International Kuznets Curve (?):

A Schumpeterian Model of the World Economy

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Abstract

The Industrial Revolution boosted growth in Britain, and as it diffuses to other countries, growth in the world economy accelerated. At the same time, inter-country inequality increased. On the other hand, the recent literature and the 2018 Maddison dataset, the Penn World Table 9.1 and the World Inequality Database suggest that inter-country inequality fell in recent decades. This implies that inter-country inequality takes an inverted U shape, i.e. an International Kuznets Curve. The aim of this paper is to build a Schumpeterian growth model of the world economy to explain this hypothesis. In the model there is a continuum of countries, and transitional dynamics involve a series of take-offs of countries which are initially backward. Simulations show that the International Kuznets Curve emerges in a transitional process of the world economy converging to the long-run equilibrium.

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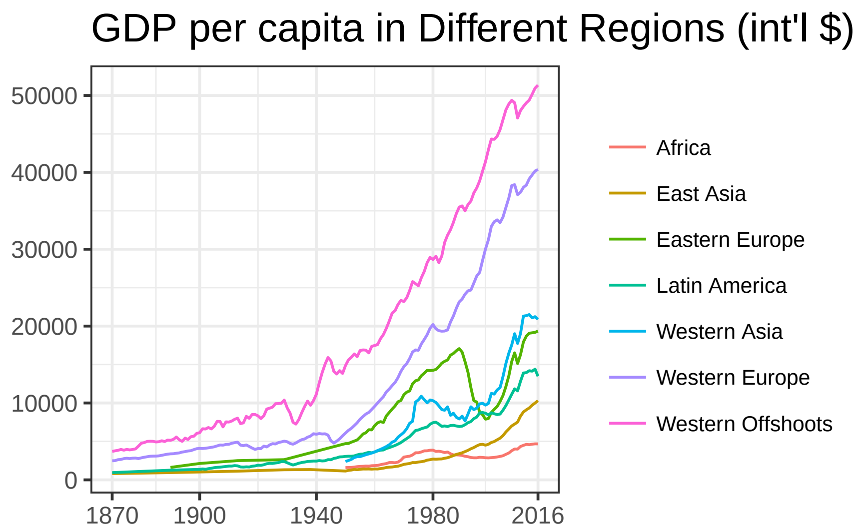

Figure 1: Source: the 2018 Maddison dataset.

1 Introduction

The Industrial Revolution in Britain in the 18-19th century was an unprecedented shock to the world economy. Its massive effects spread throughout the world only gradually, and its full effects do not seem to have been realized even today. Indeed, Crafts and O'Rourke (2014) wrote, "[The Industrial Revolution] set in train a variety of long-run adjustment processes which are still ongoing, ..."

Such adjustments manifest themselves in the form of changes in the distribution of the living standard, measured by GDP per capita, across countries in the world. Following Britain, continental European countries took off in terms of productivity improvement and new technologies introduced. The diffusion of industrialization raised income of those economies, but diffusion itself was a slow process with many countries left behind. Indeed, Figure 1 shows diverging fortunes of 7 regions in the world. It is not difficult to imagine that if data of GDP per capita of *all* countries in the world were to exist since the Industrial Revolution, such inequality measures as the Gini coefficient would have steadily increased throughout the 19th century at least. In fact, this is emphatically implied by Bourguignon and Morrisson (2002) who use data of 15 countries and 18 groups in the period of 1820-1992. They demonstrate that inter-country inequality, which assumes individuals having the same income within each country/group, steadily and dramatically increased from a very low initial level. The increasing trend continued well into the 20th century.[1]

In the 20th century, on the other hand, economic take-offs spread to more and more countries, including Japan, what used to be known as Asian Four Tiger economies, and India and China in more recent decades. The fact that more countries have been industrialized, while a growth pace of developed economies slowed down generated a downward force on the degree of inequality between countries. Indeed, Milanovic (2016) shows that the Gini coefficient of GDP per capita between countries, calculated using the 2018 Maddison dataset, has started declining around in 2000. Indeed, as the next section shows, the time series of inequality indices, such as the Gini coefficient, show an inverted U shape with a peak in 2000. In addition,

[1] In the literature, several concepts of inequality exist. For example, see Anand and Segal (2008). The present paper concerns inequality between countries with GDP per capita of country as a unit of observation. We call it here *between* inequality and inter-country inequality interchangeably.

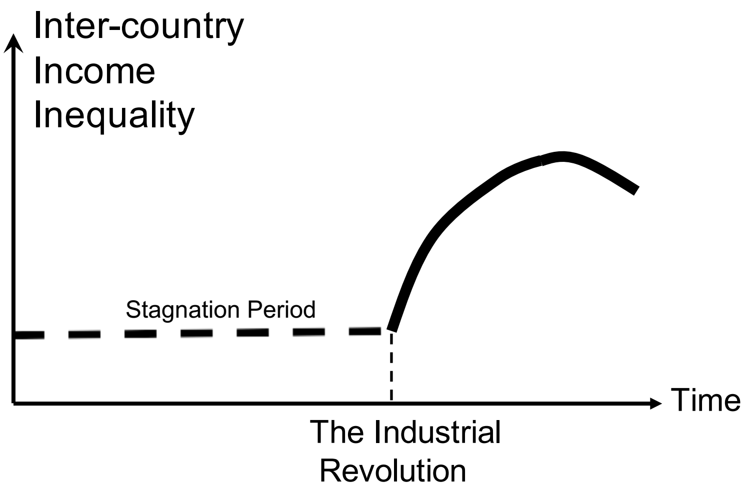

Figure 2: An image of the International Kuznets Curve hypothesis.

the same calculation using the Penn World Table 9.1 and the World Inequality Index indicate that those inequality indices have a clear downward trend since 1970 and 1950, respectively. A similar observation is made in Firebaugh (2003) and his other studies. Combined with the finding of Bourguignon and Morrisson (2002), this suggests the existence of a turning point of inter-country inequality.

The turnaround of inequality trends suggests a hypothesis of a Kuznets curve in terms of inter-country inequality in the world. Namely, as the world economy grows breaking out of the Malthusian period, inequality between countries initially increases, followed by the reversal of the inequality trend, as envisaged in Figure 2. It is no doubt that the Industrial Revolution and the subsequent gradual diffusion of industrialization in the form of sequential take-offs is a key driving force behind the emergence of what we call the *International Kuznets Curve*.[2]

Then, the following questions arise. How do we explain the International Kuznets Curve hypothesis? What is the mechanism of a gradual diffusion industrialization? What are necessary conditions for sequential take-offs to occur? What are the implications of sequential take-offs for dynamics of inequality among countries?

The main aim of the present paper is to offer answers to those questions by building a Schumpeterian growth models of the world economy. A starting point of our analysis is to take the Industrial Revolution in Britain as given. That is, we assume that in the initial period there is a single country (Britain in this case) which is already industrialized and examine a diffusion process of industrialization to other countries. By industrialization, we mean the start of producing new goods and more sophisticated products based on new technologies. In particular, using the Schumpeterian technological progress framework, we focus on knowledge accumulation in the diffusion process of industrialization. This is motivated by Jones and Romer (2010) who underline the importance of idea flows and technology adoption in the catch-up process[3]and the survey result of Caselli (2005) that differences in total factor productivity account for at least half of

[2]The word *international* is meant to indicate *between* countries, as opposed to *world* or *global*. Bourguignon and Morrisson (2002) use *world* to denote inequality *between* and *within* countries combined. In Milanovic (2016), *world* inequality means *between* inequality, and *global* indicates *between* and *within* inequality combined.

[3]Pointing out the result of Caselli and Feyrer (2007) that the marginal product of capital is remarkably similar across countries, Jones and Romer (2010) note "While the textbook transition dynamics - driven by diminishing returns to capital accumulation - are elegant and easy to explain, they are most likely not especially relevant to catch-up growth in practice." (p.232)

differences in income per capita.

To be more specific, we build a model of a world economy consisting of the base economy (Britain) and a continuum of countries. In the base economy, new knowledge is created, introducing new and better industrial products into the economy. On the other hand, other economies are initially backward and specialized in agricultural production. In the initial equilibrium, the base economy and other economies exchange high-tech and agricultural goods through international trade. Heterogeneity of countries are introduced in terms of research productivity, and the base economy has the highest productivity level. In a long-run equilibrium, a group of economies with relatively higher productivity levels engage in innovative activities and export high-tech goods, and a group of other countries remain specialized in agricultural production. We call the former the North, and the latter the South, and such division of countries into two groups is determined by equilibriating forces of international trade. Indeed, the North can consist of the base economy only if no diffusion of industrialization occurs in long run.

A more interesting aspect of the model lies in transitional dynamics. Starting from the initial condition where the base economy only is innovating, if industrialization spreads to other economies, it occurs only gradually so that Southern economies sequentially take off to join the Northern club.[4] As the world economy converges to steady state, the average Norther wages relative to the Southern wage also increase, tending to increase inter-country income inequality. In addition, the world economy can spiral around the long-run equilibrium, so that a phase of sequential take-offs and a phase of sequential falling-behind alternate, as it eventually converges to steady state. An important factor in generating different transitional patterns is what we call the *take-off externality.* This is a positive externality which captures widening opportunities for learning in research. The more countries that engage in R&D, the more likely that researchers learn different perspectives with diverse experiences. In line with this reasoning, the externality raises R&D productivity of all countries, as more and more of economies are industrialized. In the model, the take-off externality has an effect of "pulling" Southern economies out of backwardness. Indeed, if the take-off externality is weak (i.e. a weak "pulling" effect), the world economy is stuck in the initial condition with the base economy only in the North. If the externality is in the intermediate range, the diffusion of industrialization can occur (possibly with spirals) under a certain condition. With the strong externality, sequential take-offs occur (possibly with spirals) with no further qualification.

Focusing upon the last two cases, we conduct numerical simulations. A common pattern that emerges is that the diffusion of industrialization is a slow process with the initial world growth rate being low. The Gini coefficient is also low. As more and more countries take off, growth accelerates, and it decelerates as the world economy approaches steady state. That is, growth follows a sigmoid pattern. Regarding inequality, the International Kuznets Curve emerges, measured by the Gini Coefficient. The mechanism is driven by two opposing effects. Obviously, sequential take-offs increases inter-country inequality. On the other hand, take-offs are also a catch-up process. As more countries move out of backwardness, they achieve higher income levels. The Gini may start falling because the more countries that are industrialized, the less number of countries that remain backward.

Our paper is related to the literature on the long-run growth and the Big Divergence (see Pomeranz (2000) and Pritchett (1997)). In Galor and Mountford (2008) and Galor and Mountford (2006) examine the role of international trade in generating differential effects on fertility in developed and developing

[4]For example, see Williamson (2010) for take-offs of peripheral countries.

countries. In particular, trade discourages fertility in developed countries, but encourages it in developing countries via unskilled intensive production. This translates into divergence of per capita living standards. O'Rourke et al. (2019) is related to those two studies, but different in that the terms of trade is endogenized and technology transfer are introduced. Galor et al. (2009) considers the effect of land ownership concentration which affects the transition from an agricultural to an industrial economy through human capital accumulation. Lucas (2009) consider a simple model of knowledge accumulation and diffusion. The focus of those studies, however, is on the role of fertility and human capital accumulation, which quite differs from our Schumpeterian approach. Much closer to our study is Howitt and Mayer-Foulkes (2005). They adopt a Schumpeterian framework and examine implications of the arrival of modern (inherently different and scientific) R&D opportunities. Because conducting it is much costly activities, some countries starts investing in it, while others do not. As a result, countries are sorted into two groups. Divergence occurs between groups, and convergence within a group. However, there is no role of international trade, and all countries are connected only via knowledge spillovers. The role of international trade is stressed in Baldwin et al. (2001) who focus upon two countries in a core-periphery model. In Matsuyama (2004), many symmetric countries sort themselves into the rich and the poor. In a similar vein, Matsuyama (2013) also considers the Lorenz curve among ex ante symmetric countries. The last three studies are insightful in that they identify market forces which divide ex ante symmetric economies into groups of different income levels, though it means that they do not identify characteristics which make those countries richer or poorer. In contrast, our paper starts with heterogeneity in R&D productivity among countries. Because of this assumption, a transitional process of sequential take-offs endogenously divides the world into the North and the South only gradually, allowing us to examine a dynamic process of changing inter-country inequality.

Our paper is structures as follows. Section 2 briefly reviews data related to inequality, and develops a simple model which explains an intuition behind the result. Section 3 formally develops the model, and long-run equilibrium is considered. In Section 5, transitional dynamics are examined with a focus on the role of the learaning-opportunity externality. Simulation is conducted to show the emergence of the International Kuznets Curve. The last section concludes.

2 Preliminary

2.1 Data

The Industrial Revolution triggered a series of take-offs of economies reaching higher levels of income. Because it was a gradual process, inter-country inequality steadily increased. Indeed, Bourguignon and Morrisson (2002) show a clear evidence of increasing inequality since 1820. They created dataset of 15 single countries and 18 country groups based on the Maddison data and others, and in particular, income decile data within each country or group. By so doing, they examine (i) inter-country inequality, which takes individuals having the same income within each country/group, (ii) within-country inequality, which comes from inequality within each country/group, and (iii) global inequality, consisting of (i) and (ii). Figure 3 shows the between-country inequality indices, the Theil Index and the Mean Log Deviation, which steadily increase. The Gini coefficient, calculated from the "raw" data of Bourguignon and Morrisson (2002) is also shown for reference. Although not shown here, their calculation indicates that the within-

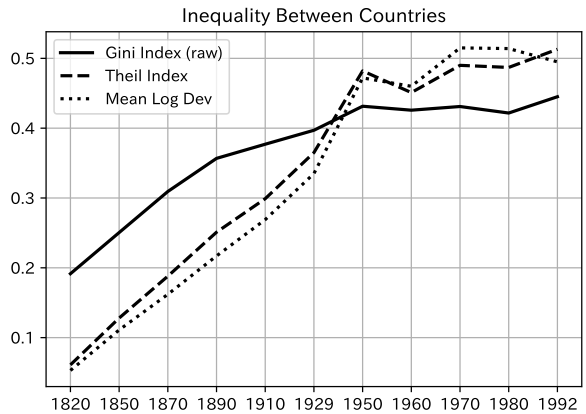

Figure 3: Between inequality is derived from the Theil index and the Mean Log Deviation of GDP per capita in the world. The Gini Coefficient is calculated using "raw" data of GDP per capita. Source: Bourguignon and Morrisson (2002)

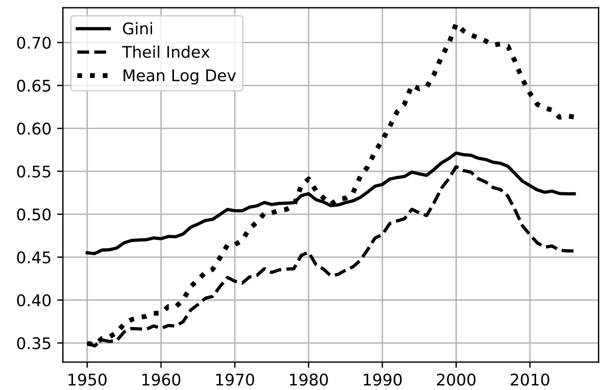

Figure 4: The Gini coefficient, the Theil index and the Mean Log Deviation of GDP per capita in the world are calculated using the "raw" data from the 2018 Maddison dataset (137 countries in 1950-2016).

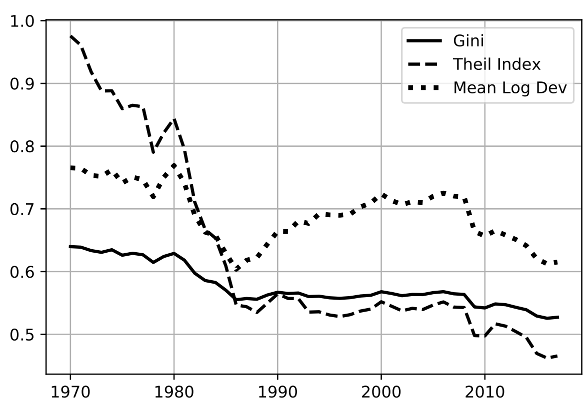

Figure 5: The Gini coefficient, the Theil index and the Mean Log Deviation of GDP per capita in the world are calculated using the "raw" data from the Penn World Table 9.1 (156 countries in 1970-2017).

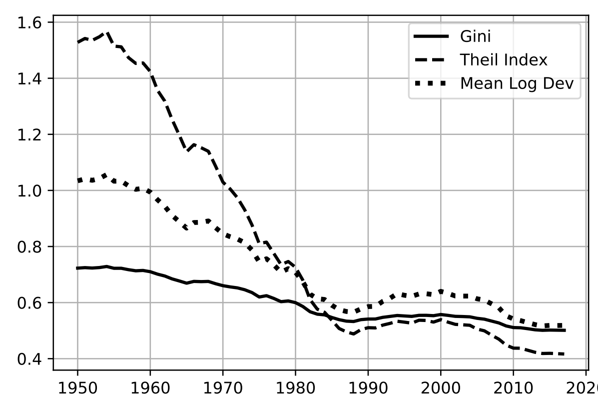

Figure 6: The Gini coefficient, the Theil index and the Mean Log Deviation of GDP per capita in the world are calculated using the "raw" data from the World Inequality Database (136 countries in 1950-2017).

inequality is high even in 1820 with its decreasing trend. It means that the relative importance of the between-country increased in the total inequality in the period considered.[5]

Now the question is what happened after a sharp increase in inter-country inequality triggered around the period of the Industrial Revolution. In Figure 3, the inequality indices appear to level off after 1950, which suggests a certain equalizing mechanism is working after the WWII. To explore this issue, we turn to datasets which allow us to use GDP per capita of individual countries (rather than groups). Figure 4 uses the 2018 Maddison data with 137 countries in 1950-2016.[6] Countries are selected so that no missing values exist.[7] All three indices show a steady increase around 2000, followed by a clear downturn in 2000. The Gini coefficient appears to exhibit a less pronounced fall compared with the other two indices. This supports the International Kuznets Curve hypothesis, and is consistent with the findings of Milanovic (2016). In Figures 5 and 6, the same indices are calculated using the Penn World Table 9.1 and the World Inequality Database. They show clear negative trends since 1970 and 1950. In fact, a similar result is also reported in, e.g. Firebaugh (2003). Given the increasing trends of the inequality indices in Figure 3, those datasets suggest that a peak of the inequality indices is likely to occur in earlier years than those figures show. Although there are discrepancies regarding the timing of the peak in Figures 4-6, they all indicate that the inter-country inequality took a U term from an increasing trend at least since the period of the Industrial Revolution.

A rise and fall in the inequality indices reflects changes in the income distribution itself. Figures 7-9 show the evolution of its shape in 5-year intervals, using Kernel density estimates. There are two common features. First, the mode is found at low income levels of income initially, and it gradually shifts rightward. Second, the distribution itself gets flattened out. This feature is much stronger in the Maddison data in Figure 7, and might have contributed to an inverted U shape in Figure 4. These visual traits can be cast in changes in skewness and kurtosis. In deed, those indices show a clear negative trends in all datasets used in Figures 7-9 (not shown here).[8] A falling skewness means that the shape of the distribution is tilting rightward, and a lower kurtosis implies that countries with extreme income levels are less in number in both tails (low and high). These pictures are consistent with an increasing number of countries catching up with developed countries, including Japan, what used to be called Asian Four Tigers, and BRICS in more recent decades, though there are countries falling back as well.[9]

Then, a more fundamental question is what drives the process of changing inequality. Although a range of factors are suggested (for example, convergence of institutional factors, human capital accumulation, labour mobility, the emergence of service sectors), we believe that the creation/use/diffusion of technology

[5] The importance of between-country inequality may also be highlighted by Milanovic (2002) who writes that between-country inequality explains 88 percent of global inequality measured by the Gini coefficient.

[6] Bourguignon and Morrisson (2002) also used its older version.

[7] We prioritize a balanced dataset covering as many countries as possible. A primary concern of using an unbalanced dataset is bias towards more equality. Countries with missing values are typically developing countries, and if they are excluded, inequality indices tend to understate the degree of inter-country inequality. In addition, as the number of countries included in a dataset changes (typically increases), such bias is not constant. In the Penn World Data 9.1, the number of countries with no missing data is 55 in 1950. It increases by 36 in 1960 and 42 in 1970, preceded and followed by a steady increase before reaching 182 in 2005. Using all countries, the calculated Gini coefficient jumps in 1960 and 1970, and in fact, it gives a much stronger image of an inverted U relationship between the index and income.

[8] Using Bourguignon and Morrisson (2002), one can show that skewness and kurtosis of GDP per capita have a positive trend up to 1950, followed by a negative trend.

[9] There are countries which fell back from a relatively high income group, as opposed to catching-up. An often-cited example is Latin American countries in the late 20th century. For example, see Edwards et al. (2007) and Jones (1997).

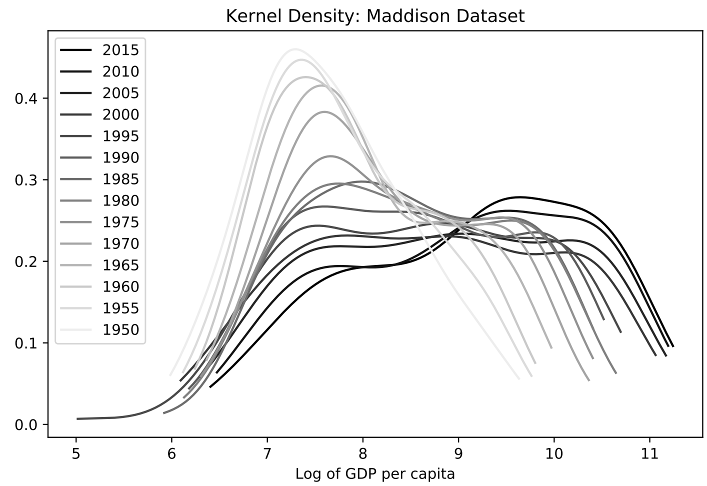

Figure 7: Kernel density estimates of the distribution of GDP per capita of 137 countries. Source: the 2018 Maddison dataset

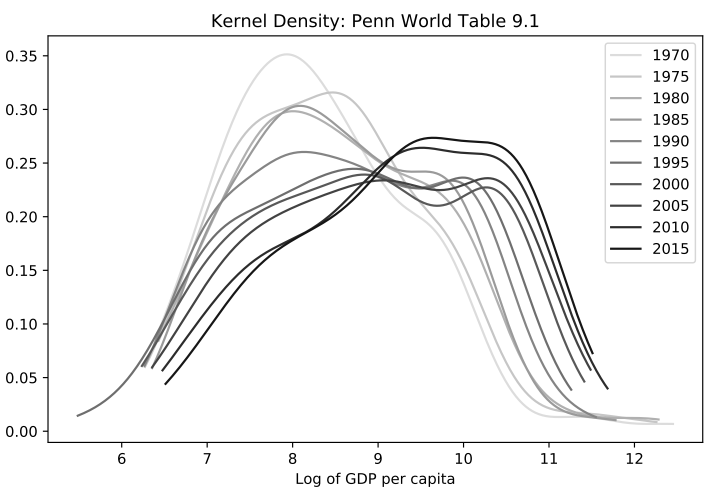

Figure 8: Kernel density estimates of the distribution of GDP per capita of 156 countries. Source: the Penn World Table 9.1

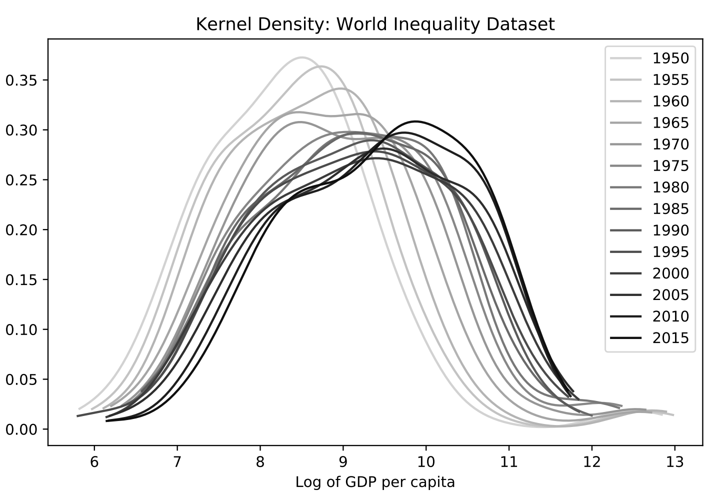

Figure 9: Kernel density estimates of the distribution of GDP per capita of 136 countries. Source: the World Inequality Database

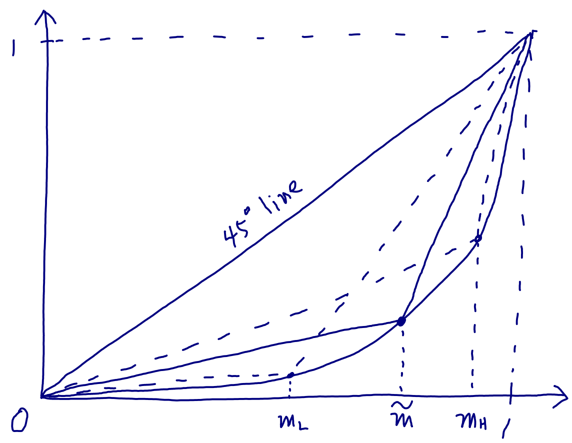

Figure 10: The Lorenz Curve for the two income groups consists of two lines. Three cases are drawn.

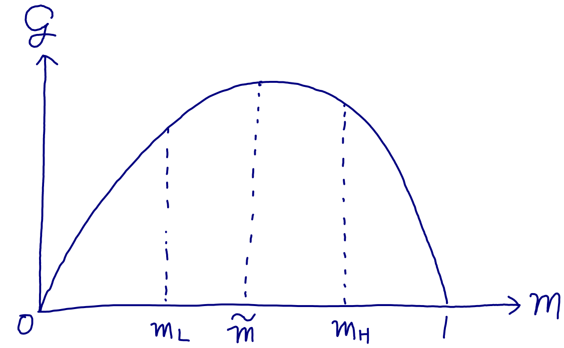

Figure 11: The Gini Coefficient takes an inverted U shape.

is the key to the understanding of a dynamic process of changing inequality between countries.[10]

2.2 International Kuznetz Curve: An Intuitive Account

In this section, we develop an intuition of the main mechanisms of our model which drives the emergence of the International Kuznets Curve. Consider a world economy with a continuum of countries, which are grouped into North and South. Countries are homogeneous within each group with income per capita y_i, $i = N, S$ and $y_S < y_N$. Assume that there are m countries in North, and $1 - m$ in South, so that the world average income is $y_W = (1 - m)\, y_S + m y_N$. Even at this stage, one can sense what is in store. If all countries belong to South, i.e. $m = 0$, the Gini coefficient is zero because of the indentical income. The same is true if all countries are in North, i.e. $m = 1$. This implies that the Gini coefficient is greater than zero for $0 < m < 1$ because $y_S < y_N$, and there must be the value of m at which its maximum is achieved.

To confirm this, let us consider the Lorenz curve for this world economy:

$$\mathcal{L}(p) = \begin{cases} \dfrac{p y_S}{y_W} & 0 \leq p \leq 1 - m \\ \dfrac{(1-m)\, y_S + [p - (1-m)]\, y_N}{y_W} & 1 - m < p \leq 1 \end{cases}$$

where p is the percentile of countries which are arranged starting from South. It is illustrated in Figure 10. Given the Lorenz curve, the Gini coefficient is now defined as

$$\mathcal{G} = 1 - 2\int_0^1 \mathcal{L}(p)\, dp = \frac{m(1-m)(y_N - y_S)}{(1-m)\, y_S + m y_N}.$$

Figure 11 depicts $\mathcal{G}$ as a function of m.

[10] See Becker et al. (2011) for the role of human capital accumulation.

Suppose that initially $m = 0$, i.e. all countries are backward with income y_S. Let us introduce the Big Divergence. Industrialization increases the number of Northern economies m from zero. As it spreads, m gradually increases with a rise in the average world income y_W. Along with it, the Gini coefficient $\mathcal{G}$ also increases. This is due to the fact that more of initially backward economies take off reaching a higher income level y_N, while other economies are left behind with a lower income y_S. This is what we call the *take-off effect* which tends to raise $\mathcal{G}$.[11] On the other hand, an increase in m also means that more of Southern countries are catching up with Northern economies, achieving a higher income. Put it differently, there are less and less number of poorer countries. This process tends to reduce $\mathcal{G}$, and is called the *catch-up effect*. When m is small, the catch-up effect is small relative to the take-off effect, so that inequality widens between countries, as in the Big Divergence period. However, as more economies are industrialized, the catch-up effect turns dominant, and $\mathcal{G}$ reaches its maximum at $\tilde{m}$ in Figure 11. As m increases further, the Gini coefficient starts falling, giving rise to the International Kuznets Curve.[12] This is the intuition for the key result of our model, developed in the following sections.

3 The Model

3.1 Consumers

Time is continuous. There are "many" heterogeneous economies, indexed by i. It consists of the *base* economy, $i = 0$, which has already been industrialized, and it plays the role of Britain in the diffusion process of industrialization. The other economies $i \in (0, M]$, $0 < M < \infty$, are initially backward and have capability to take off, though not all of them will be industrialized in equilibrium, as we will show. In each economy, there is a representative consumer who supplies constant L units of labour service inelastically. At this stage, it is instructive to clarify some details about the distribution of workers in the world. To deal with the initial situation where the base economy only is industrialized, we capture the economy at $i = 0$ using the Dirac Delta function. A consequence of this assumption is that there are $(1 + M)\, L$ workers in the world as a whole, and the cumulative distribution of workers is illustrated in Figure 12. Without loss of generality, $M = 1$ is assumed in what follows.

A representative consumer in all countries share the following instantaneous utility function:

$$u_i(t) = \beta \ln X_i(t) + (1 - \beta) \ln z_i(t)$$

where $0 < \beta < 1$ and

$$\ln X_i(t) = \int_0^{N(t)} \ln q_j(t)\, x_{ij}(t)\, dj, \qquad 0 < N(0) \leq N(t) \leq 1. \tag{1}$$

z_i denotes agricultural goods, and X_i is the composite goods consisting of industrial goods x_{ij} where the subscript $j \in [0, N(t)]$ is the industry index. We use N to denote the number of varieties of industrial goods available for consumption and q_j is the quality level of x_{ij}. In our model, two types of innovations

[11] A the risk of abusing terms, we use the term *take-off effect* here. Note that it is different from the take-off externality which increases R&D productivity as more economies take off, discussed earlier.

[12] Skewness in our simple example is given by $Skew = \frac{1-2m}{\sqrt{m(1-m)}}$. It is monotonically decreasing in m with $Skew \in [-\infty, \infty]$ and $Skew = 0$ for complete symmetry $m = 1/2$.

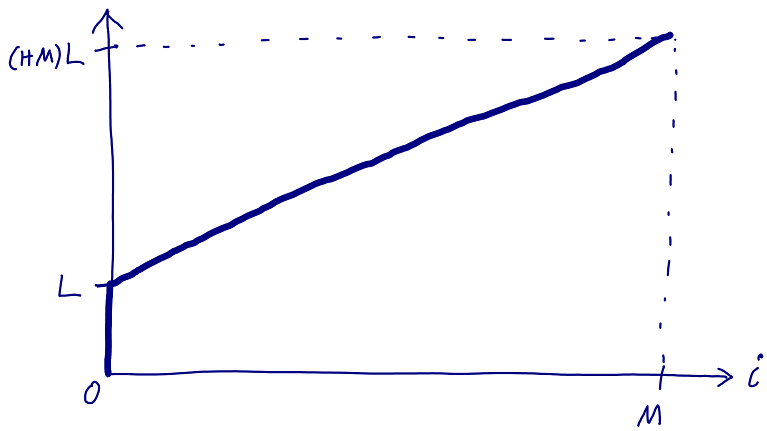

Figure 12: The cumulative distribution of workers.

coexist. The first type of innovations raises quality levels q_j of industrial goods geometrically whenever it occurs:

$$q_j(t) = \gamma_j^{c_j(t)}, \qquad \gamma_j > 1, \qquad c_j(t) = 1, 2, 3......$$

As will be explained in more detail, innovative firms optimally determine γ_j.[13] The second type of innovations expands the variety of industrial goods. It means that some industries do not initially exist until the first innovation occurs. In this sense, the first innovation in those industries is a variety innovation with the initial quality $q_j = \gamma_j$, and follow-on innovations in the same industries are of quality type. We assume that variety innovations can occur until the upper limit of $N = 1$ is reached.[14]

Given the Cobb-Douglas structure in (1), demand for each product is given by

$$x_{ij}(t) = \frac{\beta E_i(t)}{p_j(t) N(t)}, \qquad z_i(t) = \frac{(1-\beta) E_i(t)}{p_y(t)} \tag{2}$$

where E_i is consumption expenditure. It can also be easily verified that an intertemporal utility maximizing consumer choses the dynamic path of E_i which satisfies

$$\frac{\dot{E}_i(t)}{E_i(t)} = r(t) - \rho \tag{3}$$

where r is the interest rate and ρ is the common subjective rate of time preference. Now, define $E = E_0 + \int_0^M E_i di$ as the total consumption expenditure.[15] From (3), we can derive the Euler condition for

[13] This assumption follows Aghion and Howitt (1992).

[14] Early studies which model the coexistence of quality and variety innovation are Aghion and Howitt (1996) and Li (2000). However, the approach we adop is different from them and other similar studies.

[15] Formally, $E = \int_{-\varepsilon}^{+\varepsilon} E_0 \delta(0) di + \int_0^M E_i di$ where $\delta(.)$ is the Dirac Delta function. It is reduced to the above expression, as $\varepsilon \to 0$.

the world economy[16]

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \tag{4}$$

3.2 Manufacturing of Products

In characterizing equilibrium below, we will show that the world is divided into the North and the South. In the North, industrial goods are produced, and the South is specialized in the production of agricultural goods. Both types of goods are traded.[17]

Let us consider agricultural goods, which are produced under perfect competition. It is assumed that a worker is required to produce one unit of the goods. Assuming that all countries have access to the same technology, their price is given by $p_y = w_S$ where w_S is wage in South.

Industrial goods are produced by local monopoly firms which invented those goods. First, consider industrial products created through quality innovation. Suppose that a firm located in country k produces an industrial product in an industry j. Further suppose that a different firm located in country i invented a higher-quality product in the same industry. The firm charges the price

$$p_j(t) = \gamma_j w_k(t). \tag{5}$$

To understand this, note that given the Cobb-Douglas preferences in (1), quality innovation cannot be drastic in the sense that the marginal cost of the previous incumbent firm always binds the pricing decision of an innovating firm. That is why in (5) there is wage in country k where the previous incumbent firm is located. Note that $k = i$ can also happen. Regarding γ_j, it is interpreted as the quality premium over the binding marginal cost w_k.

Let us turn to variety innovations. When it occurs in country i in industry j, its quality level starts from $q_j = \gamma_j$. In addition, we assume that copycat firms emerge in a randomly chosen country k and can produce the same product with $q_j = 1$. Due to this simplifying assumption, the price charged by a firm is equivalent to (5).

It is assumed that one unit of industrial goods is produced with one worker. Therefore, a monopoly firm in country i earns profits in industry j[18]

$$\pi_{ijk}(t) = \left(1 - \frac{w_i(t)}{\gamma_j w_k(t)}\right)\frac{\beta E(t)}{N(t)}. \tag{6}$$

3.3 Uncertain Innovation Processes

There are two types of uncertainty in an innovation process. The first uncertainty is related to the choice of an industry where innovation implemented. In most of existing Schumpeterian models, firms engage in R&D in a particular industry, and if successful, innovation is implemented in that industry. In this sense, R&D is "directed". We depart from this perspective. It is assumed that R&D is "undirected" in the sense

[16]Rewrite (3) as $\dot{E}_i = E_i(r - \rho)$. Integrating it over i gives (4).

[17]For example, see Section 6.1 of Galor and Mountford (2008) for the importance of international trade in the period 18-19th centuries.

[18]Note that the subscript i in (6) refers to the country where the product is produced, whereas i in (1) and (2) denotes the country where goods are consumed.

that an industry where innovation is implemented is randomly chosen from *all* industries $i \in [0,1]$ after innovation occurs. This assumption captures in a simple way an unpredictable nature of R&D outcomes.[19]

An advantage of this assumption is that one type of R&D is enough to model both of variety and quality innovations, because a given innovation can be either of variety or quality type. For example, consider an industry which has not been chosen for innovation implementation yet. If this industry is selected for the first time, a variety innovation is implemented, creating a new variety product in a new industry which did not exist before. On the other hand, if an industry where innovation was implemented before is randomly selected, innovation takes a quality form.

The second uncertainty of innovation is related to the timing of successful innovation. This is the standard type of uncertainty in the literature, and is captured by a Poisson process of innovation. To be more specific, define R_i as the number of research workers used in country i. Then, innovation occurs with an arrival rate of $a_i R_i$ in country i where a_i is R&D productivity. a_i is the only factor that differentiates a continuum of countries in the model. Despite such simplicity, our model will exhibit rich dynamics in equilibrium, dividing the world into the North and the South with an evolving distribution of GDP per capita across countries.

3.4 The Value of Innovation

Let us define the value of innovation for an innovator in country i. First consider the ex post value after innovation occurs and is implemented in an industry j (randomly selected). A successful innovator k earns profits π_{ijk} in (9) until it is replaced with a higher quality product (i.e. the industry is selected randomly for the new successful innovation). Therefore, the ex-post value of innovation v_{ijk} is given by

$$r(t)\, v_{ijk}(t) = \pi_{ijk}(t) + \dot{v}_{ijk}(t) - g(t)\, v_{ijk}(t) \tag{7}$$

where

$$g(t) = \int_{i' \in \mathcal{N}} a_{i'}(t)\, R_{i'}(t)\, di' \tag{8}$$

and $\mathcal{N}$ is the set of countries in North. The right-hand side of (7) consists of a profit flow, capital gains and the term reflecting the risk of losing the value because of an extra innovation hitting the industry j. g in (8) is the Poisson rate of an innovation arriving in industry j, which is equivalent to the total sum of Poisson rates across the North. Note that v_{ijk} is the same for both of variety and quality innovations.

Next, consider the ex ante value of innovation. We derive it in two steps. First, we assume that an innovating firm in a country i optimally chooses the quality step γ_j. To make this explicit, we rewrite a profit flow in (6) as

$$\pi_{ik}(t) = \left(1 - \frac{w_i(t)}{\gamma_i w_k(t)}\right) \frac{\beta E(t)}{N(t)} \tag{9}$$

where γ_j is replaced with γ_i. As is shown below, the optimally chosen γ_i will be the same whichever industry is selected for implementation. Hence, it is innocuous to replace the subscript j with i in profits (9). This also allows us to rewrite the ex post value of innovation as v_{ik}.

[19] A well-known example of this type of uncertainty is a microwave oven, which was invented from radar technology for military purposes. The Internet and GPS are also byproducts of military R&D expenditure. Viagra is another example of a commercial product which was originally created for different purposes. A similar assumption is used in Kortum (1997).

Second, recall that following a successful innovation, an industry where it is implemented is randomly selected. This implies that innovation takes a quality form with the probability of N, and a variety form with $1-N$. Now suppose that an industry j is chosen and the previous incumbent firm is located in country k. Note that whichever industy is selected, profits are the same as long as the the previous incumbent firm is from the same country k. In this sense, a random selection of an industry is effectively equivalent to randomly choosing a Northern country where the previous incumbent firm in a chosen industry is located. Bearing this in mind, define n_i to be a measure of industries where production of industrial goods takes place in country k such that $N = \int_{i \in \mathcal{N}} n_k dk$. Then, in the case of quality innovation, the probability that an incumbent firm is from country k (operating in a selected industry j) is

$$\sigma_k(t) = \frac{n_k(t)}{N(t)}, \qquad k \in \mathcal{N}. \tag{10}$$

In the case of variety innovation with quality $q_j = \gamma_{ij}$, copycat firms exist, which can produce the same product with the quality level $q_j = 1$ with the marginal cost of production being wage. We assume that the probability of copycat firms being located in a country k is also given by σ_k.[20]

Now we are in position to derive the ex post value of innovation v_i. Remembering that v_{ik} is the same for variety and quality innovations, v_i is defined by

$$v_i(t) = N \int_{k \in \mathcal{N}} \sigma_k(t) v_{ik}(t) dk + (1-N) \int_{k \in \mathcal{N}} \sigma_k(t) v_{ik}(t) dk = \int_{k \in \mathcal{N}} \sigma_k(t) v_{ik}(t) dk. \tag{11}$$

The first term on the right-hand side of the first equality is the expected value of quality innovation with N being the probability of innovation being implemented as improving quality. Similarly, the second term is the expected value of variety innovation where $1-N$ is the probability of innovation being of variety type. This allows us to write the ex ante asset equation as

$$r(t) v_i(t) = \pi_i(t) + \dot{v}_i(t) - g(t) v_i(t) \tag{12}$$

where

$$\pi_i(t) = \int_{k \in \mathcal{N}} \sigma_k(t) \pi_{ik}(t) dk. \tag{13}$$

3.5 R&D

Inequality in the world economy in the present model arises due to heterogeneity of R&D productivity a_i, and this variable plays an important role in our analysis. We assume the following form:

$$a_i(t) = \kappa N(t) e^{\lambda m(t) - \eta i} \gamma_i^{-\mu}, \qquad \kappa, \lambda, \eta, \mu > 0 \tag{14}$$

where m is the number of Northern countries. (14) captures the following four assumptions:

1. a_i is decreasing in i for a given γ_i. This means that countries are arranged in a decreasing order of

[20] The crux of this explanation on an innovation implementation is that a random selection of an industry j is effectively equivalent to that of a country k where an incumbent is located. Given this, the probability that the previous incumbent firm is located in country k, given that new innovation is of quality type is $N\sigma_k$. Similarly, $(1-N)\sigma_k$ is the probability that copycat firms are located in a country k, given that new innovation is of variety type. Therefore, the probability of selecting a country k where the previous incumbent or copycats are located is $N\sigma_k + (1-N)\sigma_k = \sigma_k$.

R&D productivity. The degree of R&D heterogeneity is captured by η.

2. a_i is a decreasing function of the size of quality innovation γ_i. Noting that a profit flow in (9) is increasing in γ_i, this assumption makes it possible for research firms to optimally choose γ_i, weighing benefits in the value of innovation against costs in R&D productivity. $\mu > 0$ determines the marginal cost of increasing the size of quality innovation.

3. a_i is increasing in the number of innovative industries N. It represents a positive externality that is often found in the variety-based approach of endogenous growth, e.g. [Romer (1990)].

4. a_i is increasing in the number of Northern countries, m. This assumption captures in a simple way a positive spillover effect arising from take-offs. In each and every Northern economy, different experiences (successes and failures) are accumulated, generating diverse perspectives about research. Indeed, such varied perspectives are widely considered important in order to be successful in uncertain research. Opportunities for such learning are greater if there are more researchers and more Northern countries. In this sense, a scope for learning new technologies will be greater as more countries start R&D. λ is a parameter which governs the degree of such externality, which we call the *take-off externality*. It plays an important role in determining the nature of equilibrium.

Turning to the optimal decision of R&D, we assume that research firms decide on the number of workers employed and the size of quality innovation. That is, a representative firm in country i solves the following problem

$$\max_{R_i(t),\gamma_i} \overbrace{\frac{\int_{k\in\mathcal{N}} \sigma_k(t)\left(1-\frac{w_i(t)}{\gamma_i w_k(t)}\right)\frac{\beta E(t)}{N(t)}dk}{r(t)+g(t)-\dot{v}_i(t)/v_i(t)}}^{v_i(t)} a_i(t)R_i(t) - (1-s_i)\,w_i(t)\,R_i(t)$$

where s_i is the rate of subsidy. The first-order conditions are

$$v_i(t)\,a_i(t) = (1-s)\,w_i(t) \qquad \text{for } R_i > 0 \tag{15}$$

$$v_i(t)\,a_i(t) < (1-s)\,w_i(t) \qquad \text{for } R_i = 0 \tag{16}$$

$$a_i(t)\int_{k\in\mathcal{N}} \sigma_k(t)\frac{w_i(t)}{\gamma_i^2 w_k(t)}dk = -\frac{\partial a_i(t)}{\partial \gamma_i}\int_{k\in\mathcal{N}} \sigma_k(t)\left(1-\frac{w_i(t)}{\gamma_i w_k(t)}\right)dk \tag{17}$$

(15) and (16) are standard conditions under free entry. (17) defines the optimal size of quality innovation.[21] Its left-hand side is the marginal benefit of increasing γ_i and and the marginal cost is on the right-hand side.

Rearranging (17) gives

$$\gamma_i = \frac{1+\mu}{\mu}\frac{w_i}{w_N} \tag{18}$$

where

$$\frac{1}{w_N(t)} = \int_{i\in\mathcal{N}} \frac{\sigma_i(t)}{w_i(t)}di. \tag{19}$$

[21] The second-order condition is satisfied for $\mu > 0$, which is assumed.

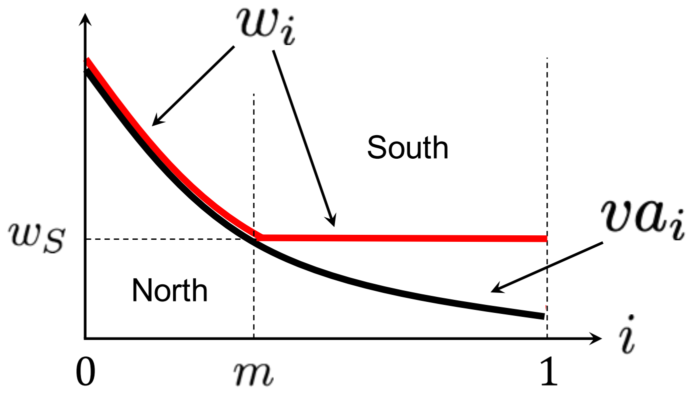

Figure 13: The endogenous division of countries into the North and the South.

(19) is the the weighted harmonic mean of Northern wages. In (18), γ_i is proportional to w_i, implying firms in a higher wage country tend to choose a greater size of quality innovation. Precisely because of this feature, profit flows will be independent of the country and industry indices. To show this, we make use of (18) to rewrite (9) as

$$\pi(t) = \pi_{ik}(t) = \frac{\beta}{1+\mu} E(t). \tag{20}$$

In addition, (20) allows us to write $v_i = v$ for all i, and hence simplify (12) as

$$r(t)\,v(t) = \pi(t) + \dot{v}(t) - g(t)\,v(t). \tag{21}$$

This equation shows that the ex ante value of innovation is identical for all countries. However, this does not mean that all countries will engage in R&D. It is because R&D productivity is too low for some countries, and the first-order condition (16) holds for firms in the South. On the other hand, (15) is satisfied for economies with higher R&D productivity in the North with active R&D. This suggests the existence of the threshold economy between the North and the South.

To demonstrate this, consider (15) which is now given by $v(t)\,a_i(t) = (1-s)\,w_i(t)$ because of $v_i = v$. By assumption, a_i is decreasing in i, and let m denote the economy with the lowest R&D productivity among countries with active R&D with $va_m = w_m$. Note that by construction m is equivalent to the measure of Northern countries. In addition, (16) or $va_i < w_S$ holds in Southern countries. Given that a_i is continuous in i, we conclude that $w_m = w_S$. Intuitively, the threshold country has the same wage as in the South, but its R&D productivity is just high enough to justify investment in research. The situation is illustrated in Figure 13. using (15) and $v_i = v$, we can determine relative wages among countries:

$$\frac{w_i(t)}{w_m(t)} = \frac{a_i(t)}{a_m(t)} \qquad \Rightarrow \qquad \frac{w_i(t)}{w_S(t)} = e^{\frac{\eta}{1+\mu}(m(t)-i)} \tag{22}$$

where $w_m = w_S$. This shows that relative wages for a Northern country i get larger as more countries take off (i.e., an increase in m). This result is intuitive because newly industrialized countries have lower productivity levels relative to a Northern country i.

To explore implications further, combining (18) and (22) yields

$$\gamma_i = \frac{1+\mu}{\mu} \cdot \frac{e^{\frac{\eta}{1+\mu}(m(t)-i)}}{\omega(t)} \tag{23}$$

where

$$\omega(t) \equiv \frac{w_N(t)}{w_S(t)} = \left(\int_{i \in \mathcal{N}} \sigma_i(t) \frac{w_S(t)}{w_i(t)} di \right)^{-1} \tag{24}$$

is interpreted as the Northern relative wages in terms of the Southern wage. (23) captures a mechanism through which take-offs induce further take-offs. As more and more countries are industrialized (i.e. a higher m), it is intuitive and indeed the case that the Norther relative wages ω tends to rise. This translates into a lower quality step γ_i chosen by firms, which in turn increases R&D productivity. This tendency prompts more countries to engage in R&D. This can be confirmed by substituting (23) into (14) to obtain

$$a_i(t) \equiv N(t)\, \alpha_i(m(t), \omega(t)) \tag{25}$$

where

$$\alpha_i(m(t), \omega(t)) \equiv \kappa \left(\frac{\mu}{1+\mu} \right)^{\mu} \omega(t)^{\mu}\, e^{\left(\lambda - \frac{\mu\eta}{1+\mu}\right) m(t) - \frac{\eta}{1+\mu} i} \tag{26}$$

$N\alpha_i(m, \omega)$ is R&D productivity expressed in (ω, m) space. (25) shows that R&D productivity a_i is increasing in ω.[22] Note that this take-off inducing mechanism embeded in (25) is realized through ω, and is different from the take-off externality which is captured by λm in (26).

3.6 Evolution of the Number of Industries

The number of innovative industries changes as the world economy grows. Recall that g is the arrival rate of innovation, and the proportion $1-N$ of innovations creates new industries. Therefore, a flow of newly created industries obeys the following equation:

$$\dot{N}(t) = g(t)(1 - N(t)). \tag{27}$$

3.7 Labour Markets

First consider the South. There are $(1-m)$ number of countries, and each country supplies L labour service, meaning that the total labour supply is $(1-m)L$. Demand for agricultural goods is given by the second equation in (2). Integrating it over i gives the total demand in the world. Therefore, workers in the South are fully employed if[23]

$$(1 - m(t))L = \frac{(1-\beta)E(t)}{w_S(t)}. \tag{28}$$

Turning to the North, consider a country i. There are R_i number of workers in R&D. To calculate labour demand for manufacturing, suppose that a monopoly firm operates in an industry j where the

[22] R&D productivity is increasing m, and it comes from the take-off externality introduced earlier.

[23] Wage of the threshold economy is the same as in the South, i.e. $w_m = w_S$. This implies that agricultural goods can be also produced there. Given its infinitesimal economy size relative to the world, we ignore its production of agricultural goods.

previous incumbent was located in country k. Then, given that one worker is required to produce one unit of industrial goods, labour demand is $\frac{\beta E}{\gamma_i w_k N}$ where $\gamma_i w_k$ is the price. Now recall that an industry is randomly chosen to implement innovation after it occurs. Therefore, the average labour demand is given by $\ell_i \equiv \int_{i\in\mathcal{N}} \sigma_k \frac{\beta E}{\gamma_i w_k N} dk = \frac{\beta E}{\gamma_i w_N N}$ using (19). In addition, n_i is the share of manufacturing industries in the country, implying that the total labour demand arising from industrial goods production is $n_i \ell_i$. Now, making use of (10) and (18), full employment of workers in country i is achieved when

$$L = R_i(t) + \sigma_i(t) \frac{\mu\beta}{1+\mu} \frac{E(t)}{w_i(t)}. \tag{29}$$

3.8 Growth Rate

In equilibrium, new innovations are generated and industrial goods are produced in the North. Those goods are traded among Northern countries and exported to the South. Agricultural goods are produced in the South and exported to the North. The North and the South both benefit from specialization and economic growth.

To calculate the growth rate, multiply both sides of the labour market condition (29) by $\alpha_i(m,\omega)$ in (26). Then, integrate the resulting equation from 0 to 1 over i, add the corresponding equation of the base economy and eliminate E by using (28) to derive the following expression

$$g(t) = N(t)\, G(m(t), \omega(t)) \tag{30}$$

where

$$G(m(t), \omega(t)) \equiv L\alpha_m(m(t), \omega(t)) \left[A(m(t)) - \frac{\mu\beta}{1+\mu} \cdot \frac{1-m(t)}{1-\beta} \right] \tag{31}$$

$$A(m(t)) \equiv e^{\frac{\eta}{1+\mu} m(t)} + \int_0^{m(t)} e^{\frac{\eta}{1+\mu}(m(t)-i)} di \tag{32}$$

$A(m)$ is equivalent to the sum of relative R&D productivity levels in terms of the threshold country.[24] Note that the growth rate in (30) reflects the world resource constraints as well as firms' incentives for production and R&D and consumers' optimizing behaviors. In particular, those incentive effects are realized through the two endogenous variables m and ω. But, in this subsection, we take m and ω parametrically to develop intuitions about the effects of changes in those two variables on $G(m,\omega)$ in (31). This helps us understand how the present model works.

First, (31) shows that G is increasing in the Northern relative wages ω through the R&D productivity of the threshold country $\alpha_m(.)$. This works via the take-off inducing channel mentioned above, through which higher Northern relative wages lead to a lower quality step chosen by firms, resulting in higher R&D productivity. [25] That is, the presence of ω in $G(.)$ comes from the endogenous determination of a quality step size.

[24] $A(m)$ is derived using the Dirac Delta function $\delta(.)$, i.e.

$$A(m(t)) = \int_{-\varepsilon}^{+\varepsilon} \frac{\alpha_i(m(t), \omega(t))}{\alpha_m(m(t), \omega(t))} \delta(0)\, di + \int_0^{m(t)} \frac{\alpha_i(m(t), \omega(t))}{\alpha_m(m(t), \omega(t))} di$$

which is reduced to the expression in the main text.

[25] See the discussion regarding (23) and (25) in Sec.3.5.

Next, let us turn to the number of Northern economies m, whose effects on G are realized via three channels. First, a higher m raises growth through the term $1-m$ inside the parentheses in (31). This is due the fact that the real consumption expenditure in terms of the Southern wage falls (see (28)), causing the reduction of manufacturing labour demand. This makes more workers available in R&D in all Northern economies. Second, a higher m raises $A(m)$, leading to a higher growth. This reflects the fact that as more countries take off, the *total* number of research workers across the North simply increases. Third, a higher m raises the R&D productivity of threshold country $\alpha_m(.)$ for $\lambda > \eta$, but it falls for $\lambda < \eta$. To explain it in more detail, suppose that the *take-off externality* is absent, i.e. $\lambda = 0$. Then, $\alpha_m(.)$ unambiguously falls with a higher m. This is due to the fact that a newly industrialized economy inevitably has a lower productivity. Now, suppose $\lambda > 0$. Then, a higher m amplifies the take-off externality effect. If this effect is sufficiently strong such that $\lambda > \eta$, a flow of take-offs raises R&D productivity of countries with a higher growth. Otherwise, growth tends to fall.

As is shown below, starting from the initial condition with $m = 0$, an equilibrium trajectory often involves an increase in ω and m both. Given the mechanisms described above, growth gradually increases (especially for $\lambda > \eta$) as the world economy evolves with sequential take-offs of economies. Having developed intuitions on growth, we will turn to the issue of how m and ω are determined in and off steady state in the ensuing sections.

4 Equilibrium and Steady State

4.1 Equilibrium Conditions

In this paper, we are interested in transitional dynamics as well as steady state. To analyse both, we need to derive the time path of σ_i which is the share of manufacturing industries in a country i. Appendix A shows

$$\dot{\sigma}_i = A_i(m(t), \omega(t)) R_i(t) - G(m(t), \omega(t)) \sigma_i(t). \tag{33}$$

Intuitively, (10) shows that the numerator is n_i which rises in proportion to the Poisson rate of innovation arriving in country i. This is captured by the first term in (33). On the other hand, the denominator of σ_i is N. Given that N increases with the world-wide Poisson rate g, the share σ_i falls in proportion to G (see (30)).

In interpreting (33), we need to pay attentions to the threshold country. To describe the issue, note that the sum of manufacturing industry shares must be one, i.e. $1 = \sigma_0 + \int_0^m \sigma_i di$. Time-differentiating it gives $\sigma_m \dot{m} = -\left(\dot{\sigma}_0 + \int_0^m \dot{\sigma}_i di\right)$. Its RHS is reduced to 0 if (33) is used. This indicates the following three possible cases:

1. $\sigma_m = \dot{m} = 0$
2. $\sigma_m > 0$ and $\dot{m} = 0$
3. $\sigma_m = 0$ and $\dot{m} \gtreqless 0$.

Note that (1) cannot arise because the threshold country conducts R&D, and hence $\sigma_m > 0$ holds in steady state. (2) holds in steady state only, and (3) is the case which is relevant to transitional dynamics. $\dot{m} > 0$ indicates that sequential take-offs occur, and $\dot{m} < 0$ means that some Northern economies are de-industrialized, reverting to the South. In both cases, the industry share must be nil. This should be intuitive. Consider $\dot{m} > 0$. More and more countries become industrialized, and the identity of the threshold country changes as time passes. Simply, those threshold countries do not initially produce industrial goods, i.e. $\sigma_m = 0$ (R&D success comes first before production). More formally, we add the following condition:

$$\dot{\sigma}_m = A_m\left(m\left(t\right), \omega\left(t\right)\right) R_m\left(t\right) \qquad \text{for } \dot{m} > 0. \tag{34}$$

For $\dot{m} < 0$, those Northern countries which revert to the South do so after losing manufacturing industries.

Now, we are ready to state the following lemma:

Lemma 1. *Equilibrium is defined by the following three differential equations:*

$$\Delta\left(m\left(t\right)\right)\dot{m}\left(t\right) - \mu\frac{\dot{\omega}\left(t\right)}{\omega\left(t\right)} = F\left(m\left(t\right), \omega\left(t\right)\right) \tag{35}$$

$$\frac{\eta}{1+\mu}\dot{m}\left(t\right) - \frac{\dot{\omega}\left(t\right)}{\omega\left(t\right)} = H\left(m\left(t\right), \omega\left(t\right)\right) \tag{36}$$

$$\dot{N}\left(t\right) = N\left(t\right)\left(1 - N\left(t\right)\right) G\left(m\left(t\right), \omega\left(t\right)\right) \tag{37}$$

where

$$F\left(m\left(t\right), \omega\left(t\right)\right) = L\alpha_m\left(m\left(t\right), \omega\left(t\right)\right)\left[A\left(m\left(t\right)\right) - B\left(1 - m\left(t\right)\right)\right] + \rho \tag{38}$$

$$H\left(m\left(t\right), \omega\left(t\right)\right) = L\alpha_m\left(m\left(t\right), \omega\left(t\right)\right)\left[\omega\left(t\right)\left(1 + m\left(t\right)\right) - A\left(m\left(t\right)\right)\right] \tag{39}$$

$$\Delta\left(m\left(t\right)\right) \equiv \frac{1}{1 - m\left(t\right)} + \eta - \lambda \tag{40}$$

$$B \equiv \left(\mu + \frac{1}{1-s}\right)\frac{\beta}{\left(1+\mu\right)\left(1-\beta\right)} \tag{41}$$

See Appendix B.

The lemma shows that equilibrium in (m, ω) can be characterized by using (35) and (36) only, given that they are independent of N. Also note that we do not need to track the changes of (33) and (34).[26] These are convenient features of this dynamical system especailly for our purpose of analyzing the time path of m.

4.2 Steady State

Before we consider transitional dynamics, we briefly consider steady state. According to (37), the number of industrial industries reaches the upper limit, i.e. $N^* = 1$ in the long run. Regarding m and ω, we require $\dot{m} = \dot{\omega} = 0$ in (50) and (51) in steady state. It means that the steady state value m^* and ω^* are

[26] Those equations will be required in simulating the Gini coefficient in transition.

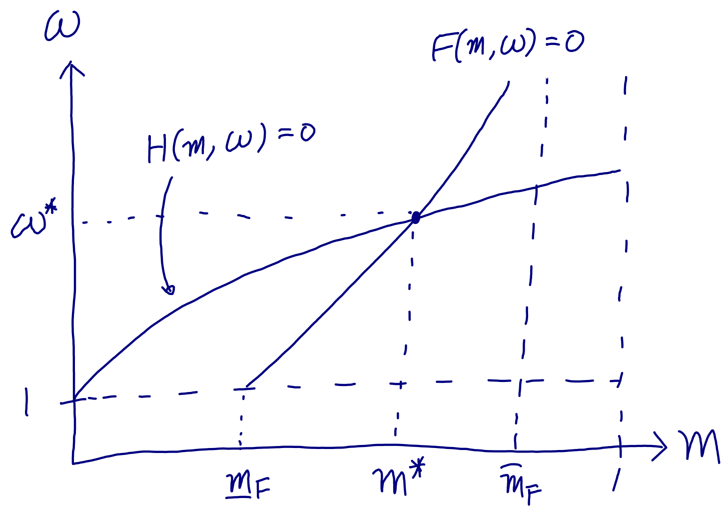

Figure 14: Steady state.

determined by the following two conditions

$$F(m^*, \omega^*) = 0 \quad \Rightarrow \quad \omega^* = \frac{1+\mu}{\mu} \left(\frac{\rho}{L\kappa e^{(\lambda-\eta)m^*} [B(1-m^*) - A(m^*)]} \right)^{\frac{1}{\mu}} \tag{42}$$

$$H(m^*, \omega^*) = 0 \quad \Rightarrow \quad \omega^* = \frac{A(m^*)}{1+m^*} \tag{43}$$

They are depicted in Figure 14 under the assumptions of

$$B > 1, \qquad \omega^*|_{F(0,\omega^*)=0} \equiv \left(\frac{1+\mu}{\mu} \right)^{\mu} \frac{\rho}{L\kappa (B-1)} < 1. \tag{44}$$

These inequalities ensure that the $F(.) = 0$ curve has a vertical intercept at $\omega < 1$, which in turn means that it starts from $m = \underline{m}_F > 0$ for $\omega = 1$, monotonically increasing in m. The following result is is clear from the figure.

Lemma 2. *The slopes of $F(m^*, \omega^*)$ and $H(m^*, \omega^*)$ in (m, ω) space are positive, and the former is always greater than the latter:*

$$-\frac{\overbrace{H_m(m^*, \omega^*)}^{(-)}}{\underbrace{H_\omega(m^*, \omega^*)}_{(+)}} < -\frac{\overbrace{F_m(m^*, \omega^*)}^{(+)}}{\underbrace{F_\omega(m^*, \omega^*)}_{(-)}}$$

Note that this inequality holds irrespective of the values of parameters under (44).

Another feature worth mention is that there is the maximum value that m can take in equilibrium. It is denoted by $\overline{m}_F$ in Figure 14. Its exact value can be found by setting the denominator of (42) to zero. This implies that not all countries can be industrialized, and Southern countries $i \in [\overline{m}_F, 1]$ are trapped in underdevelopment. This result is clear from the figure.

Proposition 1. *Countries $i \in [\overline{m}_F, 1]$ are trapped in underdevelopment.*

The following proposition summarizes comparative statics results of parameters:

Proposition 2. *In steady state, the world growth rate are given by*[27]

$$g = G = L\left(A\left(m^*\right) - \frac{\mu\beta}{1+\mu}\cdot\frac{1-m^*}{1-\beta}\right) \tag{45}$$

Comparative statics results:

1. A fall in ρ and a rise in L, κ, λ, β and s increase all of m^*, ω^* and g.

2. An increase in μ and η generates ambiguous effects on m^*, ω^* and g.

Results in (1) are intuitive. Those parameter changes are all conducive to innovation in the sense that they incentivize firms to invest more in R&D. Perhaps more interesting is how those parameters affect the countries in the underdevelopment trap.

Proposition 3. *A fall in η and a rise in μ, β and s raise $\overline{m}_F$, while it is independent of κ, λ, L and ρ.*

A lower η means a fall in the degree of heterogeneity of R&D productivity among countries. It is intuitive that the more homogeneous the countries are, the smaller the range of the underdevelopment trap. A greater μ raises the marginal cost of raising the quality step size (see (14)). The result comes from the relationship between the quality step size and R&D productivity explained in discussing the take-off inducing effect of ω pertaining to (23) and (25). Indeed, as (26) shows, a higher μ raises R&D productivity, and this effect leads to the shrinkage of the trap. What is somewhat surprising is that R&D productivity improvement κ does not make the range of the trap smaller. Neither does λ.

4.3 Gini

We first calculate GDP per capita, denoted by y_i. In the South, all workers are employed in the agricultural sector, earning w_S with no saving. Therefore, its GDP per capita is equivalent to $y_S = w_S$. In the North, workers are used in the production of manufacturing goods and R&D, earning wage w_i. In addition, monopoly firms earn profits π_i. Hence, GDP per capita is given by

$$y_i = w_i + \sigma_i \frac{\beta}{1+\mu}\frac{E}{L}, \qquad i \in \mathcal{N}. \tag{46}$$

To rewrite the industry share of industrial goods σ_i, use (33) and (45) to derive

$$\sigma_i = \frac{e^{-\frac{\eta}{1+\mu}i}}{1+\int_0^{m^*} e^{-\frac{\eta}{1+\mu}k}dk}, \qquad i \in \mathcal{N}. \tag{47}$$

It depends on the exogenous measure of industry heterogeneity η. It also depends on μ that affects the endogenous determination of the quality step size, which in turn determines the level of R&D productivity. Using (25), (46) and (47), one can easily confirm $y_i = e^{-\frac{\eta}{1+\mu}i}y_0$. This means that GDP per capita of the Northern economies relative to the total Northern GDP per capita is also equivalent to (47).

[27] The derivation of (45) is given in Appendix C.

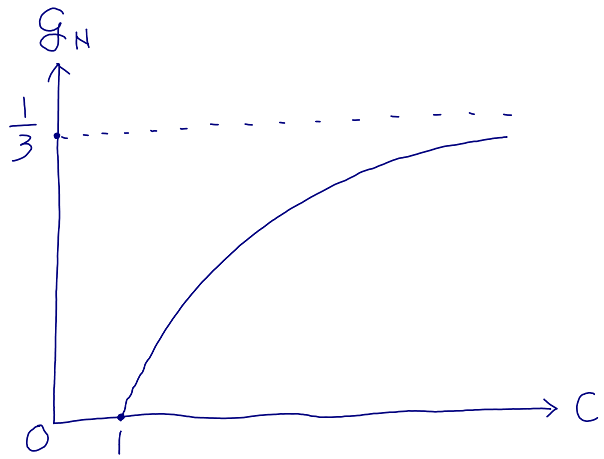

Figure 15: The Gini in the North with $C \equiv e^{\frac{\eta}{1+\mu}m^*}$.

We follow Lambert and Decoster (2005) in calculating the Gini coefficient, which we denote by $\mathcal{G}$. An advantage of their method is to allow us to decompose inequality into several components, including the Gini within the North only, denoted by $\mathcal{G}_N$. First let us consider the Northern Gini without the base economy for simplicity. In Appendix D, we show that the Lorenz curve is given by

$$\mathcal{L}_N(p) = \frac{\int_0^p \left[1 + \Phi\left(e^{\frac{\eta}{1+\mu}m^*} - 1\right)\right] d\Phi}{\int_0^1 \left[1 + \Phi\left(e^{\frac{\eta}{1+\mu}m^*} - 1\right)\right] d\Phi} \tag{48}$$

where Φ is the distribution function of y_i in the North. Hence, the Gini coefficient of the North (without the base economy) is

$$\mathcal{G}_N = 1 - 2\int_0^1 \mathcal{L}_N(p)\, dp = \frac{C-1}{3(C+1)} \qquad \text{where } C \equiv e^{\frac{\eta}{1+\mu}m^*}.$$

η and μ directly affect $\mathcal{G}_N$, which also indirectly depends on other parameters via m^*. Its convenient feature is that $\mathcal{G}_N$ depends on $C \equiv e^{\frac{\eta}{1+\mu}m^*}$ only. $\mathcal{G}_N$ is illustrated in Figure 15 where it unambiguously rises in C starting from 0 for $C = 1$ with an upper bound 1/3. A higher m^* means that countries that become industrialized have lower GDP per capita than those already in the North, and hence it increases $\mathcal{G}_N$. But the marginal effect of m^* on the Norther Gini is diminishing. An intuition is that as m^* increases, the industrial production is shared among an increased number of Northern countries, reducing σ_i for incumbent Northern countries. As a result, the marginal impact of a take-off shrinks. For example, suppose that an R&D productivity parameter κ rises, perhaps due to a major scientific discovery. We know from Proposition 2 that more countries are industrialized with a higher m^*. This translates to a rise in $\mathcal{G}_N$ according to Figure 15. Though it raises inequality among the North, its impact on inequality diminishes as κ increases further.[28]

Turning to the Gini coefficient of the world economy (excluding the base economy), it can be decomposed into[29]

$$\mathcal{G} = \xi(m^*)(1 + m^*\mathcal{G}_N(m^*)) - m^* \tag{49}$$

[28] $\mathcal{G}_N$ increases following one or combinations of the parameters in (1) of the Proposition rise, and the same implications also applies to them.

[29] See Appendix E for derivation.

where

$$\xi(m^*) = \frac{m^* y_N}{y_W}$$

is the income share of the North, y_N is the average GDP per capita in the North and y_W is the average GDP per capita in the world.[30] m^* in $\xi(m^*)$ and m^* attached to $\mathcal{G}_N$ are interpreted as the direct effect of the *take-off effect* which increases inequality. On the other hand, the last m^* in (49) represents the the *catch-up effect* of decreasing inequality. There are of course the indirect effects of a greater m via y_N, y_W and $\mathcal{G}_N$. As one can imagine, $\mathcal{G}(m^*)$ is highly non-linear in m^*, and hence, it is difficult to draw a general conclusion regarding the impacts of parameter changes on $\mathcal{G}(m^*)$.[31]

Note that the above results, e.g. Figure 15, are valid only in steady state. In transitional dynamics with sequential take-off which we are interested in, the analysis of the income distribution of Northern economies are much more involved. To cope with it, we will resort to simulation later.

5 Transitional Dynamics

5.1 The Role of the take-off Externality

In this section, we focus upon off steady state equilibrium. For this, (35) and (36) are used to derive the following system of differential equations:

Lemma 3. *Equilibrium is defined by the following two differential equations:*

$$\dot{m}(t) = W(m(t), \omega(t)) \tag{50}$$

$$\dot{\omega}(t) = U(m(t), \omega(t)) \tag{51}$$

where

$$W(m(t), \omega(t)) \equiv \frac{F(m(t), \omega(t)) - \mu H(m(t), \omega(t))}{\Delta(m(t)) - \dfrac{\eta\mu}{1+\mu}} \tag{52}$$

$$U(m(t), \omega(t)) \equiv \omega(t)\left(\frac{\eta}{1+\mu} W(m(t), \omega(t)) - H(m(t), \omega(t))\right) \tag{53}$$

Note that both (42) and (43) are required in order for $\dot{m} = 0$ and $\dot{\omega} = 0$ to hold simultaneously.

We investigate the local properties of steady state by linearizing the the system:

$$\begin{bmatrix} \dot{m} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} W(m^*, \omega^*) \\ U(m^*, \omega^*) \end{bmatrix} + J \begin{bmatrix} m - m^* \\ \omega - \omega^* \end{bmatrix} \tag{54}$$

[30] $\xi(m^*)$ is given by the following expression:

$$\xi(m^*) = \frac{\int_0^{m^*} e^{\frac{\eta}{1+\mu}k} y_m dk}{(1-m^*) w_S + \int_0^{m^*} e^{\frac{\eta}{1+\mu}k} y_m dk} = \frac{\int_0^{m^*} e^{\frac{\eta}{1+\mu}k} dk \left(1 + \frac{e^{-\frac{\eta}{1+\mu}m^*}}{1+\int_0^{m^*} e^{-\frac{\eta}{1+\mu}i} di'} \frac{\beta}{1+\mu} \frac{1-m^*}{1-\beta}\right)}{(1-m) + \int_0^{m^*} e^{\frac{\eta}{1+\mu}k} dk \left(1 + \frac{e^{-\frac{\eta}{1+\mu}m^*}}{1+\int_0^{m^*} e^{-\frac{\eta}{1+\mu}i} di'} \frac{\beta}{1+\mu} \frac{1-m^*}{1-\beta}\right)}.$$

[31] Having said this, note that $\mathcal{G}(0) = 0$ and $\mathcal{G}(1) = \mathcal{G}_N(1)$. Given Figure 15, there can be a maximum of $\mathcal{G}$ in $m^* \in (0, \overline{m}_F)$.

where

$$J = \begin{bmatrix} W_m(m^*,\omega^*) & W_\omega(m^*,\omega^*) \\ U_m(m^*,\omega^*) & U_\omega(m^*,\omega^*) \end{bmatrix} \tag{55}$$

Given this, we can establish the following lemma:

Lemma 4. *The sign of the determinant of the system (54) depends on that of* $\Delta(m^*) - \dfrac{\eta\mu}{1+\mu}$*, which in turn depends on the take-off externality* λ*:*[32]

$$Det(J) \begin{cases} < 0 & \text{for } \Delta(m^*) > \dfrac{\eta\mu}{1+\mu} \text{ with relatively low } \lambda \\ > 0 & \text{for } \Delta(m^*) < \dfrac{\eta\mu}{1+\mu} \text{ with relatively high } \lambda \end{cases}$$

Proof. See Appendix G. □

To explain the lemma, suppose that $\lambda = 0$, i.e. no take-off externality. (40) shows that $\Delta(m^*) > \frac{\eta\mu}{1+\mu}$ in this case. Now let us increase λ. Then, m^* rises, and so does $\Delta(m^*)$ initially. However, there is an upper limit $\overline{m}_F$ which m^* cannot exceed. This implies that as λ increases further, $\Delta(m^*) - \frac{\eta\mu}{1+\mu}$ will eventually turn negative. As this lemma implies, the take-off externality λ plays a crucial role in the existence of the diffusion of industrialization. To highlighte its role, we distinguish the following three cases in terms of $\Delta(m^*)$:[33]

1. A Weak Take-off Externality: $\dfrac{\eta\mu}{1+\mu} < \Delta(m^*)$
2. A Moderately Strong Take-off Externality: $\tilde{\Delta} < \Delta(m^*) < \dfrac{\eta\mu}{1+\mu}$
3. A Strong Take-off Externality: $\Delta(m^*) \leq \tilde{\Delta}$

where $\tilde{\Delta}$ is defined such that $\dfrac{\eta}{1+\mu} F_\omega(m^*,\omega^*) = \tilde{\Delta} H_\omega(m^*,\omega^*)$. It is the value of Δ which just makes $\left.\dfrac{\partial\dot{\omega}}{\partial\omega}\right|_{(m^*,\omega^*)} = \infty$. Note that $\tilde{\Delta} < 0$ because $F_\omega(m^*,\omega^*) < 0$ and $H_\omega(m^*,\omega^*) > 0$. Diagrammatically, those three cases can be distinguished in terms of the relative slopes of the $\dot{m} = 0$ and $\dot{\omega} = 0$ curves, as in Figures 16-18.[34]

5.2 A Weak Take-off Externality

Here we consider $\text{Det}(J) < 0$, which arises if the take-off externality is non-existent or relatively weak. The result is stated first.

Proposition 4. *Suppose that the take-off externality is weak with low* λ *such that* $\Delta(m^*) > \dfrac{\eta\mu}{1+\mu}$*. Then,* (m^*,ω^*) *is a saddle point equilibrium, and the diffusion of industrialization cannot occur.*

Proof. See Appendix G. □

[32] We do not consider the case of $D(m^*) = 0$ at which the determinant is undefined.

[33] The determinant $\text{Det}(J)$ is undefined for $\Delta(m^*) = \dfrac{\eta\mu}{1+\mu}$.

[34] See Appendix F for proof.

This proposition is best explained using Figure 16. In this case, the $\dot{m}=0$ curve is steeper than the $\dot{\omega}=0$ curve around the steady state. The directions of streamlines can be confirmed by the following derivatives:[35]

$$\left.\frac{d\dot{m}}{d\omega}\right|_{(m^*,\omega^*)} = \frac{\overbrace{F_\omega(m^*,\omega^*)}^{(-)} - \mu\overbrace{H_\omega(m^*,\omega^*)}^{(+)}}{\Delta(m(t)) - \frac{\eta\mu}{1+\mu}} \quad \begin{cases} <0 & \text{for } \Delta(m(t)) > \frac{\eta\mu}{1+\mu} \\ >0 & \text{for } \Delta(m(t)) < \frac{\eta\mu}{1+\mu} \end{cases} \tag{56}$$

$$\left.\frac{d\dot{\omega}}{d\omega}\right|_{(m^*,\omega^*)} = \frac{\omega^*}{\Delta(m(t)) - \frac{\eta\mu}{1+\mu}}\left(\frac{\eta}{1+\mu}\overbrace{F_\omega(m^*,\omega^*)}^{(-)} - \Delta(m^*)\overbrace{H_\omega(m^*,\omega^*)}^{(+)}\right) \tag{57}$$

The both derivatives are negative for $\Delta(m) > \frac{\eta\mu}{1+\mu}$, i.e. $\mathrm{Det}(J) < 0$. Based on this information, a saddle path is depicted in the figure.

The question we are interested in is whether or not industrialization diffuses to backward countries from the base economy. To answer this question, let us define the initial condition. We suppose that at time 0, the base economy is a sole country which has already been industrialized (i.e. $\sigma_0 = 1$), and other economies $i \in (0,1]$ are specialized in agricultural production, i.e. $m(0) = 0$. Regarding the relative wages, $w_0 = w_S$, given that R&D productivity a_i is a continuous function. In Figure 16, the initial condition is given by $(m(0), \omega(0)) = (0,1)$.

Having identified the initial condition, now the above question is rephrased as follows: Is there a transitional path from $(0,1)$ to the interior long-run equilibrium E_1? Appendix G shows the existence of $\hat{m} > 0$ in the figure at which the $\dot{m}=0$ curve hits the horizontal line at $\omega = 1$. Because the saddle path must lie below the $\dot{m}=0$ curve for $m < m^*$, it must starts from a point on the $\omega = 1$ line, e.g. $\tilde{m} \geq \hat{m}$. That is, to reach the saddle path from $(0,1)$, the world economy must somehow jump onto it. However, it is natural to interpret m as a stock variable in the sense that industrialization takes place gradually and its diffusion is a progressive process where countries take off sequentially. Therefore, with a weak take-off externality, the diffusion of industrialization does not occur and the world economy is stuck in the initial condition $(0,1)$.

An intuition goes as follows. Industrialization requires sufficiently high R&D productivity, and a dynamic process of its diffusion necessitates a sustained increase in R&D productivity to pull Southern economies with successively lower productivity levels out of backwardness. The strength of such a "pull" mechanism depends the take-off externality, which is too weak in this case. The result also implies that the take-off inducing effect through the determination of a quality step size, discussed towards the end of Sec.3.5 is not enough make set the diffusion of industrialization in motion.

5.3 A Moderately Strong Take-off Externality

We turn to the case where the strength of the take-off externality is in the intermediate range with a moderately strong "pulling" effect. The both $\dot{m}=0$ and $\dot{\omega}=0$ curves are positively sploped, but the latter is steeper now.

[35] The derivatives are evaluated at steady state to make calculation and exposition easier. Note that their signs are valid for $\left.\frac{d\dot{m}}{d\omega}\right|_{m^*}$ and $\left.\frac{d\dot{\omega}}{d\omega}\right|_{\omega^*}$ as well, otherwise different demarcation lines (differential equations) would be required.

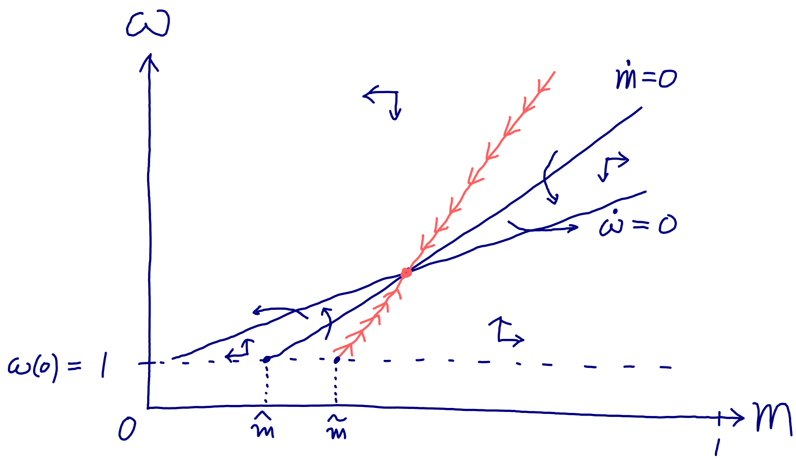

Figure 16: The case of a weak take-off externality. The steady state is a saddle with no path from $(0, \omega(0))$ to E_1.

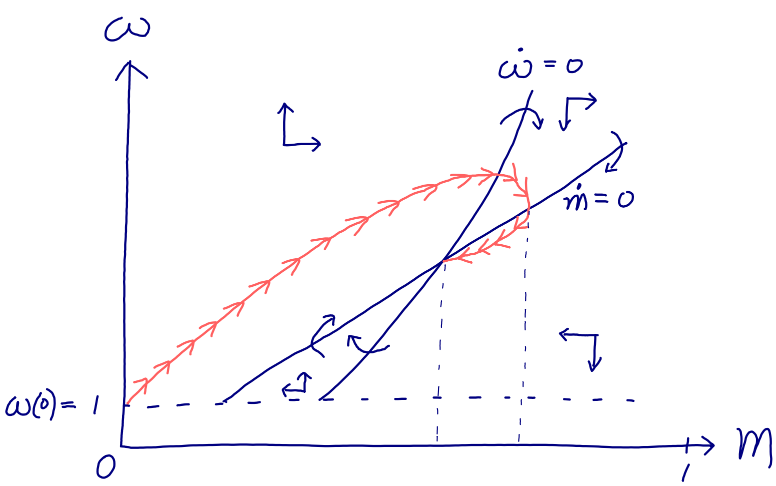

Figure 17: The case of a moderately strong take-off externality. The steady state can be a sink or a source. A path from $(0, \omega(0))$ to E_2 exists only if the steady state is a sink.

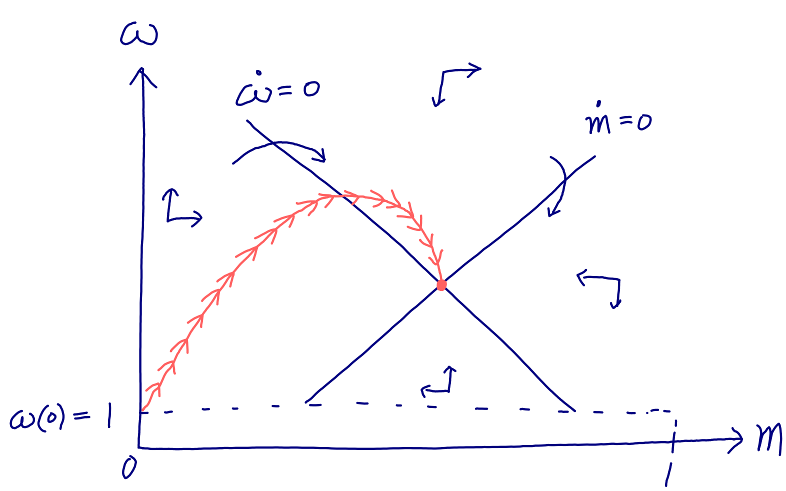

Figure 18: The case of a strong take-off externality. The steady state is a sink. A path from $(0, \omega(0))$ to E_2 always exists.

Proposition 5. *Suppose that the take-off externality is moderately strong with the value of* λ *such that* $\tilde{\Delta} < \Delta(m^*) < \frac{\eta\mu}{1+\mu}$*. Then, a sufficient condition for* (m^*, ω^*) *to be a sink is*

$$-\frac{W_m(m^*, \omega^*)}{W_\omega(m^*, \omega^*)} > \omega^* \frac{\eta}{1+\mu}, \tag{58}$$

in which case the diffusion of industrialization occurs.

Proof. See Appendix H. □

To draw a figure, let us evaluate (56) and (57) for this case:

$$\left.\frac{d\dot{m}}{d\omega}\right|_{(m^*,\omega^*)} > 0, \qquad \left.\frac{d\dot{\omega}}{d\omega}\right|_{(m^*,\omega^*)} > 0.$$

Making use of this information, we can draw Figure 17 where the steady state is totally stable and possibly spiral. In particular, starting from the initial condition $(0, 1)$, the world economy moves along a path towards long-run equilibrium E_2. In this process, industrialization gradually diffuses to economies with high R&D productivity levels and then to lower productivity levels. A steady increase in m captures the fact that more and more economies take off sequentially. As more economies join the Northern club, the relative wages rise as well. An intuition is that the take-off externality is strong enough to pull m^* Southern countries out of backwardness in the long run. Note that in this process the externality boosts R&D productivity via the take-off externality as more countries are industrialized, and this is the main driver of this diffusion process.

On the other hand, once $\tilde{E}_2$ is passed, the relative wages ω begins falling. In addition, m also starts falling after $\tilde{m}_2$ is reached. This is interpreted as follows. After $\tilde{E}_2$ some Northern economies with lower R&D productivity levels reduce investing in research, and their share of industrial industries shrinks, creating downward pressure on their wages. Consequently, the Northern relative wages steady fall. The threshold countries revert to the South after losing industrial industries, creating further downward pressure on the relative wages. Such fall-back results from the overshooting of ω and m. It is also possible that take-off and fall-back can occur alternately as the world economy spiral around the long-run equilibrium.

However, given that (58) is a sufficient condition, the steady state can become totally unstable, in which case no transitional path exists from the initial condition to E_2 and long-run equilibrium is equivalent to the initial condition. This happens if, e.g. heterogeneity of countries η is so large that the pulling effect of the take-off externality is not enough to initiate the process of sequential take-offs. In Figure 17, this case would be represented by a much flatter $\dot{m} = 0$ curve (the LHS of (58) is its slope). In this case, for example, if the world economy was initially located at $\tilde{E}_2$, it would be dragged outward, increasing the distance to E_2.

5.4 A Strong Take-off Externality

This is the case where the take-off externality is strong to the extent that the $\dot{\omega} = 0$ curve has a negative slope, as in Figure 18.

Proposition 6. *Suppose that the take-off externality is strong with λ such that $\Delta(m^*) \leq \tilde{\Delta}$. Then, (m^*, ω^*) is a sink, and hence the diffusion of industrialization occurs.*

Proof. See Appendix H. □

To check the directions of streamlines, evaluate (56) and (57):

$$\left.\frac{d\dot{m}}{d\omega}\right|_{(m^*,\omega^*)} > 0, \qquad \left.\frac{d\dot{\omega}}{d\omega}\right|_{(m^*,\omega^*)} < 0.$$

There exists a path from the initial condition to the long-run equilibrium E_3. Although an inverted U trajectory is illustrated in the figure, a monotonically increasing transitional path or a cycle are also possible. A difference from the case of a moderately strong take-off externality is the absence of a condition like (58). This is because the pulling effect of the externality is sufficiently strong.

5.5 Simulation: International Kuznets Curve

Does the International Kuznets Curve arise as the world economy moves along a transitional path? To address this question, we resort to simulation, given the highly non-linear nature of the Gini Coefficient.

We introduced the base economy in order to set the initial condition, so that we can examine the diffusion of industrialization rather than a question of why the Industrial Revolution occurred in Britain.[36] It is a useful device for our analytical purposes in the preceding sections. However, in conducting simulation analysis, the size of the base economy relative to the world is too large. The Dirac Delta function pins down the size of the base economy to one, while there is a measure one of other economies. It means that for computational purposes, the base economy is equivalent to half of the world economy. To exclude its large effect on inequality measures, we consider the Gini Index of all economies except the base economy.

We present two simulations. One with a monotonically increasing Gini, and the other with the case of the International Kuznets Curve. For the first one, we assume the following parameters:

$$\lambda = 6.0, \quad \eta = 3.0, \quad \kappa = 0.09, \quad \mu = 0.03, \quad \beta = 0.8, \quad \rho = 0.07, \quad L = 0.32, \quad s = 0.$$

Parameter values are chosen to make sure that steady state is a sink. Figure 19 demonstrates the results with $m^* = 0.14$ and $\omega^* = 1.48$. Panels (a) and (d) show (m, ω) which are monotonically increasing, i.e. countries are sequentially taking off earning higher wages. In Panel (e), two series are plotted. The variety of industrial goods N, which is set to 0.03 in the initial period, increases as innovations occur. The introduction of those goods into the world economy is indeed a slow process. It mirrors growth rate in Panel (f). σ_0 in Panel (e) is the share of the industrial goods of the base economy. It is initially one, but its share is eroded as industrialization spreads to other economies. The Gini Coefficient is shown in Panel (c), and increasing monotonically. Recall that (49) consists of three components: (1) the Gini in the North $\mathcal{G}_N$, (2) the income share of the North ξ and (3) the number of Northern countries m. The first element $\mathcal{G}_N$ takes an inverted U trajectory. This contrasts with Figure 15 which shows that $\mathcal{G}_N$ monotonically rises in m^* in C. When m is small, income differences among the Northern economies are large to the extent that $\mathcal{G}_N$ rises. But, as m increases, the industry production is shared among an increasing number

[36] For example, see Clark (2014)

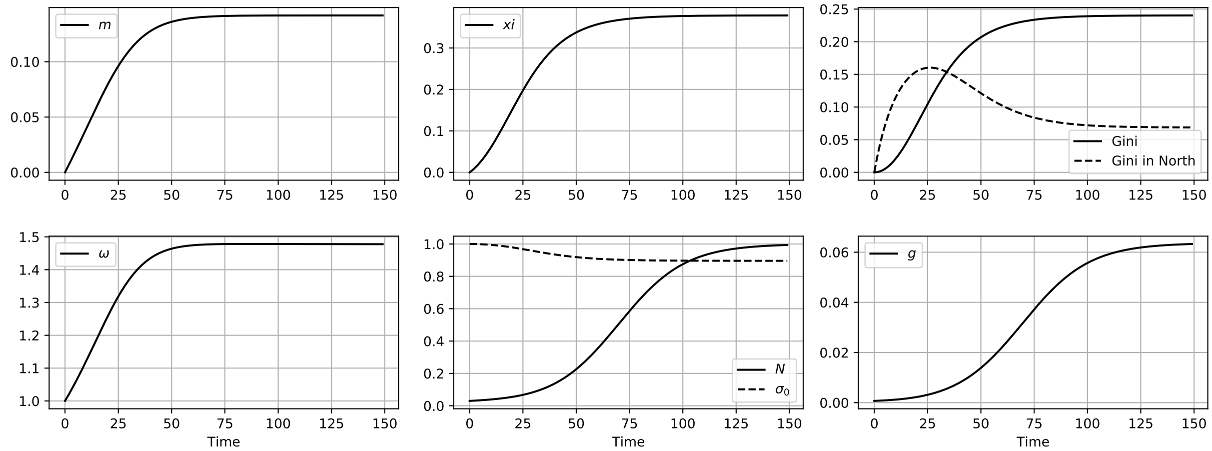

Figure 19: A monotonic increase in the Gini Coefficient of the world economy.

of Northern countries, tending to reduce income differences with a decreasing $\mathcal{G}_N$. Via these mechanisms, the Northern Gini has an upward and downward effect upon the world Gini. The next element is the Northern income share ξ. It monotonically increases in Panel (b), putting upward pressure on the world Gini. The last factor m captures the catch-up effect, which tends to reduce the Gini. Given that its steady state value is small, its effect is not large enough to offset an increasing tendency of $\mathcal{G}$.

In Figure 20, the result of another simulation is shown with the following parameters

$$\lambda = 7.0, \quad \eta = 0.6, \quad \kappa = 0.09, \quad \mu = 0.03, \quad \beta = 0.8, \quad \rho = 0.07, \quad L = 0.3, \quad s = 0.$$

As before, the parameter values are such that the steady state is a sink. Calculation shows that $m^* = 0.48$ and $\omega^* = 1.27$, and it takes shorter time for the world economy to converge to the steady state. A main difference from the previous simulation lies in the value of η, which captures heterogeneity of countries. Because it is now less pronounced, the diffusion of industrialization is more wide-spread with a higher m^*. This in turn reinforces the downward pressure on the world Gini through the last term in (49). Note that the peak of the world Gini occurs before m reaches its steady state value. This means that the critical mass of Norther economies is reached just before the steady state to reduce the world Gini. A fall in the Gini in North, i.e. income convergence among the developed countries, also plays a role in the world Gini.

6 Conclusion

The Industrial Revolution boosted growth in Britain, and as it diffuses to other countries, growth in the world economy accelerated. At the same time, inter-country inequality increased. On the other hand, the recent literature as well as the major macroeconomic datasets suggest that inter-country inequality fell in recent decades. This suggests that inter-country inequality takes an inverted U shape, i.e. the

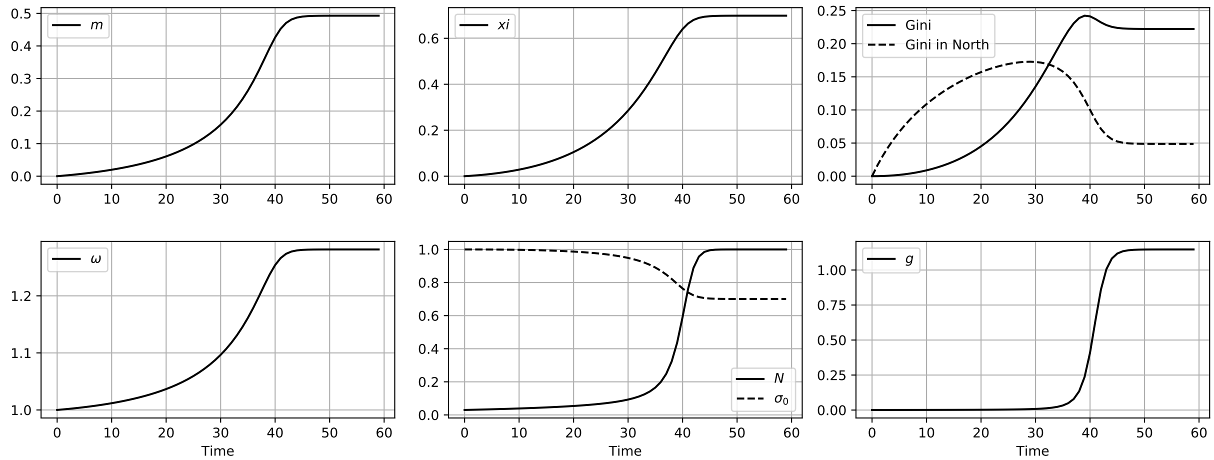

Figure 20: The case of the International Kuznets Curve.

International Kuznets Curve hypothesis. The aim of this paper was to build a Schumpeterian growth model of the world economy to explain this hypothesis. Simulations show that the International Kuznets Curve emerges in a transitional process of the world economy converging to the long-run equilibrium. Especially, it is likely to occur if heterogeneity of countries is sufficiently small. Although the model can generate the International Kuznets Curve, there still remain room for improving simulation results.

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A Derivation of (33)

n_i changes according to

$$\dot{n}_i = NA_i(m,\omega)R_i(1-n_i) - (g - NA_i(m,\omega)R_i)n_i$$

where the first term is a flow of new industries introduced and the last term is the number of industries lost. Now differentiating (10) and rewriting the resulting equation with the above equation and (30) gives (33).

B Proof of Lemma 1

Using (4), (9), (21), (15) and (25), we have

$$\frac{\dot{E}}{E} + \rho + g - \frac{\dot{v}}{v} = \frac{\beta}{(1-s)(1+\mu)}\frac{E}{w_i}\alpha_i(m,\omega). \tag{59}$$

for the threshold country. Time-differentiate (15) and rewrite the resulting equation with (25) and (22) to obtain

$$\frac{\dot{v}}{v} = -(\lambda-\eta)\,\dot{m} - \mu\frac{\dot{\omega}}{\omega} - \frac{\dot{N}}{N} + \frac{\dot{w}_S}{w_S}. \tag{60}$$

Similarly, time-differentiating (28) yields

$$\frac{\dot{E}}{E} = \frac{\dot{w}_S}{w_S} - \frac{\dot{m}}{1-m} \tag{61}$$

Using (27), (30), (28), (60) and (61), (59) can be rewritten as (35).

Using (22), rewrite (19) as $\frac{1}{\omega} = \sigma_0 e^{-\frac{\eta}{1+\mu}m} + \int_0^m \sigma_i e^{-\frac{\eta}{1+\mu}(m-i)}di$. Time-differentiating it gives

$$-\frac{\dot{\omega}}{\omega^2} = \left(\dot{\sigma}_0\frac{\alpha\,(m,\omega)}{A_0\,(m,\omega)} + \int_0^m \dot{\sigma}_i\frac{\alpha\,(m,\omega)}{A_i\,(m,\omega)}di\right) - \frac{\eta}{1+\mu}\frac{\dot{m}}{\omega}$$

where $\dot{\sigma}_m = 0$ is used. Substitute (33) into it, eliminate R_i using (29) and use (31) to derive (36).

C Derivation of (45)

In steady state (33) is reduced to $\sigma_i = \dfrac{\alpha_i\,(m,\omega)\,R_i}{G}$. Using this and (29), we have

$$L = R_i\left(1 + \frac{A_i\,(m,\omega)}{w_i}\frac{\mu\beta}{G\,(1+\mu)}E\right) = R_i\left(1 + \frac{\alpha\,(m,\omega)}{w_m}\frac{\mu\beta}{G\,(1+\mu)}E\right) \qquad \forall i$$

where the second equality uses $\dfrac{\alpha_i\,(m,\omega)}{w_i} = \dfrac{\alpha_m\,(m,\omega)}{w_m}$. This means $R_i = R$. Making use of this result, rewrite (29) by integrating the resulting equation from 0 to 1 and adding the corresponding equation of the base economy:

$$(1+m)\,L=(1+m)\,R+\frac{\mu\beta}{1+\mu}\left(\frac{\sigma_0}{w_0}+\int_0^m\frac{\sigma_i}{w_i}di\right)\frac{(M-m)\,Lw_S}{1-\beta}$$

Making use of (28) and (24), rewrite this further to ge

$$\omega^*\,(1+m^*)\,L=\omega^*\,(1+m^*)\,R+\frac{\mu\beta}{1+\mu}\frac{(1-m^*)L}{(1-\beta)}$$

Finally, substitute (43) into this equation. Rearrangement gives (45).

D Derivation of (48)

Using (25), (46) and (47), we have $y_i=e^{-\frac{\eta}{1+\mu}i}y_0$ where $y_0=w_Se^{\frac{\eta}{1+\mu}m^*}+\frac{1}{\int_0^{m^*}e^{-\frac{\eta}{1+\mu}i'}di'}\frac{\beta}{1+\mu}\frac{E}{L}$. Therefore, the density of y_i is given by $\frac{y_i}{y_0+\int_0^{m^*}y_{i'}di'}=\frac{e^{-\frac{\eta}{1+\mu}i}}{1+\int_0^{m^*}e^{-\frac{\eta}{1+\mu}i'}di'}$, which is equivalent to (47).

Now consider the North without the base economy. Its density is now given by

$$\frac{y_i}{\int_0^{m^*}y_{i'}di'}=\frac{y_me^{\frac{\eta}{1+\mu}(m^*-i)}}{y_m\int_0^{m^*}e^{\frac{\eta}{1+\mu}(m^*-i')}di'}=\frac{e^{\frac{\eta}{1+\mu}k}}{Y_N}$$

where $k=m^*-i$ and $Y_N=y_m\frac{e^{\frac{\eta}{1+\mu}m^*}-1}{\frac{\eta}{1+\mu}}=\frac{y_0-y_m}{\frac{\eta}{1+\mu}}$ is the total GDP per capita in the North without the base economy. The associated cumulative distribution is $\frac{y_m\int_0^k e^{\frac{\eta}{1+\mu}k'}dk'}{Y_N}=\frac{y_m\left(e^{\frac{\eta}{1+\mu}k}-1\right)}{Y_N\frac{\eta}{1+\mu}}=\frac{x-y_m}{y_0-y_m}=\Phi\,(x)$ where $x=y_k=y_me^{\frac{\eta}{1+\mu}k}$. This can be inverted to give $x\,(\Phi)=y_m+(y_0-y_m)\,\Phi=y_m\,[1+(C-1)\,\Phi]$. Therefore, substituting this in $\mathcal{L}\,(p)=\frac{\int_0^p x(\Phi)d\Phi}{\int_0^1 x(\Phi)d\Phi}$ gives (48).

E Sketch of Derivation of (49)

The cumulative distribution function of y_i, excluding the base economy is

$$H\,(y)=(1-m)\,F_S\,(y)+mF_N\,(y)=\begin{cases}0 & y<w_S\\ 1-m & y=w_S\\ 1-m+mF_N\,(y) & y_m\le y\le y_0\end{cases}$$

where $F_S\,(y)$ and $F_N\,(y)$ are the cumulative distribution function within each region. In particular, $F_S\,(y)=\int_0^\infty\delta\,(y-w_S)\,dy=1$ where $\delta\,(y-w_S)=\frac{d\mathcal{H}(y-w_S)}{dy}$ and $\mathcal{H}\,(y-w_S)$ is the Heaviside function. The average income in the South, the North and the world is as follows:

$$\mu_S=w_S,\qquad \mu_N=y_m+\int_{y_m}^{y_0}[1-F_N\,(y)]\,dy,\qquad \mu_Y=w_S+\int_{w_S}^{y_0}[1-H\,(y)]\,dy.$$

Note that the product of the mean income and the Gini in each region is

$$\mu_S \mathcal{G}_S = 0, \quad \mu_N \mathcal{G}_N = \int_{y_m}^{y_0} [1 - F_N(y)] F_N(y)\, dy, \quad \mu_Y \mathcal{G} = \int_{w_S}^{y_0} [1 - H(y)] H(y)\, dy.$$

Now, after tedious rearrangement, one can obtain

$$\mu_Y + \mu_Y \mathcal{G} = (1-m)^2 w_S + m\mu_N [2(1-m) + m(1+\mathcal{G}_N)]$$

which can be rearrange into (49).

F Relative Slopes of the $\dot{m} = 0$ and $\dot{\omega} = 0$ Curves

First, note that

$$\begin{aligned}
H_\omega(m^*, \omega^*) &= L\alpha_m(m^*, \omega^*)(1+m) > 0 \\
H_m(m^*, \omega^*) &= L\alpha_m(m^*, \omega^*)\left[\frac{A(m^*)}{1+m^*} - A'(m^*)\right] < 0 \\
F_\omega(m^*, \omega^*) &= -\frac{\mu}{\omega}\rho < 0 \\
F_m(m^*, \omega^*) &> 0 \qquad \text{See Figure 14} \\
W_m(m^*, \omega^*) &= \frac{F_m(m^*, \omega^*) - \mu H_m(m^*, \omega^*)}{\Delta(m^*) - \frac{\eta\mu}{1+\mu}} \qquad \begin{cases} > 0 & \text{if } \Delta(m^*) > \frac{\eta\mu}{1+\mu} \\ < 0 & \text{if } \Delta(m^*) < \frac{\eta\mu}{1+\mu} \end{cases} \\
W_\omega(m^*, \omega^*) &= \frac{F_\omega(m^*, \omega^*) - \mu H_\omega(m^*, \omega^*)}{\Delta(m^*) - \frac{\eta\mu}{1+\mu}} \qquad \begin{cases} < 0 & \text{if } \Delta(m^*) > \frac{\eta\mu}{1+\mu} \\ > 0 & \text{if } \Delta(m^*) < \frac{\eta\mu}{1+\mu} \end{cases} \\
\left.\frac{d\omega}{dm}\right|_{\dot{m}=0} &= -\frac{W_m}{W_\omega} > 0 \qquad \text{for } \Delta(m^*) \gtrless \frac{\eta\mu}{1+\mu}
\end{aligned}$$

Differentiating (51) gives

$$\left.\frac{d\omega}{dm}\right|_{\dot{\omega}=0} = -\frac{\frac{\eta}{1+\mu} F_m(m^*, \omega^*) - \Delta H_m(m^*, \omega^*)}{\frac{\eta}{1+\mu} F_\omega(m^*, \omega^*) - \Delta H_\omega(m^*, \omega^*)}. \tag{62}$$

whose sign to be determined. For this, note that

$$\frac{\partial}{\partial \Delta}\left(\left.\frac{d\omega}{dm}\right|_{\dot{\omega}=0}\right) = -\frac{\eta}{1+\mu}\frac{\overbrace{H_\omega(m^*, \omega^*)}^{(+)}\overbrace{F_\omega(m^*, \omega^*)}^{(-)}}{\left[\frac{\eta}{1+\mu} F_\omega(m^*, \omega^*) - \Delta H_\omega(m^*, \omega^*)\right]^2}\overbrace{\left(-\frac{H_m(m^*, \omega^*)}{H_\omega(m^*, \omega^*)} - \left(-\frac{F_m(m^*, \omega^*)}{F_\omega(m^*, \omega^*)}\right)\right)}^{(-)} < 0$$

after rearrangement. Therefore, $\lim_{\Delta \downarrow \tilde{\Delta}} \left.\frac{d\omega}{dm}\right|_{\dot{\omega}=0} = \infty$ and $\lim_{\Delta \uparrow \tilde{\Delta}} \left.\frac{d\omega}{dm}\right|_{\dot{\omega}=0} = -\infty$. It means that

$$\left.\frac{d\omega}{dm}\right|_{\dot{\omega}=0} < 0$$

in the case of a strong take-off externality ($\Delta(m^*) < \tilde{\Delta}$).

Turning to the remaining two cases, (62) can be rearranged into

$$\left.\frac{d\omega}{dm}\right|_{\dot{\omega}=U(m,\omega)=0} \Bigg/ \overbrace{\left.\frac{d\omega}{dm}\right|_{\dot{m}=W(m,\omega)=0}}^{(+)} = \Omega$$

where

$$\Omega \equiv \frac{\frac{\eta}{1+\mu} + \frac{1}{-\underbrace{\overbrace{\frac{F_m}{H_m}}^{(+)}}_{(-)} + \mu}\left(\Delta(m^*) - \frac{\eta\mu}{1+\mu}\right)}{\frac{\eta}{1+\mu} + \frac{1}{-\underbrace{\overbrace{\frac{F_\omega}{H_\omega}}^{(-)}}_{(+)} + \mu}\left(\Delta(m^*) - \frac{\eta\mu}{1+\mu}\right)}$$

and assume $\left.\frac{d\omega}{dm}\right|_{\dot{\omega}=U(m,\omega)=0} > 0$. Now noting Lemma 2,

$$\Delta(m^*) > \frac{\eta\mu}{1+\mu} \text{ (a weak take-off externality): } \Omega < 1,$$
$$\Delta(m^*) < \frac{\eta\mu}{1+\mu} \text{ (a moderately strong take-off externality): } \Omega > 1.$$

G Proof of Proposition 4

After tedious rearrangement, the determinant of the Jacobian (55) can be reduced to

$$\begin{aligned}\text{Det}(J) &= W_m(m^*,\omega^*)\,U_\omega(m^*,\omega^*) - W_\omega(m^*,\omega^*)\,U_m(m^*,\omega^*) \\ &= \omega^* \overbrace{\left[\left(-\frac{F_m(m^*,\omega^*)}{F_\omega(m^*,\omega^*)}\right) - \left(-\frac{H_m(m^*,\omega^*)}{H_\omega(m^*,\omega^*)}\right)\right]}^{(+)} \frac{\overbrace{H_\omega(m^*,\omega^*)}^{(+)}\overbrace{F_\omega(m^*,\omega^*)}^{(-)}}{\Delta(m^*) - \frac{\eta\mu}{1+\mu}} \qquad (63)\end{aligned}$$

It is negative for $\Delta(m^*) > \frac{\eta\mu}{1+\mu}$.

Turning to the existence of $\hat{m}$ in Figure 16, note that the $\dot{m}=0$ curve is given by $F(m,\omega) = \mu H(m,\omega)$. Define $\hat{\omega}$ such that $F(\hat{m},1) = \mu H(\hat{m},1)$, which can be rearranged to

$$\overbrace{\frac{\rho}{L\alpha_m(\hat{m},1)\left[B(1-\hat{m}) - A(\hat{m})\right]}}^{LHS(\hat{m})} = \overbrace{\mu\frac{(1+\hat{m}) - A(\hat{m})}{B(1-\hat{m}) - A(\hat{m})} + 1}^{RHS(\hat{m})}.$$

Note that $LHS(0) < 1$ due to (44), and $LHS(m^*) > 1$ because $F(m^*,\omega^*) = 0$ and $\alpha_m(m,\omega)$ is increasing in ω. Also note that $RHS(0) = 1$ and $RHS(m^*) < 1$ because $1+\hat{m} < A(\hat{m})$ and $B(1-\hat{m}) > A(\hat{m})$ for $\hat{m} \le m^*$. Therefore, there exist $0 < \hat{m} < m^*$. We conclude from this that the saddle path must start from $m \ge \hat{m} > 0$ at $\omega = 1$, as in Figure 16, given that it must lie below the $\dot{m}=0$ curve for $m < m^*$.

H Proof of Proposition 5

The trace of the Jacobian (55) is

$$\mathrm{Tr}\,(J) = \frac{\overbrace{F_m\,(m^*,\omega^*)}^{(+)} - \mu \overbrace{H_m\,(m^*,\omega^*)}^{(-)}}{\underbrace{\Delta\,(m^*) - \frac{\eta\mu}{1+\mu}}_{(-)}} + \frac{\omega^* \left[\frac{\eta}{1+\mu} F_\omega\,(m^*,\omega^*) - \Delta\,(m^*)\, H_\omega\,(m^*,\omega^*)\right]}{\underbrace{\Delta\,(m^*) - \frac{\eta\mu}{1+\mu}}_{(-)}}. \tag{64}$$

For the case of a strong take-off externality, we have

$$\frac{\eta}{1+\mu} \overbrace{F_\omega\,(m^*,\omega^*)}^{(-)} - \overbrace{\Delta\,(m^*)}^{(-)} \overbrace{H_\omega\,(m^*,\omega^*)}^{(+)} \geq 0 \tag{65}$$

because $\Delta\,(m^*) < \tilde{\Delta}$. Therefore, $\mathrm{Tr}\,(J) < 0$. To consider the case of a moderately strong take-off externality, rearrange (64) into

$$\mathrm{Tr}\,(J) = \frac{F_m - \mu H_m}{\Delta - \frac{\eta\mu}{1+\mu}} \left[1 + \omega^* \frac{\frac{\eta}{1+\mu} F_\omega - \Delta H_\omega}{F_m - \mu H_m}\right]$$

Note that $\tilde{\Delta} < \Delta\,(m^*) < \frac{\eta\mu}{1+\mu}$ for this case, and hence

$$\lim_{\Delta(m^*)\downarrow\tilde{\Delta}} \mathrm{Tr}\,(J) = \frac{F_m - \mu H_m}{\Delta - \frac{\eta\mu}{1+\mu}} < 0,$$
$$\lim_{\Delta(m^*)\uparrow\frac{\eta\mu}{1+\mu}} \mathrm{Tr}\,(J) = \frac{F_m - \mu H_m}{\Delta - \frac{\eta\mu}{1+\mu}} \left[1 + \omega^* \frac{\eta}{1+\mu} \frac{W_\omega}{W_m}\right].$$

Given that (65) is decreasing in $\Delta\,(m^*)$, (58) is sufficient for $\mathrm{Tr}\,(J) < 0$.