

Profit raising entry with upstream advertising

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Abstract

We consider a vertical market structure involving an upstream firm that invests in advertising and n downstream firms producing differentiated products. We investigate whether the entry of new firms in the downstream market can enhance the profits of incumbents. Our findings reveal that entry may increase incumbent firms' profits when the degree of product substitutability is low or the number of downstream firms is small. This result challenges the conventional view that entry is inherently detrimental to incumbents. Additionally, we demonstrate that our primary conclusions hold under persuasive upstream advertising and in Bertrand competition settings.

JEL codes: D43, L11, L13.

Keywords: entry, advertising, vertical relationship.

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1 Introduction

It is commonly believed that the entry of new firms reduces the profits of incumbent firms by increasing competition, thereby incentivizing incumbents to deter entry. However, we challenge this conventional wisdom by introducing the concept of upstream advertising.

Specifically, we consider a four-stage game played by an upstream firm and n downstream firms offering differentiated products.¹ In the first stage, the upstream firm chooses the level of advertising, which affects the size of downstream firms' markets. In the second stage, the upstream firm sets the input price to the downstream firms. In the third stage, the downstream firms compete in quantity.

We find that, in the case of informative advertising, entry may actually increase the profits of incumbent downstream firms if the degree of product substitutability or the number of downstream firms is low. This result reverses the conventional view that entry is harmful to incumbents and can be explained as follows: An upstream firm gains more from a more competitive downstream market, thus incentivizing it to invest more in advertising. Entry intensifies competition, which in turn raises the upstream firm's advertising level, benefiting the downstream firms. However, as the number of downstream firms increases, the market expansion effect of entry diminishes. Therefore, when the number of downstream firms is small, this beneficial effect is stronger. Additionally, when the degree of product substitutability is low, the negative impact of increased competition by entry is less significant. Consequently, with a low degree of product substitutability or a small number of downstream firms, the positive effect of entry through increased upstream advertising can outweigh the negative effect of increased competition, leading to higher profits for incumbents.

Additionally, we find that in the case of persuasive advertising, entry may increase the profits of incumbent firms when the advertising investment is sufficiently efficient or

¹The literature on advertising (Gross and Shapiro, 1984; Soberman, 2004; Zhang et al., 2012; Zhang et al., 2020) allows firms to sell differentiated products.

the degree of product substitutability is sufficiently low. When advertising investment is efficient, entry encourages the upstream firm to invest more, which can benefit incumbents if products are sufficiently differentiated. We also find that our main results under Cournot competition with informative upstream advertising hold true in Bertrand competition as well.

Our study is related to previous research that has challenged the well-known harmful property of entry for the incumbents in various contexts (Fanti and Buccella, 2017a,b; Ishibashi and Matsushima, 2009; Mukherjee, 2019; Mukherjee and Zhao, 2009; Naylor, 2002; Naylor and Soegaard, 2022; Pal and Sarkar, 2001; Toshimitsu, 2023; Tyagi, 1999). These studies have demonstrated that entry may raise the profits of incumbent firms. However, none have considered models with upstream firm advertising. Our work is closely related to Hu and Mizuno (2021), who examined the second-mover advantage of downstream firms in Bertrand competition by introducing upstream advertising. They found that simultaneous pricing may occur in equilibrium when product substitutability is low. Additionally, Hu et al. (2024) analyze the endogenous choice between quantity or price competition for downstream firms, considering both informative and persuasive advertising by the upstream firm. In contrast to their work, which focuses on two downstream firms, our model examines the profit-raising entry problem with n downstream firms. Thus, the mechanism of our model is distinct, and our main results and insights are closely tied to the number of downstream firms, setting us apart from Hu and Mizuno (2021) and Hu et al. (2024).

The remainder of the paper is organized as follows: Section 2 describes the basic model. Section 3 analyzes the case of informative advertising. Section 4 discusses the case of persuasive advertising and Bertrand competition. Finally, Section 5 presents our conclusions.

2 Model

We consider a market with an upstream firm and n downstream firms. The upstream firm produces input and sells it to the downstream firms at input price w . The downstream firms transform one unit of the input into one unit of output at zero marginal production cost. The products sold by the downstream firms are differentiated. We denote the output and price of the downstream firm i ($= 1, \dots, n$) as q_i and p_i , respectively. Then the aggregate final output is given by $Q = \sum_{j=1}^n q_j$ and the profit of downstream firm i is $\pi_i \equiv (p_i - w)q_i$.

The upstream firm can engage in informative advertising. We denote the level of informative advertising by θ , which represents the fraction of consumers who receive the advertisement, i.e., advertising intensity. Consumers who view the advertisement can buy either product, while those who do not view it buy none. We assume that if θ consumers receive the advertisement, the upstream firm must incur an advertising cost of $k\theta^2$, where k is a positive and constant parameter and $\theta \in [0, 1]$.² The upstream firm produces the product at a constant marginal cost of c . Consequently, the profit of the upstream firm is given by $\Pi_M \equiv (w - c)Q - k\theta^2$.

There is a unit mass of consumers. Following the established literature (Zhang et al., 2020; Hu and Mizuno, 2022), the utility function of each consumer is

$$u(\mathbf{x}, m) \equiv a \sum_{j=1}^n x_j - \frac{1}{2} \left(\sum_{j=1}^n x_j^2 + 2b \sum_{j=1}^n \sum_{h=j+1}^n x_j x_h \right) + m,$$

where $\mathbf{x} = (x_1, \dots, x_n)$, x_i is consumption level for product of downstream firm i , m is the quantity of a numeraire good, $b \in (0, 1)$ is the measure of product substitutability, and a is a positive parameter. The consumer maximizes its utility subject to the budget constraint $\sum_{j=1}^n p_j x_j + m \leq I$, where I represents income. Differentiating the utility

²We assume a quadratic advertising cost. Although some previous studies have employed linear costs and concave benefits from advertising (Nakata, 2011; Shy, 1995), quadratic advertising cost is a standard assumption in the literature (e.g., Simbanegavi, 2009; Soberman, 2004; Zhang et al., 2012; Zhang et al., 2020; Hu and Mizuno, 2021).

function with respect to x_i yields the first-order condition for the optimal consumption of good i : $a - x_i - b \sum_{j \neq i} x_j - p_i = 0$. Summing up the first-order conditions, we have $na - \sum_{j=1}^n x_j - (n-1)b \sum_{j=1}^n x_j - \sum_{j=1}^n p_j = 0$. Solving these equations for x_i and $\sum_{j=1}^n x_j$, we obtain the following consumption levels for consumers.

$$x_i = \frac{a(1-b) - [1 + b(n-1)]p_i - b \sum_{j=1}^n p_j}{(1-b)[1 + b(n-1)]}, \quad \sum_{j=1}^n x_j = \frac{na - \sum_{j=1}^n p_j}{1 + b(n-1)}.$$

When θ consumers receive the advertisement, the demand for downstream firm i is $q_i = \theta x_i$ and the aggregate demand is $Q = \theta \sum_{j=1}^n x_j$:

$$q_i = \theta \cdot \frac{a(1-b) - [1 + b(n-1)]p_i - b \sum_{j=1}^n p_j}{(1-b)[1 + b(n-1)]}, \quad Q = \theta \cdot \frac{na - \sum_{j=1}^n p_j}{1 + b(n-1)}. \quad (1)$$

Solving these demands for p_i and $\sum_{j=1}^n p_j$, we obtain the inverse demand functions as follows.

$$p_i = a - \frac{q_i + b \sum_{j \neq i} q_j}{\theta}, \quad \sum_{j=1}^n p_j = na - \frac{[1 + b(n-1)]Q}{\theta}. \quad (2)$$

Under informative advertising, the potential consumers transfer to actual ones after observing the advertising. Hence, as the advertising intensity θ increases, the demand to the downstream firms increases, which is reflected on the slope $(-1/\theta)$ of the inverse demand function in (2).

To guarantee an interior solution, we assume $k > (a-c)^2 n / [16 + 8b(n-1)]$. This assumption is obtained from the condition that the equilibrium level of informative advertising is less than 1. Additionally, this assumption is a sufficient condition for concavity of the upstream firm's profit.

The timing of this game is as follows: In the first stage, the upstream firm chooses the level of informative advertising θ . In the second stage, the upstream firm decides the wholesale price w . In the third stage, the downstream firms compete in quantity. We solve this model using backward induction.

3 Analysis

In the third stage, the downstream firms compete in quantities. Substituting the inverse demand function in (2) into the profit of downstream firm i and solving the first-order conditions, i.e., $\partial\pi_i/\partial q_i = a - (b \sum_{j \neq i} q_j + 2q_i)/\theta - w$, we have the output of downstream firm i as follows:³

$$q_i(w, \theta) = \frac{(a - w)\theta}{2 + b(n - 1)}.$$

In the second stage, the upstream firm decides the input price w . Substituting the aggregate output into the profit of the upstream firm and solving the first-order condition $\partial\Pi_M(w, \theta)/\partial w = 0$ for w , we have

$$w = \frac{a + c}{2}.$$

Note that substituting the above outcomes in the second and third stages into the downstream profit, and setting $\theta = 1$ and $k = 0$, then we can easily obtain the profit of the downstream firm with the upstream firm in a vertical structure, but *without* upstream informative advertising:

$$\pi_i^N = \frac{(a - c)^2}{4[2 + b(n - 1)]^2},$$

where the superscript N represents the case without upstream advertising. We can easily find n only exists in the denominator of π_i^N and π_i^N always decreases with n , which is correspondence to the conventional wisdom that entry always reduces the profits of the incumbents, even in a vertical structure by considering an upstream firm.

Next, we examine whether this well-known result still holds with the upstream advertising. In the first stage, the upstream firm decides the informative advertising level θ . By substituting the outcomes in the second and third stages into the profit of the upstream firm and solving the first-order condition for θ , we get equilibrium advertising

³Note the expressions that $\sum_{i=1}^n q_i = q_i + \sum_{j \neq i} q_j$ and $\sum_{i=1}^n p_i = p_i + \sum_{j \neq i} p_j$ in the process of our calculations.

level and then the equilibrium profit of the downstream firm i

$$\theta^* = \frac{(a-c)^2 n}{8k[2+b(n-1)]}, \quad \pi_i^* = \frac{(a-c)^4 n}{32k[2+b(n-1)]^3}.$$

First, we analyze the advertising level θ^* . Differentiating θ^* with respect to n for the first and second derivatives, with $0 < b < 1$, we obtain

$$\frac{\partial \theta^*}{\partial n} = \frac{(a-c)^2(2-b)}{8k[2+b(n-1)]^2} > 0, \quad \frac{\partial^2 \theta^*}{\partial n^2} = \frac{b(b-2)(a-c)^2}{4k[2+b(n-1)]^3} < 0.$$

Hence, we obtain the following result:

Lemma 1 *(i) Entry in the downstream market increases the level of the upstream informative advertising; (ii) However, this upstream informative advertising level raises less and less as the number of the downstream firms increases.*

The intuition is as follows. There will be more potential consumers transferring to actual ones after observing the advertising in a more competitive downstream market, i.e., more output and demand. For the upstream firm, an advertising gains more in a more competitive market, thus it intends to invest more in advertising in a more competitive market. Entry intensifies the competition and thus raises the upstream advertising level: $\partial \theta^* / \partial n > 0$. However, this market expansion effect of entry diminishes as the number of the downstream firms increases. This is because each additional entrant contributes less to market output expansion when competition is already high. As a result, the marginal effect of entry on advertising diminishes as well: $\partial^2 \theta^* / \partial n^2 < 0$.

Next, we examine how entry affects the incumbent downstream firms. Differentiating π_i^* with respect to n , we obtain

$$\frac{\partial \pi_i^*}{\partial n} = \frac{(a-c)^4[2-b(1+2n)]}{32k[2+b(n-1)]^4}.$$

Solving $\partial \pi_i^* / \partial n > 0$ for n , we obtain the following proposition.

Proposition 1 *With the upstream informative advertising, entry in the downstream market increases profits of the incumbent downstream firm if $b < 2/(1+2n)$.*

This result can be explained as follows. There are two opposing effects of entry on the profit of the incumbent downstream firms. The first one is the negative effect from entry, which make the market more competitive and thus harms the incumbents. When the product substitutability b is smaller, this negative effect by entry becomes less important. The second one is the beneficial effect from entry, which brings more upstream advertising as mentioned in the part (i) of Lemma 1. More upstream advertising in turn converts more potential consumers to actual ones for the downstream firms and benefits downstream profits. When the number of downstream firms n is smaller, this beneficial effect becomes stronger as mentioned in the part (ii) of Lemma 1. Therefore, when b and n are relatively small, the second beneficial effect of entry from more upstream advertising may dominate the first negative effect for more competition by entry, leading our result that entry may increase the the profits of incumbent firms. This result reverses the well-known harmful property of entry for incumbents.

4 Discussion

4.1 Persuasive advertising

While in the previous section we considered informative advertising, in this subsection, we discuss persuasive advertising, which increases the willingness to buy or marginal utility for consumers.

Model In the case of persuasive advertising, we consider a representative consumer since all consumers are aware of the products of downstream firms. We assume that the utility function of the representative consumer is

$$u(\mathbf{q}, m) = \sum_{i=1}^n q_i(a + \theta) - \frac{1}{2} \left(\sum_{i=1}^n q_i^2 + 2b \sum_{j=1}^n \sum_{h=j+1}^n q_j q_h \right) + m,$$

where θ denotes the level of persuasive advertising.⁴ From the first-order condition of the utility function, we can easily get the inverse demand function as $p_i = a + \theta - (q_i + b \sum_{j \neq i} q_j)$. The other settings are the same as those in the previous section. For concavity of the upstream firm's profit, we assume $k > n/[8 + 4b(n - 1)]$.

Calculating equilibrium In the third stage, the downstream firms choose quantities. Substituting the inverse demand function into the downstream firm's profit function and differentiating π_i with respect to q_i yields $\partial \pi_i / \partial q_i = a + \theta - 2q_i - b \sum_{j \neq i} q_j - w = 0$. Solving it for q_i , we obtain the output of downstream firm i :

$$q_i^{PA}(w, \theta) = \frac{(a + \theta - w)\theta}{2 + b(n - 1)},$$

where the superscript PA denotes the case with upstream persuasive advertising.

In the second stage, the upstream firm sets the input price w . Substituting the aggregate output into the profit of the upstream firm and solving the first-order conditions $\partial \Pi_M(w, \theta) / \partial q_i = 0$ for w , we have

$$w^{PA} = \frac{a + \theta + c}{2}.$$

Note that by substituting the above outcomes, then setting $\theta = 0$ and $k = 0$, we can have the profit of downstream firm i without upstream persuasive advertising: $\pi_i = (a + \theta - c)^2 / [4(2 + b(n - 1))^2]$, in which n only exists in the denominator and indicating the conventional wisdom that entry always reduces the profit of the incumbent firms in the downstream market.

In the first stage, the upstream firm decides the persuasive advertising level θ . By substituting the outcomes in the second and third stages into the profit of the upstream firm and solving the first-order condition for θ , we have the following equilibrium adver-

⁴The utility function under persuasive advertising is similar to Zhang et al. (2020), and Hu and Mizuno (2021).

tising level and the equilibrium profit of the downstream firm i

$$\theta^{PA} = \frac{(a-c)n}{4k[2+b(n-1)]-n}, \quad \pi_i^{PA} = \frac{4k^2(a-c)^2}{n+4k(b-bn-2)]^2}.$$

Differentiating θ^{PA} with respect to n , we obtain

$$\frac{\partial \theta^{PA}}{\partial n} = \frac{4k(2-b)(a-c)}{[n+4k(b-bn-2)]^2} > 0.$$

Hence, we obtain the following result:

Lemma 2 *Entry in the downstream market increases the level of the upstream persuasive advertising.*

The intuition is parallel to Lemma 1.

Next, differentiating π_i^{PA} with respect to n , we obtain

$$\frac{\partial \pi_i^{PA}}{\partial n} = \frac{8k^2(a-c)^2(4bk-1)}{[n+4k(b-bn-2)]^3}.$$

Note that from the second-order condition for the upstream firm, the denominator is positive. Solving $\partial \pi_i^{PA} / \partial n > 0$ for n , we obtain the following proposition.

Proposition 2 *With the upstream persuasive advertising, entry in the downstream market increases profits of the incumbent downstream firm if $b < 1/(4k)$.*

This result is intuitive. When k is small, the advertising investment is efficient, which strengthens the advertising effect by entry as mentioned in Lemma 2. Meanwhile, if the degree of product substitutability b is sufficiently low, similar to Proposition 1, competition effect becomes less important. Therefore, with small b and k , the beneficial advertising effect may dominate the negative competition effect by entry. Hence, our main result in Proposition 1 is robust, i.e., entry may also benefit the incumbents with upstream persuasive advertising.

4.2 Bertrand competition

Now, in the third stage, the downstream firms compete in prices. Substituting the demand function, (1), into the profit of the downstream firm i and solving the first-order conditions, we obtain the price of the downstream firm i and then the aggregate output.⁵

$$p_i^B(w) = \frac{a(1-b) + w + b(n-2)w}{2 + b(n-3)}, \quad Q^B(w, \theta) = \frac{[1 + b(n-2)]n(a-w)\theta}{[2 + b(n-3)][1 + b(n-1)]},$$

where the superscript B refers to the case under Bertrand competition.

In the second stage, the upstream firm decides the input price. Substituting the aggregate output into the profit of the upstream firm and solving the first-order conditions for w , we have the outcome as follows.

$$w^B = \frac{a+c}{2}.$$

In the first stage, the upstream firm decides the advertising level. By substituting the outcomes in the second and third stages into the profit of the upstream firm and solving the first-order condition for θ , we get equilibrium advertising level θ and then the profit of downstream firm i

$$\theta^B = \frac{(a-c)^2[1 + b(n-2)]n}{8k[2 + b(n-3)][1 + b(n-1)]}, \quad \pi_i^B = \frac{(1-b)(a-c)^4[1 + b(n-2)]^2n}{32k[2 + b(n-3)]^3[1 + b(n-1)]^2}.$$

To guarantee an interior solution and all equilibrium outcomes are positive, we assume

$$k > \frac{(a-c)^2[1 + b(n-2)]n}{8[2 + b(3n-5) + b^2(3-4n+n^2)]}.$$

This assumption also satisfies the condition that the equilibrium level of informative advertising is less than 1 and is a sufficient condition for concavity of the upstream firm's profit.

Now, we examine whether our main result shown in Proposition 1 holds when the product is differentiated under Bertrand competition. Since $\partial\pi_i^B/\partial n$ is very cumbersome

⁵The outcomes derivations under product differentiation are similar to Häckner (2000) and Hu and Mizuno (2023).

and the sign is not manageable, we engage in a numerical analysis. We still focus on the effect of the number of downstream firms n and the degree of product substitutability b on the influence of entry. We fix $a = 1$ and $c = 0$, which allows us to simplify the analysis without any influences on the final results, permitting algebraic tractability. We provide an example when $k = 2$ and Figure 1 presents the feasible conditions for profit raising entry, i.e., entry benefits the incumbents. It is shown that profit raising entry tends to happen if the the number of the downstream firms is small (small n) and the degree of product substitutability is small (small b), which is parallel to Proposition 1. Combining the result shown in Figure 1, we find that the main result in Proposition 1 is robust even under Bertrand competition.⁶

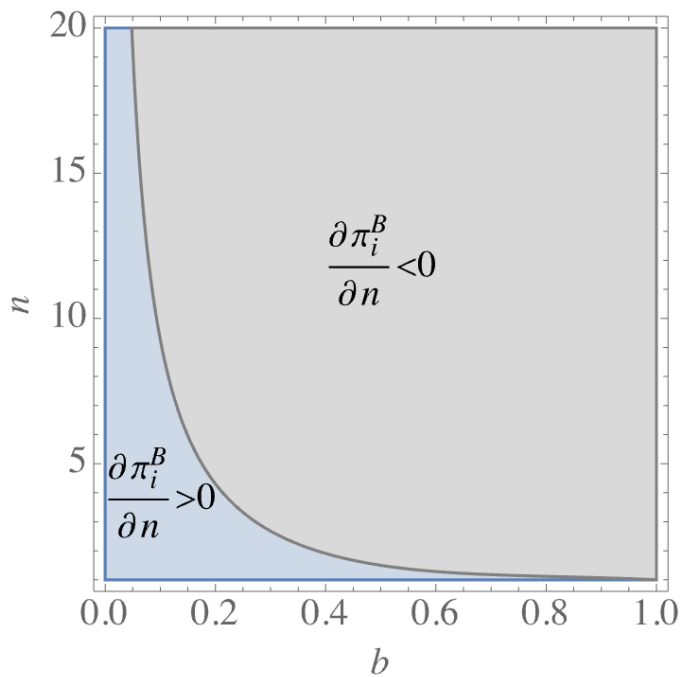


Figure 1: Condition for profit raising entry with $k = 2$ under Bertrand competition.

⁶We test the cases where $k \in \{1, 1.1, \dots, 3.9, 4.0\}$ and find the same result that profit raising entry tends to happen if the the number of the downstream firms and the degree of product substitutability are small . Thus, here we only present an example when $k = 2$.

5 Conclusions

We consider a vertical structure with an upstream firm engaging in advertising investment and n downstream firms producing differentiated products. We examine whether entry of new firms can raise the incumbent firms' profits. We find entry may increase the profits of incumbent downstream firms if the degree of product substitutability or the number of downstream firms is small, which reverses conventional wisdom of the harmful property of entry for the incumbents. We also find our main results are robust with persuasive upstream advertising and under Bertrand competition.

It is worthwhile to explore wholesale price discrimination and also advertising differentiation, in which the upstream firm provides varying levels of advertising support to downstream firms. We leave these topics for future research.

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