（前期第Ⅰ期・国際共同学修コース）

令和6(5)年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

経済理論

・第1問～第4問のすべてに日本語か英語で答えなさい。
・各問の解答は、それぞれ別の解答用紙に記入しなさい。

第1問　2人の消費者と2つの財が存在しているとする。財$i=1,2$の消費量をx_iで表すと、消費者1の効用関数は$u_1(x_1,x_2)=\sqrt{x_1}+x_2$、消費者2の効用関数は$u_2(x_1,x_2)=2\sqrt{x_1}+x_2$で表されるとする。財2の価格は1で固定されているものとする。財1の価格をp、消費者$j=1,2$の予算をI_jで表わすものとする。

(1) 各消費者の予算制約式を求めなさい。(10点)

(2) 各消費者の財1の需要関数を求めなさい。(20点)

(3) 財1の市場需要関数を求めなさい。(10 点)

(4) $p=5$のときの財1市場での消費者余剰を求めなさい。(10点)

第2問　同質財の複占市場における数量競争を考える。逆需要関数は$p=6-q_1-q_2$(ただしq_iは企業$i=1,2$の生産量)、企業iの費用はc_iq_iとする。両企業は同時に生産量を決定する。

(1) $c_1=c_2=1$のときの各企業の利潤関数を求めなさい。(10点)

(2) (1)の状況でのナッシュ均衡を求めなさい。(20点)

(3) (2)のナッシュ均衡における市場価格を求めなさい。(10点)

(4) $c_1=1,\ c_2=4$のときのナッシュ均衡を求めなさい。(10点)

第３問　債券価格と金利について以下の問いに答えなさい。なお、途中の計算式がある場合はそれも書きなさい。

(1) 額面B円、利息が年率$100 \times a$%のコンソル債券（利息が永続的に支払われる債券）の割引現在価値を求めなさい。ただし、市場金利（安全資産金利）は年率$100 \times r$%とする。（15点）

(2) 一般に、金利が上昇すると債券価格は下落することが知られている。これはなぜか説明しなさい。（15点）

(3) 市場金利が0%の経済を考える。上の問題(1)(2)を踏まえると、中央銀行が金利を上昇させたい場合にはどのような政策手段が考えられるか。また、逆に市場金利を下げたい場合にはどのような政策手段が考えられるか述べなさい。（20点）

第４問　為替レートと金利について以下の問いに答えなさい。ただし、以下の記号を使用すること：

P: 国内物価水準、P^*:外国物価水準、i: 名目国内金利、i^*：名目外国金利、

S: 名目自国通貨建て為替レート（例：１ドル=130円のとき$S=130$）。

(1) 購買力平価とは何か、簡潔に説明しなさい。（15点）

(2) 金利平価とは何か、簡潔に説明しなさい。（15点）

(3) ある２国の間で購買力平価と金利平価が成立すると仮定すると、両国の実質金利にはどのような関係が成立するか示しなさい。（20点）

（前期第Ⅰ期・国際共同学修コース）

令和6(5)年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

Economic Theory

- Answer all of the following questions either in English or in Japanese.
- Answer each question on a separate sheet.

1. There are two consumers and two goods. We denote the consumption of good $i = 1,2$ by x_i, the utility function of consumer 1 is given by $u_1(x_1, x_2) = \sqrt{x_1} + x_2$, and the utility function of consumer 2 is given by $u_2(x_1, x_2) = 2\sqrt{x_1} + x_2$. Assume the price of good 2 is fixed as 1. Denote the price of good 1 and the income of consumer $j = 1,2$ by p and I_j, respectively.
   (1) Show each consumer's budget constraint. (10 points)
   (2) Derive each consumer's demand function for good 1. (20 points)
   (3) Derive the market demand function for good 1. (10 points)
   (4) Find the consumer surplus within the market for good 1 when $p = 5$. (10 points)

2. Consider a quantity competition in a duopoly market of a homogeneous product. The inverse demand function is given by $p = 6 - q_1 - q_2$ (q_i is an output level of firm $i =$ 1,2) and firm i's cost is given by $c_i q_i$. Both firms simultaneously determine their output levels.
   (1) Show each firm's profit function when $c_1 = c_2 = 1$. (10 points)
   (2) Derive the Nash equilibrium in (1). (20 points)
   (3) Calculate the market price at the Nash equilibrium in (2). (10 points)
   (4) Derive the Nash equilibrium when $c_1 = 1, c_2 = 4$. (10 points)

3. Answer the following questions regarding bond prices and interest rates. If there are any intermediate calculations involved, include them as well.

   (1) Calculate the discounted present value of a consol bond (a bond that pays a coupon in perpetuity) with a face value of B yen and an annual coupon payment of $100 \times a$ %. Assume that the market interest rate (i.e., risk-free rate) is $100 \times r$ % annually. (15 points)

   (2) It is well known that bond prices fall when interest rates rise. Explain why this is the case. (15 points)

   (3) Consider an economy with a market interest rate of 0%. Given questions (1) and (2) above, what policy tools could the central bank use if it intends to raise interest rates? Conversely, if the central bank would like to lower market interest rates, what policy tools could be used? (20 points)

4. Answer the following questions about exchange rates and interest rates. Use the following definitions:

   P: domestic price level, P^*: foreign price level, i: nominal domestic interest rate,
   i^*: nominal foreign interest rate, S: nominal home-currency exchange rate (e.g., $S = 130$ when \$1 = 130 yen)

   (1) Explain briefly what purchasing power parity is. (15 points)

   (2) Explain briefly what interest rate parity is. (15 points)

   (3) Assuming that purchasing power parity and interest rate parity hold between two countries, show what relationship is established between the real interest rates of the two countries. (20 points)

(前期第Ⅰ期・国際共同学修コース)

令和6(5)年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

統計学

- 第1問〜第3問のすべてに日本語か英語で答えなさい。
- 各問の解答は, それぞれ別の解答用紙に記入しなさい。
- 本研究科で貸与する電卓のみ使用を認めます。
- 必要に応じて, 添付の統計分布表を利用しなさい。

第1問　2つの離散型変数 X, Y の同時確率分布が次のように与えられているとする。

X \ Y	0	1
0	0.1	a
1	0.3	b

以下の問いに答えなさい。

(1) $\mathrm{P}(X > Y)$ および $\mathrm{P}(X = 1|Y = 0)$ を求めなさい。(20点)
(2) Y の平均と分散を求めなさい。(20点)
(3) X と Y が独立になるような a と b の値を求めなさい。(20点)

第2問　連続型確率変数 X_1, X_2, X_3, X_4 は互いに独立に $\mathrm{N}(\mu, \sigma^2)$ にしたがっているとする。また $Z_i = \frac{X_i - \mu}{\sigma}$ $(i = 1, 2, 3, 4)$ であるとする。以下の問いに答えなさい。

(1) $\mathrm{P}(X_1 > \mu + 0.75\sigma)$ を求めなさい。(10点)
(2) $\mathrm{P}(Z_1^2 + Z_2^2 < a) = 0.10$ となる a の値を求めなさい。(15点)
(3) $\mathrm{P}\left(\frac{Z_3}{\sqrt{(Z_1^2+Z_2^2)/2}} > b\right) = 0.05$ となる b の値を求めなさい。(15点)
(4) 観測された x_i $(i = 1, 2, 3, 4)$ の標本平均を計算したところ $\bar{x} = \frac{\sum_{i=1}^4 x_i}{4} = 3.5$ であった。また, $\sigma^2 = 4$ は既知であるとする。帰無仮説 $\mathrm{H}_0 : \mu = 2$ を対立仮説 $\mathrm{H}_1 : \mu = 3$ に対して有意水準0.05で検定した時の検定力を求めなさい。(20点)

第３問　まず，真の回帰モデルが次のように与えられていると仮定する。

$$Y_i = \beta_0 + \beta_1 X_i + u_i, \quad i = 1, 2, \ldots, n$$

ただし，X_i は非確率変数，u_i は誤差項で，次の性質を満たしている。

$$\mathrm{E}[u_i] = 0,\ \ \mathrm{E}[u_i^2] = \sigma^2,\ \ \mathrm{E}[u_i u_j] = 0 \text{ for } i \neq j$$

ところが，分析者が誤って回帰モデルを次のように定式化した。

$$Y_i = \alpha X_i + u_i, \quad i = 1, 2, \ldots, n$$

このとき，以下の問いに答えなさい。

(1) α の最小自乗推定量 $(\widehat{\alpha})$ を導出しなさい。(20 点)
(2) $\widehat{\alpha}$ の期待値を求め、$\widehat{\alpha}$ が不偏推定量であるかどうか確かめなさい。(20 点)

次に，真の回帰モデルが次のように与えられていると仮定する。

$$Y_i = \beta X_i + v_i, \quad i = 1, 2, \ldots, n$$

ただし，X_i は非確率変数，v_i は誤差項で，次の性質を満たしている。

$$\mathrm{E}[v_i] = 0,\ \ \mathrm{E}[v_i^2] = \sigma^2,\ \ \mathrm{E}[v_i v_j] = 0 \text{ for } i \neq j$$

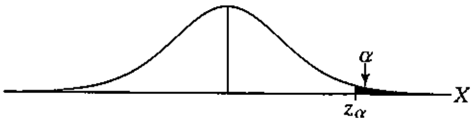
ところが，分析者が誤って回帰モデルを次のように定式化した。

$$Y_i = \alpha_0 + \alpha_1 X_i + v_i, \quad i = 1, 2, \ldots, n$$

このとき，以下の問いに答えなさい。

(3) α_1 の最小自乗推定量 $(\widehat{\alpha}_1)$ を導出しなさい。(20 点)
(4) $\widehat{\alpha}_1$ の期待値を求め、$\widehat{\alpha}_1$ が不偏推定量であるかどうか確かめなさい。(20 点)

正規分布表： $X \sim N(0,1)$

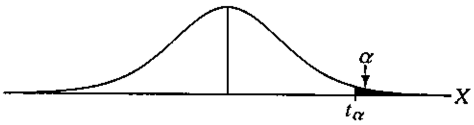


$$\alpha = P(X > z_\alpha) = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)dx$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4091	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2644	.2611	.2579	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

α	.10	.05	.025	.010	.005	.001	.0005	.0001	.00001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	4.2649

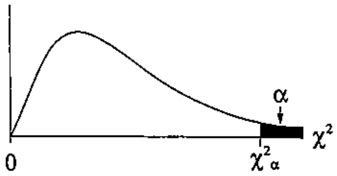
t 分布表： $X \sim t(k)$



$$\alpha = P(X > t_\alpha) = \int_{t_\alpha}^{\infty} f(x)dx$$

k (自由度) \ α	.10	.05	.025	.010	.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8187
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
120	1.2886	1.6577	1.9799	2.3578	2.6174
∞	1.2816	1.6449	1.9600	2.3263	2.5758

カイ２乗分布表： $\chi^2(m)$



$$\alpha = P(\chi^2 > \chi^2_\alpha) = \int_{\chi^2_\alpha}^{\infty} f(\chi^2)\,d\chi^2$$

α / m (自由度)	.995	.99	.975	.95	.90	.10	.05	.025	.010	.005
1	.0000393	.000157	.000982	.00393	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.06	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43	104.21
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17

(前期第Ⅰ期・国際共同学修コース)

令和 6(5) 年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

Statistics

- Answer all of the following questions either in English or in Japanese.
- Answer each question on a separate sheet.
- Applicants are authorized to use a calculator lent by our Graduate School.
- Use the statistical tables if necessary.

1. Assume that the joint distribution of two discrete random variables X, Y is given as follows:

X \ Y	0	1
0	0.1	a
1	0.3	b

Answer the following questions.

(1) Find the probabilities $P(X > Y)$ and $P(X = 1 | Y = 0)$. (20 points)

(2) Find the mean and the variance of Y. (20 points)

(3) Find the values of a and b such that X and Y are mutually independent (20 points)

2. Assume that the continuous random variables X_1, X_2, X_3, X_4 are independently and identically distributed as $N(\mu, \sigma^2)$. Define $Z_i = \frac{X_i - \mu}{\sigma}$ $(i = 1, 2, 3, 4)$. Answer the following questions.

(1) Find $P(X_1 > \mu + 0.75\sigma)$. (10 points)

(2) Find the value of a such that $P(Z_1^2 + Z_2^2 < a) = 0.10$. (15 points)

(3) Find the value of b such that $P\left(\frac{Z_3}{\sqrt{(Z_1^2+Z_2^2)/2}} > b\right) = 0.05$. (15 points)

(4) Calculating the sample mean of the observations x_i $(i = 1, 2, 3, 4)$, we obtain $\bar{x} = \frac{\sum_{i=1}^4 x_i}{4} = 3.5$. Also, assume that $\sigma^2 = 4$ is known. Find the power of the test when we test the null hypothesis $H_0 : \mu = 2$ against the alternative $H_1 : \mu = 3$ at the 0.05 significance level. (20 points)

3. First, assume that the true regression model is given as follows:

$$Y_i = \beta_0 + \beta_1 X_i + u_i, \quad i = 1, 2, \ldots, n,$$

where X_i is a non-random variable and u_i is the error term, satisfying the following properties:

$$\mathrm{E}[u_i] = 0, \ \ \mathrm{E}[u_i^2] = \sigma^2, \ \ \mathrm{E}[u_i u_j] = 0 \text{ for } i \neq j.$$

However, a researcher mistakenly specified the regression model as follows:

$$Y_i = \alpha X_i + u_i, \quad i = 1, 2, \ldots, n.$$

Answer the following questions in this case.

(1) Derive the least squares estimator ($\widehat{\alpha}$) of α. (20 points)

(2) Find the expected value of $\widehat{\alpha}$ and check if it is an unbiased estimator. (20 points)

Next, assume that the true regression model is given as follows:

$$Y_i = \beta X_i + v_i, \quad i = 1, 2, \ldots, n,$$

where X_i is a non-random variable and v_i is the error term, satisfying the following properties:

$$\mathrm{E}[v_i] = 0, \ \ \mathrm{E}[v_i^2] = \sigma^2, \ \ \mathrm{E}[v_i v_j] = 0 \text{ for } i \neq j.$$

However, a researcher mistakenly specified the regression model as follows:

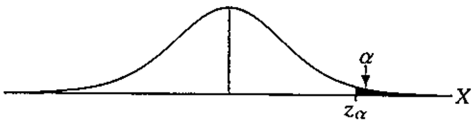
$$Y_i = \alpha_0 + \alpha_1 X_i + v_i, \quad i = 1, 2, \ldots, n.$$

Answer the following questions in this case.

(3) Derive the least squares estimator ($\widehat{\alpha}_1$) of α_1. (20 points)

(4) Find the expected value of $\widehat{\alpha}_1$ and check if it is an unbiased estimator. (20 points)

Normal distribution: $X \sim N(0,1)$

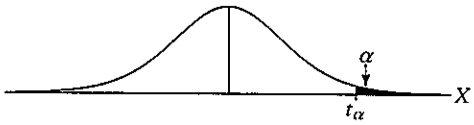


$$\alpha = P(X > z_\alpha) = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)dx$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4091	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2644	.2611	.2579	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

α	.10	.05	.025	.010	.005	.001	.0005	.0001	.00001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	4.2649

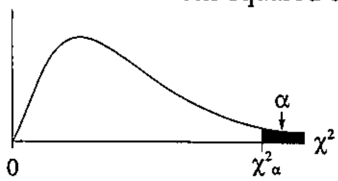
t distribution: $X \sim t(k)$



$$\alpha = P(X > t_\alpha) = \int_{t_\alpha}^{\infty} f(x)dx$$

k (Degrees of Freedom) \ α	.10	.05	.025	.010	.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8187
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
120	1.2886	1.6577	1.9799	2.3578	2.6174
∞	1.2816	1.6449	1.9600	2.3263	2.5758

Chi-squared distribution: $\chi^2(m)$



$$\alpha = P(\chi^2 > \chi^2_\alpha) = \int_{\chi^2_\alpha}^{\infty} f(\chi^2)\, d\chi^2$$

α / m (Degrees of Freedom)	.995	.99	.975	.95	.90	.10	.05	.025	.010	.005
1	.0000393	.000157	.000982	.00393	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.06	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43	104.21
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17