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Abstract

We investigate a supply chain comprising a upstream firm engaged in advertising and two downstream firms who compete with differentiated products. We find that the producer surpluses always decrease with the degree of the cross-holdings between the downstream firms, if the upstream firm engages in informative advertising. This result contradicts the conventional wisdom that cross-holdings benefit the participants because firms become less competitive with the degree of cross-holdings. In addition, we also find that our main results are robust under upstream persuasive advertising.

JEL codes: D43, L13, M21.

Keywords: cross-holdings, advertising, vertical relationship.

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1 Introduction

Cross-holdings where the acquiring firm holds financial interests of rivals are widely observed in many industries such as automobile, telecommunication, airline, etc. In a duopolistic market, since the cross-holdings participants must consider the profits of each other when they make decisions, they behave more cooperatively. It is well-known that there exists an anti-competition effect of cross-holdings if the acquiring firm has the share of its rivals. Hence, as the degree of the cross-holdings increases, the competition becomes less fiercer, leading less output but higher producer surplus (firms' profits). In this study, we challenge this well-known results by introducing upstream advertising.

We consider a vertical structure with an upstream firm providing the input to the downstream market where there are two downstream firms offering differentiated products. We assume that the downstream firms holds a share of each other, i.e. cross-holdings, and the upstream firm engages in an investment of advertising which influences the final consumers. Specifically, In the first stage, the upstream firm chooses the level of advertising¹. In the second stage, the upstream firm decides the wholesale price. In the third stage, the downstream firms compete in quantity.

We find that when the upstream firm engages informative advertising, an increase of the degree of cross-holdings, decreases the downstream producer surplus and the upstream firm's profit. The intuition can be explained as follows. As the degree of cross-holdings increases, the downstream market tends to be less competitive and thus less output. Therefore, the upstream firm has less incentive to invest advertising, which leading less final consumers and then harms the downstream profits. The less competition beneficial effect because of cross-holdings is dominated by the harmful less demand due to less advertising, leading less downstream producer surplus. By considering the upstream advertising, the well-known result that downstream producer surplus always increases with the degree of cross-holdings reverses. In addition, the weaker downstream competition leads to a lower demand to the

¹The literature on advertising (Gross and Shapiro, 1984; Soberman, 2004; Zhang et al., 2012; Zhang et al., 2020) allows firms to sell differentiated products.

upstream firm, thus reduces the upstream firm's profit. As a result, both the upstream and downstream producer surplus reduces with the degree of cross-holdings.

Our study is closely related to Hu et al. (2022) which examine how the upstream cost reducing investment affects the well-known beneficial effect of cross-holding on the producer surplus. In their model, they build a vertical model with an upstream firm and two downstream firms. The upstream firm can engage in a cost reducing investment and the downstream firms produce homogenous products. The model is similar to ours and they also provides the conditions where cross-holdings harms the downstream producer, but the upstream investment method and the intuitions are totally different from our model. In their model, the less downstream output due to cross-holdings reduces the input price thus creates a negative effect on the downstream producer. About advertising, we extended the model of Hu and Mizuno (2023), which examine how the upstream advertising affects the downstream choice between Cournot and Bertrand. Our work is closely related to that of Hu and Mizuno (2021), who investigated downstream firms' second-mover advantage property of Bertrand competition by introducing advertising by the upstream upstream firm. In our discussion, we also examine the upstream persuasive advertising, and find our main results are robust.

The remainder of the paper is organized as follows: In Section 2, we describe the basic model. In Section 3, we present the analysis under informative advertising. In Section 4, we show the case of persuasive advertising. Finally, in Section 5, we present our conclusion.

2 Model

We consider a vertical structure with an upstream firm and two downstream firms (firm i and firm j , $i, j = 1, 2$ and $i \neq j$). The upstream firm produces input and sells it to the downstream firms at input price w . The downstream firms transform one unit of the input into one unit of output at zero marginal production cost. The products sold by the downstream firms are differentiated. The price and output of downstream firm i is denoted by p_i and Q_i , respectively. We denote the operating profit of downstream firm i as $\pi_i \equiv (p_i - w)Q_i$.

We consider cross-holdings where each downstream firm holds a share $0 < s < 1/2$ of its rival's ownership in the form of passive investments with no control rights (e.g., nonvoting shares; Gilo et al., 2006). For the sake of simplicity, we consider a symmetric case and call s as cross-holding rate. Notably, there exists a chain effect (Gilo et al., 2006), by solving $V_i = \pi_i + sV_j$ for V_i , we can obtain the total value of firm i as $V_i \equiv (\pi_i + s\pi_j)/(1 - s^2)$.

The upstream firm can engage in informative advertising. We denote the level of informative advertising by θ , which represents the fraction of consumers who receive the advertisement. Consumers who view the advertisement can buy either product, while those who do not view it buy none. We assume that if θ consumers receive the advertisement, the upstream firm must incur an advertising cost of $k\theta^2$, where k is a positive and constant parameter and $\theta \in [0, 1]$.² The upstream firm produces the product at a constant marginal cost of c . Consequently, the profit of the upstream firm is given by $\Pi_M \equiv (w - c)(Q_i + Q_j) - k\theta^2$.

There is a unit mass of consumers, each with a symmetric utility function: $u(q_i, q_j, m) \equiv a(q_i + q_j) - (q_i^2 + 2\gamma q_i q_j + q_j^2)/2 + m$, where q_i and q_j are the consumption levels for products i and j , respectively; m is the quantity of a numeraire good; $\gamma \in (0, 1)$ is the measure of product substitutability; and a is a positive parameter. This utility function yields the following demand function: $q_i = [a(1 - \gamma) - p_i + \gamma p_j]/[(1 - \gamma^2)]$. As we assume that consumers must receive the advertisement to buy the products, the demand faced by the downstream firms depends on θ (indicating the fraction of consumers viewing the advertisement) and equals $Q_i(p_i, p_j) \equiv \theta q_i$ and $Q_j(p_i, p_j) \equiv \theta q_j$.³ Solving $Q_i(p_i, p_j)$ and $Q_j(p_i, p_j)$ for p_i and p_j , we obtain the inverse demand function $p_i(Q_i, Q_j) = a - (Q_i + \gamma Q_j)/\theta$. To guarantee an interior solution, we assume $k > (a - c)^2/[4(2 + \gamma + s\gamma)]$. This assumption is obtained from the condition that the equilibrium level of informative advertising is less than 1. Additionally, this assumption is a sufficient condition for concavity of the upstream firm's profitability.

²We assume a quadratic advertising cost. Although some previous studies have employed linear costs and concave benefits from advertising (Nakata, 2011; Shy, 1995), quadratic advertising cost is a standard assumption in the literature (e.g., Simbanegavi, 2009; Soberman, 2004; Zhang et al., 2012; Zhang et al., 2020; Hu and Mizuno, 2021).

³By defining a utility function of the representative consumer as $u(Q_i, Q_j, m) = a(Q_i + Q_j) - (Q_i^2 + 2\gamma Q_i Q_j + Q_j^2)/(2\theta) + m$, we can derive these demand functions. This demand function and set-up under informative advertising are similar to that in Zhang et al. (2020) and Hu and Mizuno (2021).

We assume that each downstream firm competes in quantity to maximize the total value of firm, V_i . We define the downstream producer surplus as $PS_D = \pi_i + \pi_j$. Note that $V_i + V_j = (\pi_i + \pi_j)/(1 - s) > \pi_i + \pi_j$, but the inclusion of the term $1 - s$ in the denominator creates an inflated effect which enlarges the real shareholders' total values. Hence, the downstream producer surplus, i.e., the total value to shareholders should be $(V_i + V_j)(1 - s) = \pi_i + \pi_j$ (see Shelegia and Spiegel, 2022; Jin et al. 2024). We examine how the degree of cross-holdings s affects the downstream producer surplus and the upstream firm's profit.

The timing of this game is as follows: In the first stage, the upstream firm chooses the level of informative advertising θ . In the second stage, the upstream firm decides the wholesale price w . In the third stage, the downstream firms compete in quantity. We solve this model using backward induction.

3 Analysis

3.1 Third and second stages: downstream output and input price

In the third stage, substituting the inverse demand function into the profit functions of downstream firms and solving the first-order condition $\partial V_i / \partial Q_i = 0$, we obtain the downstream firm's output as

$$Q_i(w, \theta) = \frac{\theta(a - w)}{2 + \gamma + s\gamma}. \quad (1)$$

In the second stage, the upstream firm chooses the wholesale price. Taking (1) into account, maximizing $\Pi_M = (w - c)[Q_i(w, \theta) + Q_j(w, \theta)] - k\theta^2$ for w yields

$$w = \frac{a + c}{2}. \quad (2)$$

Benchmark case without upstream advertising Here, we show how the degree of cross-holdings s affects the downstream producer surplus and the upstream firm's profit, without the upstream advertising. By substituting (1)-(2) and then $\theta = 1$, $k = 0$, we can have the following outcomes.

$$PS_D^N = \frac{(a-c)^2(1+s\gamma)}{2(2+\gamma+s\gamma)^2}, \quad \Pi_M^N = \frac{(a-c)^2}{2(2+\gamma+s\gamma)},$$

where the superscript N denotes the case without upstream advertising.

Differentiating the downstream producer surplus and the upstream profit with respect to s , respectively, we have

$$\frac{\partial PS_D^N}{\partial s} = \frac{(a-c)^2(1-s)\gamma^2}{2(2+\gamma+s\gamma)^3} > 0, \quad \frac{\partial \Pi_M^N}{\partial s} = -\frac{(a-c)^2\gamma}{2(2+\gamma+s\gamma)^2} < 0.$$

Hence, we obtain the following result.

Remark 1 *Without the upstream informative advertising, an increase of the degree of cross-holdings s , increases the downstream producer surplus but decreases the profit of the upstream firm.*

The results are well-known and the intuitions are simple. An increase in cross-holdings rate strengthens the incentive for the downstream firms to behave more cooperatively for more profits and thus softens their competition and then increases the downstream producer surplus. Meanwhile, the weaker downstream competition leads to a lower demand to the upstream firm, which reduces the upstream firm's profit.

3.2 First stage: upstream advertising level

In the first stage, the upstream firm decides the level of informative advertising. We substitute (1) and (2) into Π_M and solve the first-order condition for θ , to obtain the level of advertising, then the downstream producer surplus and the profit of the upstream firm:

$$\theta^* = \frac{(a-c)^2}{4k(2+\gamma+s\gamma)}, \quad PS_D^* = \frac{(a-c)^4(1+s\gamma)}{8k(2+\gamma+s\gamma)^3}, \quad \Pi_M^* = \frac{(a-c)^4}{16k(2+\gamma+s\gamma)^2}. \quad (3)$$

Before analyzing the effects of the cross-holdings rate on the firms' profitability, first, we examine the advertising level θ^* . From (3), we find that s is only in the denominator of the

equilibrium advertising level θ^* , and it is obvious that θ^* decreases with s . Hence, we get the following result.

Lemma 1 *With the upstream informative advertising, the level of advertising decreases with the degree of cross-holdings.*

The intuition is as follows. An advertising expenditure of the upstream firm gains less facing a less competitive downstream market. The downstream market becomes less competitive as the degree of cross-holdings increases. Therefore, the upstream firm has the incentive to advertise less with the degree of cross-holdings.

Next, differentiating the downstream producer surplus PS_D^* and the upstream profit Π_M^* with respect to s , respectively, we have:

$$\frac{\partial PS_D^*}{\partial s} = \frac{(a-c)^4[(1-2s)\gamma-1]}{8k(2+\gamma+s\gamma)^4} < 0, \quad \frac{\partial \Pi_M^*}{\partial s} = -\frac{(a-c)^4\gamma}{8k(2+\gamma+s\gamma)^3} < 0.$$

Note that with $0 < s < 1/2$ and $0 < \gamma < 1$, $\partial PS_D^*/\partial s < 0$. Then, we obtain the following result.

Proposition 1 *With the upstream informative advertising, an increase of the degree of cross-holdings s , decreases the downstream producer surplus and the upstream firm's profit.*

This result is intuitive. An increase of the degree of cross-holdings weakens downstream competition and reduces the upstream informative advertising level as explained in Lemma 1. As a result, there will be less latent consumers who are converted to actual buyers, and this demand drop harms the downstream producer surplus. In our model, this harmful effect dominates the less competition effect as mentioned in Remark 1. Meanwhile, the weaker downstream competition leads to a lower demand to the upstream firm, thus reduces the upstream firm's profit.

4 Discussion

4.1 Persuasive advertising

While in the previous section we considered informative advertising, in this subsection, we discuss persuasive advertising, which increases the willingness to buy or marginal utility for consumers.

Model We assume that the utility function of the representative consumer is $u = (a + \theta)(Q_i + Q_j) - (Q_i^2 + 2\gamma Q_i Q_j + Q_j^2)/2 + m$, where θ denotes the level of persuasive advertising.⁴ Then, the demand for each product is $Q_i = [(a + \theta)(1 - \gamma) - p_i + p_j\gamma]/[(1 - \gamma^2)]$. Solving $Q_i = [(a + \theta)(1 - \gamma) - p_i + p_j\gamma]/[(1 - \gamma^2)]$ for p_i and p_j , we can obtain the inverse demand function $p_i(Q_i, Q_j) = a + \theta - (Q_i + \gamma Q_j)$. The profits of the upstream firm and downstream firm i are $\Pi_M = (w - c)(Q_i + Q_j) - k\theta^2$ and $\pi_i = (p_i - w)Q_i$, respectively. The other settings are the same as those in the previous section. For concavity of the upstream firm's profit, we assume $k > 1/(4 + 2\gamma + 2s\gamma)$.

Calculating equilibrium In the third stage, substituting the inverse demand function into the profit function of the downstream firm i and maximizing V_i with respect to Q_i , we obtain the following output:

$$Q_i^{pa}(w, \theta) = \frac{a - w + \theta}{2 + \gamma + s\gamma}. \quad (4)$$

where the superscript pa denotes the case with upstream persuasive advertising.

In the second stage, taking (4) into account, maximizing the upstream profit with respect to w , we have

$$w^{pa}(\theta) = \frac{a + c + \theta}{2}. \quad (5)$$

In the first stage, using (4) and (5), the upstream firm chooses the level of advertising, and then we have the downstream producer surplus and the upstream profit in the equilibrium:

⁴The utility function under persuasive advertising is similar to Zhang et al. (2020), and Hu and Mizuno (2021).

$$\theta^{pa} = \frac{a - c}{2k(2 + \gamma + s\gamma) - 1}, \quad PS_D^{pa} = \frac{2(a - c)^2 k^2 (1 + s\gamma)}{[1 - 2k(2 + \gamma + s\gamma)]^2}, \quad \Pi_M^{pa} = -\frac{(a - c)^2 k}{[1 - 2k(2 + \gamma + s\gamma)]}.$$

Similar to the case of informative advertising, θ^{pa} decreases with s . Hence, we have the following result.

Lemma 2 *With the upstream persuasive advertising, the advertising level decreases with the cross-holdings rate.*

The intuition is parallel to Lemma 1. Then, differentiating the downstream producer surplus and the profit of the upstream firm with respect to s , respectively, we have the following results.

$$\frac{\partial PS_D^{pa}}{\partial s} = \frac{2(a - c)^2 k^2 \gamma [1 - 2k(1 - s)\gamma]}{[1 - 2k(2 + \gamma + s\gamma)]^3}, \quad \frac{\partial \Pi_M^{pa}}{\partial s} = -\frac{2(a - c)^2 k^2 \gamma}{[1 - 2k(2 + \gamma + s\gamma)]^2}.$$

It is easy to find that $\partial \Pi_M^{pa} / \partial s < 0$. Solving $\partial PS_D^{pa} / \partial s < 0$ for k , we obtain the following proposition.

Proposition 2 *If an upstream firm engages in persuasive advertising, an increase of the degree of cross-holdings, decreases the downstream producer surplus if $k\gamma < 1/[2(1 - s)]$, and always reduces the upstream profit.*

The intuition is as follows. When γ is small, the beneficial effect from less competition between the downstream firms becomes less important. Then, when k is small, it means advertising investment is efficient, hence the beneficial effect from advertising becomes more important. Additionally, as shown in Lemma 2, the advertising level decreases with the cross-holdings rate s . When s is sufficiently large, with small γ and k , the advertising level drop effect dominates the less competition benefit effect. As to the upstream firm, the intuition is parallel to Proposition 1.

5 Conclusions

We consider a vertical structure with an upstream firm providing the input to the downstream market where there are two downstream firms offering differentiated products. We find that when the upstream firm engages advertising, an increase of the degree of cross-holdings, may decrease the downstream producer surplus and the upstream firm's profit. As the degree of cross-holdings increases, the downstream market tends to be less competitive and thus less output. Therefore, the upstream firm has less incentive to invest advertising, which leading less final consumers and then harms the downstream profits. The less competition beneficial effect because of cross-holdings is dominated by the harmful less demand due to less advertising, leading less downstream producer surplus. In addition, the weaker downstream competition leads to a lower demand to the upstream firm, thus reduces the upstream firm's profit. As a result, both the upstream and downstream producer surplus may reduce with the degree of cross-holdings.

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