Nonlinear Effects of Public Capital in Japan: A Panel Threshold Regression Approach

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Abstract

This study examines the nonlinear effects of public capital in Japan by using a panel threshold regression method. First, our empirical results confirm the threshold effects of public capital productivity in Japan. Second, we demonstrate that rural regions gradually become low productivity regions over time. These results imply that the consideration of threshold effects is essential for understanding nonlinearities in the level of public capital and its economic effects in Japan.

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1. Introduction

When the level of public capital is low, it substantially contributes to production. However, as public capital accumulates, its marginal productivity becomes lower. This is true of the regional allocation of public investment within one country. If the allocation of public investment is lopsided in a specific area, we may observe such non-linear effects of public capital there.

To examine this issue, we focus on Japan. As shown in Figure 1, a negative correlation exists between per capita regional output and per capita public capital stock, indicating that the government has heavily allocated public investment to rural regions.[1] In this case, the nonlinear effects of public capital on productivity may be conspicuous in rural regions.

We use the fixed-effects panel threshold model developed by Hansen (1999) because it enables us to examine whether the impact on an objective variable differs in a panel data setting when a threshold variable exceeds a certain value. Our empirical results show that the impact of public capital on regional output discontinuously varies across multiple regimes. In particular, rural regions have gradually become low productivity regions over time since the 1990s.

Since the seminal paper by Mera (1973), a multitude of studies have examined the productivity effects of public capital. However, to the best of our knowledge, the study of Colletaz and Hurlin (2006) is the only one that applies the threshold regression approach to investigate the productivity effects of public capital.[2] Although Shioji (1999) examined the nonlinear effects of

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[1] Please also see Miyazaki (2018) for this point.

[2] Although Marcos and Vale (2022) examined the nonlinear relationship between public and private investment, they did not estimate the productivity effects of public capital.

public capital using regional data in Japan, the author did not use an econometric technique that considers threshold effects.

The remainder of this paper is organized as follows. Section 2 discusses the empirical framework of the study. Section 3 presents the empirical results. Finally, section 4 concludes the paper.

2. Empirical framework

2.1 Basic fixed-effects model

We consider a simple production function with public capital as follows:

$$Y = f(L, K, G),$$

where Y is the regional output, L is the labor input (i.e., number of employed people × working hours, hereafter referred to as labor), K is the private capital input (adjusted by the capital utilization ratio, hereafter referred to as private capital), and G denotes public capital.

By taking the logarithm of the production function, we write the basic fixed-effects estimation equation as follows:

$$\log Y_{it} = \mu + \alpha \log L_{it} + \beta \log K_{it} + \gamma \log G_{it} + u_i + \lambda_t + e_{it}, \quad (1)$$

where $\log Y_{it}$ is the logarithm of regional output in prefecture *i* in period *t* (year), $\log L_{it}$ is the logarithm of labor, $\log K_{it}$ is the logarithm of private capital, and $\log G_{it}$ is the logarithm of public capital. u_i is the individual effect (prefectural dummy variables), λ_t is the time effect (year dummy variables), and e_{it} is the disturbance.

For the estimation, we rewrite Equation (1) as a per capita term as follows:

$$\log y_{it} = \mu + \beta \log k_{it} + \gamma \log g_{it} + u_i + \lambda_t + e_{it}, \tag{2}$$

where y_{it} is the per capita regional output, k_{it} is the per capita private capital input, and g_{it} is the per capita public capital.

2.2 Fixed-effects panel threshold model

The specification of the fixed-effects panel threshold model is as follows:

$$y_{it} = \mu + X'_{it}\beta_1 I(q_{it} \leq \gamma) + X'_{it}\beta_2 I(q_{it} > \gamma) + u_i + e_{it}, \tag{3}$$

where X_{it} is a $K \times 1$ vector of explanatory variables and β_1 and β_2 are the $K \times 1$ vectors of the coefficients. In addition, q_{it} is the threshold variable, γ is the threshold to be estimated, u_i is the individual effect, and e_{it} is the disturbance.

The threshold value γ is then selected as the value that minimizes the residual sum of squares (RSS). The following equation represents the minimization process:

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmin}}\, s_1(\gamma), \tag{4}$$

where $s_1(\gamma)$ is the RSS of the single-threshold model under some γ.

To test whether the threshold effect is statistically significant, we follow Hansen (1999) and perform the likelihood ratio (LR) test. The null hypothesis in this case is $\beta_1 = \beta_2$, indicating that no threshold effects exist and that all coefficients are identical across regimes. The alternative hypothesis is $\beta_{1k} \neq \beta_{2k}$, meaning that a threshold effect exists for at least one coefficient. The LR statistic used to test this null hypothesis is as follows:

$$LR(\gamma) = \frac{S_0 - S_1(\gamma)}{\hat{\sigma}^2}, \tag{5}$$

where S_0 is the RSS in the no-threshold case (the linear model). $\hat{\sigma}^2$ is the estimated variance of the residuals in the single-threshold model and is calculated by dividing RSS by the sample size. If this LR statistic is significant, then a statistically significant difference exists between S_0 and $S_1(\gamma)$; this difference suggests a threshold effect. As the distribution of γ is unknown, we use a bootstrap method to evaluate the significance of the test.[3]

In addition to the single-threshold model, we consider models with multiple thresholds (double- and triple-threshold models), which divide the sample into more regimes, each characterized by distinct effects. The estimation procedure remains consistent but proceeds sequentially by comparing adjacent threshold models; for example, it compares the linear and

[3] For this test, we apply the bootstrap method proposed by Hansen (1999). The procedure consists of the following steps. First, we estimate the model under the null hypothesis and obtain the residuals. Second, we generate new residuals by using the cluster resampling method. Third, based on the resampled residuals, we construct a new data-generating process to re-estimate the model. Fourth, we calculate the LR statistics under both the null and alternative hypotheses and repeat this process through bootstrap iterations. Finally, we check the statistical significance of the threshold effect based on the empirical distribution of the bootstrapped LR statistics.

single-threshold models, then the single- and double-threshold models, and so on. An LR test is conducted at each step to determine the appropriate number of thresholds.

To address endogeneity issues, we estimate models with one-period lags for each variable as explanatory variables, as in the work of Colletaz and Hurlin (2006). We use the per capita public capital at each time point as the threshold variable to address the nonlinearity of public capital stock. In other words, we assume that the larger the size of public capital, the smaller its marginal productivity. We test this assumption by using the bootstrap method with 10,000 replications. The results are summarized in Table 1.

Table 1 presents the results of the threshold estimation. Table 1 shows that significant threshold effects are verified in several models. Based on these results, we assume the following three equations for the model, for which significant threshold effects have been tested:

$$y_{it} = \mu + X'_{it}\beta_1^{(a)} I\left(q_{it} \leq \gamma_1^{(a)}\right) + X'_{it}\beta_2^{(a)} I\left(\gamma_1^{(a)} < q_{it} \leq \gamma_2^{(a)}\right)$$

$$+X'_{it}\beta_3^{(a)} I\left(q_{it} > \gamma_2^{(a)}\right) + u_i + e_{it}, \quad (6\text{-}1)$$

$$y_{it} = \mu + X'_{it-1}\beta_1^{(b)} I\left(q_{it-1} \leq \gamma_1^{(b)}\right) + X'_{it-1}\beta_2^{(b)} I\left(\gamma_1^{(b)} < q_{it-1} \leq \gamma_2^{(b)}\right)$$

$$+X'_{it-1}\beta_3^{(b)} I\left(q_{it-1} > \gamma_2^{(b)}\right) + u_i + e_{it}, \quad (6\text{-}2)$$

$$y_{it} = \mu + X'_{it-1}\beta_1^{(c)} I\left(q_{it-2} \leq \gamma_1^{(c)}\right) + X'_{it-1}\beta_2^{(c)} I\left(\gamma_1^{(c)} < q_{it-2} \leq \gamma_2^{(c)}\right)$$

$$+X_{it-1}'\beta_3^{(c)}I\left(q_{it-2} > \gamma_2^{(c)}\right) + u_i + e_{it}, \tag{6-3}$$

where y_{it} represents the logarithm of the per capita regional output; X_{it} is the vector of explanatory variables consisting of the logarithm (or its lag) of the per capita private and public capitals; and $\beta_1^{(j)}, \beta_2^{(j)}, \beta_3^{(j)}$ $(j = a, b, c)$ is the coefficient vector estimated in each model. The threshold variable q_{it} is the logarithm of the per capita public capital (or its lag), and $\gamma_1^{(j)}, \gamma_2^{(j)}$ are the threshold values estimated in each model. Equation (6-1) is equivalent to the case with $\log g_{it}$ being a threshold variable along with two regime variables, namely, $\log k_{it}$ and $\log g_{it}$, in Table 1. Similarly, Equation (6-2) is the case where $\log g_{it-1}$ is used as a threshold variable in addition to two regime variables ($\log k_{it-1}$ and $\log g_{it-1}$). Equation (6-3) is the case with $\log g_{it-2}$ being a threshold variable along with two regime variables ($\log k_{it-1}$ and $\log g_{it-1}$) shown in Table 1.

3. Empirical results

First, we estimate the threshold values using Equation (4).[4][5] The results are summarized in Table 3.

As indicated by the results shown in Table 3, Equation (6-1) defines observations with a logarithm of per capita public capital less than 2.15 as Category 1, between 2.15 and 2.704 as Category 2, and above 2.704 as Category 3. Equations (6-2) and (6-3) employ the same categorical partitioning, and the estimated coefficients for each category are also presented in Table 3.

Table 3 shows that the coefficient of per capita public capital is estimated to be positive and statistically significant for each category in all models. However, the coefficients decrease as the category approaches 1, 2, and 3. The robustness of the results is confirmed because the same results are obtained in the model with lagged variables. The results imply that as the size of per capita public capital increases, its productivity declines. These findings suggest that considering threshold effects is crucial for understanding the relationships between the level of public capital and its economic impact.

Observations can be classified according to category. For example, Equation (6-1) has 920, 811, and 149 observations in Categories 1, 2, and 3, respectively. Regarding the regional comparison in different periods, all 47 prefectures are classified as Category 1 in the 1980s.[6]

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[4] Appendix 1 presents the data used for the estimation.

[5] The coefficients of Equation (2) are estimated to be positive but statistically insignificant. This result suggests the necessity to examine the model while considering threshold effects. The results are reported in Appendix 2.

[6] For the sake of brevity, we do not report the results for the 1980s. The results are available from the authors upon request.

For the 1990s, 43 prefectures, excluding Hokkaido, Shimane, Kochi, and Okinawa, are classified as Category 1.[7] For the period after the 2000s, Category 1 includes urban areas with high population densities, such as Tokyo, Osaka, and Aichi. Meanwhile, Category 3 includes many stagnant rural areas, such as Shimane, Kochi, and Okinawa. The prefectures classified as Category 3 in this period are almost identical to those for the period between 2010 and 2019. As Miyazaki (2018) mentioned, the Japanese government heavily allocated public investment to stagnant rural regions in the 1990s. In this regard, our results suggest that the fact that the Japanese government prioritized rural areas over urban areas with respect to the regional allocation of public investment is the root cause of the low productivity in the rural regions after the 1990s.

4. Conclusion

This study examined the nonlinear effects of public capital. To do this, we focused on Japan. We revealed the threshold effects of public capital by using Japanese prefectural data. We also demonstrated that public capital exerted nonlinear effects among regions and that such effects became salient after the 1990s, when the Japanese government heavily allocated public investment to stagnant rural regions. Our results imply that threshold effects should be considered when estimating the regional effects of public capital in Japan.

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[7] Appendix 3 shows the detailed results for the 1990s, 2000s, and 2019.

Although we addressed the endogeneity of public capital by taking a one-period lagged value for both the regime and threshold variables, the use of threshold regression with instrumental variables should still be considered in future studies. Furthermore, future research activities should include calculating and comparing the marginal productivity of private and public capitals.

Appendix 1. Data and descriptive statistics

The data on regional output, labor (number of workers), private capital stock, and public capital stock come from the database on Prefectural Economic and Fiscal Policy provided by the Cabinet Office, Japan.[8] Following Kamps (2004), we use net capital stock for public and private capitals. The data on regional output and private and public capital stocks are expressed in real terms by the 2015 deflator.

The data on working hours (offices with more than 30 employees, including overtime working hours) are obtained from the Monthly Labor Survey Report by the Ministry of Health, Labor and Welfare. The capacity utilization index data are from the Ministry of Economy, Trade and Industry. Table A1 presents the sample statistics.

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[8] Please see the website: https://www5.cao.go.jp/keizai3/database.html.

We calculate the prefectural-level capacity utilization ratio following the work of Asako and Sakamoto (1993). First, we assume the output as $Y_t = \sum_i^{47} Y_{it}$ and estimate Equation (A1) using the ordinary least squares method.

$$logY_{it} = a_i + b_i logY_t + \varepsilon_t. \quad \text{(A1)}$$

Second, using $\hat{b}_i$, we calculate the prefectural-level capacity utilization ratio for every year based on Equation (A2).

$$CU_{it} = \hat{b}_i CU_t, \quad \text{(B2)}$$

where CU_t is the national-level capacity utilization index and CU_{it} is the calculated prefectural-level capacity utilization ratio.

Appendix 2. Estimation results for Equation (2)

Table A2 reports the estimation results for Equation (2). Table A1 shows that the coefficient of the logarithm of the per capita public capital is positive but statistically insignificant.

Appendix 3. List of prefectures in each category

Tables A3–A6 present the lists of prefectures categorized in Equation (6-1). As shown in Table A3, four prefectures belonged to Category 2 in the 1990s. However, many prefectures shifted to low productivity categories in the 1990s. Notably, Hokkaido, Shimane, and Kochi belonged to Category 3 in 2000. These three prefectures remained in the same category in 2010 and 2019. Tables A5 and A6 indicate that several countries moved to Category 3 in 2010 and 2019, respectively.

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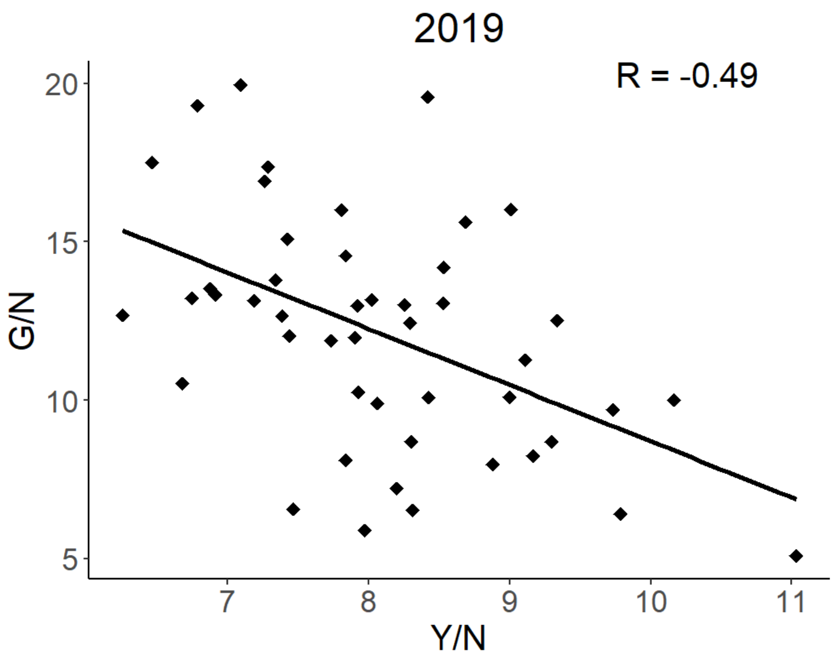
Figure 1. The correlation between per capita regional output (Y/N) and per capita public capital stock (G/N) in 2019



Note: Public capital stock is net public capital stock.

Table 1: Model selection results for each regime variable and threshold variable

Regime variable / Threshold variable	$\log k_{it}, \log g_{it}$	$\log k_{it-1}, \log g_{it-1}$
$\log g_{it}$	Double	-
$\log g_{it-1}$	Linear	Double
$\log g_{it-2}$	-	Double

Note: “Linear” and “double” indicate that the estimation is performed using a model with zero and two thresholds (three regimes), respectively.

Table 2: Threshold estimation results

Variables	Threshold	Lower	Upper
$\boldsymbol{\gamma_1^{(a)}}$	2.150	2.142	2.153
$\boldsymbol{\gamma_2^{(a)}}$	2.704	2.695	2.712
$\boldsymbol{\gamma_1^{(b)}}$	2.152	2.132	2.157
$\boldsymbol{\gamma_2^{(b)}}$	2.704	2.695	2.712
$\boldsymbol{\gamma_1^{(c)}}$	2.151	2.130	2.155
$\boldsymbol{\gamma_2^{(c)}}$	2.704	2.694	2.709

Table 3: Estimation results for Equations (6-1), (6-2), and (6-3)

Variables	Equation (6-1)	Equation (6-2)	Equation (6-3)
Intercept	−0.585***	−0.177***	−0.109*
	(0.062)	(0.064)	(0.065)
Private capital (Category 1)	0.258***	0.193***	0.185***
	(0.012)	(0.013)	(0.013)
Private capital (Category 2)	0.310***	0.245***	0.235***
	(0.011)	(0.012)	(0.012)
Private capital (Category 3)	0.367***	0.308***	0.3***
	(0.014)	(0.015)	(0.015)
Public capital (Category 1)	0.347***	0.371***	0.361***
	(0.012)	(0.013)	(0.013)
Public capital (Category 2)	0.185***	0.206***	0.209***
	(0.012)	(0.013)	(0.014)
Public capital (Category 3)	0.068**	0.073***	0.07**
	(0.025)	(0.027)	(0.027)
Within R-squared	0.923	0.907	0.897

Private and public capitals are expressed in per capita term. ***, **, and * indicate that the null hypothesis is rejected at the 1%, 5%, and 10% significance levels, respectively. The figures in parentheses indicate standard errors adjusted using the method of White (1980). For the sake of brevity, the estimation results for individual effects (prefecture dummy variables) are omitted.

Table A1: Summary statistics

Variable	NOB	Mean	Std. Dev.	Min	Max
Regional output	1880	10261414	14436654	1210828	114483109
Number of workers	1880	1374694	1545191	288464	10321365
Net private capital stock	1880	12864597	15529470	922887	105770235
Net public capital stock	1880	10766417	9008064	1679153	52577383
Working hours	1880	161.8	11.83	96.6	186

Table A2: Estimation results for Equation (2)

Variables	
Intercept	−0.896 ***
	(0.321)
Private capital	0.391 ***
	(0.016)
Public capital	0.096
	(0.074)
Adjusted R-squared	0.967

Private and public capitals are expressed in per capita term. *** indicates that the null hypothesis is rejected at the 1% significance level. For brevity, the estimation results for the individual and year effects are omitted.

Table A3: List of prefectures categorized in Equation (6-1) for the 1990s

Category 1			Category 2
Aomori	Toyama	Wakayama	Hokkaido
Iwate	Ishikawa	Tottori	Shimane
Miyagi	Fukui	Okayama	Kochi
Akita	Yamanashi	Hiroshima	Okinawa
Yamagata	Nagano	Yamaguchi	
Fukushima	Gifu	Tokushima	
Ibaraki	Shizuoka	Kagawa	
Tochigi	Aichi	Ehime	

Gunma	Mie	Fukuoka
Saitama	Shiga	Saga
Chiba	Kyoto	Nagasaki
Tokyo	Osaka	Kumamoto
Kanagawa	Hyogo	Oita
Niigata	Nara	Miyazaki
		Kagoshima

Table A4: List of prefectures categorized in Equation (6-1) for the 2000s

Category 1	Category 2		Category 3
Tochigi	Aomori	Hyogo	Hokkaido
Gunma	Iwate	Nara	Shimane
Saitama	Miyagi	Wakayama	Kochi
Chiba	Akita	Tottori	
Tokyo	Yamagata	Okayama	
Kanagawa	Fukushima	Hiroshima	
Shizuoka	Ibaraki	Yamaguchi	
Aichi	Niigata	Tokushima	

Kyoto	Toyama	Kagawa
Osaka	Ishikawa	Ehime
Fukuoka	Fukui	Saga
	Yamanashi	Nagasaki
	Nagano	Kumamoto
	Gifu	Oita
	Mie	Miyazaki
	Shiga	Kagoshima
		Okinawa

Table A5: List of prefectures categorized in Equation (6-1) in 2010

Category 1	Category 2		Category 3
Tochigi	Aomori	Hyogo	Hokkaido
Saitama	Iwate	Nara	Akita
Chiba	Miyagi	Wakayama	Niigata
Tokyo	Yamagata	Okayama	Yamanashi
Kanagawa	Fukushima	Hiroshima	Tottori
Shizuoka	Ibaraki	Yamaguchi	Shimane
Aichi	Gunma	Kagawa	Tokushima
Osaka	Toyama	Ehime	Kochi

Fukuoka	Ishikawa	Saga	
	Fukui	Nagasaki	
	Nagano	Kumamoto	
	Gifu	Oita	
	Mie	Miyazaki	
	Shiga	Kagoshima	
	Kyoto	Okinawa	

Table A6: List of prefectures categorized in Equation (6-1) in 2019

Category 1	Category 2		Category 3
Tochigi	Aomori	Nara	Hokkaido
Saitama	Miyagi	Wakayama	Iwate
Chiba	Fukushima	Okayama	Akita
Tokyo	Ibaraki	Hiroshima	Yamagata
Kanagawa	Gunma	Yamaguchi	Niigata
Shizuoka	Toyama	Kagawa	Yamanashi
Aichi	Ishikawa	Ehime	Tottori
Osaka	Fukui	Saga	Shimane

Fukuoka	Nagano	Nagasaki	Tokushima
	Gifu	Kumamoto	Kochi
	Mie	Oita	
	Shiga	Miyazaki	
	Kyoto	Kagoshima	
	Hyogo	Okinawa	