameter at a time around the baseline configuration $(a, b, c) = (1, 1, 1)$ and $\sigma = 0.2$.

■Benchmark Model (BM) In the symmetric benchmark model, private and socially optimal investment coincide. Because personalized pricing allocates consumers according to effective costs, no price-induced allocation distortion arises. Investment therefore affects welfare solely through the technological effect, i.e., the reduction in effective costs.

Figures A.1–A.4 confirm the theoretical predictions.

- Higher baseline capability (σ) or technology efficiency (a) increases equilibrium investment.
- Higher disclosure cost (b) or investment cost (c) reduces equilibrium investment.

In all cases, prices and profits move inversely with investment, reflecting intensified competition when effective costs fall. These results establish the benchmark in which investment functions purely as a competition-adjusting device and no wedge arises between private and social incentives.

■**Asymmetric Model (AM)** Under pricing asymmetry, private and socially optimal investment no longer coincide. Under the baseline parameters, private investment is zero while socially optimal (second-best) investment is strictly positive (Table 1), implying underinvestment.

This outcome directly reflects the distortion term $x^*(s)p_B^*(s)$ identified in Section 4.3. Numerically, the distortion term decreases locally at $s = 0$, implying that the allocative correction effect dominates. Hence, the social marginal return to investment is positive at the privately chosen level.

Figures B.1–B.4 show that parameter changes affect the level of socially optimal investment in the same direction as in the benchmark model:

- Higher σ or a increases socially optimal investment.
- Higher b or c decreases socially optimal investment.

However, private investment remains pinned down at zero under the baseline configuration. Accordingly, technology-enhancing parameters (σ, a) enlarge the investment gap, whereas higher cost parameters (b, c) compress it.

Importantly, the **direction** of investment responses is invariant across pricing structures, but the **level** of investment depends critically on pricing structure. Pricing asymmetry therefore transforms investment from a privately aligned competition-adjusting instrument (BM) into a source of systematic divergence between private and social incentives (AM).

Taken together, the numerical results confirm the distortion-based decomposition of Section 4.3: the investment wedge arises not from the technology itself, but from the interaction between technological cost reduction and uniform-price competition.

6 Discussion

6.1 Structural Interpretation

The analysis shows that pricing structure fundamentally governs the relationship between AI investment and welfare.

In the benchmark model (BM), symmetric personalized pricing allocates consumers according to effective costs. No price-induced allocation distortion arises, and investment affects welfare solely through reductions in effective mismatch costs. Private and socially second-best incentives therefore coincide.

Under asymmetric pricing (AM), firm B's uniform price introduces an allocation distortion.

The equilibrium boundary satisfies $\psi(s, x^*) = (1 - x^*) + p_B^*(s)$, so allocation depends on total prices rather than effective costs. The resulting wedge is captured by

$$D(s) \equiv x^*(s)p_B^*(s),$$

which measures the deviation from effective-cost minimization.

The divergence between private and social incentives is governed by

$$\frac{dW}{ds} - \frac{d\pi_A}{ds} = -\frac{d}{ds}D(s).$$

Investment reduces mismatch costs (technological effect), but may expand or contract the distorted mass of consumers through boundary and price responses (competition effect). The sign of D_s determines whether investment mitigates or amplifies the distortion.

To characterize welfare sensitivity more generally, let $W^r(s;\theta)$ denote social surplus under pricing structure $r \in \{BM, AM\}$, where $\theta \in \{\sigma, a, b, c\}$ is a structural parameter. While the welfare concept—the sum of consumer surplus and profits net of investment costs—is identical across structures, its functional form differs because allocation depends on how prices are structured.

Let $s_r^W(\theta)$ solve $\partial W^r/\partial s = 0$. By the envelope theorem,

$$\frac{d}{d\theta}W^r(s_r^W(\theta);\theta) = \left.\frac{\partial W^r}{\partial \theta}\right|_{s=s_r^W}.$$

Welfare sensitivity therefore depends only on the direct effects of parameters on effective costs and investment costs. The following proposition summarizes the implication.

Proposition 3. *Let $s_r^W(\theta)$ denote the socially second-best investment under pricing structure $r \in \{BM, AM\}$.*

(i) If $\theta \in \{\sigma, a\}$, then $s_r^W(\theta)$ increases and maximized welfare increases.

(ii) If $\theta \in \{b, c\}$, then $s_r^W(\theta)$ decreases and maximized welfare decreases.

Under asymmetric pricing, welfare changes additionally reflect the responsiveness of the distortion term $D(s)$, whereas no such distortion arises under symmetric pricing.

Parameters σ and a improve matching efficiency at the margin, raising the social return to investment. Higher disclosure or investment costs reduce the marginal gain from investment and lower maximized welfare. Pricing asymmetry influences the level of investment through $D(s)$, but not the direction of welfare sensitivity. A formal derivation is provided in Appendix C.1.

6.2 Competition Policy Implications

The central policy implication is structural. AI-investment incentives cannot be evaluated independently of the pricing structure under which firms compete.

Technology-enhancing parameters (σ, a) increase the social value of investment in both pricing structures. Under asymmetric pricing, however, private incentives may remain misaligned because investment interacts with the distortion term $D(s)$. By contrast, increases in cost parameters (b, c) reduce both socially optimal investment and welfare; a shrinking wedge in this case reflects weaker technological gains rather than improved alignment.

The distortion originates from the coexistence of personalized and uniform pricing. Policies that merely raise data frictions or investment costs do not address this structural inefficiency. Interventions that affect pricing structures—such as transparency requirements, interoperability mandates, or limits on discriminatory pricing—alter the distortion term directly and therefore reshape the investment–welfare relationship.

Digital advertising markets illustrate this mechanism. Highly personalized targeting platforms coexist with competitors relying on more uniform pricing schemes. AI investment improves matching efficiency, yet uniform pricing can distort allocation. The welfare impact of AI progress thus depends critically on the surrounding pricing structure.

More broadly, digital competition policy should consider not only data access and technological capability, but also how pricing structures translate technological improvements into market allocation. Recent policy reports in the UK, EU, and United States similarly emphasize the joint determination of AI capability, data accumulation, and market structure in digital markets.[*20]

7 Conclusion

This paper studies how pricing institutions shape AI investment incentives in data-driven markets. We develop a Hotelling framework in which effective mismatch costs are jointly determined by firm investment and consumer disclosure, and compare symmetric personalized pricing with asymmetric pricing regimes.

The analysis reveals that AI investment is institution-dependent. Under symmetric personalized pricing, consumers are allocated according to effective costs, so investment affects welfare solely through technological improvements. In this benchmark environment, private and socially second-best incentives coincide. Under asymmetric pricing, however, allocation depends on total prices rather than effective costs. Investment alters the market boundary and induces strategic price responses by the uniform-price rival, generating a distortion wedge between technological efficiency and allocation. The sign and magnitude of this wedge determine

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[*20] See, for example, Competition and Markets Authority (2019), Crémer et al. (2019), and Federal Trade Commission (2024). While these reports focus on data concentration and entry barriers, our analysis highlights a complementary mechanism: pricing structures themselves shape the investment–welfare relationship.

whether private incentives under- or over-align with social objectives.

By embedding endogenous data disclosure and investment into a spatial competition framework, the paper connects the literature on personalized pricing and privacy with the classic theory of strategic investment in oligopoly. We show that the effective mode of competition is not exogenously given by price or quantity competition, but is mediated by the pricing institution through which personalization operates. This institutional channel governs whether investment acts as a distortion-correcting or distortion-amplifying force.

The policy implication is therefore structural rather than purely technological. Interventions that focus solely on data access or investment subsidies may fail to correct welfare distortions if pricing asymmetries persist. Policies that influence the pricing environment—such as transparency requirements or rules affecting discriminatory pricing—can directly modify the allocation wedge and thereby reshape investment incentives.

Several extensions remain for future research. First, dynamic learning and multi-period data accumulation may amplify or mitigate institutional distortions over time. Second, allowing both firms to invest asymmetrically in AI capability could generate endogenous transitions between pricing regimes. Finally, incorporating richer demand heterogeneity or platform competition would further clarify how data-driven investment interacts with digital market structure.

Overall, the welfare impact of AI investment cannot be evaluated independently of the institutional structure of pricing. Understanding this interaction is essential for assessing how digital competition policy shapes innovation incentives in the AI era.

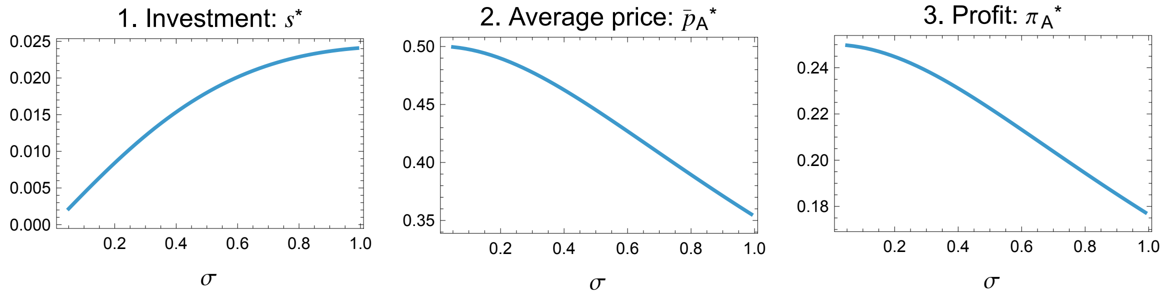
Figures

This section collects all comparative-statics figures reported in Section 5.

Figures A.1–A.4 correspond to the benchmark model (BM). Because private and socially optimal investment coincide in this model, each figure reports a single investment curve. Accordingly, throughout Figures A.1–A.4 we focus on the solid line, which represents the equilibrium investment level as well as the associated equilibrium outcomes. All remaining panels are evaluated at this common investment level.

Figures B.1–B.4 correspond to the asymmetric model (AM). In this case, private and socially optimal (second-best) investment do not coincide. Therefore, in the investment panel of each figure, the dashed line represents the privately optimal investment, whereas the solid line represents the socially second-best investment. In Panels 2–6, all equilibrium outcomes (prices, market shares, and profits) are evaluated at the corresponding investment levels shown in Panel 1, allowing for a direct comparison between private and social outcomes.

Together, these figures illustrate how pricing asymmetry generates systematic underinvestment and how the resulting investment gap varies with model parameters.

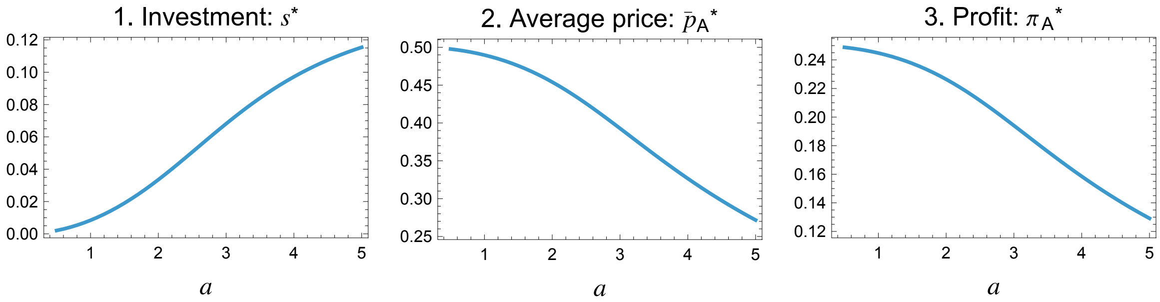


1. Investment: s^*

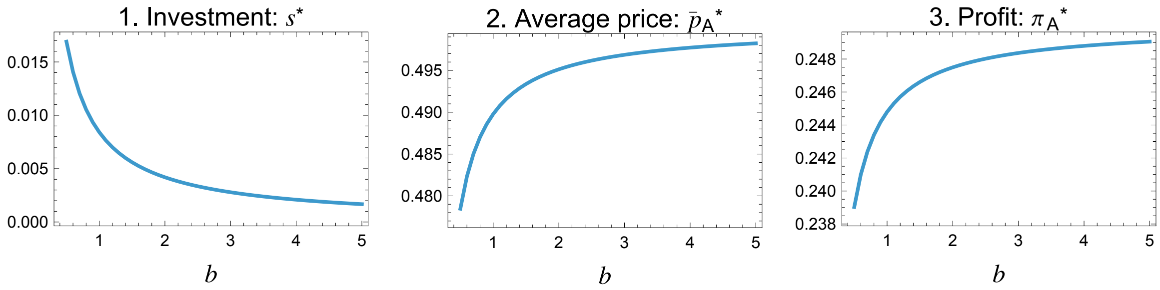
2. Average price: $\bar{p}_A{}^*$

3. Profit: $\pi_A{}^*$

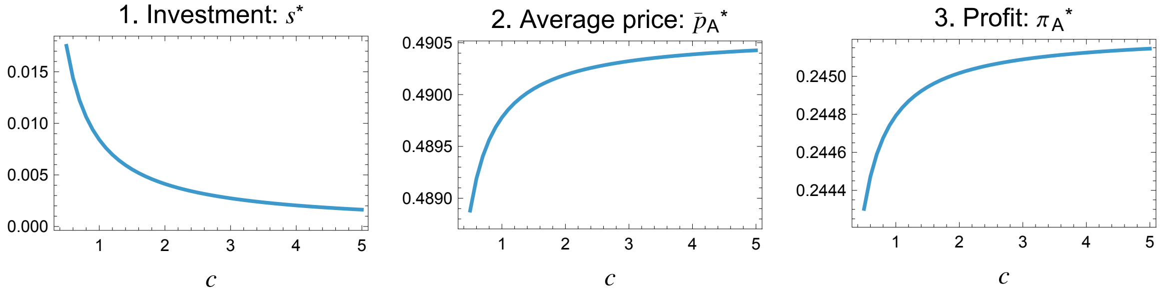
FigureA.1: Benchmark model (BM): comparative statics in σ.



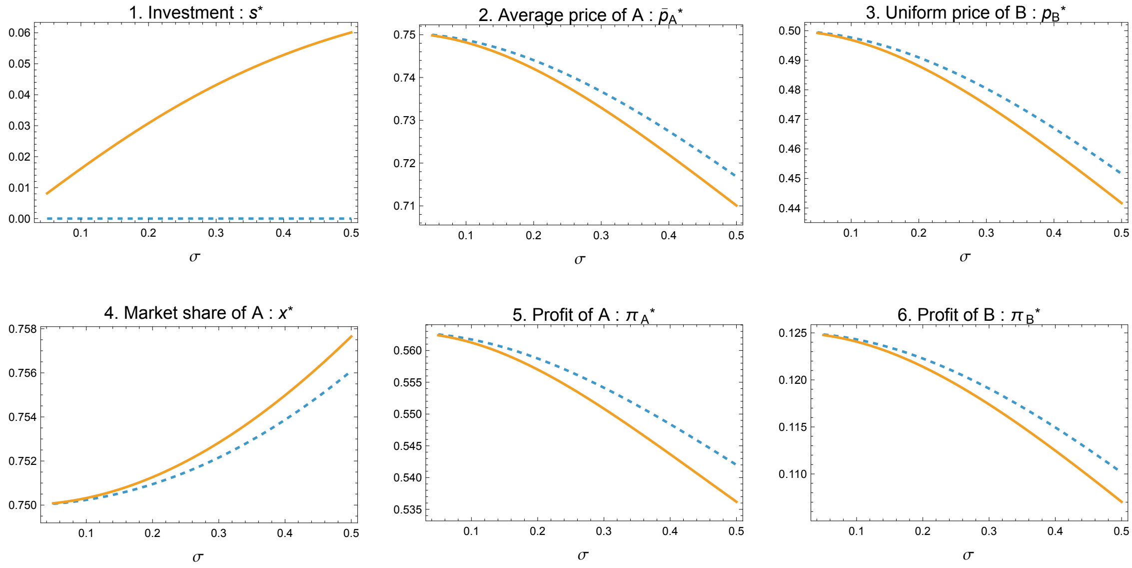
FigureA.2: Benchmark model (BM): comparative statics in a.



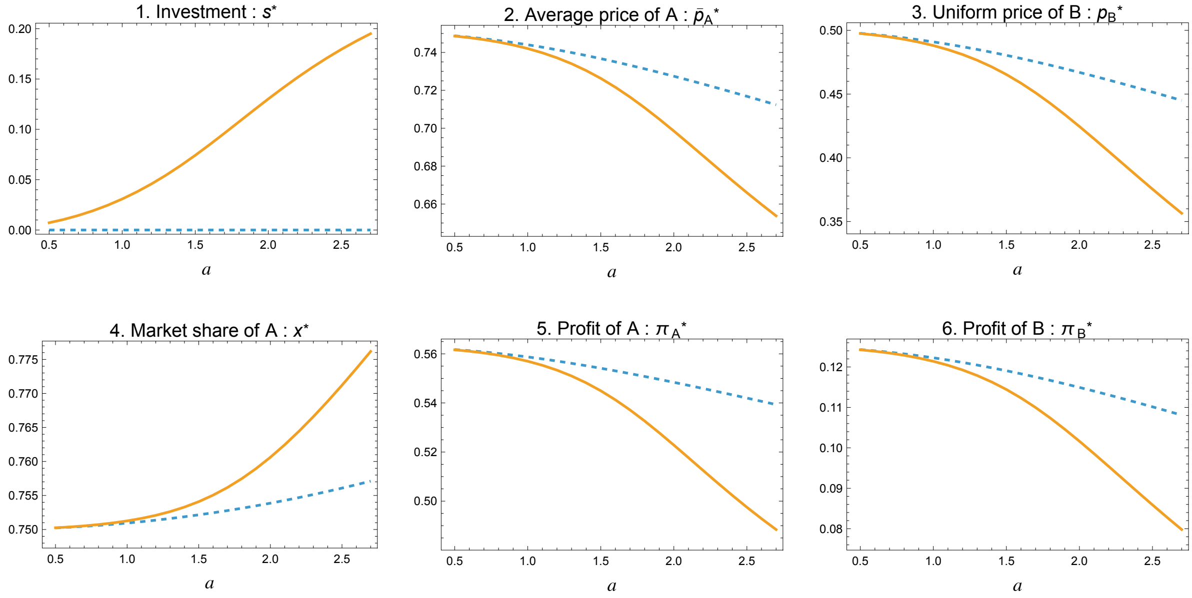
FigureA.3: Benchmark model (BM): comparative statics in b.



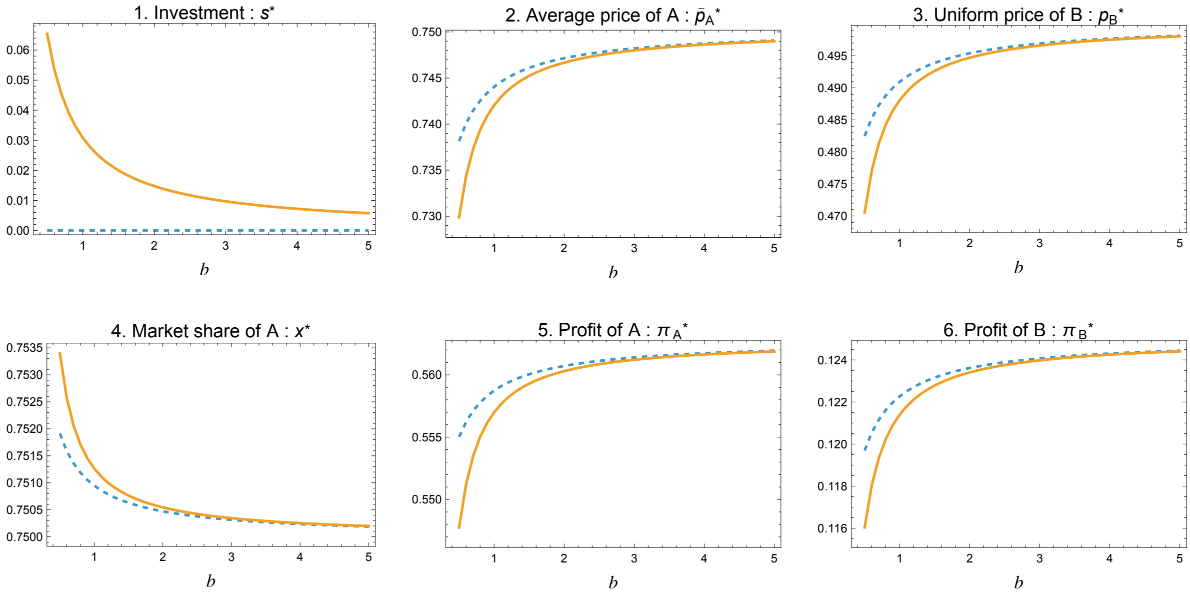
FigureA.4: Benchmark model (BM): comparative statics in c.



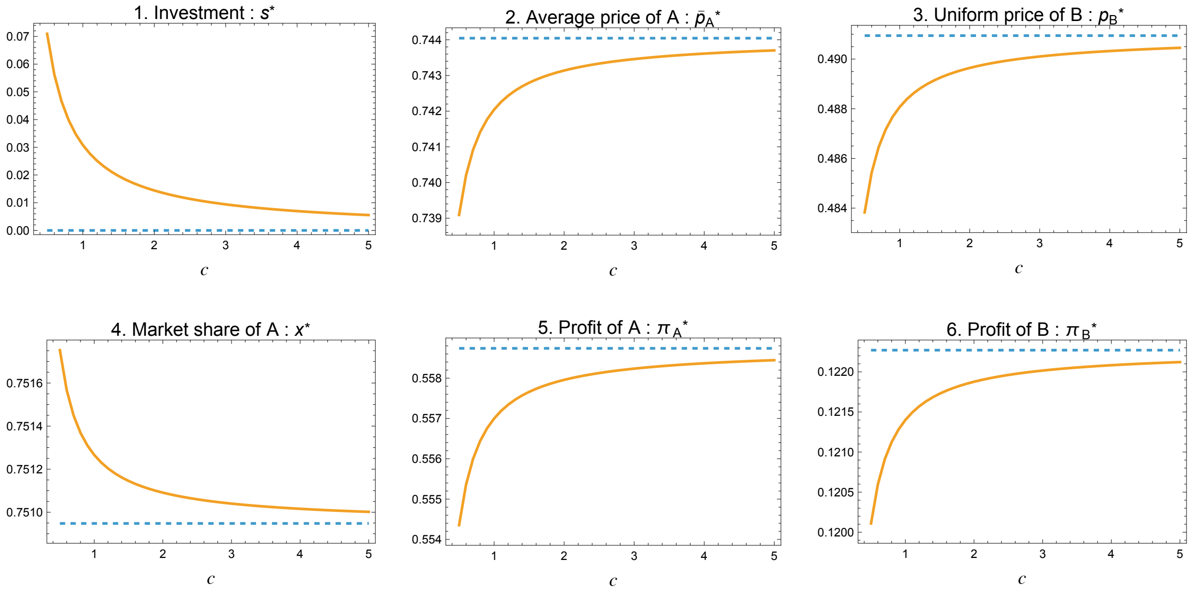
FigureB.1: Asymmetric model (AM): comparative statics in σ.



FigureB.2: Asymmetric model (AM): comparative statics in a.



FigureB.3: Asymmetric model (AM): comparative statics in b.



FigureB.4: Asymmetric model (AM): comparative statics in c.

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Appendix A Proofs for Section 2

This appendix provides formal proofs for the lemmas stated in Section 2. The assumptions imposed in the main text are chosen to be consistent with the functional forms adopted in Section 5 for numerical simulations. In the proofs below, we make explicit how these assumptions are used to establish existence, uniqueness, and comparative statics.

A.1 Proof of Lemma 1

Proof. Fix an investment level $s \geq 0$ and a consumer location $x \in [0, 1]$. Consider the consumer's disclosure problem under personalized pricing,

$$\min_{y \geq 0} \left(1 - \phi(s, y)\right) x + d(y). \tag{A.1}$$

The objective function is continuous on $y \geq 0$. As $y \to \infty$, the disclosure cost $d(y)$ diverges because $d'(y) > 0$ and $d''(y) \geq 0$. Hence, the objective function is coercive and admits a minimizer. The first derivative with respect to y is

$$\frac{\partial}{\partial y}\big[(1 - \phi(s, y))x + d(y)\big] = -\phi_y(s, y)\, x + d'(y). \tag{A.2}$$

Under the maintained assumptions, $d'(y)$ is continuous and weakly increasing in y, while $\phi_y(s, y)$ is continuous and weakly decreasing in y because $\phi_{yy}(s, y) \leq 0$. Therefore, the derivative above is strictly increasing in y, which implies that the first-order condition

$$\phi_y(s, y)\, x = d'(y) \tag{FOC-y}$$

admits a unique solution for each (s, x).

In particular, when $x = 0$, the disclosure problem reduces to

$$\min_{y \geq 0} d(y), \tag{A.3}$$

which implies $y^*(s, 0) = 0$ because $d'(y) > 0$ and $d(0) = 0$. Hence, the optimal disclosure level is well-defined on the entire domain $x \in [0, 1]$.

We now establish comparative statics. Totally differentiating (FOC-y) yields

$$d''(y^*)\, y_s^* = \phi_{sy}(s, y^*)\, x + \phi_{yy}(s, y^*)\, x\, y_s^*. \tag{A.4}$$

Rearranging gives

$$y_s^* = \frac{\phi_{sy}(s, y^*)\, x}{d''(y^*) - \phi_{yy}(s, y^*)\, x}.$$

Since $d''(y^*) \geq 0$, $\phi_{yy}(s, y^*) \leq 0$, and $\phi_{sy}(s, y^*) \geq 0$, the denominator is positive and the numerator is non-negative. Thus,

$$y_s^* \geq 0, \tag{A.5}$$

with strict inequality for $x > 0$ whenever $\phi_{sy} > 0$. Similarly,

$$y_x^* = \frac{\phi_y(s, y^*)}{d''(y^*) - \phi_{yy}(s, y^*)\, x} > 0 \quad \text{for } x > 0.$$

This establishes existence, uniqueness, and monotonicity of the optimal disclosure choice. □

A.2 Proof of Lemma 2

Proof. Recall that the effective cost is defined as

$$\psi(s, x) = (1 - \phi(s, y^*(s, x)))\, x + d(y^*(s, x)), \tag{A.6}$$

where $y^*(s, x)$ is the optimal disclosure characterized by (FOC-y).

■**First-order derivatives.** By the envelope theorem, differentiation with respect to x yields

$$\psi_x(s, x) = 1 - \phi(s, y^*(s, x)). \tag{A.7}$$

Since $\phi(s, y) < 1$ for all (s, y), it follows that

$$\psi_x(s, x) > 0. \tag{A.8}$$

Differentiating with respect to s gives

$$\psi_s(s, x) = -\phi_s(s, y^*(s, x))\, x. \tag{A.9}$$

Because $\phi_s(s, y) \geq 0$ and $x \geq 0$,

$$\psi_s(s, x) \leq 0, \tag{A.10}$$

with equality only at $x = 0$.

■**Cross-partial derivative.** Differentiating ψ_x with respect to s,

$$\psi_{sx}(s, x) = -\phi_{sy}(s, y^*(s, x))\, y_s^*(s, x). \tag{A.11}$$

From Lemma 1, $y_s^*(s, x) \geq 0$, with strict positivity for $x > 0$ when $\phi_{sy} > 0$. Since $\phi_{sy}(s, y) \geq 0$,

$$\psi_{sx}(s, x) \leq 0, \tag{A.12}$$

with strict inequality for interior consumer locations under strict complementarity.

■**Second derivative with respect to distance.** Differentiating ψ_x with respect to x,

$$\psi_{xx}(s,x) = -\phi_y(s, y^*(s,x))\, y^*_x(s,x). \tag{A.13}$$

For $x > 0$, Lemma 1 implies $y^*_x(s,x) > 0$. Since $\phi_y(s,y) > 0$,

$$\psi_{xx}(s,x) < 0 \quad \text{for } x > 0. \tag{A.14}$$

At $x = 0$, we have $y^*(s,0) = 0$ and $y^*_x(s,0) = 0$, so that $\psi_{xx}(s,0) = 0$. Hence, in general,

$$\psi_{xx}(s,x) \leq 0. \tag{A.15}$$

■**Second derivative with respect to investment.** Differentiating $\psi_s(s,x) = -\phi_s(s, y^*(s,x))\, x$ with respect to s yields

$$\psi_{ss}(s,x) = -\phi_{ss}(s,y^*)\, x - \phi_{sy}(s,y^*)\, x\, y^*_s(s,x). \tag{A.16}$$

The first term is non-negative because $\phi_{ss} \leq 0$, while the second term is non-positive because $\phi_{sy} \geq 0$ and $y^*_s \geq 0$. Therefore, the two components enter with opposing signs, and the sign of $\psi_{ss}(s,x)$ cannot be determined in general under the maintained assumptions.

Collecting the results,

$$\psi_x > 0, \quad \psi_s \leq 0, \quad \psi_{sx} \leq 0, \quad \psi_{xx} \leq 0,$$

while the curvature with respect to investment is not restricted in general. □

A.3 Economic Interpretation

Lemma 1 characterizes consumers'optimal disclosure behavior. It shows that higher AI investment by the firm and greater distance from the firm both increase the incentive for consumers to disclose information. The former reflects technological complementarity between investment and disclosure, while the latter captures the fact that consumers with a larger mismatch benefit more from effective cost reduction.

Lemma 2 summarizes the key properties of the effective cost. Effective cost is increasing in distance and decreasing in investment, while exhibiting diminishing sensitivity to distance and a non-positive cross effect between distance and investment. These properties ensure that boundary responses and price reactions in subsequent optimization problems are well behaved and do not become excessively steep.

Together, these results provide a transparent link between individual disclosure decisions, AI investment, and market-level outcomes. All comparative statics and equilibrium characterizations in Sections 3–5 can be expressed in terms of the effective cost and its properties established above.

Appendix B Proofs for Section 4

This appendix collects proofs and technical derivations for the asymmetric model analyzed in Section 4. In particular, it establishes the conditions under which the marginal consumer and firm B's price respond to investment, and clarifies why the second-order condition for firm A's investment problem cannot be signed in general form.

B.1 Proof of Lemma 3

Lemma 3 states that, in the asymmetric equilibrium, the marginal consumer $x^*(s)$ responds positively to firm A's investment provided that the technological effect dominates the distance-sensitivity effect.

Proof. In the asymmetric model with investment chosen prior to pricing, the equilibrium marginal consumer $x^*(s)$ is characterized by the reduced boundary condition

$$F(s,x) \ \equiv \ \psi(s,x) - (1-x)\big(\psi_x(s,x)+2\big) = 0, \tag{BC-red}$$

where $\psi(s,x)$ denotes the effective cost of purchasing from firm A.

Assume that F is continuously differentiable and that $F_x(s,x^*(s)) \neq 0$. By the implicit function theorem, the response of the marginal consumer to investment is given by

$$\frac{dx^*(s)}{ds} = -\frac{F_s(s,x^*(s))}{F_x(s,x^*(s))}. \tag{IFT}$$

We now examine the signs of the numerator and denominator.

Sign of the numerator. Differentiating $F(s,x)$ with respect to s yields

$$F_s(s,x^*) = \psi_s(s,x^*) - (1-x^*)\psi_{xs}(s,x^*). \tag{B.1}$$

From the maintained assumptions in the model section, investment reduces effective costs, so that $\psi_s(s,x) \le 0$, and investment increases distance sensitivity, so that $\psi_{xs}(s,x) \le 0$. Hence, the sign of $F_s(s,x^*)$ is in general ambiguous.

If, however, the technological effect dominates the distance-sensitivity effect, in the sense that

$$\psi_s(s,x^*) \le (1-x^*)\psi_{xs}(s,x^*), \tag{B.2}$$

then it follows that

$$F_s(s,x^*) \le 0. \tag{B.3}$$

Sign of the denominator. Differentiating $F(s,x)$ with respect to x yields

$$\begin{aligned} F_x(s,x^*) &= \psi_x(s,x^*) + \big(\psi_x(s,x^*)+2\big) - (1-x^*)\psi_{xx}(s,x^*) \\ &= 2\psi_x(s,x^*) - (1-x^*)\psi_{xx}(s,x^*) + 2. \end{aligned} \tag{B.4}$$

By assumption, effective costs increase with distance, $\psi_x(s,x) > 0$, and are concave in distance, $\psi_{xx}(s,x) \leq 0$. Therefore,

$$F_x(s, x^*) > 0. \tag{B.5}$$

Conclusion. Under condition (B.2), the numerator in (IFT) is weakly negative and the denominator is strictly positive. It follows that

$$\frac{dx^*(s)}{ds} = -\frac{F_s(s, x^*(s))}{F_x(s, x^*(s))} \geq 0,$$

which establishes the claim in Lemma 3. □

B.2 Proof of Lemma 4

Lemma 4 establishes that, in the asymmetric equilibrium, firm B's optimal uniform price responds negatively to firm A's investment under the same condition as in Lemma 3.

Proof. From the Stage-2 analysis, firm B's optimal price for a given investment level s can be written as

$$p_B^*(s) = (1 - x^*(s))\big(1 + \psi_x(s, x^*(s))\big), \tag{B.6}$$

where $x^*(s)$ denotes the marginal consumer determined by the boundary condition.

Differentiating $p_B^*(s)$ with respect to s yields

$$\frac{dp_B^*}{ds} = -(1 + \psi_x)\frac{dx^*}{ds} + (1 - x^*)\frac{d}{ds}\psi_x(s, x^*(s)). \tag{B.7}$$

This expression shows that firm B's price response can be decomposed into (i) a boundary-shift effect operating through $\frac{dx^*}{ds}$, and (ii) a direct effect operating through changes in distance sensitivity.

Applying the chain rule, we have

$$\frac{d}{ds}\psi_x(s, x^*(s)) = \psi_{xs}(s, x^*) + \psi_{xx}(s, x^*)\frac{dx^*}{ds}. \tag{B.8}$$

Substituting this expression back, we obtain

$$\frac{dp_B^*}{ds} = \Big[(1 - x^*)\psi_{xx}(s, x^*) - (1 + \psi_x(s, x^*))\Big]\frac{dx^*}{ds} + (1 - x^*)\psi_{xs}(s, x^*). \tag{B.9}$$

From the maintained assumptions in the model section, $\psi_{xx} \leq 0$ and $\psi_{xs} \leq 0$ hold. Moreover, under the condition of Lemma 3, the marginal consumer responds positively to investment, so that $\frac{dx^*}{ds} > 0$. Together, these imply that both terms on the right-hand side are weakly negative. Hence,

$$\frac{dp_B^*}{ds} \leq 0,$$

which establishes that firm B's equilibrium price decreases in response to firm A's investment. □

B.3 Second-Order Condition for Firm A's Investment

This subsection examines the second-order condition (SOC) for firm A's investment problem in the asymmetric model. The goal is not to establish global concavity in general form, but to clarify which curvature components are sign-ambiguous and why additional restrictions (or numerical evaluation under a specific functional form) are needed.

Recall from Section 4.2.3 that firm A's profit is

$$\pi_A(s) = \int_0^{x^*(s)} \big[p_B^*(s) + (1-x) - \psi(s,x)\big]\, dx - C(s),$$

where $x^*(s)$ and $p_B^*(s)$ are determined endogenously in Stages 2–3.

■Second derivative of profit Differentiating $d\pi_A/ds$ once more yields

$$\frac{d^2\pi_A}{ds^2} = \int_0^{x^*(s)} \big(-\psi_{ss}(s,x)\big)\, dx + x^*(s)p_{ss}^*(s) + \big(p_s^*(s) - \psi_s(s,x^*(s))\big)x_s^*(s) - C_{ss}(s). \quad \text{(B.10)}$$

Using the boundary condition $p_B^*(s) + (1 - x^*(s)) - \psi(s, x^*(s)) = 0$ and differentiating it with respect to s, we obtain the identity

$$p_s^*(s) - \psi_s(s, x^*(s)) = (1 + \psi_x(s, x^*(s)))x_s^*(s).$$

Substituting this into (B.10) gives

$$\frac{d^2\pi_A}{ds^2} = \int_0^{x^*(s)} \big(-\psi_{ss}(s,x)\big)\, dx + x^*(s)p_{ss}^*(s) + (1 + \psi_x(s,x^*(s)))\big(x_s^*(s)\big)^2 - C_{ss}(s). \quad \text{(B.11)}$$

Since $\psi_x > 0$ by Lemma 2, and under the regularity conditions ensuring differentiability of $x^*(s)$, the third term is non-negative.

■Interpretation and why the SOC is ambiguous Expression (B.11) consists of four components:

1. **Effective-cost curvature:** the term $\int_0^{x^*}(-\psi_{ss})dx$ is **not** signed in the general model, because Section 2 does not restrict the sign of ψ_{ss}.
2. **Price-response curvature:** the term $x^* p_{ss}^*$ captures the second-order response of firm B's equilibrium price to investment. Its sign is generally ambiguous because p_{ss}^* depends on higher-order responses of the marginal consumer (e.g., x_{ss}^*) and higher-order derivatives of ψ.
3. **Boundary-adjustment effect:** the non-negative term $(1 + \psi_x)(x_s^*)^2$ reflects the second-order effect of endogenous boundary movements and weakens concavity whenever x_s^* is sizable.
4. **Investment-cost curvature:** the term $-C_{ss}(s)$ strictly strengthens concavity under $C_{ss}(s) > 0$.

Because components (1)–(3) are not signed in general, the overall curvature of $\pi_A(s)$ cannot be determined without additional restrictions. Section 5 therefore evaluates the second-order condition under the seeded exponential specification, where ψ and its derivatives (and thus $x^*(s)$ and $p_B^*(s)$) can be computed explicitly. The sign of $d^2\pi_A/ds^2$ is then verified numerically at the computed equilibrium for the parameter configurations considered.

Appendix C Proof for Section 6

C.1 Proof of Proposition 3

Let $r \in \{BM, AM\}$ denote the pricing regime. Social surplus can be written as

$$W^r(s;\theta) = -\int_0^1 \psi(s,x;\theta)\,dx - C(s;c) - \mathbf{1}_{\{r=AM\}}D(s;\theta), \quad \text{up to transfers,} \tag{C.1}$$

where $D(s;\theta) = x^*(s)p_B^*(s)$ is the distortion term that arises only under asymmetric pricing.

Let $s_r^W(\theta)$ solve

$$\frac{\partial W^r}{\partial s} = 0.$$

By the envelope theorem,

$$\frac{d}{d\theta}W^r\big(s_r^W(\theta);\theta\big) = \left.\frac{\partial W^r}{\partial \theta}\right|_{s=s_r^W}. \tag{C.2}$$

Since the envelope condition holds at the welfare-maximizing investment level, only the direct parameter effect on effective costs enters the first-order welfare sensitivity.

From C.1,

$$\frac{\partial W^r}{\partial \theta} = -\int_0^1 \psi_\theta(s,x;\theta)\,dx - \mathbf{1}_{\{r=AM\}}D_\theta(s;\theta). \tag{C.3}$$

Under the seeded exponential specification,

$$\psi_\sigma < 0, \quad \psi_a < 0, \quad \psi_b > 0,$$

and $C_c > 0$.

Hence,

$$\theta \in \{\sigma, a\} \;\Rightarrow\; \frac{\partial W^r}{\partial \theta} > 0, \qquad \theta \in \{b, c\} \;\Rightarrow\; \frac{\partial W^r}{\partial \theta} < 0.$$

Therefore, parameters that increase the effectiveness of investment raise maximized welfare in both regimes, while parameters that increase costs reduce it. The asymmetric regime differs only in that welfare changes operate additionally through the distortion term $D(s;\theta)$. ☐

Appendix D Analytical Characterization under the Seeded Exponential Specification

This appendix provides an analytical characterization of the seeded exponential specification introduced in Section 2.5. The purpose of this part of the appendix is to verify that the functional form used in the numerical analysis satisfies all the assumptions imposed in the general model and reproduces the key structural properties established in Section 2. Importantly, the results derived below are not used to characterize equilibrium outcomes. Instead, they serve to illustrate how the general properties of optimal disclosure and effective cost emerge under a concrete specification.

D.1 Optimal disclosure under seeded exponential technology

We adopt the following functional forms for AI technology, disclosure cost, and investment cost:

$$\phi(s,y) = 1 - \exp\{-a(\sigma+s)y\}, \qquad d(y) = \frac{b}{2}y^2, \qquad C(s) = \frac{c}{2}s^2, \tag{D.1}$$

where $a, b, c > 0$ and $\sigma \geq 0$. These functional forms satisfy all the assumptions imposed in Section 2. In particular, ϕ is increasing in both arguments, exhibits diminishing returns to investment and disclosure, and features complementarity between investment and disclosure, while the disclosure cost is strictly convex.

Consider a consumer located at distance $x \in [0,1]$ from firm A. Under personalized pricing, the consumer chooses the disclosure level $y \geq 0$. The disclosure problem is therefore given by

$$\min_{y\geq 0} \left(1-\phi(s,y)\right)x + d(y).$$

The first-order condition characterizing the optimal disclosure choice is

$$\phi_y(s,y)\,x = d'(y). \tag{FOC-y}$$

Under the functional forms above, (FOC-y) can be written as

$$by = a(\sigma+s)x\exp\{-a(\sigma+s)y\}. \tag{D.2}$$

The left-hand side is strictly increasing in y, while the right-hand side is strictly decreasing. Hence, the first-order condition admits a unique solution, which defines the optimal disclosure level $y^*(s,x)$.

This equation can be inverted explicitly using the Lambert W function. Letting $w \equiv a(\sigma+s)y$, the first-order condition can be rewritten as

$$we^w = \frac{a^2(\sigma+s)^2}{b}x.$$

Since the argument is non-negative for $x \in [0,1]$, the Lambert W function admits a unique real solution. Hence, the optimal disclosure level is given by

$$y^*(s,x) = \frac{1}{a(\sigma+s)} W\left(\frac{a^2(\sigma+s)^2}{b} x\right). \tag{D.3}$$

This explicit representation clarifies how optimal disclosure increases with both investment s and distance x, consistent with Lemma 1.

D.2 Effective Cost and Derivatives

Substituting the optimal disclosure level $y^*(s,x)$ into the consumer's objective function yields the effective cost

$$\psi(s,x) = (1 - \phi(s, y^*(s,x)))\, x + d(y^*(s,x)).$$

Under the seeded exponential specification, the effective cost admits the closed-form representation

$$\psi(s,x) = \frac{b}{a^2(\sigma+s)^2}\left[W(m) + \frac{1}{2}W(m)^2\right], \qquad m \equiv \frac{a^2(\sigma+s)^2}{b} x, \tag{EC}$$

where $W(\cdot)$ denotes the Lambert W function.

Expression (EC) provides a concrete illustration of the effective cost function introduced in Section 2 and clarifies how investment and disclosure jointly reduce mismatch costs.

■**First-order derivatives** Using the envelope property of the disclosure problem, the derivative with respect to distance simplifies to

$$\psi_x(s,x) = 1 - \phi\big(s, y^*(s,x)\big). \tag{D.4}$$

Under the seeded exponential specification,

$$\psi_x(s,x) = e^{-W(m)}.$$

Since $W(m)e^{W(m)} = m$ implies $e^{-W(m)} = W(m)/m$, we obtain

$$\psi_x(s,x) = \frac{W(m)}{m} > 0 \quad (x > 0).$$

Differentiating (EC) with respect to investment yields

$$\psi_s(s,x) = -\frac{b}{a^2(\sigma+s)^3} W(m)^2 < 0, \quad (x > 0). \tag{D.5}$$

Thus, effective cost decreases with investment.

■**Cross-partial derivative** Differentiating ψ_x with respect to s yields

$$\psi_{sx}(s,x) = -\frac{2}{\sigma+s}\frac{W(m)^2}{m(1+W(m))} < 0 \quad (x>0), \tag{D.6}$$

confirming that the cost-reducing effect of investment is stronger for consumers located farther from the firm.

■**Second derivative with respect to distance** Differentiating ψ_x with respect to x gives

$$\psi_{xx}(s,x) = -\frac{W(m)^2}{x\,m\,(1+W(m))} < 0 \quad (x>0). \tag{D.7}$$

Hence, under the seeded exponential specification, the effective cost is concave in distance.

■**Second derivative with respect to investment** Differentiating ψ_s with respect to s yields

$$\psi_{ss}(s,x) = \frac{b}{a^2(\sigma+s)^4}W(m)^2\frac{3W(m)-1}{1+W(m)}. \tag{D.8}$$

The sign of ψ_{ss} depends on the magnitude of $W(m)$. In particular,

$$\psi_{ss}(s,x) \geq 0 \quad \text{if and only if} \quad W(m) \geq \frac{1}{3}.$$

Hence, unlike the general sign restrictions imposed in Section 2, the curvature of the effective cost with respect to investment is parameter-dependent under the specific functional form.

■**Summary** Under the seeded exponential specification, the effective cost satisfies

$$\psi_x > 0, \qquad \psi_s < 0, \qquad \psi_{xx} < 0, \qquad \psi_{sx} < 0,$$

while the sign of ψ_{ss} depends on parameter values.

These results confirm that the seeded exponential specification reproduces the key monotonicity and interaction properties of the general model, while making transparent how curvature properties depend on the strength of the technology effect.

D.3 Implications for numerical implementation

D.3.1 Boundary Consumer

In the asymmetric model, the market boundary x^* is characterized by the boundary condition (BC) derived in Section 4. Under the seeded exponential specification, the effective cost $\psi(s,x)$ is strictly increasing in x (Appendix D.2), which guarantees existence and uniqueness of the boundary consumer for all parameter configurations considered.

■**Investment incentives in the general model.** In Sections 3 and 4, firm A's investment decision is characterized by a balance between the direct cost-reducing effect of investment and the indirect competitive response through prices and market boundaries. While the sign of the net investment incentive is generally ambiguous in the abstract model, the analysis identifies the key channels through which investment affects profits.

■**Decomposition under the seeded exponential specification.** Under the seeded exponential specification, the direct effect of investment on effective cost is governed by $\psi_s(s, x)$, while its interaction with consumer location is captured by $\psi_{sx}(s, x)$.

■**Role of parameters.** The functional form highlights distinct roles played by the model parameters.

D.3.2 Parameter Dependence of Investment Incentives

This subsection clarifies how the investment incentives identified in the general analysis of Sections 3 and 4 depend on model parameters under the seeded exponential specification. While the sign and strength of investment incentives are generally ambiguous in the abstract model, the concrete functional form makes the underlying mechanisms transparent and provides a bridge to the numerical analysis in Section 5.

In the general model, firm A's investment decision reflects a balance between the direct technological benefit from data-driven investment, the indirect competitive response through firm B's pricing, and the convex cost of investment. The seeded exponential specification allows these channels to be expressed explicitly through the effective cost function and its derivatives.

The numerical analysis varies four key parameters: the baseline technology level σ, the productivity parameter a, the disclosure cost parameter b, and the investment cost parameter c. Their economic roles can be understood from how they enter the effective cost and the reduced-form investment problem.

■**Baseline technology (σ).** The parameter σ captures the exogenous data-processing capability prior to investment. A higher σ reduces the marginal impact of additional investment, as the effective cost is already low before investment. As a result, increases in σ make interior investment solutions less likely, consistent with diminishing returns to investment.

■**Productivity of data-driven investment (a).** The parameter a governs how effectively investment and consumer disclosure translate into reductions in effective cost. A higher a amplifies the direct technological gain from investment, but it also strengthens competitive pressure by inducing a stronger price response from firm B. Under the seeded exponential specification, these opposing forces become transparent, helping explain why the net effect of a on equilibrium investment is theoretically ambiguous and must be assessed numerically.

■Disclosure cost (b). The disclosure cost parameter b limits consumers'willingness to provide information. Higher disclosure costs dampen the effectiveness of data-driven investment by weakening the disclosure channel, thereby reducing the marginal benefit of investment. Consequently, increases in b tend to lower firm A's investment incentives.

■Investment cost (c). The parameter c governs the convex cost of investment. Higher values of c directly discipline firm A's investment choice and shrink the parameter region in which the first-order condition admits an interior solution. This force operates independently of market competition and provides a benchmark against which the technological and competitive effects are evaluated.

Taken together, these parameter dependencies illustrate how the general investment conditions derived in Sections 3 and 4 translate into concrete comparative statics under the seeded exponential specification. Section 5 exploits this structure to compute equilibrium outcomes numerically and to visualize how investment incentives vary across parameter configurations.