

Tax reform, unemployment, and fertility

Minoru Watanabe

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Minoru Watanabe^{1,2}

Abstract

This brief article describes the development of a simple overlapping generations model with unemployment and endogenous fertility to analyze the impact of increasing capital income tax. We find that higher capital income tax promotes employment and fertility.

JEL classification: H20, J13, J60.

Keywords: Capital income tax, Unemployment, Fertility

1. Introduction

As Fanti and Gori (2010) and Wang (2015) contend, declining fertility and rising unemployment are serious economic issues in developed countries. Becker and Barro (1988) revealed that an increase in child-rearing costs reduces fertility, while Daveri and Tabellini (2000) revealed that a high cost of labor increases unemployment.

The present study analyzes the role of capital income tax on unemployment and fertility choice. Uhlig and Yanagawa (1996) demonstrated that capital income taxation can increase economic growth in an overlapping generations model. Their study introduced a revenue-neutral tax reform in which increasing capital income reduces labor income tax. Kunze and Schuppert (2010) extended Uhlig and Yanagawa's (1996) model by considering involuntary unemployment caused by wage bargaining, showing that higher capital income tax reduces labor income tax and,

¹ Hokusei Gakuen University, 2-3-1, Ohyachi-Nishi, Atsubetsu-ku, Sapporo 004-8631, Japan
E-mail: m-watanabe@hokusei.ac.jp

² Research Fellow, Graduate School of Economics, Kobe University

consequently, reduces wages. Therefore, an increase in capital income tax was found to promote employment. However, Uhlig and Yanagawa (1996) and Kunze and Schuppert (2010) do not consider fertility choice, and Kunze and Schuppert (2010) assume a constant growth rate of population size.

Fanti and Gori (2010) noted that previous theoretical studies analyzed unemployment and fertility choice separately. Their study demonstrated that child tax improves both unemployment and fertility in an overlapping generations model. Wang (2015) extended Fanti and Gori's (2010) model by including public pensions and child allowances. However, studies on the effect of capital income tax on unemployment and fertility in an overlapping generations model are scarce. Thus, we extended the work of Fanti and Gori (2010) and Wang (2015) by adding capital income tax, showing that a tax reform, which increases capital income tax, can promote both employment and fertility.

We describe our model and the results in Section 2, and conclusions in Section 3.

2. The Model

2-1. Households

Consider a standard one-sector overlapping generations model. Households are identical and are classified into two periods (young and old). They are endowed with one unit of time devoted to labor supply and child-rearing during the young period, in accordance with Fanti and Gori (2007) and Hirazawa and Yakita (2009). The time constraint for young agents is

$$l_t + z_t = 1, \tag{1}$$

where l_t is the labor supply and z_t is the time cost for child-rearing. According to Fanti and Gori (2007), Fanti and Gori (2010), and Wang (2015), a constant minimum wage increases unemployment, and the unemployment rate is defined as a fraction of time. Note that Hirazawa and Yakita (2009) assumed full employment.

Households derive utility from consumption and number of children. We assume the following lifetime utility, similar to Fanti and Gori (2010) and Wang (2015).

$$v_t = \log c_t + \beta \log d_{t+1} + \log n_t, \tag{2}$$

where c_t and d_{t+1} represent the consumption in the young and old periods, respectively, n_t is the number of children, and $\beta \in (0,1)$ is the discount factor. We denote N_t as the population size born at t and the evolution of the population is represented by $N_{t+1} = n_t N_t$.

During the young period, households earn wage income, which is spent on consumption, savings, and childcare. In the old period, they retire and consume their savings. The budget constraints for the young and old periods are:

$$c_t + s_t + x_t = (1 - \tau_{w,t})(1 - u_t)l_t \bar{w}, \quad (3)$$

$$d_{t+1} = [1 + (1 - \tau_{r,t+1})r_{t+1}]s_t, \quad (4)$$

where s_t is the savings, x_t is the childcare cost captured by purchased final goods, $\tau_{w,t}$ is the labor income tax, u_t is the unemployment rate, $\bar{w} > 0$ is the constant minimum wage, $\tau_{r,t+1}$ is the capital income tax, and r_{t+1} is the interest rate. In equation (3), $(1 - u_t)l_t$ denotes the working period. Note that households cannot provide childcare while they supply labor, including during unemployed periods, as explained by Fanti and Gori (2007). According to Apps and Rees (2004) and Hirazawa and Yakita (2009), child-rearing requires both final goods and time costs. We thus assume the following fertility function, as put forth by Apps and Rees (2004) and Hirazawa and Yakita (2009):

$$n_t = x_t^\varepsilon z_t^{1-\varepsilon}, \quad 0 < \varepsilon < 1, \quad (5)$$

Note that if we omit the time cost for child-rearing (i.e., $z_t = 0$), and assume equation (5) as $n_t = x_t^\varepsilon$, our results do not change qualitatively.

According to Hirazawa and Yakita (2009), households choose consumption and the number of children to maximize equation (2) subject to equations (1), (3), (4), and (5). We obtain the following optimal allocations:

$$x_t = \frac{\varepsilon(1 - \tau_{w,t})(1 - u_t)\bar{w}}{2 + \beta}, \quad (6)$$

$$z_t = \frac{1 - \varepsilon}{2 + \beta}, \quad (7)$$

$$n_t = \frac{\varepsilon^\varepsilon(1 - \varepsilon)^{1-\varepsilon}[(1 - \tau_{w,t})(1 - u_t)\bar{w}]^\varepsilon}{2 + \beta}, \quad (8)$$

$$\frac{s_t}{n_t} = \frac{\beta[(1 - \tau_{w,t})(1 - u_t)\bar{w}]^{1-\varepsilon}}{\varepsilon^\varepsilon(1 - \varepsilon)^{1-\varepsilon}}. \quad (9)$$

Suppose p_t is the cost of rearing one child, $p_t \equiv [(1 - \tau_{w,t})(1 - u_t)\bar{w}z_t + x_t]/n_t$ and $(1 - \tau_{w,t})(1 - u_t)\bar{w}z_t$ is the opportunity cost for childcare. We obtain p_t as follows:

$$p_t = \frac{[(1 - \tau_{w,t})(1 - u_t)\bar{w}]^{1-\varepsilon}}{\varepsilon^\varepsilon(1 - \varepsilon)^{1-\varepsilon}}. \quad (10)$$

2-2. Firms

Firms produce final goods with capital and labor inputs in a competitive market. We thus assume the following production similar to Fanti and Gori (2010) and Wang (2015):

$$Y_t = K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (11)$$

where Y_t is the total output of the final goods, and K_t and L_t are the aggregate capital and labor inputs, respectively. The labor input is

$$L_t = (1 - u_t)l_t N_t. \quad (12)$$

As mentioned earlier, a constant minimum wage prevails in this economy. We assume no capital depreciation. The factor demand is

$$\bar{w} = (1 - \alpha)k_t^\alpha [(1 - u_t)l_t]^{-\alpha}, \quad (13)$$

$$r_t = \alpha k_t^{\alpha-1} [(1 - u_t)l_t]^{1-\alpha}, \quad (14)$$

where $k_t \equiv K_t/N_t$ is the per-capita capital. From equation (13), we obtain:

$$1 - u_t = \left(\frac{1 - \alpha}{\bar{w}}\right)^{\frac{1}{\alpha}} k_t l_t^{-1}. \quad (15)$$

Substituting equation (15) into equation (14), we obtain the following constant interest rates:

$$r_t = r = \alpha \left(\frac{1 - \alpha}{\bar{w}}\right)^{\frac{1-\alpha}{\alpha}}. \quad (16)$$

2-3. Government

The government imposes a tax on labor and capital income to provide non-productive expenditure under a balanced budget, in accordance with Uhlig and Yanagawa's

(1996) model. The government's budget constraint is:

$$\tau_{w,t}(1 - u_t)l_t\bar{w}N_t + \tau_{r,t}r_t s_{t-1}N_{t-1} = G_t, \quad (17)$$

where G_t denotes nonproductive expenditure, which contributes to no marginal utility or marginal productivity. The left-hand side of this equation denotes tax revenue from labor and capital income, respectively. We introduce a tax reform in accordance with the models by Uhlig and Yanagawa (1996) and Kunze and Schuppert (2010). Thus, we impose the following assumptions:

$$\tau_{r,t} = \tau_r, \quad 0 < \tau_r < 1, \quad (18)$$

$$G_t = \mu y_t N_t, \quad 0 < \mu < 1. \quad (19)$$

In equation (19), y_t represents the per-capita gross domestic product (GDP), where $y_t \equiv Y_t/N_t = k_t^\alpha [(1 - u_t)l_t]^{1-\alpha}$. If we assume capital income tax as an exogenous variable, the labor income tax is determined such that it balances the government's budget constraint. Equation (19) indicates that the government's expenditure is a fraction of GDP, as shown by Uhlig and Yanagawa (1996).

2-4. Equilibrium

The dynamics in this economy are

$$k_{t+1} = s_t/n_t. \quad (20)$$

Recall that $y_t = k_t^\alpha [(1 - u_t)l_t]^{1-\alpha}$; therefore, using equations (13) and (14), we arrive at

$$(1 - u_t)l_t\bar{w} = (1 - \alpha)y_t, \quad (21)$$

$$r_t k_t = \alpha y_t. \quad (22)$$

From equations (17)–(22), we obtain the constant of the labor income tax rate:

$$\tau_w = \frac{\mu - \alpha\tau_r}{1 - \alpha}. \quad (23)$$

Following Uhlig and Yanagawa (1996), we assume that $0 < \tau_w < 1$. Equation (23) indicates that increasing capital income tax reduces labor income tax. Using equations (1), (7), (9), (15), (20), and (23), we obtain

$$k_{t+1} = \rho k_t^{1-\varepsilon}. \quad (24)$$

$$\rho \equiv \frac{\beta}{\varepsilon^\varepsilon(1-\varepsilon)^{1-\varepsilon}} \left\{ \left(\frac{1-\alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{2+\beta}{1+\beta+\varepsilon} \right) [1-\alpha(1-\tau_r)-\mu] \right\}^{1-\varepsilon}. \quad (25)$$

Note that $0 < 1 - \alpha(1 - \tau_r) - \mu < 1$ holds in equation (25) because τ_w is given by equation (23), and we assume $0 < \tau_w < 1$. Thus, we obtain $\rho > 0$ in equation (25). Recall that $0 < \varepsilon < 1$ from equation (5). Equation (24) reveals the existence of a stable and unique steady state in this economy. Suppose k is the per-capita capital in the steady state. From (24) and (25), k is

$$k = \left[\frac{\beta}{\varepsilon^\varepsilon(1-\varepsilon)^{1-\varepsilon}} \right]^{\frac{1}{\varepsilon}} \left\{ \left(\frac{1-\alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{2+\beta}{1+\beta+\varepsilon} \right) [1-\alpha(1-\tau_r)-\mu] \right\}^{\frac{1-\varepsilon}{\varepsilon}}. \quad (26)$$

Clearly, we obtained $dk/d\tau_r > 0$ and $dk/d\bar{w} < 0$. Equation (23) reveals that a higher capital income tax reduces labor income tax, which increases disposable income and savings. Thus, a higher capital income tax promotes capital accumulation, as revealed by Uhlig and Yanagawa (1996). From equations (7) and (15), we denote the long-run unemployment rate as u and obtain

$$u = 1 - \left(\frac{1-\alpha}{\bar{w}} \right)^{\frac{1}{\alpha}} \left(\frac{2+\beta}{1+\beta+\varepsilon} \right) k. \quad (27)$$

Equation (27) indicates that long-run unemployment rate declines with capital accumulation as shown by Fanti and Gori (2010) and Wang (2015). Suppose k_f is the long-run per-capita capital under full employment: $u = 0$. Combining equations (1), (7), and (15), and taking $u = 0$, we describe k_f as

$$k_f = \left(\frac{\bar{w}}{1-\alpha} \right)^{\frac{1}{\alpha}} \frac{1+\beta+\varepsilon}{2+\beta}. \quad (28)$$

If $k_f > k$ holds, unemployment exists in the long run. Substituting equation (26) into equation (27), we obtain:

$$u = 1 - \sigma [1 - \alpha(1 - \tau_r) - \mu]^{\frac{1-\varepsilon}{\varepsilon}}, \quad (29)$$

$$\sigma \equiv \left(\frac{1-\alpha}{\bar{w}} \right)^{\frac{1-\alpha(1-\varepsilon)}{\alpha\varepsilon}} \left(\frac{2+\beta}{1+\beta+\varepsilon} \right)^{\frac{1}{\varepsilon}} \left[\frac{\beta}{\varepsilon^\varepsilon(1-\varepsilon)^{1-\varepsilon}} \right]^{\frac{1}{\varepsilon}} > 0. \quad (30)$$

Note that $du_t/d\bar{w} > 0$ holds for any period t . Differentiating equation (29) with

respect to τ_r , we derive

$$\frac{du}{d\tau_r} = -\frac{\alpha\sigma(1-\varepsilon)}{\varepsilon}[1-\alpha(1-\tau_r)-\mu]^{\frac{1-\varepsilon}{\varepsilon}-1} < 0. \quad (31)$$

Daveri and Tabellini (2000) found that the negative correlation between unemployment and labor income tax holds. In this article, increasing capital income tax was found to reduce unemployment because a higher capital income tax cuts the labor income tax, which increases savings and per-capita capital. As described by equation (27), capital accumulation promotes employment. Therefore, we obtain Proposition 1.

Proposition 1

An increase in capital income tax promotes capital accumulation, thereby reducing unemployment in the long run.

Suppose y is the long-run per-capita output, where $y = k^\alpha[(1-u)l]^{1-\alpha}$. Note that l is constant in equilibrium because $l = 1 - z = \frac{1+\beta+\varepsilon}{2+\beta}$ from equations (1) and (7). Recall that a rise in capital income tax increases not only per-capita capital but also employment. Thus, increasing capital income tax increases long-run per-capita output.

Next, we examine the impact of capital income tax on fertility. Assuming that n is the long-run fertility rate, from equations (8), (23), (26), and (27), under a generic form, it is

$$n = n\{\tau_w(\tau_r), u[k(\tau_r)]\}. \quad (32)$$

The total derivative of equation (32) with respect to τ_r yields

$$\frac{dn}{d\tau_r} = \underbrace{\frac{\partial n}{\partial \tau_w}}_{-} \underbrace{\frac{\partial \tau_w}{\partial \tau_r}}_{-} + \underbrace{\frac{\partial n}{\partial u}}_{+} \underbrace{\frac{\partial u}{\partial k}}_{-} \underbrace{\frac{\partial k}{\partial \tau_r}}_{+}. \quad (33)$$

Equation (33) demonstrates that the capital income tax has two effects. First, increasing it reduces the labor income tax, which leads to an increase in disposable income, thereby improving fertility. The first term on the right-hand side of equation

(33) describes this effect. Second, capital accumulation with a higher capital income tax promotes both employment and fertility. The second term of the right-hand side of equation (33) denotes this effect. Substituting equations (23), (29), and (30) into (8), we obtain:

$$n = \frac{\beta}{1 + \beta + \varepsilon} \left(\frac{1 - \alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} [1 - \alpha(1 - \tau_r) - \mu]. \quad (34)$$

By differentiating equation (34) with respect to τ_r , we derive:

$$\frac{dn}{d\tau_r} = \frac{\alpha\beta}{1 + \beta + \varepsilon} \left(\frac{1 - \alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} > 0. \quad (35)$$

Thus, we arrive at the following proposition.

Proposition 2

Higher capital income tax improves fertility in the long run.

Our results are not affected by considering the labor income tax reductions. However, many developed countries have come up against a deterioration of finances in recent years, as indicated by Ueshina (2018), and tax reduction will worsen their fiscal situation. Compared to the labor income tax reduction, the tax reform in the current study is revenue-neutral.

3. Conclusion

Many developed countries are plagued by not only a decline in fertility but also a high unemployment rate. In this study, we built an overlapping generations model and demonstrated that capital income taxation can resolve these economic issues.

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