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Cournot vs. Bertrand Competitions in a Vertical Relationship with Separate Downstream Markets*

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Abstract

Contrary to conventional wisdom, our analysis of vertical relationships involving two geographically distinct downstream markets challenges the notion that Bertrand competition yields lower profits than Cournot competition. We demonstrate that when the market size where two downstream firms compete on either quantity or price is smaller than the other downstream market, the input price in downstream Bertrand competition is lower compared to Cournot competition. Therefore, this may result in higher profits for downstream firms under Bertrand competition.

JEL codes: L13, D43,

Keywords: Cournot competition, Bertrand competition, vertical relationship

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1 Introduction

The literature on industrial organization has extensively examined the contrasts between Cournot and Bertrand competitions (Singh and Vives, 1984). Conventionally, Bertrand competition is deemed more competitive, leading to diminished profits but augmented consumer and overall surpluses. Following Singh and Vives (1984), subsequent studies, such as Arya et al. (2008) and Correa-López and Naylor (2004), analyze the outcomes of Cournot and Bertrand competitions, exploring instances where the conventional wisdom may not hold. Following this literature, we scrutinize the results on profits.

We focus on a situation where an upstream firm supplies input to two geographically separate downstream markets; we compare the outcomes of Cournot and Bertrand competitions in one downstream market. In this setting, increased competition in the downstream market may lower the input price. The reason is that the input price depends on the price elasticities of demands for the input in both downstream markets, and when competition increases in a downstream market with elastic input demand, the input price will decrease. Therefore, lower input prices can be realized in Bertrand competition, which increases the profits of downstream firms.

To incorporate the mechanism described above, we construct the following model. An upstream firm produces input and sells it to downstream firms. There are two geographically separate downstream markets with three downstream firms. In one market, two downstream firms compete, while in the other market, one downstream firm is a monopolist. We assume that the downstream markets have different sizes. In a market with two downstream firms, the downstream firms produce differentiated products and compete on either quantity or price.

Comparing equilibrium outcomes under Bertrand and Cournot competitions, we find the following results. If the size of the downstream market with two downstream firms is smaller than that with one downstream firm, the input price under Bertrand competition is smaller than that under Cournot competition. In addition, if the size of the downstream market with two downstream firms is sufficiently small, the profits of all firms and consumers and total surpluses are larger under Bertrand competition than under Cournot competition. Therefore, Bertrand competition is potentially desirable for all economic agents.

Our study relates to the literature on the comparison of the outcomes of Bertrand and Cournot competitions in vertical markets. This literature can be divided into two groups: studies that consider Nash bargaining and studies that consider asymmetries in a downstream market. Correa-López and Naylor (2004) show that downstream firms obtain substantial profits under Bertrand competition than under Cournot competition when downstream firms engage in Nash bargaining with their unions over wages. Subsequent studies show that when upstream and downstream firms engage in Nash bargaining over two-part tariffs, downstream firms earn substantial profits under Bertrand competition (Alipranti et al., 2014; Basak and Mukherjee, 2017). Furthermore, in the framework of Nash bargaining and two-part tariffs, Liu and Wang (2020) incorporate downstream firms' investment and Wang and Li (2020) consider downstream firms' relative profit maximization.

Some studies have shown that the profit under Bertrand competition is higher than under Cournot competition by considering asymmetries in the downstream market; Arya et al. (2008) and Fanti and Scrimitore (2019) discuss the situation where an upstream firm has direct and indirect sales channels, Matsuoka (2023) considers sequential contracts of input price, and Mukherjee et al. (2012) consider technological asymmetry among downstream firms. Because we do not consider Nash bargaining, our study is more related to studies that consider asymmetries between downstream firms. However, considering that our study is symmetric with downstream firms in a downstream market, we can propose a new insight into this literature.

2 Model

We consider a vertical market with one upstream firm (U) and three downstream firms (Di, $i \in \{1, 2, 3\}$). In the downstream sector, two markets exist: market X and market Y. Upstream firm U produces input and sells it to the downstream firms at input price w. To produce one unit of the final product, each downstream firm uses one unit of input. Downstream firms $D1$ and $D2$ supply their products to market X; downstream firm $D3$ supplies its product to market Y. We assume that the production costs of all firms are zero.

We assume that markets X and Y are geographically separated, and these markets are independent. We denote the output and price of Di by q_i and p_i, respectively. In market X, products produced by $D1$ and $D2$ are differentiated. We assume that consumer surpluses in markets X and market Y are

$$\begin{aligned} CS_X &= a_X(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2 - p_1 q_1 - p_2 q_2, \\ CS_Y &= a_Y q_3 - \frac{1}{2} q_3^2 - p_3 q_3, \end{aligned}$$

where $a_X, a_Y > 0$ and γ is the degree of product substitutability, which satisfies $0 < \gamma < 1$. From the consumer surpluses, demand functions are $q_1 = [a_X(1-\gamma) - p_1 + \gamma p_2]/(1-\gamma^2)$, $q_2 = [a_X(1-\gamma) - p_2 + \gamma p_1]/(1-\gamma^2)$, and $q_3 = a_Y - p_3$; inverse demand functions are $p_1 = a_X - q_1 - \gamma q_2$, $p_2 = a_X - q_2 - \gamma q_1$, and $p_3 = a_Y - q_3$. To guarantee positive outputs in equilibrium, we assume that $r_{min} < r < r_{max}$, where $r = a_X/a_Y$, $r_{min} = (2+\gamma)/(8+2\gamma)$, and $r_{max} = (10 + \gamma - \gamma^2)/4$.

From the above setting, the profits of upstream and downstream firm Di are as follows.

$$\pi_U = w(q_1 + q_2 + q_3), \quad \pi_{Di} = (p_i - w)q_i.$$

Consumer, producer, and total surpluses are $CS = CS_X + CS_Y$, $PS = \pi_U + \pi_{D1} + \pi_{D2} + \pi_{D3}$, and $TS = CS + PS$, respectively.

Using the above setting, we compare outcomes under Cournot competition with those under Bertrand competition. The timing of the game is as follows: In the first stage, the upstream firm U sets the input price. In the second stage, each downstream firm chooses its output or price. Using the backward induction, we solve this game.

3 Analysis

Cournot competition First, we consider the case of Cournot competition in the market X. Solving the first-order condition in the second stage, each downstream firm chooses the following output.

$$q_1 = q_2 = \frac{a_X - w}{2+\gamma}, \quad q_3 = \frac{a_Y - w}{2}. \tag{1}$$

Solving the first-order condition in the first stage leads to an equilibrium input price.

$$w^C = \frac{4a_X + a_Y(2+\gamma)}{2(6+\gamma)},$$

where the superscript 'C' denotes the case under Cournot competition.

The equilibrium outcomes are as follows.

$$\begin{aligned}
&\pi_{D1}^C = \pi_{D2}^C = \frac{[2a_X(4+\gamma) - a_Y(2+\gamma)]^2}{4(2+\gamma)^2(6+\gamma)^2}, \quad \pi_{D3}^C = \frac{[4a_X - a_Y(10+\gamma)]^2}{16(6+\gamma)^2}, \\
&\pi_U^C = \frac{[4a_X + a_Y(2+\gamma)]^2}{8(2+\gamma)(6+\gamma)}, \\
&CS^C = \frac{(1+\gamma)[2a_X(4+\gamma) - a_Y(2+\gamma)]}{4(2+\gamma)^2(6+\gamma)^2} + \frac{[4a_X - a_Y(10+\gamma)]^2}{32(6+\gamma)^2}, \\
&TS^C = \pi_{D1}^C + \pi_{D2}^C + \pi_{D3}^C + \pi_U^C + CS^C.
\end{aligned}$$

Bertrand competition Next, we consider the case of Bertrand competition. By solving the first-order conditions, we obtain final product prices.

$$p_1 = p_2 = \frac{a_X(1-\gamma) + w}{2-\gamma}, \quad p_3 = \frac{a_Y + w}{2}. \tag{2}$$

To maximize the profit of the upstream firm, we determine the equilibrium input price.

$$w^B = \frac{4a_X + a_Y(2+\gamma-\gamma^2)}{2(6+\gamma-\gamma^2)},$$

where the superscript 'B' denotes the case under Bertrand competition.

Subsequently, the equilibrium outcomes are as follows.

$$\begin{aligned}
\pi_{D1}^B &= \pi_{D2}^B = \frac{(1-\gamma)[2a_X(4+\gamma-\gamma^2) - a_Y(2+\gamma-\gamma^2)]^2}{4(2-\gamma)^2(1+\gamma)(6+\gamma-\gamma^2)^2},\\
\pi_{D3}^B &= \frac{[4a_X - a_Y(10+\gamma-\gamma^2)]^2}{16(6+\gamma-\gamma^2)^2},\\
\pi_U^B &= \frac{[4a_X + a_Y(2+\gamma-\gamma^2)]^2}{8(2-\gamma)(1+\gamma)(6+\gamma-\gamma^2)},\\
CS^B &= \frac{[2a_X(4+\gamma-\gamma^2) - a_Y(2+\gamma-\gamma^2)]}{4(2-\gamma)^2(1+\gamma)(6+\gamma-\gamma^2)^2} + \frac{[4a_X - a_Y(10+\gamma-\gamma^2)]^2}{32(6+\gamma-\gamma^2)^2},\\
TS^B &= \pi_{D1}^B + \pi_{D2}^B + \pi_{D3}^B + \pi_U^B + CS^B.
\end{aligned}$$

Comparison of input prices First, we analyze the difference in input prices between Cournot and Bertrand competitions. By comparing w^B with w^C, we obtain the following.

$$w^B - w^C = \frac{2(a_X - a_Y)\gamma^2}{(6+\gamma)(6+\gamma-\gamma^2)}.$$

This result directly leads to Lemma 1.

Lemma 1 *Upstream firm U selects a lower input price under Bertrand competition than under Cournot competition if the size of market X is smaller than that of market Y. Specifically, $w^B < w^C$ if $a_X < a_Y$.*

To provide insight into Lemma 1, we examine the price elasticity of input demand. From the outcomes in the second stage, (1) and (2), the price elasticities of input demands in markets X and Y are expressed as $\varepsilon_X = w/(a_X - w)$ and $\varepsilon_Y = w/(a_Y - w)$, respectively. Notably, these values remain consistent across both Cournot and Bertrand competitions. Hence, if the size of market X is smaller than that of market Y, $a_X < a_Y$, the price elasticity of input demand in market X surpasses that in market Y.

Next, we analyze the price elasticity of total input demand, derived from the weighted average of price elasticities in markets X and Y. Denoting the second-stage outputs of downstream firm Di in Cournot and Bertrand competitions as $q_i^C(w)$ and $q_i^B(w)$ respectively, we proceed to present the price elasticities of total input demand for Cournot and Bertrand competitions as follows.

$$\begin{aligned}
\varepsilon^C &= \frac{q_1^C(w)+q_2^C(w)}{q_1^C(w)+q_2^C(w)+q_3^C(w)}\cdot\varepsilon_X + \frac{q_3^C(w)}{q_1^C(w)+q_2^C(w)+q_3^C(w)}\cdot\varepsilon_Y \\
&= \frac{w(6+\gamma)}{4a_X+a_Y(2+\gamma)-w(6+\gamma)}, \\
\varepsilon^B &= \frac{q_1^B(w)+q_2^B(w)}{q_1^B(w)+q_2^B(w)+q_3^B(w)}\cdot\varepsilon_X + \frac{q_3^B(w)}{q_1^B(w)+q_2^B(w)+q_3^B(w)}\cdot\varepsilon_Y \\
&= \frac{w(3-\gamma)(2+\gamma)}{4a_X+a_Y(2+\gamma-\gamma^2)-w(6+\gamma-\gamma^2)}.
\end{aligned}$$

The coefficient of ε_X is larger under Bertrand competition than under Cournot competition because intense competition results in larger outputs. Thus, if $\varepsilon_X > \varepsilon_Y$, which is equivalent to $a_X < a_Y$, the price elasticities of total input demand are greater under Bertrand competition compared to Cournot competition: $\varepsilon^B > \varepsilon^C$. Therefore, in this case, the upstream firm U sets a lower input price under Bertrand competition than under Cournot competition.

Comparison of profits First, we consider the profit of downstream firms Di, where $i = 1, 2$. Using $r = a_X/a_Y$, we obtain the following.

$$\pi_{Di}^B - \pi_{Di}^C = \frac{a_Y^2\gamma^2(\Psi_2 r^2 + \Psi_1 r + \Psi_0)}{2(3-\gamma)^2(2-\gamma)^2(1+\gamma)(2+\gamma)^2(6+\gamma)^2},$$

where $\Psi_0 = -(2-\gamma)^2(12+17\gamma+6\gamma^2+\gamma^3)$, $\Psi_1 = 4(60+40\gamma-33\gamma^2-11\gamma^3+3\gamma^4+\gamma^5)$, and $\Psi_2 = -4(48+116\gamma-52\gamma^2-17\gamma^3+4\gamma^4+\gamma^5)$. Solving $\pi_{Di}^B - \pi_{Di}^C > 0$ for r, we obtain $r < r_D$, where

$$r_D = \frac{60+40\gamma-33\gamma^2-11\gamma^3+3\gamma^4+\gamma^5+(36-24\gamma+\gamma^2+\gamma^3)\sqrt{1-\gamma^2}}{2(48+116\gamma-52\gamma^2-17\gamma^3+4\gamma^4+\gamma^5)}.$$

Next, we compare π_{D3}^B with π_{D3}^C.

$$\pi_{D3}^B - \pi_{D3}^C = \frac{a_Y^2(1-r)\gamma^2[60+16\gamma-7\gamma^2-\gamma^3-r(24+4\gamma-2\gamma^2)]}{2(3-\gamma)^2(2+\gamma)^2(6+\gamma)^2}.$$

Solving $\pi_{D3}^B - \pi_{D3}^C > 0$ for r, we obtain $r < 1$.

Finally, comparing π_U^B with π_U^C, we obtain the following.

$$\pi_U^B - \pi_U^C = \frac{a_Y^2\gamma^2[4r^2(4-\gamma)+r(4+2\gamma-2\gamma^2)-2-\gamma+\gamma^2]}{2(3-\gamma)(2-\gamma)(1+\gamma)(2+\gamma)(6+\gamma)} > 0,$$

where from $r_{min} < r < r_{max}$, the aforementioned inequality is resolve. By summarizing these results, we obtain the following proposition.

Proposition 1 *(i) Downstream firms $D1$ and $D2$ earn larger profits under Bertrand competition than under Cournot competition if $r < r_D$. (ii) The profits of downstream firm $D3$ under Bertrand competition is larger than that under Cournot competition if $r < 1$. (iii) The profit of upstream firm U under Bertrand competition is larger than that under Cournot competition.*

From Proposition 1, under Bertrand competition, all firms obtain larger profits than under Cournot competition, if the size of market X is sufficiently smaller than that of market Y.

We explain the intuition behind Proposition 1. In a price competition between downstream firms $D1$ and $D2$, their profits are small because of heightened competition. We call this the *competition effect*. Additionally, as deduced from Lemma 1, in the scenario of $a_X < a_Y$, or equivalently, $r < 1$, the upstream firm opts for a lower input price under Bertrand competition than under Cournot competition. We refer to this effect as the *input price effect*. The input price effect becomes robust even if r is small. Thus, when r is sufficiently small, the input price effect dominates the competition effect, leading to higher profits for downstream firms $D1$ and $D2$ under Bertrand competition than under Cournot competition. Hence we obtain (i) in Proposition 1. Next, we consider

(ii) in Proposition 1. When the input price effect lowers the input price, the profit of downstream firm $D3$ increases because markets X and Y are independent. Finally, we consider (iii) in Proposition 1. Because heightened competition due to Bertrand competition partially resolved the double marginalization problem, the profit of the upstream firm U increased.

Comparison of surpluses We compare consumer and total surpluses between Cournot and Bertrand competition.

$$CS^B - CS^C = \frac{a_Y^2\gamma^2 \begin{bmatrix} 2r^2(1008 + 312\gamma - 576\gamma^2 - 14\gamma^3 + 67\gamma^4 - \gamma^5 - 2\gamma^6) \\ -r(1008 + 528\gamma - 744\gamma^2 - 132\gamma^3 + 139\gamma^4 + 2\gamma^5 - 5\gamma^6) \\ +2(2-\gamma)^2(36 + 51\gamma + 12\gamma^2 - 4\gamma^3 - \gamma^4) \end{bmatrix}}{4(3-\gamma)^2(2-\gamma)^2(1+\gamma)(2+\gamma)^2(6+\gamma)^2} > 0,$$

$$TS^B - TS^C = \frac{a_Y^2\gamma^2 \begin{bmatrix} 2r^2(1872 - 1080\gamma - 544\gamma^2 + 270\gamma^3 + 49\gamma^4 - 15\gamma^5 - 2\gamma^6) \\ -r(144 - 144\gamma^{-}280\gamma^2 + 100\gamma^3 + 77\gamma^4 - 18\gamma^5 - 3\gamma^6) \\ +2(2-\gamma)^2(36 + 45\gamma + 2\gamma^2 - 8\gamma^3 - \gamma^4) \end{bmatrix}}{4(3-\gamma)^2(2-\gamma)^2(1+\gamma)(2+\gamma)^2(6+\gamma)^2} > 0,$$

where because of $r_{min} < r < r_{max}$, the above inequalities satisfied. Thus, we obtain the following proposition.

Proposition 2 *Consumer and total surpluses under Bertrand competition are larger than those under Cournot competition.*

The intuition behind this result is similar to that behind (iii) in Proposition 1. Specifically, because Bertrand competition mitigates the double marginalization problem, consumer and total surpluses are larger under Bertrand competition than under Cournot competition.

4 Conclusions

We consider a vertical market with one upstream and three downstream firms. Two of the downstream firms compete in a downstream market and one of them is a monopolist in the other downstream market. By considering these two distinct downstream markets, we show that an increase in output in a competing downstream market increases the price elasticity of inputs, which in turn decreases input prices. Thus, although Bertrand competition increases competition, it can benefit downstream firms by lowering input prices. Our results add a new insight that profits in Bertrand competition are larger than those in Cournot competition, even when the competing firms are symmetric.

Our study has several limitations. First, we assume uniform linear contracts between upstream and downstream firms. If an upstream firm discriminates input prices or uses non-linear contracts, such as a two-part tariff, the impact of downstream market competition on input prices will be weaker and our results may not be obtained. Moreover, even if the monopolistic downstream market becomes oligopolistic, the impact of downstream market competition on input prices would be weakened. We acknowledge the significance of these considerations and propose them as areas for future research.

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