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September 2025

Discussion Paper No. 2522

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

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September 18, 2025

Abstract

Real-world cost asymmetries highlight the importance of firm heterogeneity. Studies focus on monopolistic competition owing to model tractability but overlook oligopoly settings. We analyze how the productivity gap between efficient and inefficient oligopolistic firms affects export policies within a standard third-market model. We show that a "subsidy–tax–subsidy" export policy pattern emerges depending on the degree of the productivity gap. Extending our model to multiple firms, we consider whether the gap in firm numbers affects export policies. We find that when this gap is large, one exporter may receive an export subsidy whereas the other faces an export tax.

Keywords: Export policy; Productivity gap; Subsidy–tax–subsidy; Multiple firms

JEL classification: F12; F13; D43

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1 Introduction

In the past 25 years, trade theorists and practitioners have focused on productivity gaps and cost heterogeneity among firms (e.g., Melitz, 2003; Melitz and Redding, 2014). This stream of literature continues to the present day, with many trade theorists emphasizing "horizontal monopolistic competition with firm heterogeneity." For example, as Head and Spencer (2017) indicate, a rapid increase in studies on "horizontal monopolistic competition with firm heterogeneity" has occurred because cost heterogeneity among firms is incorporated relatively easily under monopolistic competition. However, in the real world, it is not just monopolistic competition that is important. For instance, the international duopoly of the aircraft market by Boeing and Airbus is well known;[1] and *international oligopoly* is often observed. Furthermore, the productivity gap among oligopolistic firms is important because it can affect decisions on trade policies and the welfare of countries.

Within the international oligopoly literature, originating with the seminal works of Brander and Spencer (1985) and Eaton and Grossman (1986), several studies analyze cost heterogeneity between domestic (or home) and foreign firms (e.g., de Meza, 1986; Mai and Hwang, 1988; Neary, 1994). Building on the export-rivalry model of Brander and Spencer (1985), which considers Cournot duopoly, these studies demonstrate that domestic governments grant larger export subsidies to firms with lower marginal costs than to firms with higher marginal costs.[2] Most of these studies assume that firms have constant marginal costs. However, assuming

[1]The competition between Bombardier (Canada) and Embraer (Brazil) in producing mid-size aircraft (i.e., the regional aircraft market) is another popular example used in discussing export subsidies (da Costa Ramalho, 2018).

[2]Bandyopadhyay (1997) considers the effects of demand elasticities in determining export policy. He establishes the opposite result: under inelastic demand, the lower cost firm receives the smaller subsidy. Collie and de Meza (2003) note that the low cost firm can be taxed under inelastic demand and further show that the absolute value of the export subsidy/tax set by the country with the lower cost firm is larger than the country with the higher cost firm under general demand function.

that oligopolistic firms have constant marginal costs would drastically simplify their market interactions to the extent that it would be impossible to analyze rigorously how the degree of productivity gaps among domestic firms (or within an industry) affects export policy.[3] In reality, firms do not always have constant marginal costs; some have technologies involving decreasing returns to scale.[4]

This paper considers the effects of a productivity gap among domestic firms on the optimal export policy when firms have a linear-quadratic cost in a Brander and Spencer (1985)-type third-market model consisting of an importer and two exporters. For exporters, there are two types of firms: a highly productive firm and a less-productive firm. We define the coefficient of the quadratic term in the cost function of the less-productive firm as the measure of the productivity gap among the domestic firms. We show that if the productivity gap is large (small), the optimal export policy becomes a subsidy (tax). In other words, the optimal policy follows a "subsidy—tax—subsidy" pattern as the productivity gap alters.[5] This result arises despite the existence of differentiated quantity competition. Furthermore, we demonstrate that in a multiple-firm case, one exporter is offered a subsidy but the other exporter faces a tax if there is a large gap in the number of firms between the two exporting countries. Conversely, if this gap is small, the optimal export policies are "(Subsidy, Subsidy)." When the number of firms is sufficiently large, the optimal export policies become "(Tax, Tax)."

[3]See, for example, the well-known text book on game theory by Gibbons (1992), pages 75–79. Although his example concerns a symmetric two-country tariff game with constant marginal cost, the marginal cost asymmetry does not play an essential role.

[4]According to the empirical analysis by Basu and Fernald (1997), in the US, a typical two-digit industry has constant- or decreasing-returns-to-scale technologies. In particular, the transport sector is considered to have decreasing returns to scale (Xu et al., 1994).

[5]Ghosh and Saha (2008) find that the "subsidy—tax—subsidy" pattern can appear in a technology licensing model, and Mizuno and Takauchi (2020) show that the "tax—subsidy—tax" pattern can appear in a vertically-related-markets model.

We bring two new insights into the international trade policy literature. First, we demonstrate that there may be a nonmonotonic relationship between the productivity gap among domestic firms and export policy. This suggests that policy practitioners should exercise cautiousness and prudence in the formulation and implementation of export policies. This is because, in practice, the productivity levels of firms differ. If a policy-maker misjudges the productivity gap among firms in the home country, and infers a productivity gap that differs slightly from the real gap, the export policy implemented may have the opposite sign to the optimal policy (i.e., the policy-maker may impose a subsidy when a tax is optimal, or vice versa). Hence, there is a danger that welfare loss will occur. Second, we generalize the baseline model with respect to the number of firms (i.e., considering multiple firms and differing firm numbers between the two exporters). Consequently, it is possible to immediately determine the optimal policy for each exporter by examining the comparative firm numbers. For example, when there is gap between the countries in firm numbers, the home country provides an export subsidy but the foreign country imposes an export tax (see also the discussion in relation to Figure 3). When a productivity gap exists, we demonstrate the relationship between the optimal export policy and the market structure. Therefore, our analysis elucidates and extends the results of Brander and Spencer (1985).

The related studies of international oligopoly with cost asymmetry can be classified broadly into the following categories: (**a**) Cournot duopoly models assuming constant marginal costs (e.g., de Meza, 1986; Mai and Hwang, 1988; Neary, 1994; Bandyopadhyay, 1997; Collie and de Meza, 2003); (**b**) Cournot oligopoly models with multiple firms within the same country assuming: (**i**) constant marginal costs (e.g., Collie, 1993; Lahiri and Ono, 1997; Long and Soubeyran, 1997; Leahy and Montagna, 2001; Amir et al., 2022); and (**ii**) nonconstant or increasing marginal costs (e.g., Long and Soubeyran, 1999; Chang and Sugeta, 2005).

Collie (1993) identifies the rationalization effect of an export subsidy. Specifically, he shows that the export subsidy induces efficient (less-efficient) domestic firms to increase their output, which improves (worsens) the domestic welfare when demand is convex (concave). Neary (1994) emphasizes the role of the social cost of funds. Long and Soubeyran (1997) find a rationalization effect similar to Collie (1993). Furthermore, they relate the welfare effect of an export subsidy to the degree of convexity (or concavity) of demand and the degree of industry concentration measured by the Herfindahl index.[6] Lahiri and Ono (1997) focus on the effects of the entry/exit policies of firms. They establish a multi-country trade model of international oligopoly to explore whether the elimination of an inefficient firm can improve national welfare (see also the closed economy analysis of Lahiri and Ono (1988)). Leahy and Montagna (2001) introduce the social cost of funds and consider firm-specific export subsidies. They conclude that higher export subsidies are given to the more efficient domestic firms if the social cost of funds is sufficiently small. Under an asymmetric multi-country oligopoly model, Amir et al. (2022) establish that a certain country's export subsidy can improve world welfare.

Our study on cost heterogeneity is most closely related to Long and Soubeyran (1999). They assume increasing marginal costs (or convex costs) and show that the optimal policy becomes an export tax even in the case of quantity competition. They also consider a firm-specific export policy in which the government can impose an export tax (or grant an export subsidy) discriminatorily for domestic firms. They establish that the less-efficient firms are penalized by higher taxes and the more efficient firms are granted larger subsidies. Chang and Sugeta (2005) also adopt increasing marginal costs and introduce policy interactions by governments to analyze the welfare effects of firm-specific and uniform export subsidies. They show that the results in Long and Soubeyran (1999) are applicable to the more general setting.

However, none of these studies considers the productivity gap among firms and different

[6]In addition, Ueng et al. (2022) emphasize the market concentration rate and examine the effects of three different subsidy policies.

market structures. By considering these factors in our study, we derive unique results. Hence, we consider that our study substantially complements the related literature.

The remainder of the paper is organized as follows. Section 2 presents the baseline model and Section 3 provides the basic results. In Section 4, we extend the baseline model to confirm the robustness of our main result. Section 5 generalizes the number of firms and presents a major result. In Section 6, we examine differentiated export policies. Section 7 concludes.

2 Model

We consider a standard third-market model; however, each identical country $i\,(=H,F)$ has two firms and there is a productivity gap between the two firms. Each firm has a linear quadratic cost,[7] the coefficient of which we refer to as "productivity." To better understand and simplify the analysis, we fix the productivity of one firm, called firm n, at unity. The productivity of the other firm, firm b, is $\gamma > 0$. γ denotes the productivity gap between firms n and b. The profits of the firms in exporting country i are defined by:

$$\pi_b^i \equiv (p_i - (c+s_i))q_b^i - \frac{\gamma}{2}\left(q_b^i\right)^2 \quad \text{and} \quad \pi_n^i \equiv (p_i - (c+s_i))q_n^i - \frac{1}{2}\left(q_n^i\right)^2,$$

where p_i is the price in the third market, q_b^i (q_n^i) is the value of the exports of firm b (n) in exporting country i, $i \neq j$, $i,j = H,F$, $a,c > 0$, and $\alpha \equiv a - c > 0$. When $\gamma < 1$ (> 1), firm b is regarded as an efficient (inefficient) firm and thus, a smaller (larger) γ implies that the productivity gap is increasing. Note that export policy s_i is a tax (subsidy) if $s_i >(<)\, 0$.

The inverse demand in the third market is $p_i = a - (q_b^i + q_n^i) - (q_b^j + q_n^j)$.

We examine a two-stage game. First, each country i decides on the export policy level, s_i. Subsequently, firms determine their exports to the third market. The game is solved by backward induction.

[7]Takauchi et al. (2024) also assume that firms have quadratic costs and consider the effects of several trade policies.

3 Results

Let $\boldsymbol{s} = (s_i, s_j)$ for $i \neq j$, $i, j = H, F$. In Cournot competition, the second-stage exports of each firm in country i are $q_b^i(\boldsymbol{s})$ and $q_n^i(\boldsymbol{s})$. This yields the maximization problem below and the optimal export policy s_i^*:

$$\max_{s_i} \ \pi_b^i(\boldsymbol{s}) + \pi_n^i(\boldsymbol{s}) + \left[q_b^i(\boldsymbol{s}) + q_n^i(\boldsymbol{s})\right] s_i \ \Rightarrow \ s_i^* = \frac{(1-\gamma)(\gamma^2-2\gamma-7)\alpha}{11\gamma^3+83\gamma^2+177\gamma+113}. \tag{1}$$

We establish Proposition 1 from (1).

Proposition 1. *Although large productivity gaps result in export subsidies, small productivity gaps lead to export taxes. Formally,* $s_i^* < 0$, *if* $\gamma < 1$ *or* $\gamma > 1+2\sqrt{2}$; $s_i^* > 0$, *if* $1 < \gamma < 1+2\sqrt{2}$.

Proof. Solving $(1-\gamma)(\gamma^2 - 2\gamma - 7) \geq 0$ in (1) with respect to γ, we obtain Proposition 1. □

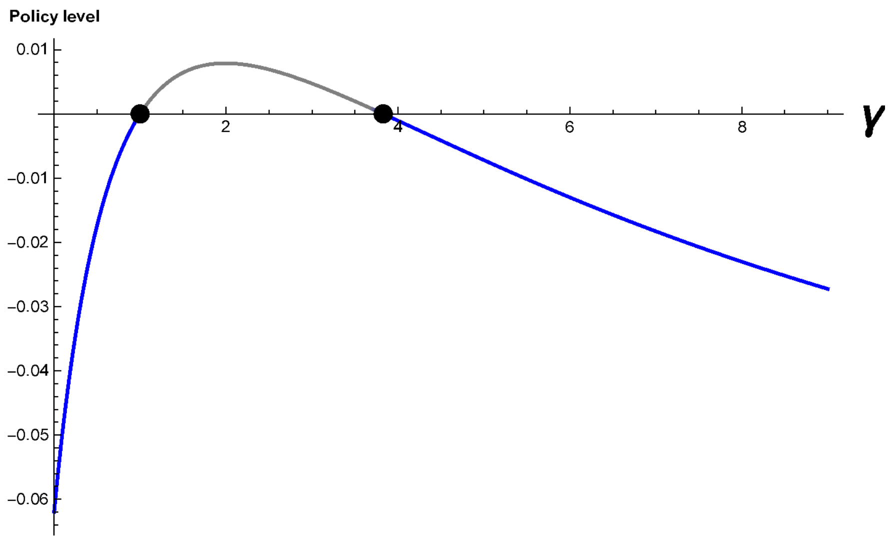

Figure 1: Optimal export policy: Blue denotes a **subsidy**; gray denotes a **tax**.

Proposition 1 is illustrated in Figure 1. The intuition is as follows. When the productivity gap is small, there is no significant difference between firms in each exporting country. If an exporting country provides a subsidy in an effort to promote exports, production cost increases rapidly because the firms' marginal costs are rising. This harms welfare in the exporting countries.

Therefore, the exporters impose a tax on their firms to raise tax revenue to increase welfare. This result is similar to that of Long and Soubeyran (1999).

In contrast, when the productivity gap is large, efficient and less-efficient firms coexist in each exporting country i. If the domestic exporter imposes a tax on its domestic firms, the ability of the less-efficient firms to export will shrink significantly. Because this diminishes the domestic firms' market share and shifts rent from domestic to foreign firms, the exporting country provides an export subsidy (Brander and Spencer, 1985).

Our contribution is to demonstrate that the productivity gap within an industry may alter the optimal export policy several times—that is, the optimal pattern may be subsidy—tax—subsidy. This implies that the optimal export policy is sensitive to the productivity gap among firms.

Next, we consider the exporting country's welfare. From (1), the welfare of exporting country i becomes as follows:

$$W_i^* = \frac{(\gamma+3)^2(3\gamma+5)(7\gamma^3+51\gamma^2+93\gamma+41)\alpha^2}{2(11\gamma^3+83\gamma^2+177\gamma+113)^2}. \tag{2}$$

Using (2), we obtain the following proposition.

Proposition 2. *Suppose that $\gamma < 1$. An improvement in an efficient firm's productivity decreases the welfare of exporters. Suppose that $\gamma > 1$. A deterioration in a less-efficient firm's productivity increases the welfare of exporters.*

Proof. Differentiating (2) with respect to γ, we have:

$$\frac{\partial W_i^*}{\partial \gamma} = \frac{8(\gamma+3)\left(2\gamma^6+7\gamma^5-21\gamma^4+174\gamma^3+1424\gamma^2+2811\gamma+1747\right)\alpha^2}{(11\gamma^3+83\gamma^2+177\gamma+113)^3} > 0.$$

This implies Proposition 2. □

An increase in γ has two effects on the exporting country's welfare. The first effect is negative, leading to production inefficiency. The second effect is positive, as less-efficient firms exit from the market (Lahiri and Ono, 1988). In our model, the negative effect is dominated by the positive effect; thus, an increase in γ always raises welfare.

Proposition 2 has an important implication. Even if domestic production becomes inefficient, shifting production from less-efficient to efficient firms may enhance welfare. This provides some rationale for an industrial policy that creates a national champion.

4 Differentiated Goods

In this section, we confirm the robustness of Proposition 1 in the case where the model is extended to differentiated goods. We first consider differentiated quantity competition followed by differentiated price competition.

4.1 Differentiated quantity competition

Here, we introduce $\beta \in [0,1]$, which is the variable representing the degree of product differentiation among products. Hence, the inverse demand in the third market is:

$$p_\ell^i = a - q_\ell^i - \beta(q_\ell^j + q_k^j + q_k^i) \quad \text{for} \quad i \neq j,\ i,j = H,F;\ \ell \neq k,\ \ell,k = b,n.$$

Note that if $\beta = 1$ (0), exporters' goods are homogenous (independent).

The optimal export policy of country $i\,(= H, F)$ is:

$$s_i^{C*} = \frac{-\alpha\beta}{E_C}\left[6\beta^4 - 10\beta^3(\gamma+5) + 6\beta^2(\gamma+5)^2 - \beta(\gamma+5)(\gamma^2+16\gamma+37) + 18(\gamma+2)^2\right], \tag{3}$$

where "C" denotes the case of differentiated Cournot competition. Note that E_C is reported in the Appendix.

We establish Proposition 3 from (3).

Proposition 3. *Suppose that $\beta > \beta_C \simeq 0.707299$. The subsidy—tax—subsidy pattern becomes optimal as γ rises.*

Proof. See the Appendix.

From the results of Propositions 3, our main result does not change when firms are engaged in differentiated Cournot competition. Figure 2 illustrates Proposition 3 in (γ, β)-space.

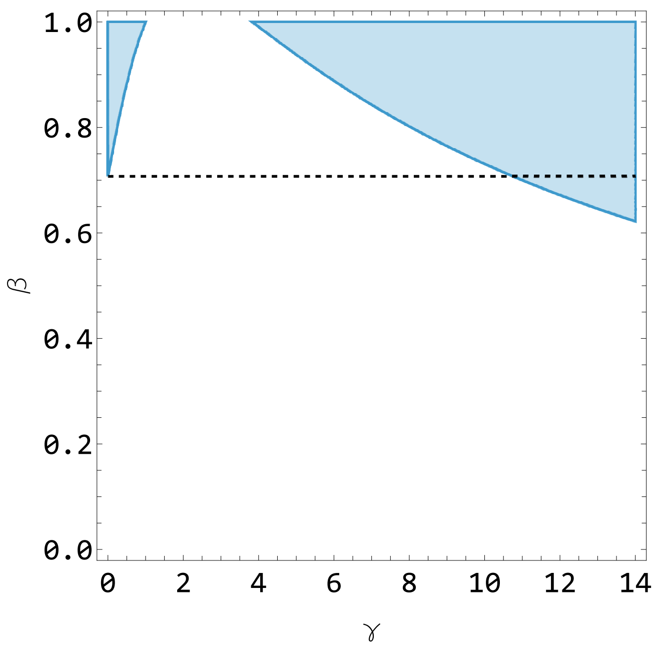

Figure 2: Optimal export policy: The shaded area indicates the export subsidy $\boldsymbol{s_i^{C*} < 0}$.

As shown in Proposition 1, the optimal export policy can be sensitive to the productivity gap γ even if differentiated Cournot competition prevails. Proposition 3 illustrates this feature of the export policy.

4.2 Differentiated price competition

The demand function for firm $\ell\,(= b, n)$ in country i is given by

$$q_\ell^i = \frac{a(1-\beta) + \beta(p_\ell^j + p_k^j + p_k^i) - 2\beta p_\ell^i - p_\ell^i}{(1-\beta)(1+3\beta)} \quad \text{for} \;\; i \neq j,\; i,j = H,F; \;\; \ell \neq k,\; \ell,k = b,n.$$

From the above demand function and the profit of firms b and n, the optimal export policy of each country $i\,(= H, F)$ becomes:

$$s_i^{B*} = \frac{\beta\left[\beta(2\beta{+}1)^3(8{+}14\beta{-}13\beta^2)\gamma^3{-}(1{-}\beta)B_1{-}(2\beta{+}1)B_2\gamma{+}3(2\beta{+}1)^2B_3\gamma^2\right]\alpha}{(2\beta{+}1)[(2\beta{+}1)^3(9{+}21\beta{-}4\beta^2{-}5\beta^3)\gamma^3{+}B_4{+}B_5\gamma{+}(2\beta{+}1)B_6\gamma^2]} > 0.$$

Note that B_h $(h = 1, \ldots, 6)$ is reported in the Appendix. From $s_i^{B*} > 0$, the optimal policy is always an export tax under differentiated price competition.

The intuition is relatively simple. The price competition means that equilibrium prices tend to decline; therefore, each country imposes an export tax and attempts to raise the product prices. This logic is basically similar to that of Eaton and Grossman (1986).

5 General Market Structure: Multiple-Firm Case

This section considers the case where without loss of generality, there are multiple firms m_i and m_j for $i \neq j = H, F$.

The inverse demand in the third market is $p = a - Q$. Total sales Q is rewritten as follows:

$$Q \equiv \left(\sum_{k=1}^{m_H} q_{b,k}^H + \sum_{k=1}^{m_H} q_{n,k}^H \right) + \left(\sum_{l=1}^{m_F} q_{b,l}^F + \sum_{l=1}^{m_F} q_{n,l}^F \right).$$

The profits are:

$$\pi_{b,k}^i \equiv p q_{b,k}^i - c q_{b,k}^i - \frac{\gamma_b}{2} \left(q_{b,k}^i\right)^2 - s_i q_{b,k}^i,$$
$$\pi_{n,k}^i \equiv p q_{n,k}^i - c q_{n,k}^i - \frac{\gamma_n}{2} \left(q_{n,k}^i\right)^2 - s_i q_{n,k}^i.$$

The cost functions are generalized to include the various cases, for instance, (i) symmetric costs: $(\gamma_b, \gamma_n) = (\gamma, \gamma)$, (ii) symmetric constant marginal costs: $(\gamma_b, \gamma_n) = (0, 0)$, (iii) asymmetric costs: $(\gamma_b, \gamma_n) = (\gamma, 1)$, and so on. The equilibrium conditions are now modified to:

$$\alpha - s_i - \left(m_i q_b^i + m_i q_n^i + m_j q_b^j + m_j q_n^j \right) = \varphi_b^{-1} q_b^i,$$
$$\alpha - s_i - \left(m_i q_b^i + m_i q_n^i + m_j q_b^j + m_j q_n^j \right) = \varphi_n^{-1} q_n^i, \quad i \neq j = H, F,$$

where $\varphi_b \equiv (\gamma_b + 1)^{-1}$ and $\varphi_n \equiv (\gamma_n + 1)^{-1}$.

Note that φ_b and φ_n are interpreted as the productivity measures for firms b and n, respectively. The Cournot–Nash equilibrium outputs and profits are derived as $q_b^i(\boldsymbol{s}) = \varphi_b z_i(\boldsymbol{s})$, $q_n^i(\boldsymbol{s}) = \varphi_n z_i(\boldsymbol{s})$, $\pi_b^i(\boldsymbol{s}) = \frac{1}{2}(\varphi_b^{-1} + 1) \left(q_b^i\right)^2$, and $\pi_n^i(\boldsymbol{s}) = \frac{1}{2}(\varphi_n^{-1} + 1) \left(q_n^i\right)^2$. Here, we define the following:

$$z_i(\boldsymbol{s}) \equiv \frac{\alpha - s_i + m_j\varphi_1(s_j - s_i)}{M\varphi_1 + 1} \quad \text{for} \quad i \neq j = H, F,$$

where $\varphi_1 \equiv \varphi_b + \varphi_n$, $\varphi_2 \equiv \varphi_b^2 + \varphi_n^2$, $M \equiv m_i + m_j$, and φ_2 appears in the next paragraph. It should be noted that $\varphi_1 - \varphi_2 > 0$ holds because $0 < \varphi_b \leq 1$ and $0 < \varphi_n \leq 1$.

The social welfare of country i is expressed as $W_i(\boldsymbol{s}) = m_i[\pi_b^i(\boldsymbol{s}) + \pi_n^i(\boldsymbol{s})] + s_i m_i[q_b^i(\boldsymbol{s}) + q_n^i(\boldsymbol{s})]$. Social welfare maximization implies maximization of the following expression:

$$w_i(\boldsymbol{s}) \equiv \frac{W_i(\boldsymbol{s})}{m_i} = \frac{\varphi_1 + \varphi_2}{2}\, z_i(\boldsymbol{s})^2 + \varphi_1 s_i z_i(\boldsymbol{s}).$$

Define $h_j \equiv m_j\varphi_1 + 1$ and $h_i \equiv m_i\varphi_1 + 1$, to express

$$z_i = \frac{\alpha + m_j\varphi_1 s_j - h_j s_i}{M\varphi_1 + 1}; \quad \frac{\partial z_i}{\partial s_i} = -\frac{h_j}{M\varphi_1 + 1}.$$

Therefore, the first-order condition for welfare maximization is[8]

$$\frac{\partial w_i(\boldsymbol{s})}{\partial s_i} = \frac{(\varphi_1^2 m_i - h_j\varphi_2)(\alpha + m_j\varphi_1 s_j) - [2\varphi_1^2 m_i + h_j(\varphi_1 - \varphi_2)]h_j s_i}{(M\varphi_1 + 1)^2} = 0.$$

The effect of an export tax on the national welfare in the free trade is determined by:

$$\left.\frac{\partial w_i(\boldsymbol{s})}{\partial s_i}\right|_{s_i=0} = \frac{(\alpha + m_j\varphi_1 s_j)\,\Phi_i}{(M\varphi_1 + 1)^2} \quad \text{and} \quad \Phi_i \equiv \varphi_1^2 m_i - (m_j\varphi_1 + 1)\varphi_2.$$

It should be noted that the positive outputs imply $\alpha + m_j\varphi_1 s_j > 0$.[9] Then, the sign of the above expression depends only on the sign of Φ_i. When foreign firms are absent from the third country's market, i.e., $m_j = 0$, we have the following:

$$\left.\frac{\partial w_i(\boldsymbol{s})}{\partial s_i}\right|_{s_i=m_j=0} = \frac{(\varphi_1^2 m_i - \varphi_2)\alpha}{(M\varphi_1 + 1)^2} > 0,$$

where the last inequality is proved as follows: From $m_i \geq 1$, $\varphi_1 \equiv \varphi_b + \varphi_n$ and $\varphi_2 \equiv \varphi_b^2 + \varphi_n^2$, we immediately obtain

$$\varphi_1^2 > \varphi_2 \quad \text{and thus} \quad \varphi_1^2 m_i - \varphi_2 \geq \varphi_1^2 - \varphi_2 > 0$$

[8]The second-order condition is reported in the Appendix.

[9]Specifically, we have $q_{i,b}(0, s_j) = \frac{1}{M\varphi_1+1}\varphi_b(\alpha + m_j\varphi_1 s_j) > 0$ and $q_{i,n}(0, s_j) = \frac{1}{M\varphi_1+1}\varphi_n(\alpha + m_j\varphi_1 s_j) > 0$.

holds. Therefore, it is optimal for the home country to impose a tax on its exports when the third country's market is served by the home firms alone. Through the export tax, the home country's government attempts to raise the export price and induces the home industry to behave like an export cartel. In the presence of foreign firms (i.e., $m_j \geq 0$), there is an incentive for the home country to grant a subsidy. Specifically, an increase in m_j implies that the foreign industry is larger than the home country and an export subsidy results in shifting more profits away from the foreign industry to the home firms. If m_j becomes sufficiently large such that Φ_i becomes negative, the optimal tax rate s_i becomes negative. In other words, the export subsidy becomes optimal for a sufficiently large m_j.

In what follows, the Nash equilibrium tax rates denoted by $(\widehat{s}^*_H, \widehat{s}^*_F)$ are derived as the functions of $\boldsymbol{m} \equiv (m_H, m_F)$. The above first-order conditions imply that $\Omega_i h_j s_i = \Phi_i\,(\alpha + m_j \varphi_1 s_j)$, $i \neq j = H, F$. By solving these two equilibrium conditions, we find the Nash equilibrium export tax rate:

$$\widehat{s}^*_i(\boldsymbol{m}) = \frac{\alpha}{\Delta}\left[\varphi_1^2 m_i - (m_j\varphi_1 + 1)\varphi_2\right]\left[(\varphi_1 - \varphi_2)(m_i\varphi_1 + 1) + m_j\varphi_1^2\right] \quad \text{for} \quad i \neq j = H, F.$$

The above Δ is reported in the Appendix.

Figure 3 illustrates our result. In (m_H, m_F)-space, we draw two curves:

$$\Phi_H \equiv \varphi_1^2 m_H - (m_F\varphi_1 + 1)\varphi_2 = 0 \quad \text{and} \quad \Phi_F \equiv \varphi_1^2 m_F - (m_H\varphi_1 + 1)\varphi_2 = 0. \tag{4}$$

The two curves intersect at $(m_H, m_F) = (m_0, m_0)$, where $m_0 \equiv \varphi_2\,/\,(\varphi_1(\varphi_1 - \varphi_2))$. If $m_0 > 1$ holds, the area in which both countries adopt an export subsidy at the Nash equilibrium appears. Otherwise, only one of the two countries adopts an export subsidy or both countries adopt an export tax.

$$m_0 > 1 \quad \Leftrightarrow \quad \varphi_b^2 + \varphi_n^2 > \frac{(\varphi_b + \varphi_n)^2}{1 + \varphi_b + \varphi_n},$$

where we use $\varphi_1 \equiv \varphi_b + \varphi_n$ and $\varphi_2 \equiv \varphi_b^2 + \varphi_n^2$.

From the above argument and (4), we establish Theorem 1.

Theorem 1. *Suppose* $m_0 \equiv \dfrac{\varphi_2}{\varphi_1(\varphi_1 - \varphi_2)} > 1$, *where* $\varphi_1 \equiv \varphi_b + \varphi_n$ *and* $\varphi_2 \equiv \varphi_b^2 + \varphi_n^2$. *Then, the optimal policies are as follows:*

(i) Subsidies for the home and foreign countries if $\boldsymbol{m} \in S_1 \equiv \{(m_H, m_F) \mid \underline{m}_F(m_H) \le m_F \le \overline{m}_F(m_H)\}$.

(ii) The tax for home and the subsidy for foreign if $\boldsymbol{m} \in S_2 \equiv \{(m_H, m_F) \mid m_F < \min\{\underline{m}_F(m_H), \overline{m}_F(m_H)\}\}$.

(iii) The subsidy for the home and the tax for foreign country if $\boldsymbol{m} \in S_3 \equiv \{(m_H, m_F) \mid m_F > \max\{\underline{m}_F(m_H), \overline{m}_F(m_H)\}\}$.

(iv) Subsidies for the home and foreign countries if $\boldsymbol{m} \in S_4 \equiv \{(m_H, m_F) \mid \overline{m}_F(m_H) \le m_F \le \underline{m}_F(m_H)\}$.

Here, $\underline{m}_F(m_H) \equiv \left(\dfrac{\varphi_1}{\varphi_2}\right) m_H - \dfrac{1}{\varphi_1}$ *and* $\overline{m}_F(m_H) \equiv \left(\dfrac{\varphi_2}{\varphi_1}\right) m_H + \dfrac{\varphi_2}{\varphi_1^2}$.

<u>Case 1</u>: **Symmetric oligopoly.** When $\gamma_b = \gamma_n = \gamma$, we have $\varphi_b = \varphi_n = (\gamma + 1)^{-1}$. Hence,

$$\Phi_i(\boldsymbol{m}) \equiv \varphi_1^2 m_i - h_j \varphi_2 = -\frac{2[(\gamma+1)(1-2m_i) + 2m_j]}{(\gamma+1)^3}.$$

By setting $\gamma = 0$, we obtain the result of Dixit's (1984) symmetric oligopoly model with constant marginal cost:

$$s_i \lesseqqgtr 0 \quad \Leftrightarrow \quad \Phi_i(\boldsymbol{m}) = -2(1 + 2m_j - 2m_i) \lesseqqgtr 0 \quad \Leftrightarrow \quad 1 + \underbrace{2m_j}_{=N} \gtreqqless \underbrace{2m_i}_{=n}$$

Note that N (n) denotes the number of foreign (home) firms in Dixit (1984) (see the initial equation on page 12 of Dixit (1984)).

<u>Case 2</u>: **Our asymmetric oligopoly.** When $\gamma_b = \gamma$ and $\gamma_n = 1$, we have $\varphi_b = (\gamma + 1)^{-1}$ and $\varphi_n = 2^{-1}$. In this case, $\Phi_i(1, m) < 0$ and $\Phi_j(1, m) > 0$ hold. This implies that the optimal policy of the home (foreign) country is an export subsidy (tax). (The detailed computation is reported in the Appendix.)

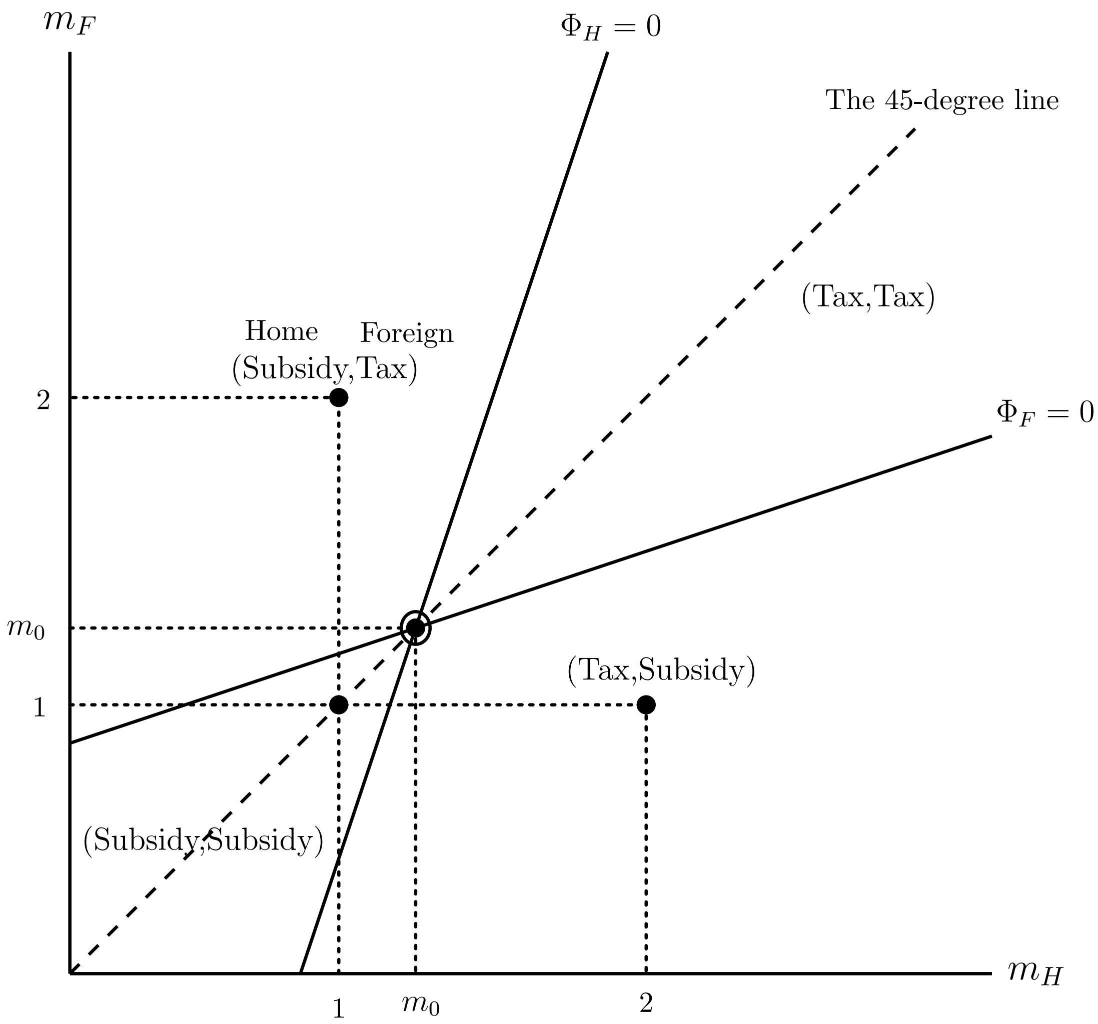

Figure 3: Market structures and optimal export policies.

6 Discussion

6.1 Differentiated tax/subsidy

In this section, we consider a situation in which both countries apply a differentiated export policy to firms b and n instead of an export policy that is common to all the firms in each country. By setting different export policy variables for the two types of firms in each country $i\,(=H,F)$, we obtain the following optimal export policies:

$$s^*_{i,b} = \frac{(2\gamma+3)(\gamma-1)\alpha}{25\gamma+11\gamma^2+12} \quad \text{and} \quad s^*_{i,n} = \frac{(\gamma+2)(1-\gamma)\alpha}{25\gamma+11\gamma^2+12}.$$

The above equation immediately yields the following.

Remark 1. *(i) Suppose that $\gamma < 1$. Firm b receives a positive subsidy, whereas an export tax is levied on firm n. (ii) Suppose that $\gamma = 1$. Free trade is realized—that is, $s^*_{i,b} = s^*_{i,n} = 0$. (iii) Suppose that $\gamma > 1$. Firm n receives a positive export subsidy, whereas an export tax is levied on firm b.*

From a welfare perspective, when the country can distinguish between firms based on their productivity differences, it is desirable for it to provide a subsidy to the efficient firm and to impose a tax on the inefficient firm. This is because by impeding the production of the inefficient firm through the imposition of the export tax, the country can raise national welfare (Lahiri and Ono, 1988). Furthermore, if both firms have the same level of productivity, differentiating export policies becomes impossible and the countries cannot influence national welfare by selecting a tax or subsidy as their export policy.

The equilibrium total surplus in country i becomes:

$$W_i^{d*} = \frac{(13\gamma + 7\gamma^2 + 4)(3\gamma + 5)(\gamma + 1)\,\alpha^2}{2(25\gamma + 11\gamma^2 + 12)^2}.$$

The above equation yields:

$$\frac{\partial W_i^{d*}}{\partial \gamma} = \frac{(319\gamma + 219\gamma^2 + 61\gamma^3 + 5\gamma^4 + 164)\,\alpha^2}{2(25\gamma + 11\gamma^2 + 12)^3} > 0.$$

This result is the same as Proposition 2 and, hence, the differentiated tax/subsidy does not alter the nature of welfare.

7 Conclusion

In this paper, we incorporate a productivity gap among (domestic) firms with convex (i.e., linear quadratic) costs into a standard third-market model and show that the optimal export policy of the exporting countries is a "subsidy—tax—subsidy" pattern that alters with the productivity gap. We also demonstrate that an improvement (deterioration) in the productivity of the efficient (less-efficient) firm reduces (increases) the exporters' welfare. Furthermore, we extend our baseline model to the multiple-firm case and show that the optimal policies of the two exporters depend on the gap in firm numbers between the exporters. For example, if firm numbers are sufficiently small in both exporting countries, the optimal policy of both exporters is an export subsidy. By contrast, if the gap in firm numbers is large, one exporter offers an export subsidy, but the other imposes an export tax. (See Figure 3 and Theorem 1.)

Our model is limited to the third-market situation. Hence, extending our model to a two-country two-way trade situation may yield interesting results. Moreover, it may be a fruitful to generalize the linear demand function. However, these topics are beyond the scope of our analysis and are left to future research.

Acknowledgements

This work is supported by the Japan Society for the Promotion of Science KAKEMHI [Grant number 24K04904] and a Grant-in-Aid from the Zengin Foundation for Studies on Economics and Finance. All errors are our own. We thank Dr Amanda Fitzgibbons from Edanz (https://jp.edanz.com/ac) for editing a draft of this manuscript.

Appendix

Proof of Proposition 3.

From (3), s_i^{C*} is a cubic function of γ. Thus, it can be solved analytically with respect to γ. The equation $s_i^{C*} = 0$ for γ has two real roots, $\gamma = \varphi_L(\beta)$ and $\gamma = \varphi_R(\beta)$. The $\varphi_L(\beta)$ intersects with the vertical axis at a certain point, β, between 0.6 and 0.8. (See Figure 2.) From a numerical calculation, $\varphi_L^{-1}(\gamma = 0)$ yields $\beta = \beta_C \simeq 0.707299$. (See the dashed-line in Figure 2.) This implies Proposition 3. $\square$

E_C of s_i^{C*} in Section 4.1.

$$E_C \equiv 5\beta^4(\gamma+5)-\beta^3(5\gamma^2+74\gamma+173)+\beta^2(\gamma+5)(\gamma^2+34\gamma+73)-3\beta(\gamma+2)(\gamma+5)^2-9(\gamma+2)^2(\gamma+5).$$

B_h $(h = 1, \ldots, 6)$ in $s_i^{B*} > 0$.

$$B_1 \equiv 5250\beta^8 - 10600\beta^7 - 4460\beta^6 + 12194\beta^5 + 5399\beta^4 - 4003\beta^3 - 3353\beta^2 - 841\beta - 72 < 0,$$

$$B_2 \equiv 2200\beta^7 - 5110\beta^6 - 24\beta^5 + 4754\beta^4 + 427\beta^3 - 1608\beta^2 - 648\beta - 72 < 0,$$

$$B_3 \equiv 95\beta^5 - 154\beta^4 - 62\beta^3 + 86\beta^2 + 47\beta + 6 > 0,$$

$$B_4 \equiv 3000\beta^9 - 11575\beta^8 + 5340\beta^7 + 17045\beta^6 - 7257\beta^5 - 12840\beta^4 + 406\beta^3 + 4273\beta^2 + 1590\beta + 180 > 0,$$

$$B_5 \equiv -2675\beta^8 + 5170\beta^7 + 6910\beta^6 - 9088\beta^5 - 9427\beta^4 + 2657\beta^3 + 5184\beta^2 + 1863\beta + 216 > 0,$$

$$B_6 \equiv 295\beta^6 - 105\beta^5 - 1209\beta^4 - 112\beta^3 + 933\beta^2 + 522\beta + 81 > 0.$$

The second-order condition for welfare maximization in Section 5.

We show that the second-order condition is satisfied as follows:

$$\frac{\partial^2 w_i(\boldsymbol{s})}{\partial s_i^2} = -\frac{\Omega_i h_j}{(M\varphi_1 + 1)^2} < 0 \quad \Leftrightarrow \quad \Omega_i \equiv 2\varphi_1^2 m_i + h_j(\varphi_1 - \varphi_2) > 0,$$

where $\Omega_i > 0$ is readily proved by $\varphi_1 > \varphi_2$.

The definition of Δ.

Δ is given by:

$$\Delta \equiv \left[(\varphi_1-\varphi_2)^2\alpha+2M\varphi_1^2(\varphi_1-\varphi_2)+m_H m_F\varphi_1^4\right](M\varphi_1+1)+m_H m_F\varphi_1^2\left[\varphi_1^2+\varphi_2^2+M\varphi_1^2(\varphi_1-\varphi_2)\right].$$

Case 2 in Section 5.

We obtain the following.

$$\varphi_1 \equiv \varphi_b + \varphi_n = \frac{1}{2}(\gamma+1)^{-1}(\gamma+3); \quad \varphi_2 \equiv \varphi_b^2 + \varphi_n^2 = \frac{1}{4}(\gamma+1)^{-2}(2\gamma+\gamma^2+5),$$
$$h_i \equiv m_i\varphi_1 + 1 = \frac{1}{2}m_i(\gamma+1)^{-1}(\gamma+3) + 1; \quad h_j \equiv m_j\varphi_1 + 1 = \frac{1}{2}m_j(\gamma+1)^{-1}(\gamma+3) + 1.$$

From this, we have:

$$\Phi_i(\boldsymbol{m}) \equiv \varphi_1^2 m_i - h_j\varphi_2 = \frac{1}{8}(\gamma+1)^{-3}\left[2m_i(\gamma+1)(\gamma+3)^2 - (2(\gamma+1)+m_j(\gamma+3))\,(2\gamma+\gamma^2+5)\right].$$

By setting $m_i = 1$ and $m_j = m \geq 2$, we obtain our result:

$$\begin{aligned}\Phi_i(1,m) &= \frac{1}{8}(\gamma+1)^{-3}\left[8(\gamma+1)^2 - m(\gamma+3)(2\gamma+\gamma^2+5)\right] \\ &\leq \frac{1}{8}(\gamma+1)^{-3}\left[8(\gamma+1)^2 - 2(\gamma+3)(2\gamma+\gamma^2+5)\right] \\ &= -\frac{1}{4}(\gamma+1)^{-3}(3\gamma+\gamma^2+\gamma^3+11) < 0.\end{aligned}$$

Hence, $\Phi_i(1,m) < 0$ implies that the home country always subsidizes its exports.

Conversely,

$$\Phi_j(\boldsymbol{m}) \equiv \varphi_1^2 m_j - h_i\varphi_2 = \frac{1}{8}(\gamma+1)^{-3}\left[2m_j(\gamma+1)(\gamma+3)^2 - (2(\gamma+1)+m_i(\gamma+3))(2\gamma+\gamma^2+5)\right].$$

By setting $m_i = 1$ and $m_j = m \geq 2$, we obtain our result:

$$\begin{aligned}\Phi_j(1,m) &= \frac{1}{8}(\gamma+1)^{-3}\left[2m(\gamma+1)(\gamma+3)^2 - (3\gamma+5)(2\gamma+\gamma^2+5)\right] \\ &\geq \frac{1}{8}(\gamma+1)^{-3}\left[4(\gamma+1)(\gamma+3)^2 - (3\gamma+5)(2\gamma+\gamma^2+5)\right] \\ &= \frac{1}{8}(\gamma+1)^{-3}\left(35\gamma+17\gamma^2+\gamma^3+11\right) > 0.\end{aligned}$$

Hence, $\Phi_j(1,m) > 0$ implies that the foreign country always taxes its exports.

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