Endogenous choice of price or quantity contract with partial vertical ownership

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Abstract

We study a supply chain that consists of an upstream firm and two downstream firms. The downstream firms are assumed to have partial ownership in the upstream firm but lack control rights. We explore how these downstream firms decide whether to compete on quantity or price. Our findings show that the outcomes vary based on the degree of partial vertical ownership and the level of product substitutability, with possible equilibria including Cournot, Bertrand, and Cournot-Bertrand competition. This contradicts the conventional wisdom that firms typically prefer Cournot competition as their primary strategy.understanding that firms tend to engage in Cournot competition as their dominant strategy.

JEL codes: D43, L13, M21.

Keywords: endogenous competition mode, vertical partial ownership, vertical relationship.

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1 Introduction

We analyze the endogenous choice between quantity or price competition. Singh and Vives (1984) analyzed the endogenous choice using Cournot, Bertrand, and Cournot-Bertrand static duopoly models within the framework of Dixit (1979). They demonstrated that when products are substitutes, Cournot competition is the dominant strategy for firms. In this study, we challenge this conventional wisdom by introducing partial vertical ownership (PVO).

Partial Vertical Ownership (PVO), a business phenomenon where downstream firms acquire partial stakes in their upstream suppliers without obtaining control rights, has been widely documented in practice (Allen & Phillips, 2000; Greenlee & Raskovich, 2006; Hunold & Stahl, 2016; Fang et al., 2022). For example, JD.com purchased an 8.8% stake in the furniture manufacturer Shangpin Home Collection, and Walgreens acquired 26% of the drug wholesaler AmerisourceBergen (Fang et al., 2022). Given its prevalence, it is natural to incorporate PVO into the analysis of firms' strategic decisions regarding whether to compete on quantity or price.

Specifically, we consider a three-stage game played by a monopolistic upstream firm and two downstream firms holding PVO of the upstream firm. In the first (pre-play) stage, the downstream firms can choose the type of market contract (quantity or price). In the second stage, the upstream firm decides the wholesale price. In the third stage, the downstream firms compete according to the market contract chosen in the first stage.

We find that downstream firms' market contract choices depend on the degree of PVO and product substitutability. Specifically, when the degree of product substitutability is sufficiently high or the degree of PVO is relatively low, both downstream firms choose to offer a quantity contract (Cournot) in equilibrium, consistent with the result in Singh and Vives (1984). When the degree of product substitutability is not too high or the degree of PVO is relatively high, there may be two asymmetric subgame perfect Nash equilibria (SPNE) in the first stage, with one downstream firm offering a price contract and the other offering a quantity contract (Cournot-Bertrand). When the degree of product substitutability is relatively low or the degree of PVO is relatively high, both downstream firms may choose to offer a price contract (Bertrand), in contrast with the result in Singh and Vives (1984).

The underlying logic can be explained as follows. PVO induces downstream firms to consider the profitability of the upstream firm. Consequently, a more competitive market environment tends to enhance the profits of the upstream firm to a greater extent. It is widely recognized that the downstream market exhibits the highest level of competition under price competition, a relatively lower level of competition under asymmetric market conditions, and the lowest level of competition under quantity competition. Therefore, price competition generates the highest output, which in turn creates the greatest demand for the upstream firm. This not only boosts the upstream firm's profits but also has the most positive impact on downstream profits. In contrast, quantity competition has the least favorable effect on the upstream firm, while asymmetric market conditions lead to an intermediate outcome. The positive effect from the upstream firm's profit gains becomes more pronounced when downstream firms place a higher value on the upstream firm, that is, when the degree of PVO is higher. Meanwhile, increased competition can harm downstream firms' profits. However, this adverse effect diminishes when the degree of product substitutability is lower. Thus, if the degree of substitutability is small or the degree of PVO is high, the positive effect from the upstream firm's profit gains outweighs the negative impact of intensified downstream competition. This leads to the outcome that downstream firms may prefer price competition (BB) and asymmetric conditions (BC and CB). This finding contradicts the conventional belief that firms always prefer to compete in quantities.

Our study is related to previous research that has examined the well-known Cournot advantage property of endogenous choice between quantity or price competition in various contexts (Arya et al., 2008; Basak and Wang, 2016; Bhattacharjee and Pal, 2013; Chirco and Scrimitore, 2013; Correa-Lòpez, 2007; Fanti and Buccella, 2018; Matsumura and Ogawa, 2012; Pal, 2015; Scrimitore, 2013; Tremblay et al., 2013). These studies have demonstrated that the subgame perfect equilibrium could be Bertrand or Cournot-Bertrand. However, all of these studies considered models without PVO. Our work also contributes to the studies about PVO (Flath, 1989; Greenlee and Raskovich, 2006; Lestage, 2021; Fang et al. 2022; Sun et al.2023), which analyze the effects related to PVO in various contexts.

The remainder of the paper is organized as follows: In Section 2, we describe the basic model. In Section 3, we analyze downstream firms' market contract choices. In Section 4, we present our conclusion.

2 Model

We consider a vertically related market with a monopolistic upstream firm M and two downstream firms (firm i and firm j, $i, j = 1, 2$ and $i \neq j$). To produce on unit of the final product, the downstream firms purchase one unit of the input from the upstream firm at wholesale price w. We assume that the marginal costs of the upstream firm and the downstream firms are normalized to zero. The final products sold by the downstream firms are horizontally differentiated (e.g., each downstream firm sells a product with differentiated service). The price and output of downstream firm i are demand p_i and q_i, respectively. The profits of downstream firm i and the upstream firm are then given by $\pi_i \equiv (p_i - w)q_i$ and $\pi_M \equiv w(q_i + q_j)$, respectively.

In our model, we consider PVO between the upstream firm and downstream firms. For simplicity, we assume that the each shareholder of the downstream firm has a same share $s \in (0, 1/4)$ of the upstream firm in the form of passive investments with no control rights (e.g., nonvoting shares; Gilo et al., 2006). Hence, the shareholder of the upstream firm owns a share $1 - 2s$ of its property. Therefore, the managers of the downstream firms and the upstream firm M engage in maximize the total values $V_i = \pi_i + s\pi_M$ and $V_M = (1 - 2s)\pi_M$, respectively.

We assume that the utility function of a representative consumer as $u(q_i, q_j, m) \equiv a(q_i + q_j) - b(q_i^2 + 2\gamma q_i q_j + q_j^2)/2 + m$, where q_i and q_j are the consumption levels for products i and j, respectively; m is the quantity of a numeraire good; $\gamma \in (0, 1)$ is the measure of product substitutability; and a and b are positive parameters. This utility function yields the following demand function: $q_i = [a(1 - \gamma) - p_i + \gamma p_j]/[b(1 - \gamma^2)]$.

The timing of this game is as follows: In the first (pre-play) stage, the downstream firms can choose the type of market contract (quantity or price). In the second stage, the upstream firm decides the wholesale price w. In the third stage, the downstream firms compete according to the market contract chosen in the first stage. We solve this model using backward induction.

3 Analysis

3.1 Third stage: downstream competition

When both downstream firms choose the price contract in the first stage, in the third stage, they choose p_i to maximize $\pi_i = (p_i - w)[a(1-\gamma) - p_i + \gamma p_j]/[b(1-\gamma^2)]$. Solving the first-order conditions $\partial V_i/\partial p_i = 0$, we obtain the prices that the downstream firms choose, and then the quantities as follows:

$$p_i^B(w) = \frac{a(1-\gamma) + w(1+\gamma s - s)}{2-\gamma}, \quad q_i^B(w) = \frac{a + (1-\gamma)sw - w}{b(2-\gamma)(\gamma+1)}, \tag{1}$$

where the superscript B denotes Bertrand competition.

Next, we consider the case in which both downstream firms choose quantities. Solving $q_i(p_i, p_j)$ and $q_j(p_i, p_j)$ for p_i and p_j, we obtain the inverse demand function $p_i(q_i, q_j) = a - b(q_i + \gamma q_j)$. Substituting the inverse demand function into the profit functions of downstream firms and solving the first-order conditions $\partial V_i/\partial q_i = 0$, we obtain the quantities chosen by the downstream firms and then the prices as follows:

$$q_i^C(w) = \frac{a - (1-s)w}{b(\gamma+2)}, \quad p_i^C(w) = \frac{a + (\gamma+1)(1-s)w}{\gamma+2}, \tag{2}$$

where the superscript C denotes Cournot competition.

Finally, we examine the asymmetric case where one downstream firm chooses a quantity and the other chooses a price. Without loss of generality, we assume that downstream firm

i chooses quantity q_i, and downstream firm j chooses price p_j. Solving $q_i(p_i, p_j) = [a(1-\gamma) - p_i + \gamma p_j]/[b(1-\gamma^2)]$ for p_i and q_j, we obtain the following demand systems in strategic variables q_i and p_j: $p_i(q_i, p_j) = a(1-\gamma) + p_j\gamma - bq_i(1-\gamma^2)$, and $q_j(q_i, p_j) = (a - p_j)/b - q_i\gamma$, respectively. Using the above demand systems, solving the first-order conditions $\partial V_i/\partial q_i = 0$ and $\partial V_j/\partial p_j = 0$, we obtain the following results:

$$q_i(w) = q^{CB}(w) = \frac{a(2-\gamma) + w[\gamma + (2-3\gamma)s - 2]}{b\,(4-3\gamma^2)}, \tag{3}$$

$$p_i(w) = p^{CB}(w) = \frac{a\,(2-\gamma-\gamma^2) - w\,[2\gamma^2 - \gamma + (2-\gamma-3\gamma^2)\,s - 2]}{4-3\gamma^2}, \tag{4}$$

$$q_j(w) = q^{BC}(w) = \frac{a\,(2-\gamma-\gamma^2) - w\,[2-\gamma^2-\gamma-(2-\gamma)s+2]}{b\,(4-3\gamma^2)}, \tag{5}$$

$$p_j(w) = p^{BC}(w) = \frac{a\,(2-\gamma-2\gamma^2+\gamma^3) - w\,[\gamma^3+\gamma^2-\gamma-(\gamma^2+\gamma-2)\,s-2]}{4-3\gamma^2}, \tag{6}$$

where the superscript CB (BC) denotes that these results are obtained when one downstream firm competes in quantity (price) while the rival competes in price (quantity).

3.2 Second stage: wholesale price

In the second stage, the upstream firm chooses the wholesale price. In the Bertrand case, maximizing $V_M = (1-2s)(w-c)[q_i^B(w) + q_j^B(w)]$ for w yields

$$w^B = \frac{a}{2(\gamma-1)s+2}. \tag{7}$$

Similarly, in the Cournot case, maximizing $V_M = (1-2s)(w-c)[q_i^C(w) + q_j^C(w)]$ with respect to w leads to

$$w^C = \frac{a}{2-2s}. \tag{8}$$

In the asymmetric Cournot-Bertrand case, from maximizing $V_M = (1-2s)(w-c)[q^{CB}(w) + q^{BC}(w)]$, we obtain

$$w^{BC} (\equiv w^{CB}) = \frac{a\,(\gamma^2+2\gamma-4)}{2\,[\gamma^2+2\gamma-4(\gamma-1)s-4]}. \tag{9}$$

3.3 First stage: endogenous choices of price or quantity

Now, we can analyze the choice of quantity versus price in the first stage. Using the outcomes in the second and third stages, we obtain the downstream profit $V^B(w^B) = [p_i^B(w^B) - w^B]q_i^B(w^B) + sw^B[q_i^B(w^B) + q_j^B(w^B)]$ under Bertrand competition; while $V^{CB}(w^{CB}) = [p^{CB}(w^{CB}) - w^{CB}]q^{CB}(w^{CB}) + sw^{CB}[q_i^{CB}(w^{CB}) + q_j^{CB}(w^{CB})]$ and $V^{BC}(w^{BC}) = [p^{BC}(w^{BC}) - w^{BC}]q^{BC}(w^{BC}) + sw^{BC}[q_i^{BC}(w^{BC}) + q_j^{BC}w^{BC})]$ in the asymmetric Cournot-Bertrand case, noting that $w^{CB} = w^{BC}$; and then $V^C(w^C) = [p_i^C(w^C) - w^C]q_i^C(w^C) + sw^C[q_i^B(w^C) + q_j^B(w^C)]$ under Cournot competition. Using these total values of downstream firms V^B, V^{CB}, V^{BC} and V^C, the payoff matrix in the first stage is presented in Table 1.

		firm j Quantity	firm j Price
firm i	Quantity	V^C, V^C	V^{CB}, V^{BC}
	Price	V^{BC}, V^{CB}	V^B, V^B

Table 1: Payoff matrix in the endogenous competition choice

We compare the total values of downstream firms and show the results in Figure 1.[1] The downstream firms tend to choose to compete in prices in region A with $V^{CB} < V^B$ and $V^C < V^{BC}$. In region B with $V^B < V^{CB}$ and $V^C < V^{BC}$, there are two asymmetric equilibria, with one competing in quantity and the other one competing in price. Finally, for the case in region C with $V^B < V^{CB}$ and $V^{BC} < V^C$, both firms choose to compete in quantities. We can see that firms tend to compete in prices when the degree of substitutability γ is small or the degree of PVO s is high from Figure 1. All proofs are shown in the Appendix. Then, we have the following proposition.

Proposition 1 *With PVO, the following can be observed:*

(i) When the degree of product substitutability is relatively high (large γ) or the degree of PVO is low (small s) , SPNE will result in both downstream firms choosing to offer quantity

[1] We also obtain the similar results as Figure 1 by comparing the profits π^B, π^{CB}, π^{BC} and π^C, when the managers of the downstream firms focus to the short-term immediate financial gains from the order of moves, for example. However, in our model, the decision-making of the order of moves is considered in the first stage, hence we compare the long-term total values of the downstream firms.

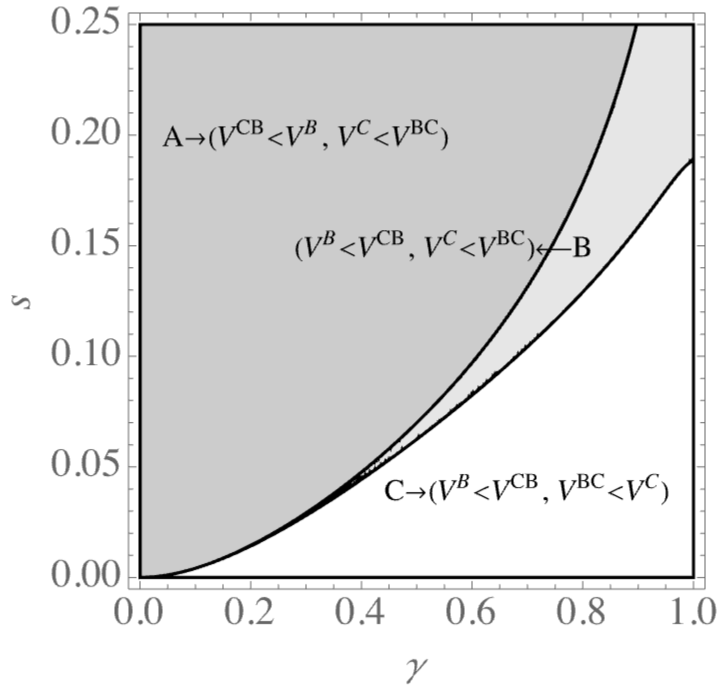

Figure 1: Total value rankings with region A: $V^{CB} < V^B$ and $V^C > V^{BC}$; region B: $V^B < V^{CB}$ and $V^C < V^{BC}$; and region C: $V^B < V^{CB}$ and $V^{CB} < V^C$.

contracts (CC).

(ii) When the degree of product substitutability is not too high or the degree of PVO is relatively high, there are two asymmetric SPNE at the first stage, with one downstream firm offering a price contract and the other offering a quantity contract (BC and CB).

(iii) When the degree of product substitutability is relatively low (small γ) or the degree of PVO is relatively high (large s), the only SPNE will result in both downstream firms choosing to offer price contracts in the first stage (BB).

The intuition is as follows. PVO makes downstream firms more concerned about the profitability of the upstream firm. As a result, a more competitive market environment has a greater positive impact on the upstream firm's profits. It is widely recognized that the downstream market is most competitive under price competition, less competitive under asymmetric conditions, and least competitive under quantity competition. Consequently, price competition generates the highest output and demand for the upstream firm, which in turn maximizes both upstream and downstream profits. In contrast, quantity competition benefits the upstream firm the least, while asymmetric competition leads to intermediate outcomes. This positive effect from the upstream firm's profit gains is amplified when downstream firms have a higher degree of PVO. Meanwhile, intense competition can harm downstream profits, but this negative impact diminishes when product substitutability is low. Therefore, when product substitutability is low or PVO is high, the benefits from increased upstream profits outweigh the drawbacks of intensified downstream competition. This leads downstream firms to prefer price competition (BB) and asymmetric market conditions (BC and CB), challenging the conventional belief that firms always favor quantity competition.

4 Conclusions

We considered a vertical structure with a upstream and two downstream firms considering PVO. We find that downstream firms' contract choices are influenced by the degree of PVO and product substitutability. When product substitutability is high or PVO is low, both firms opt for quantity contracts (Cournot), aligning with Singh and Vives (1984). However, with moderate substitutability and higher PVO, two asymmetric equilibria may emerge, with one firm choosing a price contract and the other a quantity contract (Cournot-Bertrand). When substitutability is low and PVO is high, both firms may prefer price contracts (Bertrand), contradicting Singh and Vives' findings. This pattern arises because PVO encourages downstream firms to consider upstream profitability. Price competition, which generates higher output and demand for the upstream firm, is most beneficial for both upstream and downstream profits, while quantity competition is least favorable. Asymmetric conditions yield intermediate outcomes. Higher PVO amplifies the positive impact of upstream profit gains, while lower substitutability mitigates the negative effects of intensified competition on downstream profits. Thus, when substitutability is low or PVO is high, downstream firms may favor price competition and asymmetric conditions, challenging the conventional belief that firms always prefer quantity competition.

Appendix: Proofs of the results

We prove Proposition 1 using a numerical method. We show the region A with $V^{CB} < V^{B}$ and $V^{C} < V^{BC}$, region B with $V^{B} < V^{CB}$ and $V^{C} < V^{BC}$ and region C with $V^{B} < V^{CB}$ and $V^{BC} < V^{C}$ in Figure 2, Figure 3 and Figure 4, respectively.

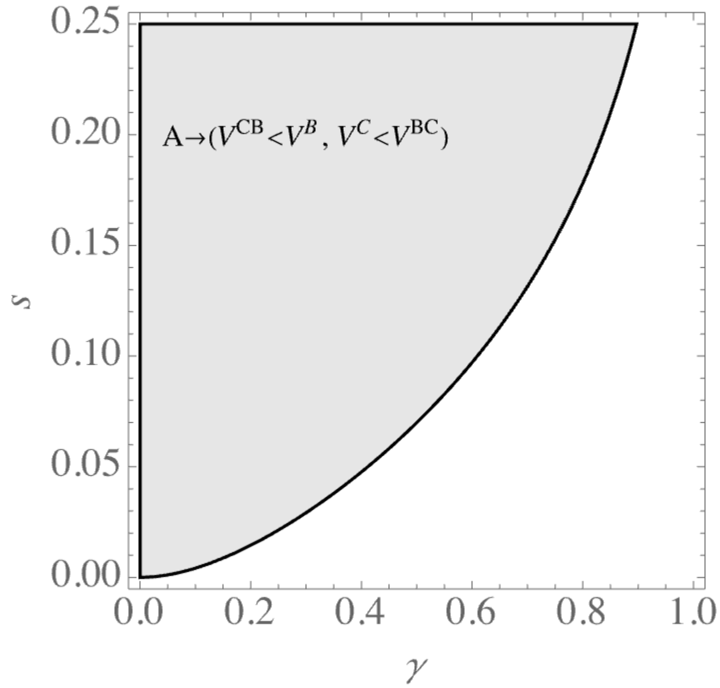

Figure 2: The region where $V^{CB} < V^{B}$ and $V^{C} < V^{BC}$.

Combining the results in Figure 2-4, we obtain the profit rankings shown in Figure 1 and complete the proof.□

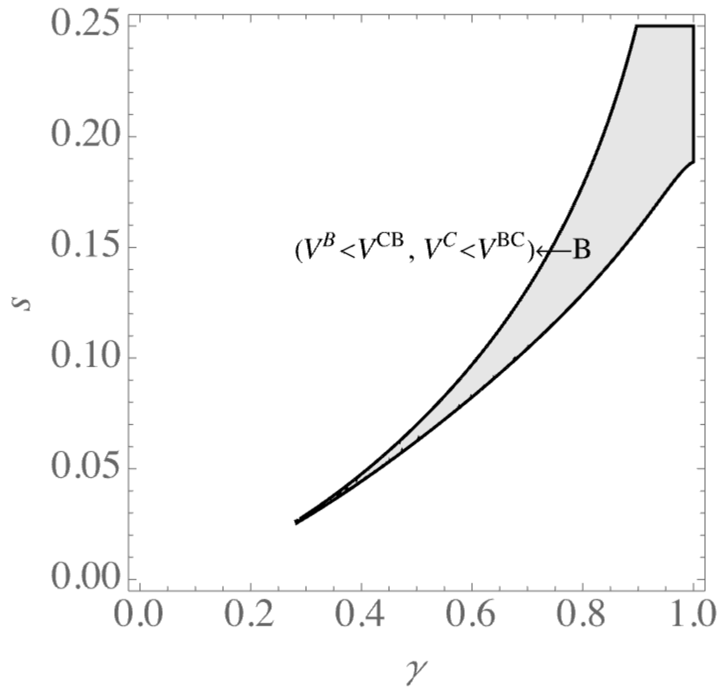

Figure 3: The region where $VB < V^{CB}$ and $V^C < V^{BC}$.

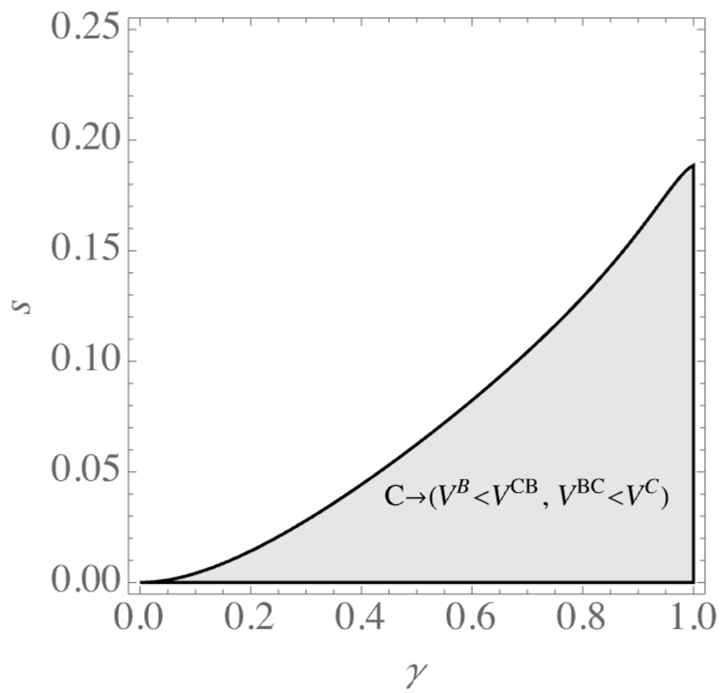

Figure 4: The region where $V^B < V^{CB}$ and $V^{BC} < V^C$.

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