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Abstract

Let us consider movement patterns in an economy based on a Ramsey type model, as do many papers in the R.B.C. school.We focus on how labor supply is determined and on how this determination affects the movement patterns. We discuss two kinds of labor supply, namely, an intensive arrangement and an extensive one. We conclude that the labor supply in both arrangements is similar from the viewpoint of a macro economy. This leads us to conjecture that the two forms of labor supply do not affect the movement pattern.

JEL classification: E20; E24; E32
Keywords; Divisible labor ; Indivisible labor; Movement patterns

1 Introduction

The R.B.C. theory has been dealing with fluctuations, especially business cycles, based on Ramsey (1928). The original Ramsey model has been transformed into a general equilibrium theory by introducing perfect foresight.[1] This means that Ramsey type models can endogenously determine several economic variables, such as savings, consumption, wage rates, rental rates of capital, and labor supply, in every period assumed. Moreover, as savings lead to investments, those models can explain how an economy moves. In particular, the question of whether business cycles can be produced or not has been a big issue. Along this line, we are interested in indeterminacy, in which many upward and downward paths in an economy can be detected. This indeterminacy has been found in many economic circumstances, such as increasing returns to scale, externality and monopoly. We can list Benhabib and Farmer (1994) as a representative and classical study. But these assumptions all depend on economic outcomes, which leads to an inevitable difficulty in managing economic policies.

However, some papers maintain that only fiscal policy can produce this indeterminacy. Fiscal policy here specifically means income taxation and a balanced budget rule. Schmitt and Uribe (1997) is a significant and controversial essay based on this line of reasoning and on an important assumption in relation to labor supply, that is, an indivisible labor supply. Thus the paper discusses what conditions cause indeterminacy in an an economy with an indivisible labor supply. On the other hand, Takata (2017) offers a mathematical proof concerning the conditions for indeterminacy in an economy with a divisible labor supply. But both papers derive similar conclusions, because in the former model, the disutility of a household is expressed in terms of an expected disutility function with two arguments, one of which is inevitably null, while in the latter model, the disutility of a household is not an expected disutility, and the disutility function consists of an argument. This is why the same conclusion is derived on appearance. Additionally, the reason one argument in the disutility function is null is derived from the assumption that some parts of households do not work. Naturally, this does not cause any disutility.

In this paper, we assume that an indivisible labor supply shows an extensive arrangement of the labor force, that is, some parts in the fixed working population work and others do not in every period, while a divisible labor supply shows an intensive arrangement, that is, the whole working popu-

[1] See Attanasio (2015) for subsequent contributions and extensions to Ramsey (1928).

lation works in every period, but changes its working hours, which is its utilization.

Why do both economies follow the same movement pattern? As mentioned above, the similarity seems to depend on the normalization of disutility, which is supposed to be null when the potential labor force is out of the labor market. Is this absolutely true? The two papers cited above apparently share a similar ambiguity concerning how they deal with a macroeconomy from the viewpoint of the two types of labor supply. Both models assume a representative agency. Does this not entail any problems?

To this question, it is appropriate to investigate Mulligan (1999), .which analyzes the relationship between the labor supply determined by the two arrangements, focusing on the relationship between the micro and macro sides of an economy.

Mulligan (1999) concludes that the labor supply in both arrangements is similar from the macroeconomic viewpoint. But this demonstration is only about the labor supply; it doesn't analyze how the economy moves.

In this paper, based on Mulligan (1999), we derive a conjecture that the same movement pattern in an economy occurs in an economy in the two cases of labor supply, i.e. of an intensive arrangement and an extensive one.

In the next section we investigate Mulligan (1999) in detail, in the cases of both divisible and indivisible labor. In what follows, in section 3, we explore an aggregate labor supply in terms of an extensive arrangement. Subsequently, in section 4, we demonstrate the similarity between the two labor supply arrangements. We demonstrate that when the Mulligan model is extended to an analysis of economic movements, its conclusions are the same as those in Takata (2017). Finally, section 5 is devoted to concluding remarks, in which we mention that the difference between the two arrangements does not affect the movement pattern in an economy.

2 The Mulligan models

2.1 Optimal divisible labor

First, Mulligan (1999) introduces a divisible labor environment as

$$\max_{C\{N_t\}_{t=1}^{T}} \left[C\ \lambda - \sum_{t=1}^{T}\ e^{-\rho(t-1)} g_t\ \nu(N_t) \right],\ \lambda > 0,\ g_t > 0, \tag{1}$$

$$s.t,\ N_t \in [0,\ n_{\max}],\ C = a + \sum_{t=1}^{T} e^{-\rho(t-1)}\ Q_t\ w_t\ N_t.$$

The divisible environment shows that representative individual works range anywhere from 0 to n_{max} units of time. During time interval t, a wage is paid at a rate w_t for time worked N_t. The present value of lifetime consumption C is financed out of lifetime earnings and an initial asset stock a.

Here, the definitions of notations are as follows: $U(C) = \lambda\ C$ is the consumption value function and λ constant; ρ is the rate of time preference; g_t is a preference parameter, which shows different degrees of disutility in a lifetime; Q_t is the period interest rate factor; $\nu(N)$ is the disutility of work.

Furthermore, Mulligan (1999) makes assumptions concerning disutility such as

$$\nu^{'} > 0 \tag{2}$$

$$\nu^{''} > 0 \tag{3}$$

$$\int_0^{n_{\max}}\ \text{In}\ \nu'(N)\ dN = 0. \tag{4}$$

We need to analyze (4) in detail, utilizing a specific disutility function such as

$$\nu(N) = \frac{N^{1-\chi}}{1-\chi}. \tag{5}$$

Here, $\chi < 0$. Based on the above function, (4) is transformed as

$$\int_0^{n_{max}} \log(N)\ dN = 0.$$

Solving the above integration, we obtain $(-1 + \log(n_{max}))n_{max} = 0$. Therefore, $n_{max} = E$ [2] and $\nu^{'}(1) = 1$ prevail. The latter relationship implies that in $0 < N \leq 1$, the marginal disutility in relation to N is less than or

[2] E means a base logarithm.

is equal to unity, while in $1 < N < E$, it is more than unity. Thus, in this model, the disutility in relation to N increases as N does, and reaches its maximum $\nu(E)$. So Mulligan (1999) assumes that the function $\nu(N)$ is constant over time, but the marginal disutility schedule varies as does g_t over time. Based on these assumptions, Mulligan (1999) deduces the optimal labor supply as

$$g_t \ \nu^{'}(N_t) = \lambda \ Q_t \ w_t. \ \ t = 1, 2, \cdots, T. \tag{6}$$

In short, the labor supply is determined to satisfy the condition that the disutility in relation to labor is equal to the discounted wage multiplied by λ.

2.2 Indivisible labor

Second, Mulligan discusses indivisible labor.

Individuals choose whether to work or not, during a time interval. Wage w_t is payed to labor during time interval t. The labor supply is determined to satisfy the following relationships:

$$\max_{C\{n_t\}_{t=1}^{T}} \left[\lambda \ C \ - \sum_{t=1}^{T} \ e^{-\rho(t-1)} \ \gamma_t \ \ n_t \right], \tag{7}$$

$$s.t, \ n_t \in \ \{0, \bar{n}\}, \ C \leqslant a + \sum_{t=1}^{T} e^{-\rho(t-1)} \ Q_t \ w_t \ n_t.$$

Here, γ_t is a preference parameter for labor and n_t is labor input in time interval t, respectively.

Moreover, the disutility function of labor $\nu(n)$ is defined as

$$\nu(\bar{n}) = \gamma_t \ \bar{n}$$

and is normalized as $\nu_t(0) = 0$. This shows that when an individual doesn't work, which means the least disutility, disutility is normalized as 0. Additionally, an individual is assumed to offer fixed time $\bar{n}$. Although the individual is engaged in a fixed time of work, disutility here is affected by the preference for labor in a lifetime.[3] Based on (7), we can obtain some

[3]This postulation is supposed to depend on the fact that an individual decides whether to participate in the labor market or not. However, Hansen (1985) and Rogerson (1988) offer a model in which this decision is determined by lotteries which firms offer. According their postulations, this risk appears as a fixed cost for the firms, because the allowances from the lotteries are payed even to unemployed workers.

conditions regarding the way decisions are made, as follows: Consider a maximization problem such as

$$\xi = \lambda \left(a + \sum_{t=1}^{T} e^{-\rho(t-1)} \; Q_t \; w_t \; n_t \right) - \sum_{t=1}^{T} \; e^{-\rho(t-1)} \; \gamma_t \;\; n_t.$$

Here, the following prevails:

$$\frac{\partial \xi}{\partial n_t} = e^{-\rho(t-1)} \left(\lambda Q_t w_t - \gamma_t \right)$$

Based on this, we can see that an individual chooses whether to work or not according to the following conditions:

$$n_t = \bar{n} \;\; as \;\; \lambda \; Q_t \; w_t \geqslant \; \gamma_t, \quad t = 1, 2, \cdots, T.$$
$$n_t = 0 \quad as \;\; \lambda \; Q_t \; w_t \; \leqslant \gamma_t.$$

The above conditions imply that an individual chooses to work when the marginal utility of consumption is larger than the disutility of labor, and vice versa.

3 Aggregation

We have discussed an individual's behavior based on (7) in an economy with many heterogeneous individuals. They naturally have heterogeneous preferences for labor in their own lives, and the differences are revealed as the life-cycle disutility of the work profile.

In this section, we intend to aggregate the results of the labor supply in system (7). In this situation, Mulligan (1999) assumes that the geometric mean preference in period t, g_t varies over time, but the distribution F of preferences around that mean is the same in every period. In short, these assumptions can be approximately expressed as

$$\log \left(\frac{\gamma_t}{g_t} \right) = F(x),$$
$$\int x \; dF(x) = 0.$$

We can know the primitive function of $F(x)$ as $f(x) = 1/x$, because $F(x) = \log(\gamma_t/g_t) = \log(x)$. Moreover, let us investigate the second relationship intensively. We can easily deduce the following:

$$\int x f(x) \, dx = 0,$$

which implies that the expectation of x is null.

Mulligan (1999) assumes that the continuum of agents equals population frequencies in terms of probability. As a result, if (γ_t/g_t) is small, we can say that the participation rate in the labor market is large, and vice versa. In other words, (γ_t/g_t) can be an indication in relation to the labor supply. Then, since $F\left(\log\left(\frac{\gamma_t}{g_t}\right)\right)$ can be a criterion concerning the fraction of consumers working at time t, Π_t, can be described as

$$\Pi_t = F\left(\log\left(\frac{\gamma_t}{g_t}\right)\right).$$

Additionally, based on $\lambda\ Q_t\ w_t \geqslant\ \gamma_t$ or $\lambda\ Q_t\ w_t \leqslant \gamma_t$, consumers determine whether to enter the labor market or not. In light of this, we can replace γ_t with $\lambda\ Q_t\ w_t$. Therefore, since the above criterion can be written as

$$\Pi_t = F\left(\log\left(\frac{\lambda\ Q_t\ w_t}{g_t}\right)\right),$$

the average labor supply at time t, N_t, is expressed as

$$N_t = F\left(\log\left(\frac{\lambda\ Q_t\ w_t}{g_t}\right)\right)\ \bar{n}. \tag{8}$$

On the other hand, the average budget constraint is obtained by averaging the budget constraint in (7) across consumers, as follows:

$$C = a + \sum_{t=1}^{T} e^{-\rho(t-1)}\ Q_t\ w_t\ N_t.$$

In addition, C means the present value of lifetime consumption.

4 The similarity between the divisible and indivisible models

Here we face an interesting outcome, that is, the two types of labor supply produce a similar supply on the whole. This can be summarized by the following propositions.

The basic idea is as follows: a general equilibrium system consists of endogenous and exogenous variables.. Obviously, the endogenous variables (solutions) are expressed in terms of exogenous variables, that is, parameters. Assume that we investigate whether system 1 and system 2 are identical or not. Then, assume that the parameters in system 1 are replaced by those in the system 2. In this situation, if the solutions in system 1 are identical to those in system 2, and vice versa, we can say that the two systems are identical.

Proposition 1 *The whole labor supply determined by extensive arrangement becomes similar to that determined by intensive arrangement.*

Here we begin to prove *Proposition 1.*

Proof 1 *(i) Choose an arbitrary set of parameters ($a,\ \rho,\ \lambda,\ \bar{n}, \{w_t\ g_t\ Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+2}$ for the extensive arrangement.*

(ii) In a whole economy, the average labor supply N_t satisfies (8).

(iii) Choose the same arbitrary set of parameters in the case of the intensive arrangement, $(a, \rho, \lambda,\ n_{\max}, \{w_t\ g_t\ Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+2}$, and choose the disutility function $\nu(N)$ satisfying

$$\nu^{'}(N) \equiv e^{F^{-1}\left(\frac{N}{\bar{n}}\right)}. \tag{9}$$

Additionally, ν has properties such as $\nu^{'} > 0,\ \nu^{''} > 0$, and these satisfy (4). These properties are completely coincident with those defined in (2) $\sim$ (4).

(iv) In light of (6),

$$\nu^{'}(N) = \left(\frac{\lambda\ Q_t\ w_t}{g_t}\right) = e^{F^{-1}\left(\frac{N}{\bar{n}}\right)}$$

prevails. The above relationship shows

$$\frac{N}{\bar{n}} = F\left(\log(\nu^{'}(N))\right) = F\left(\log\left(\frac{\lambda\ Q_t\ w_t}{g_t}\right)\right).$$

Moreover, noticing $\lambda = U^{'}(C)$, we can obtain a conclusion that eventually the average labor supply equals (8). ■

Next, let us consider the reverse process. Then, the second proposition prevails:

Proposition 2 *The aggregate labor supply expressed in terms of the parameters in the economy with an intensive arrangement is equal to that expressed in terms of the same parameters in the economy with an extensive arrangement.*

Proof 2 *(i) Choose an arbitrary set of parameters for the economy with an intensive arrangement:* $(a, \rho, \lambda,\ n_{\max}, \{w_t\ g_t\ Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+2}$.

(ii) Based on the arguments in subsection 2.1, the labor supply is denoted as (6).

(iii) Choose the same parameters for the economy with an extensive arrangement: $(\ a,\ \rho,\ \lambda,\ \bar{n}, \{w_t\ g_t\ Q_t\}) \in \mathbb{R}^2 \times \mathbb{R}_+^{3T+2}$. *Furthermore, define a distribution function such as*

$$F(x) \equiv \begin{cases} 0 & \textit{if } x < \log(\nu^{'}(0)), \\ \dfrac{\phi(e^x)}{n_{max}} & \textit{if } x \in [\log(\nu^{'}(0)),\ \log(\nu^{'}(n_{max}))], \\ 1 & \textit{if } x \geqslant \log(\nu^{'}(n_{max})). \end{cases}$$

Here, $F(x)$ *has properties such as* $F \in [0,\ 1]$, $F(x) > 0$, *and the expectation of the primitive function of* $F(x)$ *is null.*

At this stage, we need to specify F. *We assumed a disutility function (5),*

$$\nu(N) = \frac{N^{1-\chi}}{1-\chi}.$$

Based on the above assumption, since $\nu^{'}(N) = N^{-\chi},\ \log\left(\nu'(N)\right) = \log(N^{-\chi}) = x$ *prevail. Moreover, since we denote* $\log\left(\nu'(N)\right) = x$, *it is obvious that* $N = e^{\frac{x}{-\chi}}$. *On the other hand,*

$$\frac{N}{\bar{n}} \equiv F\left(\log\left(\nu'(N)\right)\right) = F(x)$$

can be defined. Thus, function $F(x)$ *can be specified as*

$$F(x) = \frac{N}{\bar{n}} = \frac{e^{\frac{x}{-\chi}}}{\bar{n}}, \tag{10}$$

which shows that the distribution $F(x)$ *is the product of exponential functions concerning* x. *Before proceeding, we need to confirm some properties* *in relation to $F(x)$. Since N_{max} is E, it is obvious that $-\infty < x < 1$. Therefore, $F \in [0,1]$ if $\bar{n} = e^{\frac{-1}{\chi}}$, and $F(x) > 0$.*

Moreover, ϕ shows a reciprocal of the disutility of labor, which can be easily proved, as follows: Based on our definition, $\nu^{'}(N) = e^x$, we can deduce

$$N = \nu^{'^{-1}}(e^x) \equiv \phi(e^x).$$

In other words, by definition $\log(\nu'(N)) = x$, so we can deduce labor supply as a mapping of the marginal disutility of labor, and this mapping is a reciprocal of the marginal disutility of labor.

(iv) In total, the average labor supply satisfies (8), and in light of the definition of F, this results in equality with (6). We can confirm this logic as

$$N_t = F\left(\log \nu^{'}(N)\right)\ \bar{n} = F\left(\log\left(\frac{\lambda\ Q_t\ w_t}{g_t}\right)\right)\bar{n} = (x)\ \bar{n}.$$

■

In the above proof, notice that $F\left(\log\left(\frac{\lambda\ Q_t\ w_t}{g_t}\right)\right)$ shows the participation rate in the labor market.

5 Concluding remarks

Mulligan (1999) demonstrates that the indivisible model is equal to the divisible model, if in the two models the marginal utility of income across the population is equal. This equality is proved only in relation to aggregated labor.

However, in this paper, we implicitly refer to Hansen (1985) and Rogerson (1988) as providing an indivisible model, respectively established on the basis of a linear disutility function. Their postulations imply an extensive arrangement in the labor supply, and therefore an expected disutility in relation to the aggregated labor supply. In their papers, it is assumed that people choose whether to work or not in their lifetime. In this sense, in their model some parts of a labor population are necessarily unemployed.. In this situation, the work force arrangement comprises a group newly entering or quitting the labor market. Furthermore, when a person chooses not to work, this person is compensated by firms in terms of allowances in lotteries. Additionally, this mechanism is repeated over the person's lifetime. If a person opts to be idle, this person's disutility concerning labor is assumed to be at a maximum and null. This normalization is a way to produce a linear expected disutility function.

On the other hand, Mulligan introduces a divisible model, and discusses how the labor supply is determined. In the divisible economy, the labor supply arrangement is determined by intensity in terms of labor hours, that is, in terms of utilization. All members of a labor population are supposed to participate in the labor market, changing utilization over a lifetime.

Mulligan concludes that there is no difference in the labor supply between the two economies. His scope in this demonstration is limited to the labor supply, but we can conjecture a scenario in which, in a general equilibrium, the two economies can move in a parallel pattern. In short, we can demonstrate that the difference between the two labor supply arrangements, one intensive and one extensive, does not affect the movement pattern of the two economies.

We are especially interested in indeterminacy in an economy based on a balanced budget rule. Schmitt and Uribe (1997) discusses this significant issue based on Hansen (1985). On the other hand, Takata (2017) offers a proof regarding an economy with an intensive arrangement. Both papers were written without referring to Mulligan (1999). Our conjecture is true.

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References

[1] Attanasio, P. O.,. (2015), "FRANK RAMSEY'S MATHEMATICAL THEORY OF SAVING," *The Economic Journal*, Vol. 125, No. 583, 269-277.

[2] Benhabib, J. and Farmer, R. F., (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, 63, 19-41.

[3] Hansen, G. D., (1985), "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16, 309-327.

[4] Mino, K., (2017). *Growth and Business Cycles with Equilibrium Indeterminacy* (Tokyo: Springer) *Advances in Japanese Business and Economics*, 13.

[5] Mulligan C., (1999), "Microfoundations and Macro Implications of Indivisible Labor," *NBER Working Paper*, No. 7116.

[6] Ramsey, F. P., (1928), "A Mathematical Theory of Saving," *The Economic Journal*, 38, No. 152 (Dec.), 543-559.

[7] Rogerson R., (1988), "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21, 3-16.

[8] Schmitt-Grohé, S. and Uribe, M., (1997), "Balanced-Budget Rules, Distortionary Taxes and Aggregation Instability,"*Journal of Political Economy*, Vol. 105, No. 5, 976-1000.

[9] Takata, F., (2017),"Two Steady States and Two Movement Patterns under the Balanced Budget Rule-An Economy with Divisible Labor," *Discussion Paper No.1702, January 2017, Kobe University.*