# Common Ownership with Downstream Firm's Voluntary Investment

**Qing Hu**
**Ryo Masuyama**
**Tomomichi Mizuno**

**September 2025**

**Discussion Paper No. 2520**

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

# Common Ownership with Downstream Firm's Voluntary Investment*

Qing Hu[†ac], Ryo Masuyama[‡bc], Tomomichi Mizuno[§c]

[a] Faculty of Business and Commerce, Kansai University

[b] Faculty of Economics, Kushiro Public University of Economics

[c] Graduate School of Economics, Kobe University

August 29, 2025

**Abstract**

It is well known that common ownership lessens competition, which tends to decrease consumer and total surpluses. This study challenges this well known result by introducing downstream firms' voluntary investment. We consider a vertical market with one upstream firm and two downstream firms, where the downstream firms engage in voluntary investment that can reduce the upstream firm's marginal cost. We show that common ownership may increase the consumer and total surpluses if the upstream marginal cost without investment is sufficiently high and the investment is sufficiently efficient. We also find our results are robust even in the market with two supply chains.

**JEL codes**: D43, L10, L13

**Keywords**: common ownership, cost-reducing investment, vertical relationship.

---

*The authors gratefully acknowledge the financial support of the Japan Society for the Promotion of Science (JSPS), KAKENHI Grant Numbers JP22H00043, JP23H00764, 24K04863, and 25K21888. The usual disclaimer applies.

†Qing Hu, Faculty of Business and Commerce, Kansai University, 3-3-35 Yamate-cho, Suita-shi, Osaka, 564-8680, Japan. Phone: (81)-6-6368-1121. Fax: (81)-78-803-7289. E-mail: huqing549@gmail.com

‡Ryo Masuyama, Faculty of Economics, Kushiro Public University of Economics, Ashino 4-1-1, Kushiro-shi, Hokkaido, 085-8585, Japan. E-mail: ry.masuyama@gmail.com

§Tomomichi Mizuno. Graduate School of Economics, Kobe University, 2-1 Rokkodai, Nada, Kobe, Hyogo, 657-8501, Japan. Phone: (81)-78-803-6802. E-mail: mizuno@econ.kobe-u.ac.jp

# 1 Introduction

Common ownership, holding shares of firms in a market by outside institutional investors, has been observed in many industries. For example, in the smartphone industry, an institutional investor, Vanguard, owns shares of competing firms, Google and Samsung. In addition, we observe common ownership in the airline and pharmaceutical industries (Azar et al. 2018), as well as in the automobile and steel industries (Xu et al. 2023). These observations have led many researchers to focus on common ownership (Chen et al. 2021; López and Vives 2019; Lømo 2024).

In the literature on common ownership, it is well known that common ownership harms welfare. This is because common ownership forces firms to consider their rivals' profits, resulting in less intense competition. In contrast, several studies have challenged this conventional wisdom by considering various factors (Brito et al. 2020; Chen et al. 2021; López and Vives 2019; Lømo 2024; Shy and Stenbacka 2020). In line with these studies, we reconsider the result of common ownership. Our focus is on common ownership in a vertical market with voluntary cost-reducing investments initiated by downstream firms.

Voluntary cost-reducing investments initiated by downstream firms have been observed in various industries–for instance, in the smartphone industry.[1] Chips are essential for smartphone production, and Qualcomm is a common chip supplier for Google and Samsung, both of which manufacture smartphones. Furthermore, Google supports Qualcomm's chip development, which makes it easier for Qualcomm to supply chips. Similarly, Samsung also contributes to Qualcomm's chip development. In this way, both Google and Samsung make voluntary investments that reduce Qualcomm's marginal cost.

In light of the above situation, we consider the following model. We consider one

[1] Similarly, the investments have also been observed in the apparel and watch industries (Hamamura and Zennyo 2021).

upstream firm and two downstream firms. In the main model, we assume that the upstream firm acts as the monopolistic supplier. Actually, Qualcomm has monopoly power in the upstream chip market, which supplies a critical input for smartphone production (Arai and Matsushima 2023). Both downstream firms are under common ownership, and each downstream firm considers the rival downstream firm's profit in addition to its own profit. Furthermore, each downstream firm can invest in reducing the marginal cost of the upstream firm. After that, the upstream firm sells inputs to each downstream firm, and then the downstream firms compete with each other.

Our findings are as follows. Common ownership improves consumer and total surpluses when the upstream marginal cost without investment is sufficiently high and cost-reducing investment is sufficiently efficient. The intuition behind this is as follows. A higher degree of common ownership has two effects on welfare. The first effect is that competition reduces and thus welfare worsens, which has been observed in previous studies. Let us call this effect the *competition reduction effect*. Meanwhile, each downstream firm considers the profit of its downstream rival firm, leading to more investment to the upstream firm, which lowers the marginal cost of the upstream firm and thus the input price. The reduction of the input price eases the double marginalization problem. This is the second effect and we call it *input price effect*. We find that the input price effect is large enough to dominate the competition reduction effect when the upstream marginal cost without investment (the original upstream firm's marginal cost) is sufficiently high and cost-reducing investment is sufficiently efficient. Hence, in our model, we find the bright side of common ownership, showing that common ownership may increase the consumer and total surpluses. We also extend our model to a market with two supply chains and find that our main results are robust.

This study contributes to the literature on non-controlling ownership among competing firms. This literature includes common ownership and horizontal shareholding. In the literature, it is well known that horizontal non-controlling ownership reduces competition and welfare. However, several studies have challenged this conventional wisdom by considering various factors: location choice in a Hotelling market (Li and Zhang 2021); differentiating investment (Fang et al. 2024; Banerjee et al. 2025); free entry (Sato and Matsumura 2020; Vives and Vravosinos 2025); mixed oligopoly (Xing et al. 2024; Cheng et al. 2023); firm size heterogeneity (Farrell and Shapiro 1990; Ma et al. 2021); R&D spillover (López and Vives 2019); endogenous quality (Brito et al. 2020); asymmetric ownership (Bayona and López 2018); and cost uncertainty (Shy and Stenbacka 2020). Our study focuses on vertical relationships, which these studies do not consider.

Similar to our study, several papers examine horizontal non-controlling ownership in vertical markets. Chen et al. (2021) and Lømo (2024) focus on settings with an upstream monopoly.[2] Chen et al. (2021) consider common ownership involving upstream and downstream firms and show that it can enhance welfare. Lømo (2024) investigates the role of demand curvature and identifies conditions under which common ownership increases welfare. Shuai et al. (2022) consider various upstream market structures and show that the welfare effects of horizontal shareholding depend on the market structure. None of these studies focus on voluntary cost-reducing investments by downstream firms.

The rest of the paper is organized as follows. Section 2 describes the basic model. In section 3, we presents main results. Section 4 extends our basic model. Finally, we provide conclusions in section 5.

# 2 Basic Model

We consider a vertical market with one upstream firm, firm $U$, and two downstream firms, firm $Di$ ($i = 1, 2$). Firm $U$ produces an input with a marginal cost, $c_U$, and sells it to the downstream firms at an input price, $w$. Each downstream firm uses one unit of the input to produce one unit of the final good sold to consumers. The inverse demand

[2] In addition, some studies that assume upstream monopoly show that horizontal non-controlling ownership worsens welfare (Hu et al. 2021; Jin et al. 2024; Monden and Mizuno 2023).

function for firm $Di$ is as follows:

$$p_i = 1 - q_i - \gamma q_j,$$

where $j = 1, 2$ and $j \neq i$; $p_i$ and $q_i$ are the price and quantity of firm $Di$, respectively; $\gamma \in [0, 1]$ measures the degree of the product substitutability.

We assume that firm $Di$ can invest to reduce the upstream firm's marginal cost $c_U$. We define $e_i$ as the investment level of firm $Di$. Additionally, we suppose that firm $Di$ incurs the investment cost $ke_i^2/2$, where $k$ measures the investment inefficiency. Therefore, the upstream marginal cost is $c_U = c - e_i - e_j$, where $c < 1$ is the marginal cost without investment. We define the profits of the upstream firm $U$ and downstream firm $Di$ as follows:

$$\pi_{Di} = (p_i - w)q_i - k\frac{e_i^2}{2}, \quad \pi_U = [w - (c - e_i - e_j)]\,(q_i + q_j).$$

Furthermore, to guarantee the positive outcomes in the equilibrium and keep a positive upstream marginal cost, $c_U$, we assume $k > \underline{k} = [(1+\alpha)(1+\alpha\gamma)]/[c(2+\gamma+\alpha\gamma)^2]$.

In our model, we consider common ownership in the downstream market. Following the literature on common ownership (Chen et al. 2024; López and Vives 2019; Xu et al. 2023), the objective function of each downstream firm is as follows:

$$V_{Di} = \pi_{Di} + \alpha\pi_{Dj}.$$

Here, $\alpha \in [0, 1)$ denotes the degree of common ownership.

We define consumer, producer, and total surpluses are as follows, respectively.

$$CS = \frac{q_i^2 + 2\gamma q_i q_j + q_j^2}{2}, \quad PS = \pi_U + \pi_{Di} + \pi_{Dj}, \quad TS = CS + PS.$$

The timing of the game is as follows. At stage 1, firm $Di$ chooses the investment level $e_i$ so as to maximize $V_{Di}$, which establishes the upstream marginal cost, $c_U$. At stage 2, firm $U$ sets its input price $w$. At stage 3, firm $Di$ chooses its quantity $q_i$ so as to maximize $V_{Di}$. We solve this game using backward induction.

# 3 Results

At stage 3, firm $Di$ chooses its quantity $q_i$ to maximize $V_{Di}$. From the first-order condition $\partial V_{D_i}/\partial q_i = 0$, we obtain the output $q_i$ of this stage as follows:

$$q_i(w) = \frac{1-w}{2+\gamma+\alpha\gamma}.$$

At stage 2, firm $U$ sets its input price $w$. Substituting $q_i(w)$ into $\pi_U$, the first-order condition on $w$ leads to the following outcome as

$$w(e_i, e_j) = \frac{1+c-e_i-e_j}{2}.$$

At stage 1, firm $Di$ chooses the investment level $e_i$ to maximize $V_{Di}$. Substituting the outcomes in the second and third stages into $V_{Di}$, the first-order condition on $e_i$ yields

$$e_i^* = \frac{(1-c)(1+\alpha)(1+\alpha\gamma)}{-2(1+\alpha)(1+\alpha\gamma)+2k(2+\gamma+\alpha\gamma)^2}.$$

Using the above outcomes, we obtain the input price, profit of each firm, consumer, producer, and total surpluses in equilibrium as follows:

$$w^* = \frac{-2(1+\alpha)(1+\alpha\gamma)+k(1+c)(2+\gamma+\alpha\gamma)^2}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]},$$

$$\pi_{Di}^* = \frac{k(1-c)^2(1+\alpha\gamma)[-(1+\alpha)^2(1+\alpha\gamma)+2k(2+\gamma+\alpha\gamma)^2]}{8[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2},$$

$$\pi_U^* = \frac{k^2(1-c)^2(2+\gamma+\alpha\gamma)^3}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2},$$

$$CS^* = \frac{k^2(1-c)^2(1+\gamma)(2+\gamma+\alpha\gamma)^2}{4[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2},$$

$$PS^* = \frac{k(1-c)^2[-(1+\alpha)^2(1+\alpha\gamma)^2+2k(2+\gamma+\alpha\gamma)^2(3+\gamma+2\alpha\gamma)]}{4[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2},$$

$$TS^* = \frac{k(1-c)^2[-(1+\alpha)^2(1+\alpha\gamma)^2+k(2+\gamma+\alpha\gamma)^2(7+3\gamma+4\alpha\gamma)]}{4[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2}.$$

First, we examine the effect of common ownership on the downstream investment level. Differentiating $e_i^*$ with respect to $\alpha$, we obtain the following Lemma 1.

**Lemma 1** *When the degree of common ownership is higher, downstream firms make larger cost-reducing investments.*

**Proof.**

$$\frac{\partial e_i^*}{\partial \alpha} = \frac{k(1-c)(2+\gamma+\alpha\gamma)[2+(1+3\alpha)\gamma+(1+\alpha)\gamma^2]}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2} > 0.\square$$

The intuition behind Lemma 1 is as follows. With a higher degree of common ownership, firm $Di$ considers the profit of its rival firm $Dj$ more. Therefore, firm $Di$ seeks to increase the profit of firm $Dj$ by increasing its own investment.

Next, differentiating $w^*$ with respect to $\alpha$, we obtain Lemma 2.

**Lemma 2** *When the degree of common ownership is higher, the upstream firm sets a lower input price.*

**Proof.**

$$\frac{\partial w^*}{\partial \alpha} = -\frac{k(1-c)(2+\gamma+\alpha\gamma)[2+(1+3\alpha)\gamma+(1+\alpha)\gamma^2]}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^2} < 0.\square$$

The intuition for Lemma 2 is as follows. According to Lemma 2, as common ownership progresses, the downstream firms engage in larger cost-reducing investments, thereby lowering the upstream marginal cost. Consequently, the reduction of the input price mitigates the problem of double marginalization.

Now, we can analyze how common ownership in the downstream market with cost-reducing investment to the upstream affects the consumer surplus. Differentiating $CS^*$ with respect to $\alpha$, we obtain the following proposition.

**Proposition 1** *Common ownership increases the consumer surplus when the upstream marginal cost without investment is sufficiently high and downstream cost-reducing investment is sufficiently efficient:* $c_1 < c < 1$ *and* $\underline{k} < k < k_1$*, where* $c_1 = [(1+\alpha)\gamma(1+$

*$\alpha\gamma)]/[2+(2+4\alpha)\gamma+(1+\alpha)^2\gamma^2]$ and $k_1 = [2+(2+4\alpha)\gamma+(1+\alpha)^2\gamma^2]/[\gamma(2+\gamma+\alpha\gamma)^2]$. Otherwise, common ownership decreases the consumer surplus.*

**Proof.** See Appendix.

Common ownership has two effects on the consumer surplus. The first effect is the *competition reduction effect*, where competition becomes less intense. The competition reduction effect decreases the consumer surplus. However, at the same time, common ownership promotes downstream cost-reducing investments, as we have seen in Lemma 1. This leads to a lower input price, as shown in Lemma 2, consequently mitigating the double-marginalization problem. Thus, when considering voluntary cost-reducing investments by downstream firms, common ownership has a second effect: the *input price effect*. This effect is more significant when the upstream marginal cost without investment is sufficiently high and the investment is sufficiently efficient. Therefore, common ownership is desirable for consumers because the input price effect dominates the competition reduction effect.

Next, we analyze the effect of common ownership on the profit of each firm and the producer surplus. Differentiating $\pi^*_{Di}$, $\pi^*_U$ and $PS^*$ with respect to $\alpha$, respectively, we obtain the following Proposition 2.

**Proposition 2** *(i) Common ownership always increases the downstream firms' profits. (ii) Common ownership increases the upstream firm's profit when the upstream marginal cost without investment is sufficiently high and the downstream cost-reducing investment is sufficiently efficient: $c_2 < c < 1$ and $\underline{k} < k < k_2$, where $c_2 = [(1+\alpha)\gamma(1+\alpha\gamma)]/[4+(3+7\alpha)\gamma+(2+3\alpha+\alpha^2)\gamma^2]$ and $k_2 = [4+(3+7\alpha)\gamma+(2+3\alpha+\alpha^2)\gamma^2]/[\gamma(2+\gamma+\alpha\gamma)^2]$. Otherwise, common ownership decreases the upstream firm's profit. (iii) Common ownership increases the producer surplus when the upstream marginal cost without investment is sufficiently high and the downstream cost-reducing investment is sufficiently efficient: $c_3 < c < 1$ and $\underline{k} < k < k_3$, where $c_3 = 2(1+\alpha)\gamma(1+\alpha\gamma)^2/\Phi_1$, $k_3 = \Phi_1/[2\gamma(1+\alpha\gamma)(2+\gamma+\alpha\gamma)^2]$*

*and* $\Phi_1 = 10-2\alpha+(11+22\alpha-5\alpha^2)\gamma+(7+17\alpha+11\alpha^2-3\alpha^3)\gamma^2+(2+5\alpha+4\alpha^2+\alpha^3)\gamma^3$. *Otherwise, common ownership decreases the producer surplus.*

**Proof.** See Appendix.

The intuition behind Proposition 2 is as follows. First, we consider Proposition 2 (i). As common ownership progresses, competition reduces. As a result, common ownership is always desirable for the downstream firms. Second, therefore, common ownership tends to be undesirable for the upstream firm. However, common ownership causes the downstream firms to invest more aggressively in cost-reducing, as seen in Lemma 1, and thus it has a positive effect on the upstream firm's profit. In addition, this aspect is more pronounced when the upstream marginal cost without investment is sufficiently high and the downstream cost-reducing investment is sufficiently efficient. In this situation, common ownership increases the profit of the upstream firm. This is the intuition behind Proposition 2 (ii). Finally, from the aforementioned discussion, we also obtain the argument of Proposition 2 (iii).

Finally, we focus on the effect of common ownership on the total surplus. By differentiating $TS^*$ with respect to $\alpha$, we have Proposition 3. The intuition behind Proposition 3 is obvious from Propositions 1 and 2.

**Proposition 3** *Common ownership increases the total surplus when the upstream marginal cost without investment is sufficiently high and downstream cost-reducing investment is sufficiently efficient:* $c_4 < c < 1$ *and* $\underline{k} < k < k_4$*, where* $c_4 = (1+\alpha)\gamma(1+\alpha\gamma)(3+\gamma+2\alpha\gamma)/\Phi_2$*,* $k_4 = \Phi_2/[\gamma(2+\gamma+\alpha\gamma)^2(3+\gamma+2\alpha\gamma)]$ *and* $\Phi_2 = 12-2\alpha+(15+26\alpha-5\alpha^2)\gamma+(10+23\alpha+12\alpha^2-3\alpha^3)\gamma^2+(3+7\alpha+5\alpha^2+\alpha^3)\gamma^3$. *Otherwise, common ownership decreases the total surplus.*

**Proof.** See Appendix.

The intuition behind this proposition is the same as that in Proposition 1.

# 4 Discussions

In the previous sections, we focused on a monopolist in the upstream market. However, in some industries (e.g., automobiles and soft drinks), the assumption of an upstream monopoly may not hold. Therefore, here, we extend our model to examine whether our main results obtained above also hold considering duopolistic vertical supply chains.

## 4.1 Model with two supply chains

Specifically, we extend to a model of two identical vertical supply chains, each composed of a downstream firm $D_i$ and its firm-specific upstream input supplier $U_i$. To produce one unit of the product, the downstream $D_i$ purchases one unit of the input from its firm-specific upstream supplier $U_i$ at input price $w_i$. The definition of the output, price and the inverse demand of the final good are the same to the previous section 2, i.e., $p_i = 1 - q_i - \gamma q_j$.[3]

Here, the downstream firms can only make investments into their firm-specific suppliers' production technology. We assume that the two supply chains are symmetric and the original marginal cost of both suppliers is $c$. To reduce the production cost of its supplier from $c$ to $c - e_i$, the downstream firm $D_i$ incurs the investment cost $ke_i^2/2$. Hence, the profits of the downstream firm and its supplier are $\pi_{Di} = (p_i - w_i)q_i - ke_i^2/2$ and $\pi_{Ui} = [w_i - (c - e_i)]q_i$, respectively.

Same to the previous sections, we consider common ownership in the downstream market thus each downstream firm competes in quantity to maximize its total value, i.e., the objective function $V_{Di} = \pi_{Di} + \alpha\pi_{Dj}$. In this case, the consumer surplus is same to the previous setting, but the total surplus is $TS = CS + \pi_{Di} + \pi_{Dj} + \pi_{Ui} + \pi_{Uj}$. To ensure all the outcomes in the equilibrium and the upstream marginal cost are positive,

[3] We assume the sufficient condition, $\gamma \in [0, 1/2)$, to keep a not very high degree of product substitutability to avoid zero profit of the upstream firms due to the fierce price competition. We will explain this assumption later in details.

we assume

$$k > \frac{2\Phi^2}{(1+\alpha\gamma)[64 - 20\gamma^2(1+\alpha)^2 + \gamma^4(1+\alpha)^4]^2} = \underline{k}^S, \quad c > \frac{4\Phi}{\Theta} = \underline{c}^S,$$

where $\Phi = -16 - 4\alpha(1+\alpha)\gamma + 2(1+\alpha)^3\gamma^2 + \alpha(1+\alpha)^3\gamma^3 < 0$, $\Theta = -k[8 + 2(1+\alpha)\gamma - (1+\alpha)^2\gamma^2]^2[8 - 2(1+\alpha)\gamma - (1+\alpha)^2\gamma^2] < 0$, and the superscript $S$ denotes the case of duopolistic supply chains.[4]

The timing of this game is as follows: At stage 1, the downstream firms choose the level of investments. At stage 2, the upstream suppliers decide input prices simultaneously. At stage 3, the downstream firms compete in quantity. We solve this model using backward induction.

## 4.2 Results

At stage 3, the downstream firms decide the quantities. Using the first-order condition $\partial V_{D_i}/\partial q_i = 0$, we obtain the output of downstream firm $i$

$$q_i^S(w_i, w_j) = \frac{2 - 2w_i - (1 - w_j)(1+\alpha)\gamma}{4 - (1+\alpha)^2\gamma^2}.$$

At stage 2, the upstream suppliers decide input prices. Substituting of the outputs obtained in the third stage, the first-order conditions $\partial \pi_{Ui}(w_i, w_j)/\partial w_i = 0$ yield

$$w_i^S(e_i, e_j) = \frac{8 + 8c - 8e_i - (2 - 2c + 2e_j + 2\alpha - 2c\alpha + 2e_j\alpha)\gamma - (1 + 2\alpha + \alpha^2)\gamma^2}{16 - \gamma^2(\alpha+1)^2}.$$

At stage 1, the downstream firms decide the level of investments. Using the outcomes obtained in the second and third stages, the downstream firm $i$ maximizes the total value $V_{Di}$ with respect to $e_i$, then we have

$$e_i^S = \frac{4(1-c)\Phi}{\Theta - 4\Phi}.$$

Substituting all back, we have the main outcomes in the equilibrium as follows.

[4]With $\gamma \in [0, 1/2)$ and $\alpha \in [0, 1)$, it is easy to find the signs of $\Phi$ and $\Theta$ are both negative. In addition, since $c < 1$, we can get $\Theta < 4\Phi$.

$$w_i^S = \frac{\begin{bmatrix} 4\Phi + k[32 - 16(1+\alpha)\gamma - 2(1+\alpha)^2\gamma^2 + (1+\alpha)^3\gamma^3] \\ (2+\gamma+\alpha\gamma)^2[2+2c-(1+\alpha)\gamma] \end{bmatrix}}{4\Phi - \Theta},$$

$$CS^S = \frac{4(1-c)^2k^2(1+\gamma)(2-\gamma-\alpha\gamma)^2(2+\gamma+\alpha\gamma)^2[16-(1+\alpha)^2\gamma^2]^2}{(4\Phi-\Theta)^2},$$

$$TS^S = \frac{4(1-c)^2k \begin{bmatrix} -4\Phi^2 + k[7+(1+2\alpha)\gamma - (1+\alpha)^2\gamma^2][64- \\ 20(1+\alpha)^2\gamma^2 + (1+\alpha)^4\gamma^4]^2 \end{bmatrix}}{(4\Phi-\Theta)^2}.$$

First, we examine how the degree of common ownership affects the downstream investment and input prices. Differentiating $e_i^S$ and $w_i^S$ with respect to $\alpha$, respectively, we get $\partial e_i^S/\partial\alpha > 0$ and $\partial w_i^S/\partial\alpha < 0$. All the proofs are shown in the Appendix. Hence, we obtain the following results.

**Lemma 3** *When the degree of common ownership is higher, the upstream firm sets a lower input price while the downstream firms have a larger cost-reducing investments, even in the market with two supply chains.*

The intuition can be explained as follows. An increase of the investment of a downstream firm will reduce its firm-specific upstream supplier's production cost thus lowers its input price. Meanwhile, a reduction of the input price of any upstream supplier will trigger the other one to lower the price, i.e., a price competition between the suppliers happens. Recall that we set the assumption with upper bound of the degree of product substitutability, i.e., $\gamma \in [0, 1/2)$. This is to avoid a much fierce price competition which leads to zero profit of the upstream firms. As the degree of common ownership increases, the downstream firms care more about the profits of each other. Therefore, the downstream firm $Di$ will invest more to lower the cost and then the input price of its firm-specific upstream supplier, which triggers the other upstream supplier to lower the input price, then benefiting the profit of the downstream firm $Dj$. This input price effect also mitigates the double marginalization problem. However, the input price competition between the upstream market will strengthen the incentive for the upstream firms to lower the input prices. Hence, compared with the case with a monopoly upstream firm in the previous sections, the input price effect may appear more obvious in the case with two supply chains.

Then, differentiating $CS^S$ with respect to $\alpha$, we have the following proposition.

**Proposition 4** *In the market with two supply chains, common ownership increases the consumer surplus when the downstream cost-reducing investment is sufficiently efficient:* $\underline{k}^S < k < k_5$*, where*

$$k_5 = \frac{2\left[\begin{array}{l} 256(1+2\alpha)+128(2-\alpha-3\alpha^2)\gamma-16(9+16\alpha)(1+\alpha)^2\gamma^2 \\ -8\gamma^3(3-5\alpha)(1+\alpha)^3+8(3+4\alpha)(1+\alpha)^4\gamma^4+2(1+\alpha)^6\gamma^5-(1+\alpha)^6\gamma^6 \end{array}\right]}{(1-\gamma-\alpha\gamma)[64-20(1+\alpha)^2\gamma^2+(1+\alpha)^4]^2\gamma^4}$$

The intuition is parallel to Proposition 1.

Next, differentiating $TS^S$ with respect to $\alpha$, we have the following proposition.

**Proposition 5** *In the market with two supply chains, common ownership increases the consumer surplus when the downstream cost-reducing investment is sufficiently efficient:* $\underline{k}^S < k < k_6$*, where*

$$k_6 = \frac{4\left[\begin{array}{l} 2048(3+10\alpha)+512[27-\alpha(7+34\alpha+4\alpha^2)]\gamma-128(1+\alpha)[71+\alpha(185+68\alpha \\ -22\alpha^2)]\gamma^2-32(1+\alpha)^2[231-\alpha(57+298\alpha+6\alpha^2)]\gamma^3+16(1+\alpha)^3[232+\alpha(497 \\ +303\alpha-48\alpha^2)]\gamma^4+8(1+\alpha)^4[151+\alpha(14-151\alpha-20\alpha^2)]\gamma^5-4(1+\alpha)^5[153+ \\ \alpha(278+147\alpha+2\alpha^2)]\gamma^6-2(1+\alpha)^7(2+\alpha)(23-8\alpha)\gamma^7+(1+\alpha)^7[43+\alpha(53+ \\ 20\alpha+12\alpha^2)]\gamma^8+(1+\alpha)^8[3+2\alpha(4+3\alpha+\alpha^2)]\gamma^9-(1+\alpha)^{10}(1-\alpha)\gamma^{10} \end{array}\right]}{(4-\gamma-\alpha\gamma)^2(2-\gamma-\alpha\gamma)^3(1-\gamma-\alpha\gamma)(2+\gamma+\alpha\gamma)^2(4+\gamma+\alpha\gamma)^3[6+2\alpha\gamma-(1+\alpha)^2\gamma^2]}$$

The intuition is also parallel to Proposition 3.

## 5 Conclusions

We consider a vertical market with one upstream firm and two downstream firms, where the downstream firms can invest to reduce the upstream firm's marginal cost. We show that common ownership may increase the consumer and total surpluses when the upstream marginal cost without investment is sufficiently high and the investment is sufficiently efficient, which reverses the well known result that common ownership tends to decrease consumer and total surpluses since it reduces competition. We also find our results are robust even in the market with two supply chains.

## References


Arai, Y. & Matsushima, N. (2023) The impacts of suppliers and mutual outsourcing on organizational forms. *Canadian Journal of Economics*, 56(1), 114–132.

Azar, J., Schmalz, M. C., & Tecu, I. (2018) Anticompetitive effects of common ownership. *Journal of Finance*, 73(4), 1513–1565.

Banerjee, S., Mukherjee, A., & Poddar, S. (2025) Product differentiation, demand expansion and the welfare effects of cross-ownership. *Canadian Journal of Economics*, 58(1), 193–226.

Bayona, A., & López, Á. L. (2018) Silent financial interests and product innovation. *Economics letters*, 170, 109–112.

Brito, D., Ribeiro, R., & Vasconcelos, H. (2020) Overlapping ownership, endogenous quality, and welfare. *Economics Letters*, 190, 109074.

Chen, L., Matsumura, T., & Zeng, C. (2024) Welfare consequence of common ownership in a vertically related market. *Journal of Industrial Economics*, 72(2), 996–1004.

Cheng, H., Ding, X., & Zeng, C. (2024) Cross-ownership on R&D and welfare in a mixed oligopoly: revisiting with convex cost. *Economics of Innovation and New Technology*, 33(6), 819–829.

Fang, J., Huang, J., & Zeng, C. (2024) Passive cross-holdings, horizontal differentiation, and welfare. *Bulletin of Economic Research*, 76(2), 508–528.

Farrell, J. & Shapiro, C. (1990) Asset ownership and market structure in oligopoly. *RAND Journal of Economics*, 275–292.

Hamamura, J. & Zennyo, Y. (2021) Retailer voluntary investment against a threat of manufacturer encroachment. *Marketing Letters*, 32(4), 379–395.

Hu, Q., Monden, A., & Mizuno, T. (2022) Downstream cross-holdings and upstream R&D. *Journal of Industrial Economics*, 70(3), 775–789.

Jin, Y., Mukherjee, A., & Zeng, C. (2024) Downstream Cross-Holdings and Upstream R&D: A Comment. *Journal of Industrial Economics*, 72(4), 1360–1368.

Li, Y. & Zhang, J. (2021) Product positioning with overlapping ownership. *Economics Letters*, 208, 110058.

Lømo, T. L. (2024) Overlapping ownership and input prices. *International Journal of Industrial Organization*, 94, 103067.

López, Á. L. & Vives, X. (2019) Overlapping ownership, R&D spillovers, and antitrust policy. *Journal of Political Economy*, 127(5), 2394–2437.

Ma, H., Qin, C., & Zeng, C. (2024) Competitive and welfare implications of cross-holdings in oligopoly. *Economic Theory*, 77, 975–997.

Monden, A. & Mizuno, T. (2023) Unprofitable Common Ownership with Asymmetric Distribution Channels. Discussion Papers 2317, Graduate School of Economics, Kobe University.

Sato, S. & Matsumura, T. (2020) Free entry under common ownership. *Economics Letters*, 195, 109489.

Shuai, J., Xia, M., & Zeng, C. (2023) Upstream market structure and downstream partial ownership. *Journal of Economics & Management Strategy*, 32(1), 22–47.

Shy, O. & Stenbacka, R. (2020) Common ownership, institutional investors, and welfare. *Journal of Economics & Management Strategy*, 29(3), 706–723.

Vives, X. & Vravosinos, O. (2025) Free entry in a Cournot market with overlapping ownership. *American Economic Journal: Microeconomics*, 17(2), 292–320.

Xing, M., Wang, L. F., & Zhou, C. (2024) Cross-ownership on R&D and social welfare in mixed oligopoly. *Economics of Innovation and New Technology*, 33(2), 206–217.

Xu, L., Zhang, Y., & Matsumura, T. (2023) Competition mode and common ownership in a mixed oligopoly. *Manchester School*, 91(4), 306–319.

# Appendix

## A.1 Proof of Proposition 1.

Differentiating $CS^*$ with respect to $\alpha$ yields the following result:

$$\frac{\partial CS^*}{\partial \alpha} = \frac{(1-c)^2(1+\gamma)(2+\gamma+\alpha\gamma)k^2 \begin{bmatrix} 2+(2+4\alpha)\gamma+(1+\alpha)^2\gamma^2 \\ -k[\gamma(2+\gamma+\alpha\gamma)^2] \end{bmatrix}}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^3}.$$

From $k > \underline{k}$, the denominator is positive. Therefore, the sign of $\partial CS^*/\partial\alpha$ is consistent with that of the square bracket in the numerator. Therefore, we obtain $\partial CS^*/\partial\alpha > 0$ if $k < k_1 = [2 + (2 + 4\alpha)\gamma + (1 + \alpha)^2\gamma^2]/[\gamma(2 + \gamma + \alpha\gamma)^2]$. Finally, $k_1 > \underline{k}$ requires $c > c_1 = [(1 + \alpha)\gamma(1 + \alpha\gamma)]/[2 + (2 + 4\alpha)\gamma + (1 + \alpha)^2\gamma^2]$. From the above, we obtain Proposition 1.□

## A.2 Proof of Proposition 2.

First, let us consider Proposition 2 (i). Calculating $\partial\pi^*_{Di}/\partial\alpha$, we obtain the following result.

$$\frac{\partial \pi^*_{Di}}{\partial \alpha} = \frac{(1-c)^2(1-\alpha)(2+\gamma+\alpha\gamma)k^2 \begin{bmatrix} 2+(1+5\alpha)\gamma+\alpha(1+3\alpha)\gamma^2 \\ +k[4\gamma^2+4(1+\alpha)\gamma^3+(1+\alpha)^2\gamma^4] \end{bmatrix}}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^3}.$$

Therefore, we have $\partial\pi^*_{Di}/\partial\alpha > 0$.

Next, we consider Proposition 2 (ii). Calculating $\partial\pi^*_U/\partial\alpha$ leads to the following result.

$$\frac{\partial \pi^*_{U}}{\partial \alpha} = \frac{(1-c)^2(2+\gamma+\alpha\gamma)^2k^2 \begin{bmatrix} 4+(3+7\alpha)\gamma+(2+3\alpha+\alpha^2)\gamma^2 \\ -k[\gamma(2+\gamma+\alpha\gamma)^2] \end{bmatrix}}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^3}.$$

Therefore, if $k < k_2 = [4+(3+7\alpha)\gamma+(2+3\alpha+\alpha^2)\gamma^2]/[\gamma(2+\gamma+\alpha\gamma)^2]$, we obtain $\partial\pi^*_U/\partial\alpha > 0$. Furthermore, $k_2 > \underline{k}$ requires $c > c_2 = [(1+\alpha)\gamma(1+\alpha\gamma)]/[4+(3+7\alpha)\gamma+(2+3\alpha+\alpha^2)\gamma^2]$.

Finally, we consider Proposition 2 (iii). Calculating $\partial PS^*/\partial\alpha$ leads to the following result.

$$\frac{\partial PS^*}{\partial \alpha} = \frac{(1-c)^2(2+\gamma+\alpha\gamma)k^2\begin{bmatrix} 10-2\alpha+(11+22\alpha-5\alpha^2)\gamma+(7+17\alpha \\ +11\alpha^2-3\alpha^3)\gamma^2+(2+5\alpha+4\alpha^2+\alpha^3)\gamma^3 \\ -k[8\gamma+(8+16\alpha)\gamma^2+(2+12\alpha+10\alpha^2)\gamma^3 \\ +(2\alpha+4\alpha^2+2\alpha^3)\gamma^4] \end{bmatrix}}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^3}.$$

Thus, for $k < k_3 = [10-2\alpha+(11+22\alpha-5\alpha^2)\gamma+(7+17\alpha+11\alpha^2-3\alpha^3)\gamma^2+(2+5\alpha+4\alpha^2+\alpha^3)\gamma^3]/[2\gamma(1+\alpha\gamma)(2+\gamma+\alpha\gamma)^2]$ , we have $\partial PS^*/\partial\alpha > 0$. Additionally, $k_3 > \underline{k}$ requires $c > c_3 = [2(1+\alpha)\gamma(1+\alpha\gamma)^2]/[10-2\alpha+(11+22\alpha-5\alpha^2)\gamma+(7+17\alpha+11\alpha^2-3\alpha^3)\gamma^2+(2+5\alpha+4\alpha^2+\alpha^3)\gamma^3]$ . From the above, we obtain Proposition 2.□

## A.3 Proof of Proposition 3.

Differentiating $TS^*$ with respect to $\alpha$ yields the following result.

$$\frac{\partial TS^*}{\partial \alpha} = \frac{(1-c)^2(2+\gamma+\alpha\gamma)k^2\begin{bmatrix} 12-2\alpha+(15+26\alpha-5\alpha^2)\gamma+(10+23\alpha \\ +12\alpha^2-3\alpha^3)\gamma^2+(3+7\alpha+5\alpha^2+\alpha^3)\gamma^3 \\ -k[12\gamma+(16+20\alpha)\gamma^2+(7+18\alpha+11\alpha^2)\gamma^3 \\ +(1+4\alpha+5\alpha^2+2\alpha^3)\gamma^4] \end{bmatrix}}{2[-(1+\alpha)(1+\alpha\gamma)+k(2+\gamma+\alpha\gamma)^2]^3}.$$

Therefore, if $k < k_4 = [12-2\alpha+(15+26\alpha-5\alpha^2)\gamma+(10+23\alpha+12\alpha^2-3\alpha^3)\gamma^2+(3+7\alpha+5\alpha^2+\alpha^3)\gamma^3]/[\gamma(2+\gamma+\alpha\gamma)^2(3+\gamma+2\alpha\gamma)]$, we obtain $\partial TS^*/\partial\alpha > 0$. In addition, $k_4 > \underline{k}$ requires $c > c_4 = [(1+\alpha)\gamma(1+\alpha\gamma)(3+\gamma+2\alpha\gamma)]/[12-2\alpha+(15+26\alpha-5\alpha^2)\gamma+(10+23\alpha+12\alpha^2-3\alpha^3)\gamma^2+(3+7\alpha+5\alpha^2+\alpha^3)\gamma^3]$. Thus, we have Proposition 3.□

## A.4 Proof of Lemma 3.

Differentiating $e_i^S$ with respect to $\alpha$ yields the following result.

$$\frac{\partial e_i^S}{\partial \alpha} = \frac{4(1-c)\left[\begin{array}{l} -\Theta[4+8\alpha-6(1+\alpha)^2\gamma-(1+\alpha)^2(1+4\alpha)\gamma^2]+2k\Phi(4-\gamma-\alpha\gamma)(2+\gamma \\ +\alpha\gamma)[8-30(1+\alpha)\gamma+(1+\alpha)^2\gamma^2+3(1+\alpha)^3\gamma^3] \end{array}\right]}{(4\Phi-\Theta)^2}.$$

Combining with our assumptions for positive outcomes, we have $\partial e_i^S/\partial\alpha > 0$.

Next, differentiating $w_i^S$ with respect to $\alpha$ yields the following result.

$$\frac{\partial w_i^S}{\partial \alpha} = \frac{2(1-c)\gamma k\left[\begin{array}{l} 4[512(1-2\alpha)-256(3+\alpha-2\alpha^2)\gamma-32(1+\alpha)^2(13-24\alpha)\gamma^2+32(1 \\ +\alpha)^3(6+\alpha)\gamma^3+32(1+\alpha)^4(1-2\alpha)\gamma^4-4(1+\alpha)^5(7+3\alpha)\gamma^5-2(1+ \\ \alpha)^6(1+2\alpha)\gamma^6+\gamma^7(1-\alpha)(1+\alpha)^7]-k(4-\gamma-\alpha\gamma)^2(2-\gamma-\alpha\gamma)^2(4 \\ +\gamma+\alpha\gamma)^2(2+\gamma+\alpha\gamma)^4 \end{array}\right]}{(4\Phi-\Theta)^2}.$$

Combining with our assumptions for positive outcomes, we have $\partial w_i^S/\partial\alpha < 0$. Thus, we have Lemma 4.□

## A.5 Proof of Proposition 4.

Differentiating $CS^S$ with respect to $\alpha$ yields the following result.

$$\frac{\partial CS^S}{\partial \alpha} = \frac{\Psi k^2\left[\begin{array}{l} 2[256(1+2\alpha)-128(3\alpha^2+\alpha-2)\gamma-16(16\alpha+9)(\alpha+1)^2\gamma^2+8(5\alpha \\ -3)(\alpha+1)^3\gamma^3+8(4\alpha+3)(\alpha+1)^4\gamma^4+2(\alpha+1)^6\gamma^5-(\alpha+1)^6\gamma^6] \\ -k(1-\gamma+\alpha\gamma)[64-20(1+\alpha)^2\gamma^2+(1+\alpha)^4\gamma^4]^2 \end{array}\right]}{(4\Phi-\Theta)^3},$$

where $\Psi = 16(1-c)^2\gamma(1+\gamma)(2-\gamma-\alpha\gamma)(2+\gamma+\alpha\gamma)[16-(1+\alpha)^2\gamma^2]$. Combining with our assumptions for positive outcomes, solving $\partial CS^S/\partial\alpha > 0$ for $k$, we have $\underline{k}^S < k < k_5$, where

$$k_5 = \frac{2\left[\begin{array}{l} 256(1+2\alpha) - 128(3\alpha^2+\alpha-2)\gamma - 16(16\alpha+9)(\alpha+1)^2\gamma^2 + 8(5\alpha \\ -3)(\alpha+1)^3\gamma^3 + 8(4\alpha+3)(\alpha+1)^4\gamma^4 + 2(\alpha+1)^6\gamma^5 - (\alpha+1)^6\gamma^6 \end{array}\right]}{(1-\gamma-\alpha\gamma)[64-20(\alpha+1)^2\gamma^2+(\alpha+1)^4\gamma^4]^2}$$

Thus, we have Propostition 4.□

## A.6 Proof of Proposition 5.

Differentiating $TS^S$ with respect to $\alpha$ yields the following result.

$$\frac{\partial TS^S}{\partial \alpha} = \frac{4(1-c)^2k\left[\begin{array}{l} 8\gamma(4-6\gamma-\gamma^2-4\alpha^3\gamma^2-3\alpha^2\gamma(2+3\gamma)+2\alpha(4-6\gamma-3\gamma^2))(4\Phi \\ -\Theta)\Phi - 32\gamma(4-6\gamma-\gamma^2-4\alpha^3\gamma^2-3\alpha^2\gamma(2+3\gamma)+2\alpha(4-6\gamma- \\ 3\gamma^2))\Phi^2 + k[8\gamma(7+(1+2\alpha)\gamma-(1+\alpha)^2\gamma^2)(64-20(1+\alpha)^2\gamma^2+ \\ (1+\alpha)^4\gamma^4)^2(4-6\gamma-\gamma^2-4\alpha^3\gamma^2-3\alpha^2\gamma(2+3\gamma)+2\alpha(4-6\gamma- \\ 3\gamma^2)) - 2(7+(1+2\alpha)\gamma-(1+\alpha)^2\gamma^2)(40(1+\alpha)\gamma^2-4(1+\alpha)^3\gamma^4) \\ (64-20(1+\alpha)^2\gamma^2+(1+\alpha)^4\gamma^4)(4\Phi-\Theta)+2\gamma(1-\gamma-\alpha\gamma)(64 \\ -20(1+\alpha)^2\gamma^2+(1+\alpha)^4\gamma^4)^2(4\Phi-\Theta)+16\gamma(64-224(1+\alpha)\gamma- \\ 60(1+\alpha)^2\gamma^2+56(1+\alpha)^3\gamma^3+5(1+\alpha)^4\gamma^4-3(1+\alpha)^5\gamma^5)\Phi^2] - k^2[4 \\ \gamma(7+(1+2\alpha)\gamma-(1+\alpha)^2\gamma^2)(64-20(1+\alpha)^2\gamma^2+(1+\alpha)^4\gamma^4)^2(64 \\ -224(1+\alpha)\gamma-60(1+\alpha)^2\gamma^2+56(1+\alpha)^3\gamma^3+5(1+\alpha)^4\gamma^4-3(1+ \\ \alpha)^5\gamma^5)] \end{array}\right]}{(4\Phi-\Theta)^3}.$$

Combining with our assumptions for positive outcomes, solving $\partial TS^S/\partial\alpha > 0$ for $k$, we have $\underline{k}^S < k < k_6$, where

$$k_6 = \frac{4\left[\begin{array}{l} 2048(3+10\alpha)+512[27-\alpha(7+34\alpha+4\alpha^2)]\gamma-128(1+\alpha)[71+\alpha(185+68\alpha \\ -22\alpha^2)]\gamma^2-32(1+\alpha)^2[231-\alpha(57+298\alpha+6\alpha^2)]\gamma^3+16(1+\alpha)^3[232+\alpha(497 \\ +303\alpha-48\alpha^2)]\gamma^4+8(1+\alpha)^4[151+\alpha(14-151\alpha-20\alpha^2)]\gamma^5-4(1+\alpha)^5[153+ \\ \alpha(278+147\alpha+2\alpha^2)]\gamma^6-2(1+\alpha)^7(1+\alpha)(23-8\alpha)\gamma^7+(1+\alpha)^7[43+\alpha(53+ \\ 20\alpha+12\alpha^2)]\gamma^8+(1+\alpha)^8[3+2\alpha(4+3\alpha+\alpha^2)]\gamma^9-(1+\alpha)^{10}(1-\alpha)\gamma^{10} \end{array}\right]}{(4-\gamma-\alpha\gamma)^2(2-\gamma-\alpha\gamma)^3(1-\gamma-\alpha\gamma)(2+\gamma+\alpha\gamma)^2(4+\gamma+\alpha\gamma)^3[6+2\alpha\gamma-(1+\alpha)^2\gamma^2]}$$

Thus, we have Proposition 5.$\square$