

**Multi-dimensional skills and matching: implications
for international trade and wage inequality**

Chihiro Inaba

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GRADUATE SCHOOL OF ECONOMICS
KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

Multi-dimensional skills and matching: implications for international trade and wage inequality*

Chihiro Inaba[†]

Graduate School of Economics, Kobe University

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Abstract

Individuals have various skills and abilities. The amount of each skill or ability is heterogeneous among individuals, and these individuals work in jobs by exerting these skills and abilities. Firms also have demands for various skill and abilities dependent on the characteristics of its industry. Considering these multi-dimensional skill sets, it can be difficult to accurately match a worker's skill set with a firm's demand. Consequently, a mismatch of the skill set between workers and firms can occur, and may reduce worker wage income and firm productivity. This mismatch generates both inter- and intra-industry wage inequalities.

This study considers how this skill mismatch affects the structure of production, wage inequalities, and international trade. I assume a two-dimensional skill set that is distributed among the workers by a normal probability distribution. I also assume a two-country economy with two industries per country. Cross-country differences in skill distribution can create differences in the structure of production and can be a source of international trade. When international trade commences, a portion of the workers employed in an importing industry will move to an exporting industry since they can earn a higher wage income than by remaining in the importing industry. However, the moving workers who are matched with less appropriate firms may suffer from a lower wage income than under autarky conditions.

Keywords: Multi-dimensional skills, International trade, Wage inequality

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[†]2-1, Rokkodai-cho, Nada-ku, Kobe, Hyogo, 657-8501, Japan. TEL: +81-78-803-7247, FAX:+81-78-803-6877, Email: chihiro.inaba01@gmail.com

1 Introduction

The skills and abilities of a person are often measured by the person's level of education (e.g., high school, university, graduate school, and so on). Reflecting this method of measurement, the skills or abilities of a person are expressed by a one-dimensional variable in economic analysis. In the labor market, a high-skilled person can obtain a job with a high salary, while a low-skilled person is often only able to obtain a job with a low salary. However, there are various types of skills and each person engages in a job by exerting these skills. For example, a salesperson should possess strong communication skills, an information technology specialist should have strong quantitative skills, and an artist should have a sensationally creative abilities. Hence, I cannot determine which of these individuals has the strongest skills and abilities if the individual's skills and abilities are judged using only one dimension. Each skill that an individual possesses has different dimensions or directions. Therefore, a person's skill is not measured by just one dimension; rather, it is measured by multiple dimensions.

Each worker has a set of skills and abilities and, accordingly, searches for a job that complements their individual combination of skills and abilities. Firms also demand certain types of skills, depending on the characteristics of the industry. The firm will search for the worker who best meets its required skill set. Since skill is intangible, it is difficult to accurately match a worker's skill set with a firm's demand. Consequently, a mismatch between workers and firms can occur. This mismatch may not encourage a worker's ability, and may affect the productivity of the firm and the income of the worker.

My Study focuses on the multi-dimensional aspects of skills and the possibility of a skill mismatch between firms and employees. For simplicity, I consider two types of skill sets: communication skills and quantitative skills. Individuals are assumed to have both skill sets, but the bundle or combination of skills is different for each individual. The skill bundle that an individual has is assumed to be inseparable and non-additive. For example one individual may have stronger communication skills than another individual, but weaker quantitative skills. Therefore, there will be a comparative advantage on two skills.

I assume that firms demand both skill sets for production. I consider two industries, each with a different requirement for skill combination, and that firms seek workers with the combination of skill to matching its demand. However, since the number of ideal workers is limited, firms may have to employ workers with a less desirable skill combination, which, in turn, reduces the productivity of the firms. In my study, I analyze the matching pattern between firms and workers, industry structures, and the wage inequalities between and within industries.

The distribution of skill sets is also different between countries, and this difference in

skill distribution can also cause the differences in industry structure between countries. Consequently, cross-country differences in industry structure can be a source of international trade. If free trade were allowed among countries that have different skill distributions, how would economic structures change? I also consider international trade between two symmetric countries that share the same technology, preferences, and size of labor; however, the two countries have different distribution of skill sets.

My study relates to the literature on the multiple dimensions of skill sets and the research on the market imperfections and international trade. First, regarding the multi-dimensional skill sets. Roy (1950) and Rosen (1978) have shown that multiple dimensions of skill affect wage distribution and industry structure. However, they have not examined the implications of international trade.

Ohnsorge and Trefler (2007) is the first work that analyzes the relationship between two-dimensional skill sets and international trade. They have shown that the two-dimensional heterogeneity of workers' skills affects job choice and that cross-country differences in the distribution of skills determine patterns of international trade. This paper mostly follows the work of Ohnsorge and Trefler (2007); henceforth, I will refer to Ohnsorge and Trefler (2007) as O&T. O&T considered two-types of skills, communication skills and quantitative skills, and that the skill bundle is heterogeneous among workers. They considered that there is a continuum of industry, and that each industry has different demands for skill bundles. Workers seek jobs and choose the one offering the highest wage. O&T showed that the comparative advantage of the two skills determines the pattern of worker sorting. When a worker has relatively stronger quantitative skills than communication skills, the worker engages in a more quantitative-intensive industry. Due to the continuum of industry, all workers can select the job that is better suited to them. O&T also analyzed how cross-country differences in the distribution of skills affect international trade. They assumed that the distribution of skills was the normal probability distribution, and examined the roles of the first, second, and high-dimensional moments. O&T showed that the differences in the distribution generated the Heckscher-Ohlin theorem. However, O&T did not consider the possibility of the mismatch of skill demand and supply between firms and workers. In their model, the production level of an industry was based on the worker with the highest productivity. Therefore, there was no mismatch between the labor demand and supply. My study intends to introduce the problem of the demand for and supply of skill combinations clearly, and will analyze how the mismatch affects industry output, wage inequality, and the role of international trade.

Second, there is extensive research concerning the relationship between international trade and the labor market. Traditional trade theories have paid less attention to the role

of the labor market on international trade. Recently, the number of the studies focusing on the role of the labor market in the context of international trade has begun to grow; these studies have shown that the problems of labor markets affect output, wage inequality, and international trade. Some researchers have focused on labor market friction in job seeking (e.g., Helpman et al. 2010a, 2010b), while others have introduced the concept of fairness into the context of international trade (e.g., Egger and Kreickemeier 2009, 2012). Although some of this research have considered the heterogeneity of workers' skill sets, the skill sets analyzed have been one-dimensional. Unlike this previous research, my study introduces two dimensional skill sets into the model and analyzes the relationship between the labor market and international trade.

The remainder of the paper is organized as follows. Section 2 explains the base of the model and the distribution of skills. Section 3 analyzes the relationship between the distribution of skills and international trade. Section 4 concludes with a summary and a discussion on future research.

2 The Model

2.1 Production

I consider an economy with comprised of two countries. In each country, two goods can be produced using one factor of production (i.e., labor). Both goods and labor markets are perfectly competitive. An individual worker has two abilities in the workplace, H and L . Following O&T, I consider that attribute H is quantitative abilities and attribute L is communication skills or teamwork kills. H and L are heterogeneously distributed among workers.

The two industries require different combinations of skills (H, L) for production. Suppose that Industry 1 uses L intensively and Industry 2 uses H intensively. I assume that y_i has a Leontief technology such as

$$y_i = \min \left\{ \frac{H}{\beta_i}, \frac{L}{1 - \beta_i} \right\}, \quad i = 1, 2, \quad (1)$$

where $\beta_1 < \beta_2$ from the assumption of industry intensity. Although O&T did not specify the product function in their model,¹ it is important to assume the Leontief technology for considering the mismatch in this study.

I assume that a firm cannot unbundle a worker's attributes, is concerned only with y_i , and pays wages to a worker based on his/her skill combination of (H, L). From the cost

¹O&T assume only constant-returns-to-scale technology.

minimization, I can derive the conditional demand for skills: $H = \beta_i y_i$ and $L = (1 - \beta_i) y_i$. Due to perfect competition, the wage for a worker is $W_i = P_i y_i$, where P_i is the price of good i . Figure 1 plots the relationship between the individual skill set and production pattern. The vertical line denotes quantitative ability H , and the horizontal line denotes communication ability L . Workers A and B have the skill bundles described by Points A and B. The half line l_i , $i = 1, 2$ denotes the locus of production implicated by the Leontief production function. Let us consider Worker A. Worker A has a production possibility area of $\square OH_A A L_A$, and the worker can produce any amount in that area. Since I do not consider the options of unemployment and leisure, the worker produces the goods on $H_A A L_A$. From Eq. (1), Worker A produces Good 1 on Point a and produces Good 2 on Point a' . Similarly, Worker B has a production possibility area of $\square OH_B B L_B$, and produces Good 1 in Point b and Good 2 in Point b' .

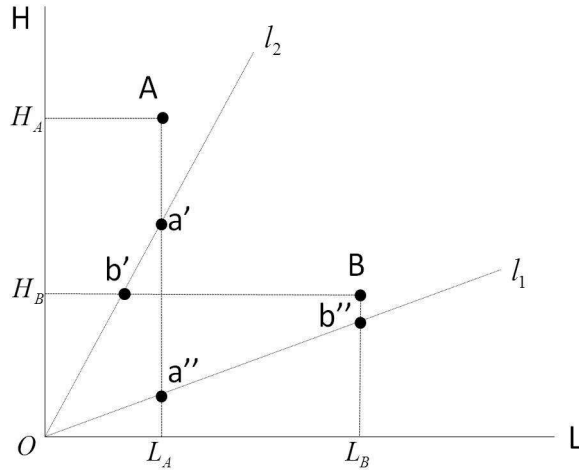


Figure 1: Relationship between individual skill set and production pattern

2.2 Distribution of workers

Following O&T, I take logarithms of the variables as well as define new variables:

$$\begin{aligned}
 l &\equiv \ln L, \\
 h &\equiv \ln H, \\
 s &\equiv \ln(H/L) \\
 p_i &\equiv \ln P_i \\
 \omega_i &\equiv \ln W_i = p_i + \ln y_i.
 \end{aligned} \tag{2}$$

Further, I assume that s and l are subject to the bivariate normal distribution (i.e., $F_{sl}(s, l)$).

$$\begin{bmatrix} s \\ l \end{bmatrix} \sim N \left(\begin{bmatrix} \mu \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \rho\sigma_s\sigma_l \\ \rho\sigma_s\sigma_l & \sigma_l^2 \end{bmatrix} \right) \quad (3)$$

where $|\rho| < 1$ is the correlation coefficient between s and l , μ is the mean of s , and σ_j , $j = s, l$ is the variance of s and l . O&T have empirically shown that $\rho < 0$ in advanced countries. I set $\sigma_s = \sigma_l = 1$ when variance plays no role.

2.3 Sorting of worker

I assume that there is a mass of workers and set the scale of workers as a numéraire. All workers must decide the sectors in which they will seek employment. Due to our assumption in the production function, firms in each industry use a specific skill ratio of H and L . I refer to this ratio as the best skill combination, and the best combination is solved by $\frac{H}{\beta_i} = \frac{L}{1-\beta_i}$, $i = 1, 2$. Using Eq. (1), I obtain the best combination of skills in Industry i :

$$s_i^* \equiv \ln \frac{H}{L} \Big|_{\frac{H}{\beta_i} = \frac{L}{1-\beta_i}} = \ln \beta_i - \ln(1 - \beta_i), \quad i = 1, 2. \quad (4)$$

Since the best skill combination is expressed by the ratio of H to L , the workers' skill combinations can be measured by $s = \ln H/L$. Firms in Industry i seek to employ workers with the skill combination s_i^* . Suppose that firms cannot identify the workers' skill combinations, but only can identify the output of the workers: $\ln y_i$, $i = 1, 2$. The firms observe the production activity of workers and pay salary ω_i to workers. Even if a worker has a substantial amount of both abilities, divergence from the best skill combination decreases productivity and wages. I consider this labor mismatch and the cost of this mismatch reduces wages. All workers are aware of labor market mismatches but cannot change their skill combinations (e.g., education or training). Workers will select jobs based on expected wage.

Within industries, production levels and wages are different as workers have different skill combinations s . If a worker has a skill combination $s < s_1^*$, the worker can produce $s + l - \ln \beta_1$ in Industry 1 and $s + l - \ln \beta_2$ in Industry 2. On the other hand, workers with $s_1^* < s$ produce $l + \ln(1 - \beta_1)$ in Industry 1 and produce $l - \ln \beta_2$ in Industry 2. Workers with $s > s_2^*$ produce $l - \ln(1 - \beta_1)$ in Industry 1 while they produces $l - \ln(1 - \beta_2)$ in Industry 2. The industries in which workers choose to be employed depend on expected wages. Since

production patterns depend on s , I consider the expected wage ω_i conditioned on s :

$$\begin{aligned} E[\omega_1(s, l)|s] &= \begin{cases} p_1 + s + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s - \mu) - \ln \beta_1, & \text{if } s < s_1^* \\ p_1 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s - \mu) - \ln(1 - \beta_1) & \text{if } s_1^* < s \end{cases} \\ E[\omega_2(s, l)|s] &= \begin{cases} p_2 + s + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s - \mu) - \ln \beta_2 & \text{if } s < s_2^* \\ p_2 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right)(s - \mu) - \ln(1 - \beta_2) & \text{if } s_2^* < s \end{cases} \end{aligned} \quad (5)$$

Differentiating the above expressions with respect to s , I obtain

$$\begin{aligned} \frac{\partial E[\omega_1|s]}{\partial s} &= \begin{cases} 1 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) > 0 & \text{if } s < s_1^* \\ \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) < 0 & \text{if } s_1^* < s \end{cases} \\ \frac{\partial E[\omega_2|s]}{\partial s} &= \begin{cases} 1 + \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) > 0 & \text{if } s < s_2^* \\ \rho \cdot \left(\frac{\sigma_l}{\sigma_s}\right) < 0 & \text{if } s_2^* < s \end{cases} \end{aligned} \quad (6)$$

where the signs are obtained when I assume $\sigma_s = \sigma_l = 1$ and $\rho < 0$. Figure 2 describes the relationship between the expected wage conditioned on s and s . From the nature of Eq. (6), the slope of ω_1 is positive when $s < s_1^*$ but becomes negative when $s_1^* < s$. Similarly, the slope of ω_2 is positive when $s < s_2^*$ and becomes negative when $s_2^* < s$. If a worker with skill combination s' earns more in Industry 1 than in Industry 2, $E[\omega_1(s')] > E[\omega_2(s')]$, and the worker chooses to work in Industry 1. In an equilibrium, there must be threshold \hat{s} where the worker is indifferent between working in Industry 1 or in Industry 2.

$$\hat{s} \equiv \ln \frac{H}{L} \Big|_{E[\omega_1]=E[\omega_2]} = \ln \frac{\beta_2}{1 - \beta_1} \frac{P_1}{P_2}. \quad (7)$$

A worker with $s < \hat{s}$ works in Industry 1 and the worker with $s > \hat{s}$ works in Industry 2. Note that the level of \hat{s} is located between s_1^* and s_2^* whenever both goods are produced. From Eq. (7), \hat{s} depends on the relative price of goods: P_1/P_2 . Figure 3 presents the case where P_1/P_2 is higher. Since the expected wage for Industry 1 is higher than Industry 2 at all s , all workers flow to Industry 1. This situation causes the production of Industry 2 to be zero and I do not have an \hat{s} . I can confirm that the case of a much lower relative price P_1/P_2 zeros the production of Industry 1. Therefore, the positive production of both industries requires $s_1^* < \hat{s} < s_2^*$, and I consider this case throughout this analysis.

Even if individuals work in the same industries, there is within-industry income inequality. In Industry 1, for example, the expected wage in the skill combination range of $s < s_1^*$ increases with s , while it decreases in the range of $s_1^* < s < \hat{s}$. Similarly, in Industry 2, as s increases, the wages that labor with $\hat{s} < s < s_2^*$ earns increases, while it decreases with the group $s_2^* < s$. This type of within-industry income inequality stems not only from absolute skill level, h and l , but also from skill mismatching between firms and the labor supply.

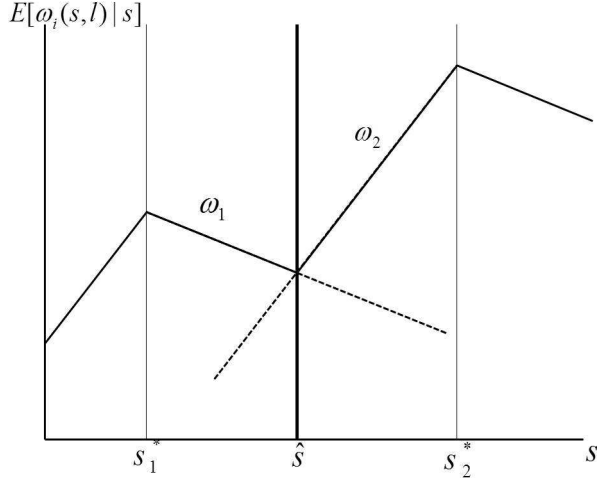


Figure 2: Workers' occupation choice

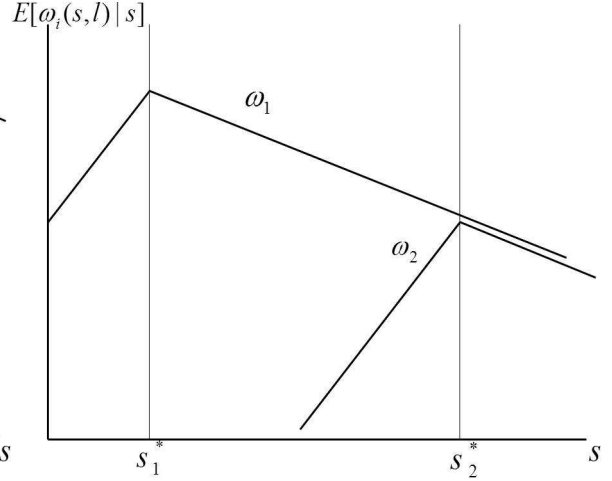


Figure 3: A case of no equilibrium of \hat{s}

Finally, I describe the industry output of two industries considering occupation choice. Industry output is the sum of the goods that each labor supply creates with its skill combination, y_i . Each industry output can be expressed as follows:

$$\begin{aligned}
 Y_1 &\equiv \int_{-\infty}^{\hat{s}} \int_{-\infty}^{\infty} e^{\ln y_1} f_{sl}(s, l) dl ds, \\
 &= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} e^{s+l-\ln \beta_1} f_{sl}(s, l) dl ds + \int_{s_1^*}^{\hat{s}} \int_{-\infty}^{\infty} e^{l-\ln(1-\beta_1)} f_{sl}(s, l) dl ds
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 Y_2 &\equiv \int_{\hat{s}}^{\infty} \int_{-\infty}^{\infty} e^{\ln y_2} f_{sl}(s, l) dl ds \\
 &= \int_{\hat{s}}^{s_2^*} \int_{-\infty}^{\infty} e^{s+l-\ln \beta_2} f_{sl}(s, l) dl ds + \int_{s_2^*}^{\infty} \int_{-\infty}^{\infty} e^{l-\ln(1-\beta_2)} f_{sl}(s, l) dl ds
 \end{aligned} \tag{9}$$

where $f_{sl}(s, l)$ is the probability density function (for more details or calculations, see the Appendix).

3 The role of distribution and international trade

I suppose that there are two countries, a home country and a foreign country. Preferences, production technologies, and factor endowments (H and L) are the same between countries. There are no barriers to trade and consumers in both countries face the same prices P_i , $i = 1, 2$. The differences between the home and foreign countries are the factors of the distribution: ρ , μ , and σ_j , $j = s, l$. These differences affect the structure of both countries, and become a source of international trade. This study decomposes the effects of the factors, making the role of each factor clear. Section 3.1 analyzes the effects of ρ on industry structure and wages. Section 3.2 examines the effects of μ , and Section 3.3 focuses on the effects of σ_j $j = s, l$.

3.1 Industry structure and ρ

T&O refers to ρ as the correlation between comparative advantage and absolute advantage. The parameter $s \equiv \ln H/L$ represents the comparative skill advantage a worker has for the H -intensive industry (Industry 2) over the L -intensive industry (Industry 1). For a given s , a large l suggests that the worker has an absolute skill advantage for both industries. T&O have empirically shown that $\rho < 0$ in advanced countries. In other words, laborers with an absolute advantage tend to work in an L -intensive industry, while those with a comparative advantage work in an H -intensive industry. T&O showed that a country with a large ρ exports H -intensive goods and imports L -intensive goods. Differentiating Y_i with respect to ρ in Eq. (8) and (9), I obtain

$$\frac{\partial Y_i}{\partial \rho} = \int_{\mathbb{S}_i} \frac{1}{\sqrt{2\pi}\sigma_s} \left[-\rho\sigma_l^2 + \sigma_l \frac{s-\mu}{\sigma_s} \right] \times \exp \left[\frac{\sigma_l^2}{2} (1 - \rho^s) + \ln y_i(s) + \sigma_l \rho \frac{s-\mu}{\sigma_s} - \frac{1}{2} \left(\frac{s-\mu}{\sigma_s} \right)^2 \right] ds. \quad (10)$$

If $\sigma_s = \sigma_l = 1$, the content of the first bracket becomes $-\rho + s - \mu$. Since their model assumes a continuum of industries, the effects of ρ on industry output are summarized in $-\rho + s - \mu$. H -intensive industries that satisfy $s > \rho + \mu$ expand production while others decrease production.

In O&T's model, due to a continuum of industries, workers can choose among many occupations. A worker with skill combination s' obtains the job in Industry i' paying the highest wages to skill combination s' , while another worker with skill combination s'' obtains a different job in Industry i'' , which pays the highest wages to skill combination s'' . The setting of O&T's model reduces the possibility that workers with different skill combinations s engage in the same industry. In other words, there is no mismatch between firm demand and the supply of skill combinations s . On the other hand, I consider a situation where there is a shortage of high productivity workers for Industry i . Each industry employs not only the best workers with s_i^* , but also workers with other types of skill combinations. This mismatch affects worker productivity and the structure of the industry. The effects of a change in ρ on production are different among the various groups of s within each industry. Since industry output is the sum of y_i that each worker produces in Industry i , the overall effects of ρ on industry output may be ambiguous. Therefore, I must consider the role of ρ in not only the first bracket of Eq. (10), but also with the whole of Eq. (10).

To investigate the relationship between ρ and industry output, I conduct numerical simulations and observe changes to the product possibility frontier (PPF) for the two industries. The PPF describes the industry structure in an economy and provides an intuitive interpretation of international trade. Suppose that two countries have different configurations of the

PPF, and the PPF of the home country, and not the foreign country, is biased towards Industry 2. I assume that the preferences are homogenous between the two countries. International trade theory states that free trade between the two countries induces the home country to export Industry 2 and induces the foreign country to export Industry 1. Comparing the differences in the PPF between the two countries informs us of the direction of international trade. If the cross-country difference in ρ generates a difference in the configuration of the PPF, I can argue the role of ρ on international trade.

To engage in a numerical simulation, I set specific values for certain parameters. I use the following parameters provided each parameter does not play a role in the analysis.

$$\begin{aligned}
\beta_1 &= 0.2 \\
\beta_2 &= 0.8 \\
\mu &= 0 \\
\rho &= -0.5 \\
\sigma_s &= \sigma_l = 1
\end{aligned} \tag{11}$$

Following T&O, I assume that ρ is negative. Their empirical results show that the value of ρ is different in advanced countries. To analyze the relationship between the value of ρ and industry output, I examine three cases where $\rho = -0.1$, $\rho = -0.5$, and $\rho = -0.9$.

The results of the numerical simulations are shown in Figure 4. There are three PPF lines, reflecting the patterns of the value ρ . The PPF line located in the lowest position corresponds to $\rho = -0.9$, the PPF line in the highest position corresponds to $\rho = -0.1$, and the PPF line located in the middle corresponds to $\rho = -0.5$. An increase in ρ leads to expanding Industry 2 over Industry 1. This result reflects the first bracket term in Eq. (10). Since Industry 2 is relatively H -intensive, the workers employed in Industry 2 have a relatively higher value of s . Therefore, the effect of ρ is positive in Industry 2.

The difference in ρ affects the direction of international trade. When all variables except for ρ are the same in the two countries, the cross-country difference in ρ generates the cross-country difference in industry output. Suppose that the home country has a higher ρ than the foreign country. From the above analysis, the home country will have a PPF biased to Industry 2. Since preferences and production technologies are the same in both countries, free trade induces the home country to export goods from Industry 2 to the foreign country, or conversely, the foreign country to export goods from Industry 1 to the home country. This effect is similar to the well-known Rybczynski theorem, which states that an increase in one factor endowment, without changing the prices of the goods, expands the output that intensively uses the factor of production. In my model, I do not change the skill endowment; rather, I change the correlation between s and l . Therefore, an increase in ρ functions as an

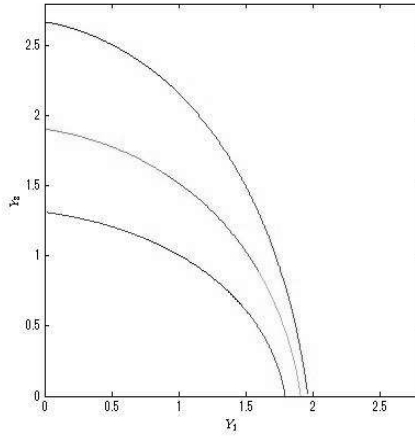


Figure 4: PPF and ρ

increase in endowment H . The increase in ρ from a negative value suggests a weakening of the trade-off between comparative advantage s and absolute advantage l . Given s , a larger ρ increases the amount of a worker's l . If the relative price is kept constant (i.e., no change in \hat{s}), the worker who works in Industry 2 increases his/her production, or, conversely, the worker who works in Industry 1 decreases his/her production. As a result, an increase in ρ begets changes to the industry structure, similar to the Rybczynski theorem.

The difference in ρ also affects wage inequality. First, let us consider a situation where ρ changes while the relative price is kept constant. Figure 5 describes the relationship between ρ and the expected wage conditioned on s . The dash lines express the expected wage after ρ increases. In $s < s_1^*$ and $\hat{s} < s < s_2^*$, the slope of $E[w_i(s, l)|s]$, $i = 1, 2$ becomes steeper as ρ increases. Since the gap in the expected wages expands between a high s and a low s , the within-industry wage inequality is worse in $s < s_1^*$ and $\hat{s} < s < s_2^*$ by an increase in ρ . On the other hand, the wage inequality in $s_1^* < s < \hat{s}$ and $s_2^* < s$ contracts. An increase in ρ reduces the slope of $E[w_i(s, l)|s]$ among these groups and then narrows the wage gap between a high s and a low s .

International trade between the home country and foreign country affects within-industry wage inequality. Since the home country has a higher correlation between s and l , the autarkic relative price p^h in the home country is larger than that of the foreign country p^f . When international trade begins, the world relative price is determined to be between p^h and p^f . The world relative price p^w affects the threshold level of a worker's choice of occupation. Free trade provides both countries the same threshold \hat{s}^w . Since $p^f < p^w < p^h$ is achieved, I obtain the relationship of $\hat{s}^f < \hat{s}^w < \hat{s}^h$. Figures 6 and 7 describe the expected wage profiles in the home and foreign countries. I assume that the home country has $\rho^h > 0.5$ and the foreign has $\rho^f < \rho^h$. The solid line expresses the autarkic situation and the dash line expresses an

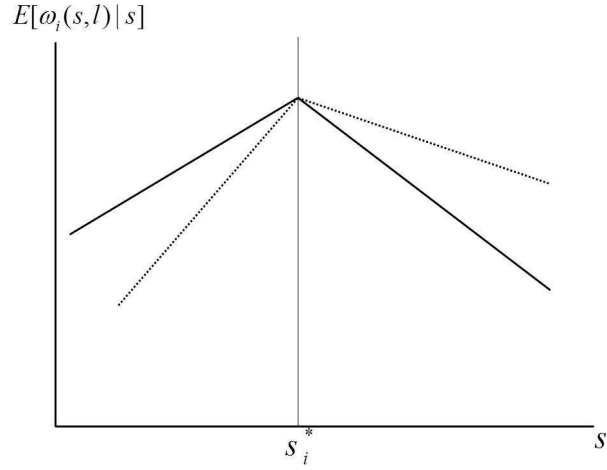


Figure 5: The expected wage conditioned on s and ρ

open economy. In the home country, as international trade increases relative prices, the number of workers who work in Industry 1 increases while number of workers who work in Industry 2 decreases. The change in relative price also increases the wages of Industry 2, but decreases those of Industry 1. The effects on wages mirror the Stolper-Samuelson theorem, which argues that free trade raises the value of the factors used intensively in the exporting sector and reduces the value of the factors used intensively in the importing sector. Similarly, in the foreign country, the wage of those employed in Industry 2 increases while the wages of those employed in Industry 1 decrease. In other words, free trade expands cross-industry industry income inequality.

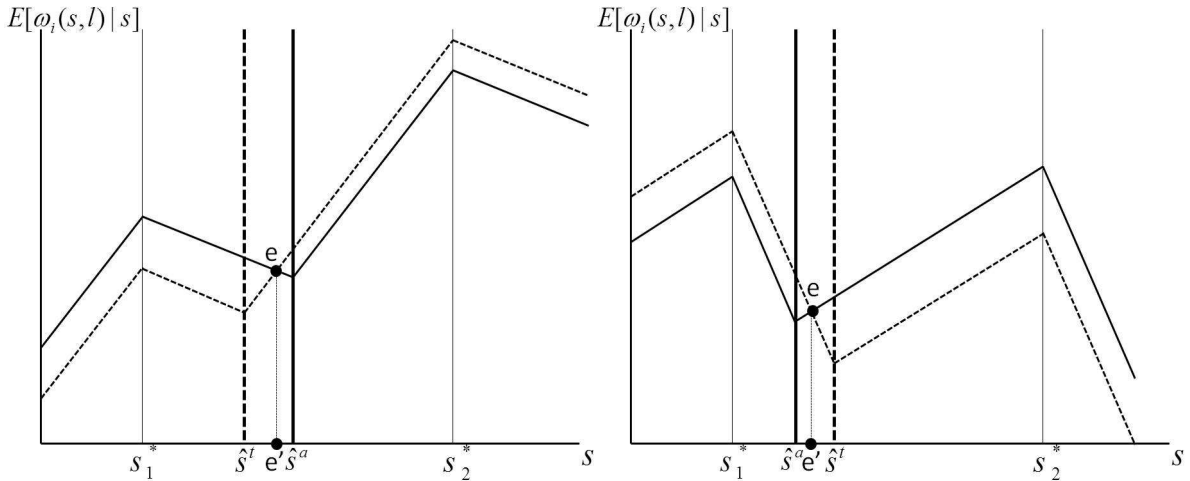


Figure 6: Wage profile in the home country Figure 7: Wage profile in the foreign country

International trade also affects the wages for workers who change occupations. Figure 6 illustrates the home country's wage profile. Shifting from autarky to free trade, the income of workers with $\hat{s}^t < s < \hat{s}^h$ becomes higher in Industry 2 than in Industry 1. Accord-

ingly, a worker will switch from Industry 1 to Industry 2. However, international trade has asymmetric effects on workers with $\hat{s}^f < s < \hat{s}^h$. In Figure 6, Point e is the intersection of wage profiles in autarky (solid line) and an open economy (dash line), and Point e' is the skill combination that parallels e . Compared with an autarky, the income of workers with $\hat{s}^f < s < e'$ decreases in open economy. On other hand, the income of workers with $e' < s < \hat{s}^h$ increases. Workers with $\hat{s}^f < s < e'$ have the greatest choice of opportunity to change occupations from Industry 1 to Industry 2 in an open economy. If these workers remained in Industry 1 in an open economy, their wages would decrease. The reason for the asymmetric effects on occupation changes is related to the mismatching of skill combinations between firms and workers. The workers with skill combination $\hat{s}^f < s < e'$ are more communication-intensive and, accordingly, are better suited for Industry 1, as opposed to those with skill combination $e' < s < \hat{s}^h$. Since the home country has a comparative advantage in Industry 2, workers with $\hat{s}^f < s < e'$ change occupations to Industry 2. However, Industry 2 is not better suited to workers with $\hat{s}^f < s < e'$. The mismatch between firms in Industry 2 and workers increases. Therefore, the income of the group with $\hat{s}^f < s < e'$ decreases. Similarly, the foreign country has winners and losers among those who change occupations in an open economy. Since the foreign country has a comparative advantage in Industry 1, workers with $\hat{s}^f < s < \hat{s}^f$ shift occupations to Industry 1. As seen in Figure 7, under an open economy, the income of workers with $\hat{s}^f < s < e'$ decreases and the income of those with $e' < s < \hat{s}^f$ increases, as compared to in an autarky. This result is similar to that of the home country. Hence, I find that international trade can cause an increase in the skill combination mismatch between firms and workers, which results a decrease in wage income for a portion of the shifting workers.

3.2 Role of μ

This section analyzes the role of the first moment of the normal distribution, μ on industry structure. Since μ is the mean of s , the country with the higher μ will have more workers with a higher proportion of s -related skills. I can consider that this country is more productive in the s -intensive industry, that is Industry 2. Figure 8 plots the comparison of the PPF with respect to μ . To describe the graph, I specify the parameters as $\beta_1 = 0, 2$, $\beta_2 = 0.8$, $\sigma_s = \sigma_l = 1$, and $\rho = -0.5$. The higher the μ is, the more the PPF curves shift outside. The expansion of the PPF is biased towards Industry 2. The effects of μ are similar to the Rybczynski theorem as well as to the role of ρ .

When two countries display different means of s , the industrial structure in both countries will substantially differ. The cross-country difference in μ is a source of international between among countries.

Suppose that the home country has a higher μ than the foreign country. Figure 8 shows that the home country engages in production activity biased in favor of Industry 2, while the foreign country is biased towards Industry 1. As indicated by the Heckscher-Ohlin theorem, the home country will export Industry 2 and import Industry 1, while the foreign country will export Industry 1 and import Industry 2. This result is consistent with the wide literature on international trade.

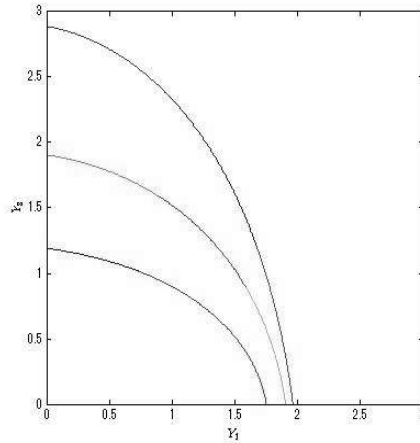


Figure 8: PPF and μ

Next, I examine the effects of μ on wage inequality. From Eq. (5), a change in μ affects the quantity of the expected wage. In a country with a high μ , wage income is higher than in other countries. Since μ also affects relative price p through international trade, worker occupations and wage inequality are affected by μ . Figures 9 and 10 describe the relationship between the expected wage profile and μ . Figure 9 indicates the wage profile of the home country when the home has a higher μ than the foreign country. Since the home country has a higher μ , the autarkic relative price p^h in the home country is larger than that of the foreign country, at p^f . When international trade commences, the world relative price p^w is determined to be between p^f and p^h . In the home country, international trade with the foreign country decreases the relative price. The number of workers in Industry 2 increases, while that of Industry 1 decreases. Moreover, the reduction in the relative price raises the wage income in Industry 2, but decreases that of Industry 1. On other hand, the foreign country experiences an increase in relative price due to free trade. It increases the number of workers in Industry 1 and the relative wage income of workers in Industry 1 as compared to Industry 2.

Similar to the analysis of ρ , free trade asymmetrically affects wage income for workers who change occupations. The workers with $\hat{s}^f < s < \hat{s}^h$ in Figure 9 are those who change jobs after free trade. Among them, some experience an increase in wage income, while oth-

ers receive a lower wage income. The workers whose income decreases are those who have the comparative advantage in communication skill l . Although shifting to Industry 2 is the best choice for them under free trade, it increases the skill combination mismatch between firms in Industry 2 and workers. In the foreign country, the same situation occurs. As shown in Figure 10, international trade induces workers with $\hat{s}^f < s < \hat{s}^t$ to shift to Industry 1. However, some of the shifting workers have a comparative advantage in quantitative skill h . The shifting of occupation may increase the divergence between the firm's demand and the worker's skill set. Again, I find that a portion of the shifting workers will see a reduction in earnings as compared to an autarky.

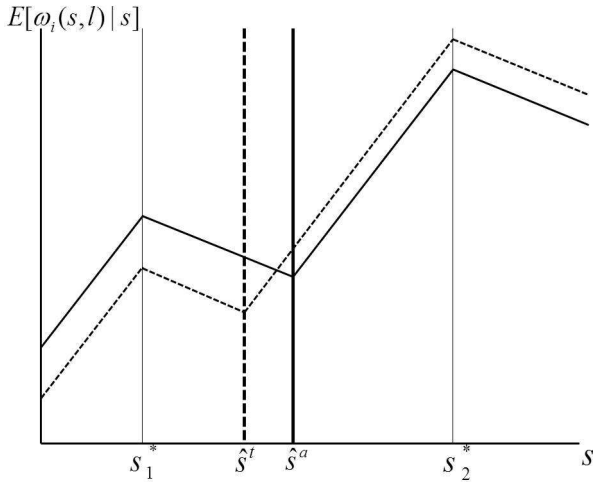


Figure 9: Wage profile under large μ

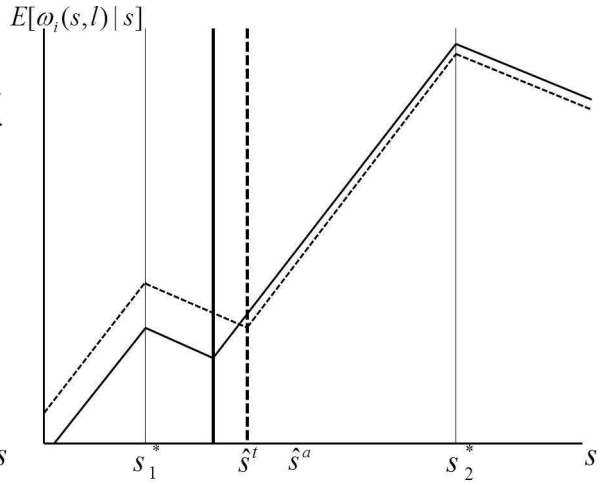


Figure 10: Wage profile under small μ

3.3 The roles of σ_s and σ_l

Next, I analyze the effect of the second moment, σ_s and σ_l , on international trade. There are two situations where I observe the role of σ_j , $j = s, l$. The first is the absolute change of σ_j , in which the other parameters of the normal distribution do not change. This change also affects covariance $\sigma_s \sigma_l \rho$, which changes the relationship between s and l . Therefore, the absolute change of σ_j has two effects on industry output, and, consequently, I cannot obtain a clear result for σ_j . The second is the adjusted change of σ_j , where not only σ_j , but also ρ , change to sustain the level of the covariance at the level prior to the change. In this case, I observe the change of σ_j on industry output. I use the second case of σ_j , $j = s, l$ to observe the role of the change in variance.

First, I observe the role of σ_s . σ_s measures the workers' amount of s . I engage in a numerical calculation to assess the effects of σ_s . I set the parameters as Eq. (11), except for σ_j , $j = s, l$ and ρ . To maintain the covariance at the same level, I set the benchmark of the

covariance as $\sigma_s \sigma_l \rho = -0.5$. Under the covariance, I choose three combinations of ρ and σ_j , $j = s, l$: (i) $\sigma_s = \sigma_l = 1$, $\rho = -0.5$, (ii) $\sigma_s = 0.8$, $\sigma_l = 1$, and $\rho = -5/8$, (iii) $\sigma_s = 1.2$, $\sigma_l = 1$, and $\rho = -5/12$. Figure 11 shows the relationship between the PPF and σ_s . The PPF located at the lowest position corresponds to (ii), the PPF at the highest position corresponds to (i), and the PPF located in the middle corresponds to (iii). The figure indicates that the PPF is biased in favor of Industry 2 as σ_s increases. The effect of σ_s is a similar to that of the Rybczynski theorem. An expansion in σ_s increases the amount of the workers' s , and, subsequently, the number of workers who can engage in an s -intensive industry increases. As a result, the output of Industry 2 expands and the PPF curve shifts biased in favor of Industry 2. If free trade is allowed between two countries with different σ_s , the country with the larger σ_s exports Industry 2, while the other country exports Industry 1.

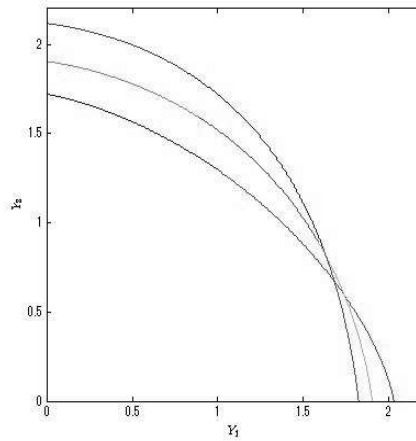


Figure 11: PPF and σ_s with control of ρ

The effects of the adjusted change in σ_s on wage income are qualitatively similar to the analysis of ρ . A country with a larger adjusted σ_s has a higher relative price than the other. When international trade commences, the country with the higher adjusted σ_s experiences a decrease in relative price. Figure 6 in Section 3.1 describes the wage profile of this country. Shifting from autarky to free trade, the wage income of workers with $\hat{s}^l < s < \hat{s}^h$ becomes higher in Industry 2 than in Industry 1. Therefore, workers will change jobs from Industry 1 to Industry 2. Similar to the analysis of ρ , a portion of the workers changing jobs will suffer from reduced wage income under free trade as compared to under an autarky. Hence, Industry 2 is not better suited to these workers and the skill mismatching between firms and workers increases.

Next, I consider the effect of σ_l . σ_l measures the workers' amount of l . A larger σ_l suggests that there are many workers with a larger l . I engage in a numerical calculation to assess the effects of σ_l . Similar to the analysis of σ_s , I set the parameters as $\beta_1 = 0.2$,

$\beta_2 = 0.8$, and $\mu = 0$. To maintain the covariance at the same level, I also set the benchmark of the covariance as $\sigma_s\sigma_l\rho = -0.5$. Under the covariance, I choose three combinations of ρ and σ_j , $j = s, l$: (i) $\sigma_s = \sigma_l = 1$, $\rho = -0.5$, (ii) $\sigma_s = 1$, $\sigma_l = 0.8$, and $\rho = -5/8$, (iii) $\sigma_s = 1$, $\sigma_l = 1.2$, and $\rho = -5/12$. Figure 12 shows the relationship between the PPF and σ_l . The PPF located at the lowest position corresponds to (ii), the PPF at the highest position corresponds to (i), and the PPF located in the middle corresponds to (iii). An increase in σ_l shifts the PPF right-upward and does not generate the bias of the PPF. The output of both industries expands equivalently as σ_l increases. σ_l affects the absolute amount of l but does not change the comparative amount of s . As the workers' choice of occupation depends on s , the adjusted change in σ_l does not affect choice of occupation.

The within-industry wage inequality is also not affected by the change in σ_l . From Eq. (5), the change in σ_s is offset by the adjusted change in ρ . The expected wages conditioned on s in all s do not change. However, the real wage ω increases in all s as σ_l expands as all workers increase the amount of l .

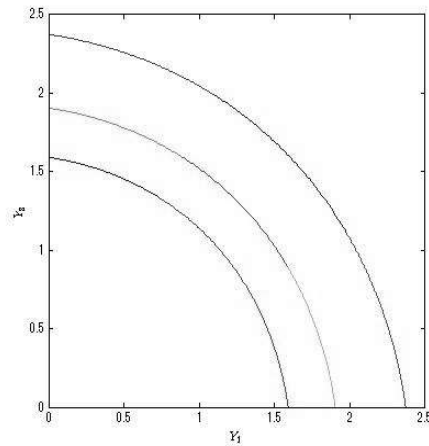


Figure 12: PPF and σ_l with control of ρ

4 Conclusion

In this study, I analyzed the role of two-dimensional skills and skill matching on international trade and wage inequality. I assumed that individuals have two primary skills and that each individual has a different combination of the two-dimensional skill set. I also assumed that the patterns of these skills combinations were normally distributed. Each industry had a demand for a certain skill combination; however, it was difficult for firms to attract workers with the required skill combination and so firms could employ the workers with other skill combinations. I referred to this as a mismatch in the labor market. The distribution of skills

also affected industry structure and within-industry income inequality. I also considered two countries with different skill distributions and examined the effects of free trade. The differences in the cross-country distributions of skills served as the causes of international trade. When international trade began, a portion of the workers employed in an importing industry moved to an exporting industry, as they could earn higher wages than if they remained in the importing industry. However, I found that some of the workers who shifted occupations were matched with less appropriate firms and suffered from lower wages than in an autarky.

There is little literature focused on multi-dimensional skills and international trade. My analysis provides additional insights to this existing literature. I also elucidate the role of the labor market on international trade and can provide implications for more effective labor policies. Future area for research should focus on the introduction of unemployment and other labor market imperfections to this type of analysis.

Appendix

This section shows the detailed calculations of Y_1 and Y_2 . As the early part of these calculations follows O&T, I refer to them only in this Appendix. First, I integrate with respect to l using the marginal distribution of s , f_s . Y_1 and Y_2 have two terms of integration. Since each integration is complex, I calculate each term individually until I am no longer able to do so. The first term of Eq. (8) becomes

$$\begin{aligned}
\text{First term of } Y_1 &= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} e^{s+l-\ln\beta_1} \frac{f_{sl}}{f_s} f_s dl ds, \\
&= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} e^l \frac{f_{sl}}{f_s} dl \cdot f_s e^{s-\ln\beta_1} ds, \\
&= \int_{-\infty}^{s_1^*} \int_{-\infty}^{\infty} e^l \frac{1}{\sqrt{2\pi}\sigma_l \sqrt{1-\rho^2}} \exp\left(-\frac{\left[l - \rho\left(\frac{\sigma_l}{\sigma_s}\right)(s-\mu)\right]^2}{2\sigma_l^2(1-\rho^2)}\right) dl f_s e^{s-\ln\beta_1} ds \\
&= \int_{-\infty}^{s_1^*} \frac{1}{\sqrt{2\pi}\sigma_l \sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{\left[l - \frac{\sigma_l}{\sigma_s}\{\rho(s-\mu) + \sigma_s\sigma_l(1-\rho^2)\}\right]^2}{2\sigma_l^2(1-\rho^2)}\right) dl \\
&\quad \cdot \exp\left(\frac{\sigma_l}{\sigma_s}\rho(s-\mu) + \frac{\sigma_l^2}{2}(1-\rho^2)\right) f_s e^{s-\ln\beta_1} ds.
\end{aligned}$$

Since $\frac{1}{\sqrt{2\pi}\sigma_l\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{[l-\frac{\sigma_l}{\sigma_s}\{\rho(s-\mu)+\sigma_s\sigma_l(1-\rho^2)\}]^2}{2\sigma_l^2(1-\rho^2)}\right) dl$ is a normal probability distribution which has a mean of $\sigma_s\sigma_l(1-\rho^2)$ and variance of $\sigma_l(1-\rho^2)$, I set this part as 1. Hence,

$$\begin{aligned} \text{First term of } Y_1 &= \int_{-\infty}^{s_1^*} \exp\left(\frac{\sigma_l}{\sigma_s}\rho(s-\mu) + s - \ln\beta_1 + \frac{\sigma_l^2}{2}(1-\rho^2)\right) \cdot f_s ds, \\ &= \frac{1}{\sqrt{2\pi}\sigma_s} \int_{-\infty}^{s_1^*} \exp\left(s - \ln\beta_1 + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2\right) ds. \end{aligned}$$

Similarly,

$$\text{Second term of } Y_1 = \frac{1}{\sqrt{2\pi}\sigma_s} \int_{s_1^*}^{\hat{s}} \exp\left(-\ln(1-\beta_1) + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2\right) ds,$$

$$\text{First term of } Y_2 = \frac{1}{\sqrt{2\pi}\sigma_s} \int_{\hat{s}}^{s_2^*} \exp\left(s - \ln\beta_2 + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2\right) ds,$$

$$\text{Second term of } Y_2 = \frac{1}{\sqrt{2\pi}\sigma_s} \int_{s_2^*}^{\infty} \exp\left(-\ln(1-\beta_2) + \frac{\sigma_l^2}{2}(1-\rho^2) + \frac{s-\mu}{\sigma_s}\sigma_l\rho - \frac{1}{2}\left(\frac{s-\mu}{\sigma_s}\right)^2\right) ds.$$

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