Capital income taxation, public capital and matching frictions in an overlapping generations model

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Abstract

This study develops a standard overlapping generations model to investigate the role of capital income tax within an endogenous growth framework. It introduces capital income tax to finance public investment as a source of endogenous growth. Furthermore, we assume that matching frictions give rise to unemployment. It was found that an increase in capital income tax can contribute to improvements in economic growth, employment, and welfare.

JEL classification: H51, H20, and J51.

Keywords: Capital income taxation, Endogenous growth, Unemployment

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1. Introduction

This study investigated the role of capital income tax on economic growth, employment, and welfare. Traditionally, capital income taxation has been controversial. As noted by Judd (1985), Chamley (1986), and Lucas (1990), the optimal capital income tax is zero in the Ramsey model. If the government imposes a tax on capital income, it directly reduces household interest income, and simultaneously reduces savings. The decline in savings inhibits capital accumulation and economic growth. Conversely, Bertola (1996), Uhlig and Yanagawa (1996), and Caballé (1998) found that increasing capital income tax can enhance economic growth in an overlapping generations (OLG) model. Households decide to save during young periods and receive the return from savings during old periods in a standard overlapping generation framework. In this case, even if the government imposes a tax on interest income, savings, and economic growth may not be inhibited. For example, Uhlig and Yanagawa (1996) introduced a tax reform that involved cutting labor income tax with a higher capital income tax. Their study revealed that such tax reforms promote savings and economic growth. These studies testify to the impact of capital income tax on economic growth within a theoretical framework. According to Chen et al. (2017), some studies empirically demonstrated that capital income tax inhibits economic growth, whereas others show that capital income tax is neutral or contributes to economic growth. Thus, there is a disagreement regarding the impact of capital income taxation on economic growth.

Although Bertola (1996), Uhlig and Yanagawa (1996), and Caballé (1998) demonstrated that capital income taxation contributes to economic growth, they assume full employment. Kunze and Schuppert (2010) insist that several developed countries face a combination of sluggish growth and rising unemployment rates. Thus, Kunze and Schuppert (2010) considered tax reform, as did Uhlig and Yanagawa (1996), and involuntary unemployment caused by wage bargaining, as explained by Layard et al. (1991) and Bräuninger (2005). They found that such a tax reform could contribute to improvements in economic growth, employment, and welfare in an OLG model.

Studies examining the impact of capital income tax on economic growth, employment, and welfare using an endogenous growth model are insufficient. This is because several previous studies that incorporated endogenous growth assumed full employment. Therefore, the current study develops a standard OLG model to investigate the impact of capital income tax on macroeconomic performance building as in Kunze and Schuppert (2010). Compared to Kunze and Schuppert (2010), there are two major differences in our study. First, following Barro (1990) and Futagami et al. (1993), we assume that public spending generates endogenous growth[3]: Numerous empirical studies have testified to the impact of government spending on employment and output. Using US data, Monacelli et al. (2010) showed that public spending promotes employment. Kato and Miyamoto (2013) revealed that fiscal stimulus policies reduce unemployment in Japan. Abiad et al. (2016) demonstrated that public investment increases output and reduces unemployment in advanced countries. Petrović et al. (2021) found that public investment generates positive effects on outputs, employment, and wages in Central and East European countries. Second, we assume that matching frictions lead to unemployment,[4] as in Batyra et al. (2019) and Zhang et al. (2022). If we introduce involuntary unemployment, as Kunze and Schuppert (2010), unemployment may not improve unless wages decline. However, Zhang et al. (2022) revealed that rising wages and improved employment are compatible.

Following conventional wisdom, capital income tax is a controversial system. This study develops a standard OLG model that includes endogenous growth and matching frictions. The contributions of this study are summarized as follows: First, this study reveals that a rise in capital income tax without tax reform promotes not only economic growth and employment but also welfare, although tax reform is the definitive assumption in Kunze and Schuppert (2010). Second, this study demonstrates that increasing capital income tax can improve welfare for the current, subsequent, and present old generations. In Kunze and Schuppert (2010), old generations uniquely experience welfare losses with a higher capital income tax. Third, several empirical studies have found that public spending can contribute to output and employment. The results of this study reinforce the findings of previous empirical studies.

The remainder of this paper is organized as follows. Section 2 describes the proposed model. Section 3 discusses welfare effects. Section 4 presents numerical examples. Section 5 concludes the paper.

2. Model

Consider a standard OLG model with discrete times. Agents in this economy are households, firms, and the government. The identical households experience two

[3] Kunze and Schuppert (2010) considered productive externality as a source of endogenous growth as shown by Romer (1986).

[4] Raurich and Sorolla (2003) and Greiner and Flaschel (2010) constructed an endogenous growth model incorporating public capital and involuntary unemployment caused by union wage setting.

periods: young and old. Households endow one unit of labor and supply it due to labor market inelasticity. The population size is constant. Identical firms input capital and labor to provide final goods in a competitive economy. The government imposes a tax on capital income to provide for public investment, which increases economic growth, as shown by Barro (1990) and Futagami et al. (1993).

2.1 Matchings

Following Zhang et al. (2022), all young households are unemployed at the beginning of the period. They search for jobs and experience matching frictions. Young individuals who are unsuccessful in finding jobs remain unemployed. When the households become old, they retire. Following Den Haan et al. (2000) and Hashimoto et al. (2020), the matching technology in period t is given by

$$M(v_tN,N)=\frac{(v_tN)N}{[(v_tN)^{\sigma}+N^{\sigma}]^{\frac{1}{\sigma}}},\sigma>0 \tag{1}$$

where v_t is the vacant jobs per workforce, and N is the constant population size. Note that N represents the workforce and number of unemployed workers at the beginning of the period. Thus, v_tN denotes the total number of vacant jobs. Suppose θ is the labor market tightness and is defined as follows.

$$\theta_t \equiv v_tN/N=v_t. \tag{2}$$

where θ_t is the ratio of the total number of vacancies to the total number of applicants. If vacant jobs increase, the tightness of the labor market increases. We denote q_t as the probability that firms hire workers and is described as follows:

$$q_t=\frac{M(v_tN,N)}{v_tN}=(\theta_t^{\sigma}+1)^{\frac{-1}{\sigma}}. \tag{3}$$

Suppose l_t is the employment rate and the employment rate equals total matching, that is, $l_tN=M(v_tN,N)$. From Equations (1)–(3), we obtain

$$l_t=(1+\theta_t^{-\sigma})^{\frac{-1}{\sigma}}. \tag{4}$$

In this equation, an increase in labor market tightness increases employment, as in the standard search theory literature[5]. We assume that the population size is unity for the remainder of the article; that is, $N=1$.

2.2 Households

Young individuals search for jobs and earn a wage income if they find jobs. Employed

[5] See Pissarides (2000).

households split it between consumption in the young age and savings. Households that fail to find jobs remain unemployed. We consider "large household" in line with Lu (2015) and Zhang et al. (2022). This assumption indicates that households share their income. When households become old, they retire and consume their savings. Households derive utility from consumption in both periods. As denoted by Josten (2006) and Kunze (2010), The log utility function is empirically supported. Thus, the utility function can be expressed as follows:

$$U_t = \log c_t + \beta \log d_{t+1}, 0 < \beta < 1 \tag{5}$$

In this equation, c_t and d_{t+1} are the consumption in young periods and old periods respectively, and β is the discount factor. Kunze and Schuppert (2010) assume the CES utility function. The budget constraints for households are as follows:

$$c_t + s_t = w_t l_t, \tag{6}$$

$$d_{t+1} = [1 + (1-\tau) r_{t+1}] s_t, \tag{7}$$

where s_t is the savings, w_t is the wage, l_t is the employment rate, r_{t+1} is the interest rate and τ is the capital income tax to finance public capital. The optimal allocation is as follows:

$$c_t = \frac{w_t l_t}{(1+\beta)}, \tag{8}$$

$$d_{t+1} = \frac{\beta[1 + (1-\tau) r_{t+1}] w_t l_t}{(1+\beta)}. \tag{9}$$

$$s_t = \frac{\beta w_t l_t}{1+\beta} \tag{10}$$

For later use, we derive the values for households with additional employment in line with Batyra et al. (2019). Using Equations (5)–(7), we obtain:

$$\frac{\partial U_t}{\partial l_t} = U_{c,t} w_t. \tag{11}$$

In this equation, $U_{c,t}$ denotes the marginal utility of consumption during the young periods. Differing from Batyra et al. (2019), households work only during young periods and retire in old periods in the present study[6].

2.3 Firms

The basic structure of a firm's behavior is similar to that of Zhang et al. (2022). Firms are identical and employ both labor and capital to provide final goods in a competitive market. The production technology is as follows:

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[6] Batyra et al. (2019) constructed three periods' OLG model, namely, young, senior, and old. In their study, households work during young and senior periods.

$$y_t = Ak_t^{\alpha}(e_t l_t)^{1-\alpha},\ A > 0,\ \alpha \in (0,1) \tag{12}$$

where, y_t is the total output, k_t and l_t are the total capital stocks and total labor inputs, respectively, and e_t is labor efficiency. In other words, e_t indicates a positive externality that gives rise to endogenous growth. Differentiating y_t with respect to k_t and l_t, we obtain:

$$y_{k,t} = \alpha A k_t^{\alpha-1}(e_t l_t)^{1-\alpha}, \tag{13}$$

$$y_{l,t} = (1-\alpha) A k_t^{\alpha} e_t^{1-\alpha} l_t^{-\alpha}. \tag{14}$$

respectively, where $y_{k,t}$ and $y_{l,t}$ are the marginal products of capital and labor, respectively. Firms incur the cost of opening up jobs. Denoting λ_t as the vacancy costs, such as advertising and hiring costs. Then, the total amount of vacancy cost is described as $\lambda_t v_t$. Assuming no capital depreciation, in line with Kunze and Schuppert (2010), profits are given by

$$y_t - w_t l_t - r_t k_t - \lambda_t v_t. \tag{15}$$

Following Lu (2015) and Chen et al. (2016), we assume[7] the following vacancy costs.

$$\lambda_t = \mu \bar{k}_t, \qquad \mu > 0 \tag{16}$$

where $\bar{k}_t$ is average capital. Equation (16) captures the vacancy cost as a proportion of average capital[8]. As we assume that the population size is unity, $\bar{k}_t = k_t$ holds in the equilibrium. The firms choose k_t and v_t to maximize their profits. Recall that q_t is the probability that a firm fills vacancies. The employment rate is expressed as follows:

$$l_t = q_t v_t. \tag{17}$$

Using Equations (13)–(17), the first-order conditions for k_t and v_t are given as follows:

$$y_{k,t} = r_t, \tag{18}$$

$$y_{l,t} q_t = w_t q_t + \mu \bar{k}_t, \tag{19}$$

The left-hand sides of Equations (18) and (19) indicate the marginal revenue with additional capital and vacancies, respectively. The right sides of Equations (18) and (19) capture the marginal costs of additional capital and vacancies, respectively.

Following Kalaitzidakis and Kalyvitis (2004) and Yakita (2008), labor efficiency is given by

$$e_t = \frac{k_t^{\delta} \rho_t^{1-\delta}}{l_t}, \delta \in (0,1) \tag{20}$$

where ρ_t is public capital. If $\delta \to 1$, the labor efficiency boils down to $e_t = k_t / l_t$ as

[7] As quoted by Chen et al. (2016), "This setup is natural the more the economy uses capital, the more the firms compete for resources and the greater vacancy posting cost will be."

[8] Hagedorn and Manovskii (2008) assumed vacancy cost is the liner to capital.

assumed by Kunze and Schuppert (2010) and Ono (2010). In this case, government expenditure does not contribute to productivity or welfare, as assumed by Chamley (1986) and Uhlig and Yanagawa (1996). On the contrary, if $\delta \to 0$, the labor efficiency boils down to $e_t = \rho_t / l_t$ as assumed by Shioji (2001) and Maebayashi (2013). Note that if we assume labor efficiency as $e_t = \rho_t / l_t$, our analytical results do not change qualitatively.

2.4 Wage bargaining

When firms and job-seekers meet, they negotiate wages: Following Zhang et al. (2022), the objective function is expressed as

$$(1-\gamma)\log(y_{l,t} - w_t) + \gamma \log \frac{\partial U_t}{\partial l_t}, \quad 0 < \gamma < 1. \tag{21}$$

γ and $1-\gamma$ are the bargaining power for workers and firms respectively. $y_{l,t} - w_t$ is the firm's marginal profit with additional employment. Firms and workers decide their wages to maximize Equation (21). The negotiated wage[9] is given as follows:

$$w_t = \gamma y_{l,t}. \tag{22}$$

This expression is consistent with the results reported by Zhang et al. (2022). The negotiated wage is a fraction of the marginal product of labor.

2.5 Government

The government provides public investments financed by capital income taxes under a balanced budget. The government's budget constraint in period t is as follows:

$$\tau r_t s_{t-1} = \rho_t. \tag{23}$$

The left-hand side of this equation indicates tax revenue. The right-hand side of this equation denotes expenditure on public investment. Kunze and Schuppert (2010) considered labor and capital income taxes to finance unemployment benefits and public pensions under a balanced budget. Their study assumed unemployment benefits and public pension is a fraction of wages.

2.6 Equilibrium

The clearing condition of the capital market is given as follows.

$$k_{t+1} = s_t. \tag{24}$$

See the Appendix for the derivation of this equation. Suppose g_t is the growth rate and defined as $g_t \equiv y_{t+1}/y_t = k_{t+1}/k_t$. Using Equations (11), (14), (15), (19), (23),

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[9] Substituting Equation (11) into Equation (21), the first order condition with respect to wage is given by $\frac{1-\gamma}{y_{l,t} - w_t} = \frac{\gamma}{w_t}$. Solve this equation with w_t we obtain Equation (22).

(24), and (25), we obtain the following constant growth rate:

$$g = \frac{\beta\gamma\bar{A}(\alpha\tau)^{\varphi}}{1+\beta}, \tag{25}$$

where $\bar{A} \equiv (1-\alpha)A^{\frac{1}{1-(1-\delta)(1-\alpha)}}$ and $\varphi \equiv \frac{(1-\delta)(1-\alpha)}{1-(1-\delta)(1-\alpha)} > 0$. From this equation, transitional dynamics does not occur. We assume enough large A to ensure sustained growth. By differentiating this equation with respect to τ, we obtain

$$\frac{dg}{d\tau} = \frac{\beta\gamma\varphi\bar{A}\alpha^{\varphi}\tau^{\varphi-1}}{1+\beta} > 0. \tag{26}$$

A rise in capital income tax increases public capital and, hence, increases working income and savings. Therefore, higher capital income tax can lead to faster economic growth. If the public investment is financed by the labor income tax, a growth-maximizing tax rate exists, as explained by Barro (1990). Higher labor income tax for financing public investment has two opposite effects on savings. First, wages increase with higher labor income tax. It promotes savings. Second, an increase in the labor income tax directly reduces disposable income. It reduces savings. Whether higher labor income tax promotes economic growth depends on the level of labor income tax.

Next, we analyze how capital income taxation affects unemployment. By substituting Equation (22) into Equation (19), the first-order condition with respect to v_t can be rewritten as

$$y_{l,t}q_t = \gamma y_{l,t}q_t + \mu\bar{k}_t. \tag{27}$$

The left-hand side of the equation indicates the marginal revenue with additional vacancies, while the right-hand side of this equation denotes the marginal cost with additional vacancies. Using Equations (2), (4), (14), (17), (25), and (27), we obtain the following tightness of the labor market and employment rate:

$$\theta = \frac{(1-\gamma)\bar{A}(\alpha\tau)^{\varphi}}{\mu}, \tag{28}$$

$$l = \left\{1 + \left[\frac{\mu}{(1-\gamma)\bar{A}(\alpha\tau)^{\varphi}}\right]^{\sigma}\right\}^{\frac{-1}{\sigma}}, \tag{29}$$

From Equations (28) and (29), we derive

$$\frac{dl}{d\tau} = \frac{dl}{d\theta}\frac{d\theta}{d\tau} = (\theta^{\sigma}+1)^{\frac{-(\sigma+1)}{\sigma}}\frac{\varphi}{\tau}\theta > 0. \tag{30}$$

An increase in capital income tax promotes public investment. It improves the marginal products of labor, that is, $y_{l,t}$, and hence improves marginal revenue, as explained by the left-hand side of Equation (27). Conversely, a higher capital income tax raises the marginal cost of opening a new job, as explained by the right-hand side of Equation (27). As the former effect uniquely dominates the latter, firms create vacant jobs. Labor market tightness increases with an increase in vacant jobs, which improves employment, as shown in Equation (4). Consequently, an increase in capital income tax promotes employment.

Public investment increased wage levels in this study. The wage level in period t is given by

$$w_t = \gamma y_{l,t},$$

$$= \gamma \bar{A} \alpha^{\varphi} \left\{ \tau^{\varphi\sigma} + \left[\frac{\mu}{(1-\gamma)\bar{A}\alpha^{\varphi}} \right]^{\sigma} \right\}^{\frac{1}{\sigma}} k_t. \tag{31}$$

As $\sigma > 0$ and $\varphi > 0$ hold, we have $\partial w_t / \partial \tau > 0$ from this equation. Therefore, a higher capital income tax increases both employment and wages. These results are similar to those reported by Zhang et al. (2022). Zhang et al. (2022) introduced automated capital and robotic taxes[10]. Their study demonstrated that robot taxes increase both employment and wages. Therefore, we obtain the following proposition.

Proposition 1

A rise in capital income tax promotes not only economic growth but also employment.

Kunze and Schuppert (2010) found that cutting labor income tax with a higher capital income tax promotes savings. Further, such a tax reform moderates bargained wages and, hence, reduces unemployment. Therefore, an increase in capital income tax promotes economic growth and employment, although wages decline with a higher capital income tax. In contrast, this study demonstrates that a rise in capital income tax promotes economic growth and employment, while wages rise with higher capital income tax.

3. Effects on the welfare

Following Kunze and Schuppert (2010), this section analyzes how capital income tax affects welfare for the current, subsequent, and present old generations. First, we verified the welfare effects for the current generation. Using Equations (8), (9),

[10] Zhang et al. (2022) introduced two types of capital, i.e. traditional physical capital and automation capital. Automation capital is the perfect substitute to labor inputs and robot tax is imposed on the use of automation capital.

(13), (14), (18), (20), (22), (23), and (24), we have the following consumption levels during the young and old periods:

$$c_t = \frac{\gamma \bar{A}(\alpha\tau)^{\varphi} k_t}{1+\beta}, \tag{32}$$

$$d_{t+1} = \frac{\beta[1 + (1-\tau)\psi\tau^{\varphi}]\gamma \bar{A}(\alpha\tau)^{\varphi} k_t}{1+\beta}, \tag{33}$$

respectively, where $\psi \equiv (\alpha A)^{\frac{1}{1-(1-\delta)(1-\alpha)}}$. Note that the interest rate is constant, that is $r_{t+1} = r = \psi\tau^{\varphi}$ in this study although the equilibrium interest rate is independent of the tax rate in Kunze and Schuppert (2010). Denoting $\widehat{U}_t$ as the indirect utility for the current generations, we obtain the following indirect utility for the current generations using Equations (5), (32), and (33):

$$\begin{aligned} \widehat{U}_t &= \log c_t + \beta \log d_{t+1}, \\ &= (1+\beta)\varphi \log \tau + \beta \log[1 + (1-\tau)\psi\tau^{\varphi}] + H_t, \end{aligned} \tag{34}$$

where $H_t \equiv (1+\beta)\log\frac{\gamma\bar{A}\alpha^{\varphi}}{1+\beta} + \beta\log\beta + (1+\beta)\log k_t$ and H_t is independent of the tax rate. As previously mentioned, we assume that there are large households. Thus, Equation (34) is an economy-wide indirect utility. Kunze and Schuppert (2010) delivered the indirect utility for employed and unemployed agents separately. Differentiating Equation (34) with respect to τ, we have

$$\frac{\partial \widehat{U}_t}{\partial \tau} = \underbrace{\frac{(1+\beta)\varphi}{\tau}}_{(*1)} + \frac{\beta\psi\tau^{\varphi}}{1+(1-\tau)\psi\tau^{\varphi}}\left[\underbrace{\frac{(1-\tau)\varphi}{\tau}}_{(*2)} - \underbrace{1}_{(*3)}\right]. \tag{35}$$

If $\partial\widehat{U}_t/\partial\tau > 0$ holds, that a higher capital income tax enhances the welfare of current generations and this intuition is explained as follows. A higher capital income tax has three effects on indirect utility. First, a rise in capital income tax raises wages[11] under given capital stocks and hence promotes consumption during the young and old periods. This effect improves welfare. Second, as increasing capital income tax raises the interest rate, it increases consumption during old periods and thereby improves welfare. Third, interest income is directly reduced by a higher capital income tax. This reduces the old period's consumption and hence worsens welfare. The terms $(*1)$-$(*3)$ in Equation (35) capture the first, second, and third effects, respectively. If the first and second effects dominate the third, capital income taxation improves the welfare of the current generation.

Next, we focus on the subsequent generations. Denoting $\widehat{U}_{t+j}$ as the indirect utility

[11] From Equation (31), wage rises with a higher capital income tax

for subsequent generations[12], the indirect utility for subsequent generations is given by

$$\begin{aligned}\widehat{U}_{t+j} &= \log c_{t+j} + \beta \log d_{t+j},\\ &= (1+j)(1+\beta)\varphi \log \tau + \beta \log[1+(1-\tau)\psi\tau^{\varphi}] + H_{t+j},\end{aligned} \tag{36}$$

where $H_{t+j} \equiv (1+j)(1+\beta)\log\frac{\gamma\bar{A}\alpha^{\varphi}}{1+\beta} + [j+\beta(1+j)]\log\beta + (1+\beta)\log k_t$ and H_{t+j} is also independent of the tax rate. It is noteworthy that $k_{t+j} = g^j k_t$ holds. From (36), the following equation is obtained:

$$\frac{\partial \widehat{U}_{t+j}}{\partial \tau} = \frac{\left(1+\overbrace{j}^{(*4)}\right)(1+\beta)\varphi}{\tau} + \frac{\beta\psi\tau^{\varphi}}{1+(1-\tau)\psi\tau^{\varphi}}\left[\frac{(1-\tau)\varphi}{\tau} - 1\right]. \tag{37}$$

This expression is similar to Equation (35). From equations (35) and (37), if $\partial\widehat{U}_t/\partial\tau > 0$ holds, then $\partial\widehat{U}_{t+j}/\partial\tau > 0$ holds simultaneously. Thus, increasing capital income tax can be enhanced for subsequent generations. This intuition is qualitatively consistent with the case of the current generation, except for the growth effect. An increase in capital income tax directly raises wage levels and thus promotes capital accumulation. This effect can enhance the welfare of subsequent generations. Term $(*4)$ in Equation (37) represents the growth effect.

Finally, we investigate the welfare effects for the old generations. We denote $\widehat{U}_{o,t}$ as the indirect utility for the current old generation in period t and $\widehat{U}_{o,t}$ is given as follows:

$$\widehat{U}_{o,t} = \beta \log d_t = \beta \log[1+(1-\tau)\psi\tau^{\varphi}]\, s_{t-1} \tag{38}$$

Note that s_{t-1} is decided for period $t-1$. Differentiating this equation with respect to τ, it makes

$$\frac{\partial \widehat{U}_{o,t}}{\partial \tau} = \frac{\beta\psi\tau^{\varphi}}{1+(1-\tau)\psi\tau^{\varphi}}\left[\frac{(1-\tau)\varphi}{\tau} - 1\right]. \tag{39}$$

A higher capital income tax raises the interest rate in period t, which not only increases interest income but also consumption for old generations. It enhances welfare. Conversely, a higher capital income tax reduces interest income and, hence, reduces consumption for the current old generations. This effect worsens welfare. From Equations (35) and (39), if $\partial\widehat{U}_{o,t}/\partial\tau > 0$ holds, then $\partial\widehat{U}_t/\partial\tau > 0$ holds. Thus, we arrive at the following proposition.

Proposition 2

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[12] We assume j as $j = 1, 2, 3, \cdots$.

If $\frac{(1-\tau)\varphi}{\tau} - 1 > 0$ holds a higher capital income tax improves welfare for the current, subsequent, and presently old generations.

According to Kunze and Schuppert (2010), a higher capital income tax can improve welfare for the current and subsequent generations, while old generations experience welfare loss. A higher capital income tax directly reduces interest income for old generations; hence, it worsens welfare in their study. In contrast, the present study demonstrates that increasing capital income tax improves welfare for the current, subsequent, and old generations.

4. Numerical example

In this section, we provide a numerical example to investigate the impacts of capital income tax on the growth rate, labor market tightness, employment rate, and welfare for the current and old generations. The present model is composed by eight parameters, that is $\alpha, \beta, \gamma, \delta, \mu, \sigma, A$ and τ.We assume tax rate from 0.1 to 0.5. The remaining parameters[13] are set as follows: $\alpha = 0.36, \beta = 0.45, \gamma = 0.5, \delta = 0.5, \mu = 2.7, \sigma = 4$ and $A = 14$. The results are summarized in Table 1.

[Table 1: Impacts of capital income tax]

In this table, an increase in capital income tax promotes not only economic growth but also employment. Further, welfare for both current and present old generations improves with a higher capital income tax if the capital income tax is not high.

5. Conclusions

According to conventional wisdom, the optimal capital income tax is zero. This is because the capital income tax inhibits savings and economic growth in the Ramsey model. Conversely, capital income tax can accelerate economic growth within an OLG model.

This study constructs an OLG model that incorporates endogenous growth and involuntary unemployment to investigate how capital income tax affects economic growth, employment, and welfare. This study assumes that public investment arises from endogenous growth and that matching frictions cause unemployment.

The results of this study are summarized as follows. First, a rise in capital income

[13] Den Haan et al. (2000) and Hashimoto et al. (2020) set $\alpha = 0.36$. Hashimoto et al. (2020) set $\sigma = 4$.

tax increases public capital and, hence, contributes to improvements in economic growth, employment, and welfare. Furthermore, wages rise with higher capital income taxes. Compared to Kunze and Schuppert (2010), tax reform is not essential for obtaining the above results. Second, a higher capital income tax improves welfare for the current, subsequent, and present old generations, in this study, whereas increasing capital income tax leads to welfare loss for the present old generations in Kunze and Schuppert (2010). This study complements Kunze and Schuppert (2010). Furthermore, our results reinforce the empirical study that testifies to the impact of public spending. It shows that financing government spending through capital income taxes is a good idea.

Appendix

We derive Equation (24) as follows. Using Equations (17)–(19), we obtain:

$$y_{k,t}k_t = r_t k_t, \tag{A.1}$$

$$y_{l,t}l_t = w_t l_t + \mu\bar{k}_t v_t. \tag{A.2}$$

From Equations (12)-(14), we have

$$y_t = y_{k,t}k_t + y_{l,t}l_t. \tag{A.3}$$

Substituting Equations (A.1) and (A.2) into Equation (A.3), we obtain

$$y_t = w_t l_t + r_t k_t + \mu\bar{k}_t v_t. \tag{A.4}$$

Denoting $\tilde{c}_t$ and i_t as the total consumption and investment in period t, respectively, we have $\tilde{c}_t = c_t + d_t$ and $k_{t+1} = i_t + k_t$. Recall that we assume the population size to be unity with zero depreciation. In this economy, the market-clearing condition of the goods market is

$$\begin{aligned} y_t &= \tilde{c}_t + i_t + \mu\bar{k}_t v_t + \rho_t, \\ &= c_t + d_t + k_{t+1} - k_t + \mu\bar{k}_t v_t + \rho_t, \\ &= w_t l_t - s_t + [1 + (1-\tau)r_t]s_{t-1} + k_{t+1} - k_t + \mu\bar{k}_t v_t + \rho_t. \end{aligned} \tag{A.5}$$

From Equations (23), (A.4), and (A.5), we have:

$$(1 + r_t)(k_t - s_{t-1}) = k_{t+1} - s_t. \tag{A.6}$$

To satisfy Equation (A.6) for any period t, Equation (24) should hold. Even if we assume full depreciation, we can obtain Equation (A.6).

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Table 1: Impacts of capital income tax

τ	0.1	0.2	0.3	0.4	0.5
g	1.007	1.396	1.689	1.934	2.148
θ	1.202	1.665	2.016	2.308	2.563
l	0.907	0.970	0.985	0.991	0.994
$\partial \widehat{U}_t / \partial \tau$	8.064	3.810	2.326	1.528	0.986
$\partial \widehat{U}_{o,t} / \partial \tau$	1.240	0.398	0.051	-0.178	-0.379

$\alpha = 0.36, \beta = 0.45, \gamma = 0.5, \delta = 0.5, \mu = 2.7, \sigma = 4$ and $A = 14$