# Online Appendices for "The role of local currency pricing in the international transmission effects of a government spending shock in an economy with vertical production linkage and foreign direct investment" by Kohjiro Dohwa

## Appendix A

In this appendix, we specify the price indexes that are defined using the specification of production functions in the main text. We also derive the demand of home-located final goods firms for intermediate inputs required in production activities. Further, we specify the resource constraints in home and foreign intermediate inputs used by home and foreign final goods firms.

### Price indexes

The unit costs of producing home and foreign final goods are shown as follows:

$$\Lambda_t = \tilde{P}_{h,t}^{\frac{1}{2}} \tilde{P}_{f,t}^{\frac{1}{2}}, \tag{A.1}$$

$$\Lambda_t^* = \tilde{P}_{h,t}^{*\frac{1}{2}} \tilde{P}_{f,t}^{*\frac{1}{2}}, \tag{A.2}$$

where

$$\tilde{P}_{f,t} = \left( \frac{\eta^*}{2} \left( \tilde{P}_{f,t}^{LCP} \right)^{1-\sigma} + \frac{1-\eta^*}{2} \left( \tilde{P}_{f,t}^{PCP} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{A.3}$$

$$\tilde{P}_{h,t}^* = \left( \frac{\eta}{2} \left( \tilde{P}_{h,t}^{*LCP} \right)^{1-\sigma} + \frac{1-\eta}{2} \left( \tilde{P}_{h,t}^{*PCP} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{A.4}$$

and

$$\tilde{P}_{h,t} = \left( \int_0^{\frac{1}{2}} \tilde{p}_{h,t}(z_{I|D})^{1-\sigma} dz_{I|D} \right)^{\frac{1}{1-\sigma}}, \ \tilde{P}_{f,t}^* = \left( \int_0^{\frac{1}{2}} \tilde{p}_{f,t}^*(z_{I|D}^*)^{1-\sigma} dz_{I|D}^* \right)^{\frac{1}{1-\sigma}}, \tag{A.5}$$

$$\tilde{P}^{LCP}_{f,t} = \left( \frac{2}{\eta^*} \int_{\frac{1}{2}}^{\frac{1+\eta^*}{2}} \tilde{p}^{LCP}_{f,t}(z^*_{I|X})^{1-\sigma} dz^*_{I|X} \right)^{\frac{1}{1-\sigma}}, \ \tilde{P}^{*LCP}_{h,t} = \left( \frac{2}{\eta} \int_{\frac{1}{2}}^{\frac{1+\eta}{2}} \tilde{p}^{*LCP}_{h,t}(z_{I|X})^{1-\sigma} dz_{I|X} \right)^{\frac{1}{1-\sigma}}, \tag{A.6}$$

$$\tilde{P}^{PCP}_{f,t} = \left( \frac{2}{1-\eta^*} \int_{\frac{1+\eta^*}{2}}^{1} \tilde{p}^{PCP}_{f,t}(z^*_{I|X})^{1-\sigma} dz^*_{I|X} \right)^{\frac{1}{1-\sigma}}, \ \tilde{P}^{*PCP}_{h,t} = \left( \frac{2}{1-\eta} \int_{\frac{1+\eta}{2}}^{1} \tilde{p}^{*PCP}_{h,t}(z_{I|X})^{1-\sigma} dz_{I|X} \right)^{\frac{1}{1-\sigma}}. \tag{A.7}$$

In Eqs.(A.1) and (A.2), $\tilde{P}_{h,t}$ ($\tilde{P}^*_{h,t}$) is the home (foreign) price index that corresponds to a composite of the inputs produced by domestic market (export) firms in the home intermediate goods sector, $\tilde{P}_{f,t}$ ($\tilde{P}^*_{f,t}$) is the home (foreign) price index that corresponds to a composite of the inputs produced by export (domestic market) firms in the foreign intermediate goods sector. The import price indexes of home- and foreign-located final goods firms are given in Eqs.(A.3) and (A.4), where $\tilde{P}^{PCP}_{f,t}$ ($\tilde{P}^{LCP}_{f,t}$) is the home price index that corresponds to a composite of the inputs produced by PCP (LCP) export firms in the foreign intermediate goods sector, and $\tilde{P}^{*PCP}_{h,t}$ ($\tilde{P}^{*LCP}_{h,t}$) is the foreign price index that corresponds to a composite of the inputs produced by PCP (LCP) export firms in the home intermediate goods sector. In Eqs.(A.5)−(A.7), $\tilde{p}_{h,t}(z_{I|D})$ ($\tilde{p}^*_{f,t}(z^*_{I|D})$) is the home (foreign)-currency price of the input produced by domestic market firm $z_{I|D}$ ($z^*_{I|D}$) in the home (foreign) intermediate goods sector, $\tilde{p}^{PCP}_{f,t}(z^*_{I|X})$ ($\tilde{p}^{LCP}_{f,t}(z^*_{I|X})$) is the home-currency price of the input produced by PCP (LCP) export firm $z^*_{I|X}$ in the foreign intermediate goods sector, and $\tilde{p}^{*PCP}_{h,t}(z_{I|X})$ ($\tilde{p}^{*LCP}_{h,t}(z_{I|X})$) is the foreign-currency price of the input produced by PCP (LCP) export firm $z_{I|X}$ in the home intermediate goods sector.

## Demand

By solving the final goods firms' expenditure minimization problem, we can derive the demand for intermediate inputs required in production activities. To begin with, subject to Eq.(1), the home-located final goods firm $z_{F|j}$ minimizes $\Lambda_t Y_t(z_{F|j}) = \tilde{P}_{h,t} Y_{h,t}(z_{F|j}) + \tilde{P}_{f,t} Y_{f,t}(z_{F|j})$. Thus, the demands of the home-located final goods firm $z_{F|j}$ for $Y_{h,t}(z_{F|j})$ and $Y_{f,t}(z_{F|j})$ are derived as follows:

$$Y_{h,t}(z_{F|j}) = \frac{1}{2} \left( \frac{\tilde{P}_{h,t}}{\Lambda_t} \right)^{-1} Y_t(z_{F|j}), \tag{A.8}$$

$$Y_{f,t}(z_{F|j}) = \frac{1}{2} \left( \frac{\tilde{P}_{f,t}}{\Lambda_t} \right)^{-1} Y_t(z_{F|j}). \tag{A.9}$$

Next, let us consider the home-located final goods firm $z_{F|j}$'s demand for the input produced by home intermediate goods firm $z_{I|D}$. Here, a composite of the inputs produced by home intermediate goods firms that exist continuously in the interval $[0,\frac{1}{2}]$ is given by Eq.(2), and the home-located final goods firm $z_{F|j}$'s nominal expenditure for the inputs produced by these firms is formulated as $\tilde{P}_{h,t}Y_{h,t}(z_{F|j}) = \int_0^{\frac{1}{2}} \tilde{p}_{h,t}(z_{I|D})Y_{h,t}(z_{F|j}, z_{I|D})dz_{I|D}$. Subject to Eq.(2), the home-located final goods firm $z_{F|j}$ determines $Y_{h,t}(z_{F|j}, z_{I|D})$ in order to minimize this expenditure. Thus, the home-located final goods firm $z_{F|j}$'s demand for the input produced by home intermediate goods firm $z_{I|D}$ is derived as follows:

$$Y_{h,t}(z_{F|j}, z_{I|D}) = \left(\frac{\tilde{p}_{h,t}(z_{I|D})}{\tilde{P}_{h,t}}\right)^{-\sigma} Y_{h,t}(z_{F|j}). \tag{A.10}$$

Similarly, the home-located final goods firm $z_{F|j}$'s demands for the inputs produced by foreign PCP intermediate goods firm $z^*_{I|X}$ and foreign LCP intermediate goods firm $z^*_{I|X}$ can be calculated as follows:

$$Y^{PCP}_{f,t}(z_{F|j}, z^*_{I|X}) = \left(\frac{\tilde{p}^{PCP}_{f,t}(z^*_{I|X})}{\tilde{P}^{PCP}_{f,t}}\right)^{-\sigma} \left(\frac{\tilde{P}^{PCP}_{f,t}}{\tilde{P}_{f,t}}\right)^{-\sigma} Y_{f,t}(z_{F|j}), \tag{A.11}$$

$$Y^{LCP}_{f,t}(z_{F|j}, z^*_{I|X}) = \left(\frac{\tilde{p}^{LCP}_{f,t}(z^*_{I|X})}{\tilde{P}^{LCP}_{f,t}}\right)^{-\sigma} \left(\frac{\tilde{P}^{LCP}_{f,t}}{\tilde{P}_{f,t}}\right)^{-\sigma} Y_{f,t}(z_{F|j}). \tag{A.12}$$

Combining Eqs.(A.8) and (A.10), the home-located final goods firm $z_{F|j}$'s demand for the input produced by home intermediate goods firm $z_{I|D}$ is derived in the following exact form:

$$Y_{h,t}(z_{F|j}, z_{I|D}) = \frac{1}{2}\left(\frac{\tilde{p}_{h,t}(z_{I|D})}{\tilde{P}_{h,t}}\right)^{-\sigma} \left(\frac{\tilde{P}_{h,t}}{\Lambda_t}\right)^{-1} Y_t(z_{F|j}). \tag{A.13}$$

Similarly, the home-located final goods firm $z_{F|j}$'s demands for the inputs produced by foreign PCP intermediate goods firm $z^*_{I|X}$ and foreign LCP intermediate goods firm $z^*_{I|X}$ are derived in the exact form as follows:

$$Y^{PCP}_{f,t}(z_{F|j}, z^*_{I|X}) = \frac{1}{2}\left(\frac{\tilde{p}^{PCP}_{f,t}(z^*_{I|X})}{\tilde{P}^{PCP}_{f,t}}\right)^{-\sigma} \left(\frac{\tilde{P}^{PCP}_{f,t}}{\tilde{P}_{f,t}}\right)^{-\sigma} \left(\frac{\tilde{P}_{f,t}}{\Lambda_t}\right)^{-1} Y_t(z_{F|j}), \tag{A.14}$$

$$Y^{LCP}_{f,t}(z_{F|j}, z^*_{I|X}) = \frac{1}{2}\left(\frac{\tilde{p}^{LCP}_{f,t}(z^*_{I|X})}{\tilde{P}^{LCP}_{f,t}}\right)^{-\sigma} \left(\frac{\tilde{P}^{LCP}_{f,t}}{\tilde{P}_{f,t}}\right)^{-\sigma} \left(\frac{\tilde{P}_{f,t}}{\Lambda_t}\right)^{-1} Y_t(z_{F|j}). \tag{A.15}$$

## Resource constraints

The resource constraints in home and foreign intermediate inputs used by home and foreign final goods firms are represented as follows:

$$Y_{h,t} \geq \frac{1}{2}\left(\frac{\tilde{P}_{h,t}}{\Lambda_t}\right)^{-1} \left(\sum_{j''=D,X} \int_0^{n_{j'',t}} Y_t(z_{F|j''})dz_{F|j''} + \int_0^{n^*_{MN,t}} Y_t(z_{F|MN})dz_{F|MN}\right)$$

$$+\left(\sum_{j''=D,X} n_{j'',t}^{1+\gamma} + n_{MN,t}^{*1+\gamma}\right), \tag{A.16}$$

$$Y_{f,t} \geq \frac{1}{2}\left(\frac{\tilde{P}_{f,t}}{\Lambda_t}\right)^{-1}\left(\sum_{j''=D,X}\int_0^{n_{j'',t}} Y_t(z_{F|j''})dz_{F|j''} + \int_0^{n^*_{MN,t}} Y_t(z_{F|MN})dz_{F|MN}\right)$$

$$+\left(\sum_{j''=D,X} n_{j'',t}^{1+\gamma} + n_{MN,t}^{*1+\gamma}\right), \tag{A.17}$$

$$Y^*_{h,t} \geq \frac{1}{2}\left(\frac{\tilde{P}^*_{h,t}}{\Lambda^*_t}\right)^{-1}\left(\sum_{j''=D,X}\int_0^{n^*_{j'',t}} Y^*_t(z^*_{F|j''})dz^*_{F|j''} + \int_0^{n_{MN,t}} Y^*_t(z^*_{F|MN})dz^*_{F|MN}\right)$$

$$+\left(\sum_{j''=D,X} n_{j'',t}^{*1+\gamma} + n_{MN,t}^{1+\gamma}\right), \tag{A.18}$$

$$Y^*_{f,t} \geq \frac{1}{2}\left(\frac{\tilde{P}^*_{f,t}}{\Lambda^*_t}\right)^{-1}\left(\sum_{j''=D,X}\int_0^{n^*_{j'',t}} Y^*_t(z^*_{F|j''})dz^*_{F|j''} + \int_0^{n_{MN,t}} Y^*_t(z^*_{F|MN})dz^*_{F|MN}\right)$$

$$+\left(\sum_{j''=D,X} n_{j'',t}^{*1+\gamma} + n_{MN,t}^{1+\gamma}\right), \tag{A.19}$$

where $Y^*_{h,t}$ ($Y^*_{f,t}$) is a composite of the inputs produced by export (domestic market) firms in the home (foreign) intermediate goods sector.

# Appendix B

In this appendix, we specify the price indexes that are defined using the specification of consumption indexes in the main text. We also derive the demand of the home household $x$ for final goods.

## Price indexes

The home and foreign CPIs are shown as follows:

$$P_t = P_{T,t}^{\delta} P_{N,t}^{1-\delta}, \tag{B.1}$$

$$P^*_t = P_{T,t}^{*\delta} P_{N,t}^{*1-\delta}, \tag{B.2}$$

where

$$P_{T,t} = \left(n_{D,t}(P_{h,t})^{1-\rho} + n^*_{X,t}(P_{f,t})^{1-\rho}\right)^{\frac{1}{1-\rho}}, \tag{B.3}$$

$$P^*_{T,t} = \left(n^*_{D,t}(P^*_{f,t})^{1-\rho} + n_{X,t}(P^*_{h,t})^{1-\rho}\right)^{\frac{1}{1-\rho}}, \tag{B.4}$$

$$P_{N,t} = \left(n^*_{MN,t}(P_{h|f'sMN,t})^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{B.5}$$

$$P^*_{N,t} = \left(n_{MN,t}(P^*_{f|h'sMN,t})^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{B.6}$$

and

$$P_{h,t} = \left(\frac{1}{n_{D,t}}\int_0^{n_{D,t}} p_{h,t}(z_{F|D})^{1-\theta}dz_{F|D}\right)^{\frac{1}{1-\theta}},\ P_{f,t} = \left(\frac{1}{n^*_{X,t}}\int_0^{n^*_{X,t}} p_{f,t}(z^*_{F|X})^{1-\theta}dz^*_{F|X}\right)^{\frac{1}{1-\theta}}, \tag{B.7}$$

$$P^*_{f,t} = \left(\frac{1}{n^*_{D,t}}\int_0^{n^*_{D,t}} p^*_{f,t}(z^*_{F|D})^{1-\theta}dz^*_{F|D}\right)^{\frac{1}{1-\theta}},\ P^*_{h,t} = \left(\frac{1}{n_{X,t}}\int_0^{n_{X,t}} p^*_{h,t}(z_{F|X})^{1-\theta}dz_{F|X}\right)^{\frac{1}{1-\theta}}, \tag{B.8}$$

$$P_{h|f'sMN,t} = \left(\frac{1}{n^*_{MN,t}}\int_0^{n^*_{MN,t}} p_{h,t}(z_{F|MN})^{1-\theta}dz_{F|MN}\right)^{\frac{1}{1-\theta}}, \tag{B.9}$$

$$P^*_{f|h'sMN,t} = \left(\frac{1}{n_{MN,t}}\int_0^{n_{MN,t}} p^*_{f,t}(z^*_{F|MN})^{1-\theta}dz^*_{F|MN}\right)^{\frac{1}{1-\theta}}. \tag{B.10}$$

In Eqs.(B.1) and (B.2), $P_t$ ($P^*_t$) is the CPI of the home (foreign) country, $P_{T,t}$ ($P^*_{T,t}$) is the home (foreign) price index of final tradable goods, and $P_{N,t}$ ($P^*_{N,t}$) is the home (foreign) price index of final non-tradable goods. In Eqs.(B.3)−(B.6), $P_{h,t}$ ($P^*_{h,t}$) is the home (foreign) price index of the goods produced by home-located final goods firm $z_{F|D}$ ($z_{F|X}$), $P_{f,t}$ ($P^*_{f,t}$) is the home (foreign) price index of the goods produced by foreign-located final goods firm $z^*_{F|X}$ ($z^*_{F|D}$), and $P_{h|f'sMN,t}$ ($P^*_{f|h'sMN,t}$) is the home (foreign) price index of the goods produced by home (foreign)-located final goods firm $z_{F|MN}$ ($z^*_{F|MN}$). In Eqs.(B.7)−(B.10), $p_{h,t}(z_{F|D})$ ($p^*_{f,t}(z^*_{F|D})$) is the home (foreign)-currency price of the goods produced by home (foreign)-located final goods firm $z_{F|D}$ ($z^*_{F|D}$), $p_{f,t}(z^*_{F|X})$ ($p^*_{h,t}(z_{F|X})$) is the home (foreign)-currency price of the goods produced by foreign (home)-located final goods firm $z^*_{F|X}$ ($z_{F|X}$), and $p_{h,t}(z_{F|MN})$ ($p^*_{f,t}(z^*_{F|MN})$) is the home (foreign)-currency price of the goods produced by home (foreign)-located final goods firm $z_{F|MN}$ ($z^*_{F|MN}$).

## Demand

Let us consider the optimal consumption demands for $C_{h,t}(z_{F|D}, x)$, $C_{f,t}(z^*_{F|X}, x)$ and $C_{h,t}(z_{F|MN}, x)$. To begin with, the home household $x$'s expenditure for the sum of $C_{T,t}(x)$ and $C_{N,t}(x)$ is given as follows:

$$P_t C_t(x) = P_{T,t} C_{T,t}(x) + P_{N,t} C_{N,t}(x). \tag{B.11}$$

Subject to Eq.(B.11), the home household $x$ maximizes Eq.(16). Thus, the demands of the home household $x$ for $C_{T,t}(x)$ and $C_{N,t}(x)$ are derived as follows:

$$C_{T,t}(x) = \delta \left( \frac{P_{T,t}}{P_t} \right)^{-1} C_t(x), \tag{B.12}$$

$$C_{N,t}(x) = (1-\delta) \left( \frac{P_{N,t}}{P_t} \right)^{-1} C_t(x). \tag{B.13}$$

In a similar manner, the optimal demands for $C_{h,t}(x)$, $C_{f,t}(x)$ and $C_{h|f'sMN,t}(x)$ can be calculated as follows:

$$C_{h,t}(x) = n_{D,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\rho} C_{T,t}(x), \tag{B.14}$$

$$C_{f,t}(x) = n^*_{X,t} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\rho} C_{T,t}(x), \tag{B.15}$$

$$C_{h|f'sMN,t}(x) = n^*_{MN,t} \left( \frac{P_{h|f'sMN,t}}{P_{N,t}} \right)^{-\theta} C_{N,t}(x). \tag{B.16}$$

Next, let us consider the home household $x$'s demand for the home final good $z_{F|D}$. Here, $C_{h,t}(x)$ is given by Eq.(19), and the nominal consumption expenditure that corresponds to $C_{h,t}(x)$, is defined as $P_{h,t}C_{h,t}(x) \equiv \int_0^{n_{D,t}} p_{h,t}(z_{F|D}) C_{h,t}(z_{F|D}, x) dz_{F|D}$. Subject to this definition, the agent determines $C_{h,t}(z_{F|D}, x)$ in order to maximize Eq.(19). Thus, the optimal consumption demand for $C_{h,t}(z_{F|D}, x)$ is derived as follows:

$$C_{h,t}(z_{F|D}, x) = \frac{1}{n_{D,t}} \left( \frac{p_{h,t}(z_{F|D})}{P_{h,t}} \right)^{-\theta} C_{h,t}(x). \tag{B.17}$$

Similarly, the optimal consumption demands for $C_{f,t}(z^*_{F|X}, x)$ and $C_{h,t}(z_{F|MN}, x)$ can be calculated as follows:

$$C_{f,t}(z^*_{F|X}, x) = \frac{1}{n^*_{X,t}} \left( \frac{p_{f,t}(z^*_{F|X})}{P_{f,t}} \right)^{-\theta} C_{f,t}(x), \tag{B.18}$$

$$C_{h,t}(z_{F|MN},x) = \frac{1}{n^*_{MN,t}} \left( \frac{p_{h,t}(z_{F|MN})}{P_{h|f'sMN,t}} \right)^{-\theta} C_{h|f'sMN,t}(x). \tag{B.19}$$

From Eqs.(B.12), (B.14) and (B.17), the optimal consumption demand for $C_{h,t}(z_{F|D},x)$ is derived in the following exact form:

$$C_{h,t}(z_{F|D},x) = \delta \left( \frac{p_{h,t}(z_{F|D})}{P_{h,t}} \right)^{-\theta} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\rho} \left( \frac{P_{T,t}}{P_t} \right)^{-1} C_t(x). \tag{B.20}$$

Similarly, the optimal consumption demands for $C_{f,t}(z^*_{F|X},x)$ and $C_{h,t}(z_{F|MN},x)$ are derived in the exact form as follows:

$$C_{f,t}(z^*_{F|X},x) = \delta \left( \frac{p_{f,t}(z^*_{F|X})}{P_{f,t}} \right)^{-\theta} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\rho} \left( \frac{P_{T,t}}{P_t} \right)^{-1} C_t(x), \tag{B.21}$$

$$C_{h,t}(z_{F|MN},x) = (1-\delta) \left( \frac{p_{h,t}(z_{F|MN})}{P_{h|f'sMN,t}} \right)^{-\theta} \left( \frac{P_{h|f'sMN,t}}{P_{N,t}} \right)^{-\theta} \left( \frac{P_{N,t}}{P_t} \right)^{-1} C_t(x). \tag{B.22}$$

From Eqs.(B.20)−(B.22), the aggregate home consumption demand for goods produced by the domestic market firm $z_{F|D}$, which belongs to the home-located tradable sector final goods firms, the export firm $z^*_{F|X}$, which belongs to the foreign-located tradable sector final goods firms and the foreign multinational firm $z_{F|MN}$, which belongs to the home-located non-tradable sector final goods firms are given as follows:

$$\int_0^1 C_{h,t}(z_{F|D},x)dx \equiv C_{h,t}(z_{F|D}) = \delta \left( \frac{p_{h,t}(z_{F|D})}{P_{h,t}} \right)^{-\theta} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\rho} \left( \frac{P_{T,t}}{P_t} \right)^{-1} C_t, \tag{B.23}$$

$$\int_0^1 C_{f,t}(z^*_{F|X},x)dx \equiv C_{f,t}(z^*_{F|X}) = \delta \left( \frac{p_{f,t}(z^*_{F|X})}{P_{f,t}} \right)^{-\theta} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\rho} \left( \frac{P_{T,t}}{P_t} \right)^{-1} C_t, \tag{B.24}$$

$$\int_0^1 C_{h,t}(z_{F|MN},x)dx \equiv (1-\delta) \left( \frac{p_{h,t}(z_{F|MN})}{P_{h|f'sMN,t}} \right)^{-\theta} \left( \frac{P_{h|f'sMN,t}}{P_{N,t}} \right)^{-\theta} \left( \frac{P_{N,t}}{P_t} \right)^{-1} C_t. \tag{B.25}$$

# Appendix C

This Appendix illustrates the public demand for goods produced by the home-located domestic market firm $z_{F|D}$, the foreign-located export firm $z^*_{F|X}$, and the home-located foreign multinational firm $z_{F|MN}$. Using the process of analysis adopted in Appendix B, the public demand for these goods is derived as follows:

$$G_{h,t}(z_{F|D}) = \delta \left( \frac{p_{h,t}(z_{F|D})}{P_{h,t}} \right)^{-\theta} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\rho} \left( \frac{P_{T,t}}{P_t} \right)^{-1} G_t, \tag{C.1}$$

$$G_{f,t}(z^*_{F|X}) = \delta \left( \frac{p_{f,t}(z^*_{F|X})}{P_{f,t}} \right)^{-\theta} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\rho} \left( \frac{P_{T,t}}{P_t} \right)^{-1} G_t, \tag{C.2}$$

$$G_{h,t}(z_{F|MN}) = (1-\delta) \left( \frac{p_{h,t}(z_{F|MN})}{P_{h|f'sMN,t}} \right)^{-\theta} \left( \frac{P_{h|f'sMN,t}}{P_{N,t}} \right)^{-\theta} \left( \frac{P_{N,t}}{P_t} \right)^{-1} G_t. \tag{C.3}$$

# Appendix D

This Appendix illustrates closed form solutions derived in the initial steady state with $G = G^* = 0$, $B_H = B_H^* = B_F = B_F^* = 0$ and $\mu = \mu^* = 1$.[1]

To begin with, from Eq.(30) and its foreign analog, we derive:

$$W = W^* = \frac{\xi \kappa}{\xi - 1}. \tag{D.1}$$

Next, from the two conditions of $PC = P^*C^*$ and $P_T C_T = \varepsilon P_T^* C_T^*$, we derive:

$$\varepsilon = 1. \tag{D.2}$$

Further, from Eqs.(D.1) and (D.2), and the relationships of $\tilde{p}_h^*(z_{I|X}) = \hat{\tilde{p}}_h(z_{I|X})/\varepsilon$, $\tilde{p}_f(z^*_{I|X}) = \varepsilon \hat{\tilde{p}}_f^*(z^*_{I|X})$, $\tilde{p}_h(z_{I|D}) = \frac{\sigma}{\sigma-1} W$, $\tilde{p}_f^*(z^*_{I|D}) = \frac{\sigma}{\sigma-1} W^*$, $\hat{\tilde{p}}_h(z_{I|X}) = \tilde{p}_h(z_{I|D})$ and $\hat{\tilde{p}}_f^*(z^*_{I|X}) = \tilde{p}_f^*(z^*_{I|D})$, we derive:[2]

$$\tilde{p}_h(z_{I|D}) = \tilde{p}_h^*(z_{I|X}) = \tilde{p}_f^*(z^*_{I|D}) = \tilde{p}_f(z^*_{I|X}) = \frac{\sigma}{\sigma - 1} \frac{\xi \kappa}{\xi - 1}. \tag{D.3}$$

Moreover, from Eqs.(14), (A.1), (A.2), (A.5), and (D.3), we derive:

$$\Lambda = \Lambda^* = 2^{\frac{1}{\sigma-1}} \frac{\sigma}{\sigma - 1} \frac{\xi \kappa}{\xi - 1}. \tag{D.4}$$

Therefore, from Eqs.(31)−(35), (D.2) and (D.4), we derive:

$$p_h(z_{F|D}) = p_h^*(z_{F|X}) = p_h(z_{F|MN}) = p_f^*(z^*_{F|D}) = p_f(z^*_{F|X})$$

$$= p_f^*(z^*_{F|MN}) = 2^{\frac{1}{\sigma-1}} \frac{\theta}{\theta - 1} \frac{\sigma}{\sigma - 1} \frac{\xi \kappa}{\xi - 1}. \tag{D.5}$$

Here, from Eqs.(44), (45), (47), (48), (50), (D.2), and (D.5); the condition $P_T = \varepsilon P_T^*$; and the definitions of $\Lambda_t$ and $\Lambda_t^*$ we derive:

$$n_D = n_X = n_D^* = n_X^*. \tag{D.6}$$

[1] The initial steady-state levels of home and foreign money supply are assumed to be $M = M^* = \chi(1-\beta)^{-1}$.

[2] $\hat{\tilde{p}}_h(z_{I|X})$ $\left(\hat{\tilde{p}}_f^*(z^*_{I|X})\right)$ is the predetermined component of the foreign (home)-currency price of input produced by each home (foreign) export firm in the intermediate goods sector.

Therefore, from Eqs.(14), (38), (44), (A.5), (D.2), (D.3) and (D.6), we derive:

$$n_D(= n_X = n_D^* = n_X^*) = \left( \frac{\delta}{2^{\frac{2\sigma-1}{\sigma-1}}\theta} \frac{\sigma-1}{\sigma} \frac{\xi-1}{\xi\kappa} \right)^{\frac{1}{1+\gamma}}. \tag{D.7}$$

Similarly, from Eqs.(14), (46), (49), (A.5), (D.2), and (D.3), the number of foreign (home) multinational firms among home (foreign)-located final goods firms is derived as follows:

$$n_{MN}^*(= n_{MN}) = \left( \frac{1-\delta}{2^{\frac{\sigma}{\sigma-1}}\theta} \frac{\sigma-1}{\sigma} \frac{\xi-1}{\xi\kappa} \right)^{\frac{1}{1+\gamma}}. \tag{D.8}$$

Finally, from Eqs.(14), (23), (44)−(49), (A.5), (A.8), (A.9), and (D.4)−(D.8), $\Pi_F(z_{F|j}) = \frac{p_h Y(z_{F|j})}{\theta}$, and their foreign analogs, the home and foreign labor services are derived as follows:

$$\ell = \ell^* = \frac{\sigma-1}{\sigma} \frac{\xi-1}{\xi\kappa}. \tag{D.9}$$