（前期第Ⅱ期・国際共同学修コース（対面）・社会人）

令和７年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

経済理論

・第１問～第４問のすべてに日本語か英語で答えなさい。
・各問の解答は、それぞれ別の解答用紙に記入しなさい。

第１問　以下のゲームにおいて、プレイヤーXの戦略はT、SおよびB、プレイヤーYの戦略はL、MおよびRであり、表中の数字は（Xの利得，Yの利得）を表す。

		Y		
		L	M	R
X	T	(1, 1)	(0, 0)	(0, 0)
	S	(0, 0)	(2, 2)	(0, 0)
	B	(0, 0)	(0, 0)	(3, 3)

(1) 全ての純粋戦略ナッシュ均衡を求めなさい。(20 点)
(2) 全ての混合戦略ナッシュ均衡を求めなさい。(20 点)

第２問　同質財を３企業S、T、およびUが生産・販売する寡占市場を考える。最初に企業Sが生産量x_Sを決定し、すべての企業がこれを観察する。次に企業Tが生産量x_Tを決定し、すべての企業がこれを観察する。最後に企業Uが生産量x_Uを決定し、財価格Pが$P = 9 - x_S - x_T - x_U$として決定される。企業i（$i = S, T, U$）の費用関数は$C_i = x_i$である。

(1) 企業Uの利潤を、x_Sとx_Tとx_Uを使って表現しなさい。(10 点)
(2) 企業Uの最適反応関数を、x_Sとx_Tの関数として求めなさい。(10 点)
(3) 企業Tの最適反応関数を、x_Sの関数として求めなさい。(10 点)
(4) 均衡における企業S、T、およびUの生産量を求めなさい。(30 点)

第 3 問　開放経済において、自由な資本移動、固定相場制、独立した金融政策の全てを同時に実現することができるか否か議論しなさい。(50 点)

第 4 問　次の用語について説明しなさい。

(1) 信用創造と信用収縮（10 点）

(2) 時間非整合性（10 点）

(3) 付加価値と帰属価値（10 点）

(4) 小国開放経済（10 点）

(5) ラスパイレス価格指数とパーシェ価格指数（10 点）

（前期第Ⅱ期・国際共同学修コース（対面）・社会人）

令和７年度　神戸大学大学院経済学研究科
博 士 課 程 前 期 課 程 入 学 試 験 問 題

Economic Theory

・Answer all of the following questions either in English or in Japanese.
・Answer each question on a separate sheet.

1. Consider the following game. The strategies of player *X* are *T*, *S*, and *B*. The strategies of player *Y* are *L*, *M*, and *R*. The numbers in the matrix show (*X*'s payoff, *Y*'s payoff).

		Y		
		L	*M*	*R*
X	*T*	(1, 1)	(0, 0)	(0, 0)
	S	(0, 0)	(2, 2)	(0, 0)
	B	(0, 0)	(0, 0)	(3, 3)

(1) Calculate all pure strategy Nash equilibria. (20 points)
(2) Calculate all mixed strategy Nash equilibria. (20 points)

2. Consider an oligopoly market where a homogeneous good is produced and sold by three firms, identified by the letters *S*, *T*, and *U*. First, firm *S* chooses its output x_S, and every firm observes it. Next, firm *T* chooses its output x_T, and every firm observes it. Finally, firm *U* chooses its output x_U, and the price of this good, P, is determined as $P = 9 - x_S - x_T - x_U$. The cost function of firm i $(i = S, T, \text{and } U)$ is $C_i = x_i$.
(1) Describe the profit of firm *U* as a function of x_S, x_T, and x_U. (10 points)
(2) Calculate the best response function of firm *U* as a function of x_S and x_T. (10 points)
(3) Calculate the best response function of firm *T* as a function of x_S. (10 points)
(4) Calculate the equilibrium output of firms *S*, *T*, and *U*. (30 points)

3. Discuss whether or not it is possible for an open economy to achieve all of the following at the same time: free capital flows, a fixed exchange rate and independent monetary policy. (50 points)

4. Explain the following terminologies.

   (a) Credit creation and credit crunch (10 points)

   (b) Time inconsistency (10 points)

   (c) Value added and inputed value (10 points)

   (d) A small open economy (10 points)

   (e) Laspeyres price index and Paasche price index (10 points)

（前期第Ⅱ期・国際共同学修コース（対面）・社会人）

令和７年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

経　済　史

・第１問～第４問のすべてに、日本語か英語で答えなさい。
・各問の解答は、それぞれ別の解答用紙に記入しなさい。

第１問　15～18 世紀の日本における鉱山開発と鉱産資源の商業的利用について具体例を挙げて説明しなさい。(50 点)

第２問　近世の飢饉に対する幕府の危機管理政策について説明しなさい。(50 点)

第３問　幕末・明治以降の日本における技術導入の特徴について、具体的な事例を示しながら説明しなさい。さらに、日本の技術導入の経験は、発展途上国に対してどのようなインプリケーションを持つか論じなさい。(50 点)

第４問　グローバル経済史研究は、ここ 30 年の間に国際的に盛んになってきた。以下の二つの問題に答えなさい。

（１）「グローバル経済史」とは何か。従来の経済史と比較しながら説明しなさい。(25 点)

（２）日本経済史研究は、グローバル経済史研究の発展にどのような貢献ができるかを論じなさい。(25 点)

（前期第Ⅱ期・国際共同学修コース（対面）・社会人）

令和７年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

経　済　史

Economic History

・Answer all of the following four questions either in English or in Japanese.
・Answer each question on a separate sheet.

1. Explain the development of mines and the commercial use of mineral resources in Japan in the 15~18th centuries with specific examples.

2. Explain the Shogunate's crisis management policy regarding famine in the early modern period.

3. Explain what type of foreign technology was introduced into Japan since the end of Tokugawa period and beginning of the and Meiji period, by showing particular example(s). In addition, discuss what kind of implications developing countries could draw from Japan's experience with new foreign technology. (50 points)

4. The study of global economic history has flourished internationally in the past thirty years. Answer the below two questions.
   (1) What is 'global economic history'? Explain it by comparing with traditional economic history. (25 points)
   (2) Discuss how research on the economic history of Japan could contribute to the literature on global economic history. (25 points)

(前期第II期・国際共同学修コース(対面)・社会人)

令和7年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

統計学

- 第1問〜第3問のすべてに日本語か英語で答えなさい。
- 各問の解答は，それぞれ別の解答用紙に記入しなさい。
- 本研究科で貸与する電卓のみ使用を認めます。
- 必要に応じて，添付の統計分布表を利用しなさい。

第1問　離散型確率変数 X, Y について，X は 0 か 1，Y は 1 か 2 を実現値として取るとする。また，$\mathrm{P}(X=0)=\mathrm{P}(X=1)=0.5$, $\mathrm{P}(Y=1|X=0)=0.4$, $\mathrm{P}(Y=2|X=0)=a$, $\mathrm{P}(Y=1|X=1)=0.8$, $\mathrm{P}(Y=2|X=1)=b$ であるとする。以下の問いに答えなさい。

(1) a と b の値を求めなさい。(10点)
(2) Y の平均と分散を求めなさい。(10点)
(3) X と Y の共分散を求めなさい。(20点)
(4) $Z=X-Y$ とする時，Z の平均と分散を求めなさい。(20点)

第2問　連続型確率変数 X は確率密度関数

$$f(x)=\begin{cases}\frac{2x}{a^2} & 0\leqq x\leqq a \text{ の時}\\ 0 & \text{それ以外}\end{cases}$$

を持つ分布にしたがっているとする。ただし a は正の定数である。以下の問いに答えなさい。

(1) X の分布関数を求めなさい。(10点)
(2) X の中央値を求めなさい。(10点)
(3) X の平均，分散を求めなさい。(20点)
(4) この分布から得られた大きさ1の標本 X を用いて，帰無仮説 $\mathrm{H}_0: a=1$ を対立仮説 $\mathrm{H}_1: a=2$ に対して検定したい。$\{x|x>c\}$ をこの検定の棄却域とするとき，有意水準 α の検定に対する c の値を求めなさい。また，有意水準 α の検定の検定力を求めなさい。(20点)

第 3 問　以下のような生産関数を考える。

$$Y = AL^{\beta_1}K^{\beta_2} \qquad [1]$$

Y は生産水準, L は労働投入量, K は資本ストック水準を示している。また A, β_1, β_2 は未知のパラメータを表している。

線形回帰モデルによってパラメータを推定するため, [1] 式の両辺に対数を取り, [2] 式を推定する。

$$lY_i = \beta_0 + \beta_1 lL_i + \beta_2 lK_i + u_i \qquad [2]$$

ここで $lY_i = \log(Y_i)$, $lL_i = \log(L_i)$, $lK_i = \log(K_i)$, $\beta_0 = \log(A)$ を示している。また u_i は誤差項であり, 互いに独立に $N(0, \sigma^2)$ に従うとする。

(1) [2] 式においてパラメータ (β_1, β_2) は何を意味するのか, 計算プロセスを示しながら答えなさい。(10 点)

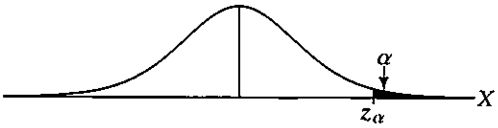
(2) 生産構造が規模に関して収穫一定（全ての生産要素を n 倍した時, 生産水準が n 倍になる状態）であるか否かを, [2] 式を推定した後, 検定を行うことによって確かめたい。具体的な検定の手続きを, 以下の語句を用いて詳しく説明しなさい。(30 点)

　　制約付き回帰モデル　無制約回帰モデル

(3) (2) の検定において, サンプル数が 16 のもとで, 制約付き回帰モデルの OLS 推定の基づく残差平方和が 0.002, 無制約回帰モデルの OLS 推定の基づく残差平方和が 0.0006 であったとする。この時 (2) の検定における検定統計量の値を計算しなさい（小数点 2 桁目を切り下げ)。(20 点)

(4) (3) で求めた検定統計量の値をもとに, 有意水準 1 %のもとで (2) で示した検定を行いなさい。(20 点)

正規分布表： $X \sim N(0,1)$

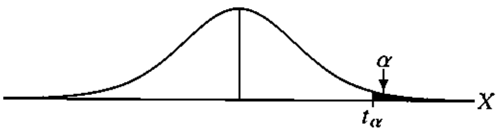


$$\alpha = P(X > z_\alpha) = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)dx$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4091	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2644	.2611	.2579	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

α	.10	.05	.025	.010	.005	.001	.0005	.0001	.00001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	4.2649

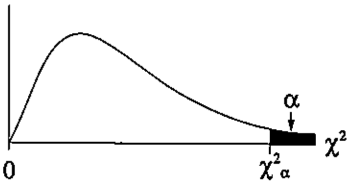
t 分布表： $X \sim t(k)$



$$\alpha = P(X > t_\alpha) = \int_{t_\alpha}^{\infty} f(x)\mathrm{d}x$$

k（自由度） \ α	.10	.05	.025	.010	.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8187
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
120	1.2886	1.6577	1.9799	2.3578	2.6174
∞	1.2816	1.6449	1.9600	2.3263	2.5758

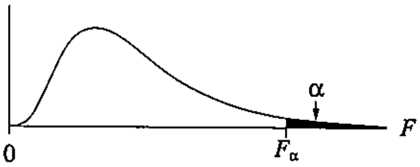
カイ２乗分布表： $\chi^2(m)$



$$\alpha = P(\chi^2 > \chi^2_\alpha) = \int_{\chi^2_\alpha}^{\infty} f(\chi^2)\,d\chi^2$$

α / m (自由度)	.995	.99	.975	.95	.90	.10	.05	.025	.010	.005
1	.0000393	.000157	.000982	.00393	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.06	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43	104.21
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17

F 分布表 (1% 点, $\alpha = 0.01$)： $F(m_1, m_2)$



$\alpha = P(F > F_\alpha) = \int_{F_\alpha}^{\infty} f(F)\,dF$

m_1=分子の自由度
m_2=分母の自由度

m_2 \ m_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	17	20	25	30	40
1	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6083	6106	6126	6143	6157	6181	6209	6240	6261	6287
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1	27.0	26.9	26.9	26.8	26.7	26.6	26.5	26.4
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.5	14.4	14.3	14.2	14.2	14.1	14.0	13.9	13.8	13.7
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89	9.82	9.77	9.72	9.64	9.55	9.45	9.38	9.29
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.66	7.60	7.56	7.48	7.40	7.30	7.23	7.14
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47	6.41	6.36	6.31	6.24	6.16	6.06	5.99	5.91
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67	5.61	5.56	5.52	5.44	5.36	5.26	5.20	5.12
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	5.05	5.01	4.96	4.89	4.81	4.71	4.65	4.57
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.65	4.60	4.56	4.49	4.41	4.31	4.25	4.17
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40	4.34	4.29	4.25	4.18	4.10	4.01	3.94	3.86
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16	4.10	4.05	4.01	3.94	3.86	3.76	3.70	3.62
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.91	3.86	3.82	3.75	3.66	3.57	3.51	3.43
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.75	3.70	3.66	3.59	3.51	3.41	3.35	3.27
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.61	3.56	3.52	3.45	3.37	3.28	3.21	3.13
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55	3.50	3.45	3.41	3.34	3.26	3.16	3.10	3.02
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46	3.40	3.35	3.31	3.24	3.16	3.07	3.00	2.92
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37	3.32	3.27	3.23	3.16	3.08	2.98	2.92	2.84
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.24	3.19	3.15	3.08	3.00	2.91	2.84	2.76
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.18	3.13	3.09	3.02	2.94	2.84	2.78	2.69
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.24	3.17	3.12	3.07	3.03	2.96	2.88	2.79	2.72	2.64
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.07	3.02	2.98	2.91	2.83	2.73	2.67	2.58
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	3.02	2.97	2.93	2.86	2.78	2.69	2.62	2.54
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.98	2.93	2.89	2.82	2.74	2.64	2.58	2.49
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	3.06	2.99	2.94	2.89	2.85	2.78	2.70	2.60	2.54	2.45
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	2.90	2.86	2.81	2.75	2.66	2.57	2.50	2.42
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.99	2.93	2.87	2.82	2.78	2.71	2.63	2.54	2.47	2.38
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	2.84	2.79	2.75	2.68	2.60	2.51	2.44	2.35
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93	2.87	2.81	2.77	2.73	2.66	2.57	2.48	2.41	2.33
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84	2.79	2.74	2.70	2.63	2.55	2.45	2.39	2.30
31	7.53	5.36	4.48	3.99	3.67	3.45	3.28	3.15	3.04	2.96	2.88	2.82	2.77	2.72	2.68	2.61	2.52	2.43	2.36	2.27
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02	2.93	2.86	2.80	2.74	2.70	2.65	2.58	2.50	2.41	2.34	2.25
33	7.47	5.31	4.44	3.95	3.63	3.41	3.24	3.11	3.00	2.91	2.84	2.78	2.72	2.68	2.63	2.56	2.48	2.39	2.32	2.23
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89	2.82	2.76	2.70	2.66	2.61	2.54	2.46	2.37	2.30	2.21
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.80	2.74	2.69	2.64	2.60	2.53	2.44	2.35	2.28	2.19
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	2.52	2.45	2.37	2.27	2.20	2.11
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.67	2.61	2.55	2.51	2.46	2.39	2.31	2.21	2.14	2.05
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.63	2.56	2.51	2.46	2.42	2.35	2.27	2.17	2.10	2.01
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.44	2.39	2.35	2.28	2.20	2.10	2.03	1.94
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.51	2.45	2.40	2.35	2.31	2.23	2.15	2.05	1.98	1.89
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.48	2.42	2.36	2.31	2.27	2.20	2.12	2.01	1.94	1.85
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.45	2.39	2.33	2.29	2.24	2.17	2.09	1.99	1.92	1.82
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.43	2.37	2.31	2.27	2.22	2.15	2.07	1.97	1.89	1.80
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.37	2.31	2.25	2.20	2.16	2.09	2.00	1.90	1.83	1.73
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27	2.22	2.17	2.13	2.06	1.97	1.87	1.79	1.69
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18	2.13	2.08	2.04	1.97	1.88	1.77	1.70	1.59

(前期第 II 期・国際共同学修コース (対面)・社会人)

令和 7 年度　神戸大学大学院経済学研究科

博士課程前期課程入学試験問題

Statistics

· Answer all of the following questions either in English or in Japanese.

· Answer each question on a separate sheet.

· Applicants are authorized to use a calculator lent by our Graduate School.

· Use the statistical tables if necessary.

1. Let X, Y be discrete random variables such that X takes a value of either 0 or 1, and Y takes a value of either 1 or 2. Suppose that the following conditions hold: $\mathrm{P}(X=0)=\mathrm{P}(X=1)=0.5$, $\mathrm{P}(Y=1|X=0)=0.4$, $\mathrm{P}(Y=2|X=0)=a$, $\mathrm{P}(Y=1|X=1)=0.8$ and $\mathrm{P}(Y=2|X=1)=b$. Answer the following questions.

(1) Find the values of a and b. (10 points)

(2) Find the mean and the variance of Y. (10 points)

(3) Find the covariance of X and Y. (20 points)

(4) Let $Z = X - Y$. Find the mean and the variance of Z. (20 points)

2. Suppose that a continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{2x}{a^2} & 0 \leqq x \leqq a, \\ 0 & \text{otherwise}, \end{cases}$$

where a is a positive constant. Answer the following questions.

(1) Find the distribution function of X. (10 points)

(2) Find the median of X. (10 points)

(3) Find the mean and the variance of X. (20 points)

(4) Consider testing the null hypothesis $\mathrm{H}_0 : a = 1$ against the alternative $\mathrm{H}_1 : a = 2$ using a sample X of size 1 from this distribution. Let $\{x|x > c\}$ be the rejection region of this test. Find the value of c for the test of the α significance level. Also, find the power of the test of the α significance level. (20 points)

3. Consider the following production function:

$$Y = AL^{\beta_1}K^{\beta_2}, \qquad [1]$$

where Y is the production level, L is the labor input, and K is the capital stock level. A, β_1 and β_2 represent unknown parameters.

To estimate the parameters by a linear regression model, we take the logarithm on both sides of equation [1] and estimate equation [2]:

$$lY_i = \beta_0 + \beta_1 lL_i + \beta_2 lK_i + u_i, \qquad [2]$$

where $lY_i = \log(Y_i), lL_i = \log(L_i), lK_i = \log(K_i), \beta_0 = \log(A)$. u_i is the error term and assume that u_i is independently distributed as $N(0, \sigma^2)$.

(1) Explain what the parameters (β_1, β_2) mean in equation [2], showing the calculation process. (10 points)

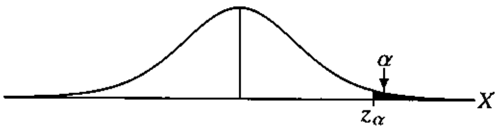
(2) After estimating equation [2], we will test whether the production structure exhibits constant returns to scale (i.e., the production level is n times the original production level when all factors of production change by a factor of n. Explain in detail the specific test procedure using all of the following terms (30 points):

constrained regression model, unconstrained regression model

(3) In the test of (2), suppose that the residual sum of squares based on OLS estimation for the constrained regression model is 0.002 and the residual sum of squares based on OLS estimation for the unconstrained regression model is 0.0006 under a sample size of 16. Calculate the value of the test statistic (round down to the second decimal place). (20 points)

(4) Based on the value of test statistic obtained in (3), perform the test indicated in (2) at the 1% significance level . (20 points)

Normal distribution: $X \sim N(0,1)$

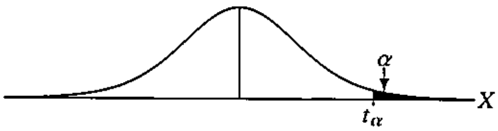


$$\alpha = P(X > z_\alpha) = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)dx$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4091	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2644	.2611	.2579	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

α	.10	.05	.025	.010	.005	.001	.0005	.0001	.00001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	4.2649

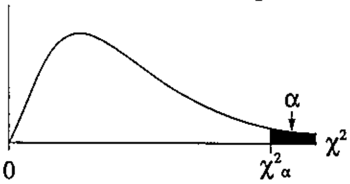
t distribution: $X \sim t(k)$



$$\alpha = P(X > t_\alpha) = \int_{t_\alpha}^{\infty} f(x)\,dx$$

k (Degrees of Freedom) \ α	.10	.05	.025	.010	.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8187
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
120	1.2886	1.6577	1.9799	2.3578	2.6174
∞	1.2816	1.6449	1.9600	2.3263	2.5758

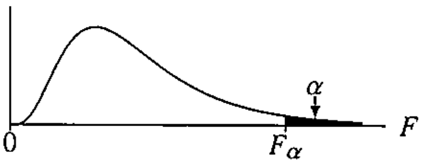
Chi-squared distribution: $\chi^2(m)$



$$\alpha = P(\chi^2 > \chi^2_\alpha) = \int_{\chi^2_\alpha}^{\infty} f(\chi^2)\,d\chi^2$$

α / m (Degrees of Freedom)	.995	.99	.975	.95	.90	.10	.05	.025	.010	.005
1	.0000393	.000157	.000982	.00393	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.06	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43	104.21
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17

F distribution (1% point, $\alpha = 0.01$) : $F(m_1, m_2)$



$$\alpha = P(F > F_\alpha) = \int_{F_\alpha}^{\infty} f(F)\, dF$$

m_1: Degrees of freedom for the numerator
m_2: Degrees of freedom for the denominator

m_2 \ m_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	17	20	25	30	40
1	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6083	6106	6126	6143	6157	6181	6209	6240	6261	6287
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	27.1	27.0	26.9	26.9	26.8	26.7	26.6	26.5	26.4
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.5	14.4	14.3	14.2	14.2	14.1	14.0	13.9	13.8	13.7
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.96	9.89	9.82	9.77	9.72	9.64	9.55	9.45	9.38	9.29
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.66	7.60	7.56	7.48	7.40	7.30	7.23	7.14
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.54	6.47	6.41	6.36	6.31	6.24	6.16	6.06	5.99	5.91
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.73	5.67	5.61	5.56	5.52	5.44	5.36	5.26	5.20	5.12
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.18	5.11	5.05	5.01	4.96	4.89	4.81	4.71	4.65	4.57
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.77	4.71	4.65	4.60	4.56	4.49	4.41	4.31	4.25	4.17
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.46	4.40	4.34	4.29	4.25	4.18	4.10	4.01	3.94	3.86
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.22	4.16	4.10	4.05	4.01	3.94	3.86	3.76	3.70	3.62
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.91	3.86	3.82	3.75	3.66	3.57	3.51	3.43
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.75	3.70	3.66	3.59	3.51	3.41	3.35	3.27
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.61	3.56	3.52	3.45	3.37	3.28	3.21	3.13
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.62	3.55	3.50	3.45	3.41	3.34	3.26	3.16	3.10	3.02
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.46	3.40	3.35	3.31	3.24	3.16	3.07	3.00	2.92
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.43	3.37	3.32	3.27	3.23	3.16	3.08	2.98	2.92	2.84
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.24	3.19	3.15	3.08	3.00	2.91	2.84	2.76
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.29	3.23	3.18	3.13	3.09	3.02	2.94	2.84	2.78	2.69
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.24	3.17	3.12	3.07	3.03	2.96	2.88	2.79	2.72	2.64
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.07	3.02	2.98	2.91	2.83	2.73	2.67	2.58
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	3.02	2.97	2.93	2.86	2.78	2.69	2.62	2.54
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.09	3.03	2.98	2.93	2.89	2.82	2.74	2.64	2.58	2.49
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	3.06	2.99	2.94	2.89	2.85	2.78	2.70	2.60	2.54	2.45
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	3.02	2.96	2.90	2.86	2.81	2.75	2.66	2.57	2.50	2.42
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.99	2.93	2.87	2.82	2.78	2.71	2.63	2.54	2.47	2.38
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.96	2.90	2.84	2.79	2.75	2.68	2.60	2.51	2.44	2.35
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.93	2.87	2.81	2.77	2.73	2.66	2.57	2.48	2.41	2.33
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.91	2.84	2.79	2.74	2.70	2.63	2.55	2.45	2.39	2.30
31	7.53	5.36	4.48	3.99	3.67	3.45	3.28	3.15	3.04	2.96	2.88	2.82	2.77	2.72	2.68	2.61	2.52	2.43	2.36	2.27
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02	2.93	2.86	2.80	2.74	2.70	2.65	2.58	2.50	2.41	2.34	2.25
33	7.47	5.31	4.44	3.95	3.63	3.41	3.24	3.11	3.00	2.91	2.84	2.78	2.72	2.68	2.63	2.56	2.48	2.39	2.32	2.23
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89	2.82	2.76	2.70	2.66	2.61	2.54	2.46	2.37	2.30	2.21
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.80	2.74	2.69	2.64	2.60	2.53	2.44	2.35	2.28	2.19
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.73	2.66	2.61	2.56	2.52	2.45	2.37	2.27	2.20	2.11
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.67	2.61	2.55	2.51	2.46	2.39	2.31	2.21	2.14	2.05
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.63	2.56	2.51	2.46	2.42	2.35	2.27	2.17	2.10	2.01
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.44	2.39	2.35	2.28	2.20	2.10	2.03	1.94
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.51	2.45	2.40	2.35	2.31	2.23	2.15	2.05	1.98	1.89
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.48	2.42	2.36	2.31	2.27	2.20	2.12	2.01	1.94	1.85
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.45	2.39	2.33	2.29	2.24	2.17	2.09	1.99	1.92	1.82
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.43	2.37	2.31	2.27	2.22	2.15	2.07	1.97	1.89	1.80
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.37	2.31	2.25	2.20	2.16	2.09	2.00	1.90	1.83	1.73
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.34	2.27	2.22	2.17	2.13	2.06	1.97	1.87	1.79	1.69
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.25	2.18	2.13	2.08	2.04	1.97	1.88	1.77	1.70	1.59