

**A dynamic theory of the Feldstein-Horioka puzzle and
financial frictions: Re-estimation of the saving
retention coefficient**

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A dynamic theory of the Feldstein–Horioka puzzle

and financial frictions:

Re-estimation of the saving retention coefficient

by

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Abstract

Many empirical studies of the Feldstein-Horioka Puzzle (FHP) conducted so far have common drawbacks; among others, those estimates are free from economic models, and their specification errors are not avoidable. In other words, statistically significant estimates of the saving retention coefficient are not known yet. Taking a complete different approach from earlier studies to elucidate FHP, I re-estimate the savings retention coefficient (called “beta”) indirectly based on the perfect-foresight saddle-path dynamics of investment under convex adjustment cost. I assume that the error term of the regression model will represent various shocks and effect of financial frictions. Our main empirical results are; (1) Replication of the FHP in the 16 OECD countries during 1964–1974 yields 0.52 as the estimate of the beta, which is much smaller than the estimate of Feldstein and Horioka (1980). (2) Estimates using samples for every 10 years during 1960–1999 exhibit that the beta decreases gradually. (3) During 2000–2008, the FHP temporarily disappeared and then the higher beta returned after 2008.

(167 words).

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1. Introduction

In their controversial paper, Feldstein and Horioka (1980; hereinafter, FH) studied the relation between domestic investment and domestic saving shares of GDP among 16 OECD countries. Their empirical studies revealed that the estimated correlation, 0.89, was very high across countries. They interpreted their findings as an indication of a high degree of financial friction. They concluded that they were inconsistent with the perfect capital mobility assumption commonly made for international finance studies. This problem has often been designated as the “Feldstein–Horioka puzzle” (hereinafter, FHP). I will leave earlier research of the FHP for the detailed survey by Coakley et al. (1998) and, Apergis and Tsoumas (2009), which will mainly focus on the most recent research results.

Recently Bai and Zhang (2010) specifically examined financial friction to solve the puzzle theoretically. They considered financial friction of two kinds: “limited enforcement” and “limited spanning” and found that the calibrated model with friction of both types produced a saving–investment correlation close to value found from the data: 0.52. Recent empirical studies examining FHP also mainly emphasize estimation of the savings retention coefficient as the indices of globalization and capital mobility. Younas and Nandwa (2010) introduced not only the financial openness index for the OECD countries but also the foreign aid index for developing countries as omitted variables. Applying the dynamic panel estimation, they obtained much lower estimation values of savings retention coefficient (“beta”) than that of FH. Furthermore, Younas and Chakraborty (2011) used the globalization index instead of the financial openness index to re-estimate the beta and yielded the lower estimation values too. Both papers, however, share a common defect. The estimators of the savings retention coefficient depend on the values of indices. Therefore, some ambiguity persists. Georgopoulos and Hejazi (2005) introduced a time trend interaction term in the panel data model and reported that the value of savings retention coefficient fell over time by demonstrating that the estimated coefficient of the time trend interaction term was negative. That result implies that the economy will become more open and financially integrated over time. Because the trend interaction term is assumed as linear, financial integration will become perfect over time: beta will approach and eventually reach zero. But and Morley (2017) examined the time-varying beta across the OECD using recursive estimations and found that the estimated beta for OECD countries had a negative trend. Furthermore, they found that the FHP disappeared briefly prior to the 2008 crisis.

It is important to note that the majority of preceding empirical researches share the

common drawbacks; among others, they were not based on economic models. In other words, specification errors of their estimates will not be avoidable. Taking a completely different approach from earlier empirical studies to elucidate FHP, I estimate the savings retention coefficient indirectly. To do so, I start from setting up the representative firm's investment behavior model. According to the System of National Accounts (SNA), gross domestic investment (GDI) comprises housing investment, gross private investment, and public investment. Among them, gross private investment predominates and serves a crucial role in the behavior of GDI. Therefore, I shall start from particularly addressing a representative firm's investment behavior and then based on the perfect-foresight saddle-path dynamics of investment under convex adjustment costs. Because the optimal investment behavior includes lagged investment variables, adding domestic saving variables as independent variables to estimate the beta, the dynamic panel regression method will be employed to avoid a biased estimate problem.

My main empirical findings are the following:

1. Replication of the FHP in the 16 OECD countries during 1964–1974 yields 0.52 as the estimate of the beta, which is much smaller than the estimate of 0.89 by FH (1980).
2. Estimates using samples for every 10 years during 1960–1999 exhibit that the beta decreases gradually from 0.57 to 0.32.
3. During 2000–2014, the estimate of beta is not statistically significant from 2000 through 2008. It becomes statistically significant again after 2008, where the estimated beta is 0.51, which is similar to the result for 1960s and 1970s. In other words, the FHP temporarily disappeared, then the higher beta returned after 2008.

The results described above clearly support the theoretical solution of the FHP presented by Bai and Zhang (2010). Additionally, it is noteworthy that our estimated beta value closely approximates that obtained from their calibration: 0.52. Globalization and financial integration facilitated the cross-border movements of portfolio and direct investment in 1980s and 1990s. Therefore, I am able to observe that the saving–investment correlation estimated by the beta was gradually declining until 1999 because some degree of financial friction (e.g. home country bias) persists and the estimate of the beta remained significant. One might say that the FHP was robust during the observation period of 1960–1999 if one considers these estimated beta values as high. Then, during 2000–2008, I observed that beta turned out to be statistically not significant and that the FHP had temporarily disappeared. Worldwide trends of financial deregulation in the late 1990s caused these results. Indeed, after the 2008 financial crisis,

tougher financial regulations were implemented again² and higher beta of 0.51 came back. These results firmly support Bai and Zhang's financial friction solution for the FHP.

Each section is organized as follows: Section 2 is assigned for presenting a representative firm's investment behavior model. I derive the estimation equation by solving the firm's optimal problem. All the mathematical derivations are demonstrated in Appendix. In Section 3, setting up the regression model based on the firm's investment function derived in Section 2, I employ the dynamic panel method to estimate the saving retention rate. I will report and discuss the results. In Section 4, I will briefly summarize and conclude my work.

2. The Model and Assumptions

Let us consider the following representative firm investment behavior model:

$$\begin{aligned} & \max_{\{I_t\}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} (R(k_t) - I_t - C(I_t)) \\ & s.t. k_t = I_t + (1-\delta)k_{t-1} \text{ and } k_0. \\ & \text{where} \\ & r \quad : \text{discount rate } (0 < r < 1), \\ & R(k_t) : \text{revenue function ,} \\ & I_t \quad : \text{gross investment,} \\ & C(k_t) : \text{adjustment cost function .} \end{aligned}$$

The model implies that the representative firm will choose its investment level to maximize the total discounted pure revenue flow where investment will increase capital stock and raise the revenue, but it will pay costs of investment adjustment as well as investment itself. Showing the saddle-path stability of the model, I derive the following investment share equation as demonstrated in Mathematical Appendix.

$$\tilde{I}_{it} = \beta_0 \tilde{I}_{it-1} + \beta_1 \tilde{I}_{it-2} \quad (1)$$

where \tilde{I}_{it} is the national investment share in the period t , $\beta_0 = \frac{(1+\lambda_1)}{(1+g)}$ and

² For example, the Dodd-Frank Wall Street Reform and Consumer Protection Act enacted on July 21, 2010.

$$\beta_1 = -\frac{\lambda_1}{(1+g)^2} .$$

Note that due to the derivation of Eq. (1) demonstrated in Appendix, β_0 and β_1 should satisfy the following sign conditions: $\beta_0 > 0$ and $-1 < \beta_1 < 0$.

3. Estimation and Results

Based on the optimal investment behavior presented as Eq. (1) and adding variables of the national saving share, the following regression model can be finally set up:

$$(*) \tilde{I}_{it} = \beta_0 \tilde{I}_{it-1} + \beta_1 \tilde{I}_{it-2} + \beta_2 S_{it} + \beta_3 S_{it-1} + \gamma_i + u_{it}$$

where i = country index, t = time index,

\tilde{I}_{it} : national investment share of GDP,

S_{it} : national saving share of GDP,

u_{it} : error term (shocks and effect of frictions),

γ_i : fixed effect of the country i .

Since the deterministic part of the above model derived based on the perfect-foresight firm's model, the error term exhibits various fundamental shocks³ and effect of frictions explained in Section 1. The fixed effect of the country will stand for the home country bias, as discussed by Kraay et al. (2000) and Kraay and Ventura (2000).

It is also noteworthy that signs of coefficients of (*) must satisfy the following restrictions because of the theoretical model discussed in Appendix:

Condition I:

β_0 and β_1 are statistically significant, and they satisfy (i) $0 < \beta_0$, (ii) $-1 < \beta_1 < 0$.

Because I assume that the investment share and the saving share should satisfy the equation; $\tilde{I}_{it} = \alpha + \beta \tilde{S}_{it}$, it follows that $\tilde{I}_{it} - \tilde{I}_{it-1} = \beta (\tilde{S}_{it} - \tilde{S}_{it-1}) = \beta \tilde{S}_{it} + (-\beta) \tilde{S}_{it-1}$.

Thus the following additional sign condition should be also satisfied for the coefficients of the saving shares:

³ They indicate temporary changes in the terms of trade, transfers from abroad, and fluctuations in production.

Condition II:

β_2 and β_3 are statistically significant, and (iii) $\beta_2 = -\beta_3$.

If condition (iii) would not hold, then the estimate of β_2 should not be regarded as the savings retention coefficient.

The regression model (*) includes the lagged dependent variables as independent variables as well as the fixed effect. Therefore, the standard ordinary least squares (OLS) estimates will exhibit strong bias. To avoid that difficulty I apply dynamic panel estimation (DPE) developed by Arellano and Bond (1998) and Arellano and Bover (1996). Although these estimation methods are powerful, one must be careful about weak instrument and over-identification problems. Actually, DPE is well-known to generate a huge number of instruments easily. To examine these problems, two important statistics will be reported in our tables: for the weak-instrument problem, Arellano–Bond AR(1) (A-B AR(1) test) and Arellano–Bond AR(2) tests (A-B AR(2) test) for first and second serial correlations, and for the over-identification problem, the Hansen tests for over-identification of the instruments. To avoid the weak instrument problem, the null hypothesis of “no serial correlation” must be rejected in the A-B AR(1) test. Conversely, in the A-B AR(2) test the null should be accepted. For the over-identification problem, the null of the Hansen J-test “not over-identified” should be accepted. For system GMM, result of the Difference-in-Hansen test for the validity of the additional moment restriction will be reported.

I examine the following three cases:

- **Case 1:** Replication studies of the Feldstein–Horioka Puzzle
Data: Gross national investment GDP share and gross saving GDP share
Period: 1960–1974
Countries: 21 OECD countries

- **Case 2:** Ten-year sub-samples
Data: Same as Case 1
Period: 1960–1969, 1970–1979, 1980–1989, 1990–1999
Countries: 28 OECD countries

- **Case 3:** Breaks in 2000s
Data: Same as Case 1
Period: 2000–2008, 2009–2014, 2000–2014

Countries: 28 OECD countries

All the data come from the National Account Data included in the Penn World Table Ver.9⁴.

The estimation results of Case 1 are reported in Table 1. I apply one-step and two-step difference GMM with collapsed instruments for this case with and without a constant term. The results are reported as Columns (1) and (2), and one-step and two-step system GMM are reported as Columns (3) and (4), respectively in Table 1. The Arellano–Bond AR(1) and AR(2) tests exhibit a weak-instrument problem for Columns (1) and (2). All the over-identification test results are satisfactory. I will choose Column (3) of Table 1 as our final result because all sign conditions *i*) through *iii*) derived from the theoretical model are satisfied. Consequently, the replicated estimated savings retention coefficient is 0.52, which is much smaller than the value estimated by Feldstein and Horioka (1980): 0.89. Our estimated value coincides with the value calibrated by Bai and Zhang (2010). I can regard 0.52 as a benchmark value of the savings retention coefficient.

The estimation results of Case 2 are presented in Table 2 below as Columns (1) through (3), where one-step difference or one-step system GMM is employed. For the observation period of the 1990s, the one-step system GMM is applied and reported as Column (4) in Table 2. Furthermore, the two-step system GMM is applied to Case 2. Its results are reported in Table 3. For results of Columns (2) and (3) in Table 2, the sign of \tilde{S}_{it-1} is not statistically significant. Consequently, the estimates in Columns (2) and (3) of the beta cannot be regarded as savings retention coefficients. However, all coefficient-sign conditions of *i*) through *ii*) are satisfied in Table 3. Condition *iii*) also holds except the estimates in Column (3). Thus, overall required sign conditions are satisfied and robust. Furthermore the weak-instrument tests and over-identification tests present favorable results. To sum up, our test statistics hit at a proper specification.

Table 4 reported the estimation results from 2000 to 2014, and 2008 is the break year. All the estimates are also statistically significant. It is important to emphasize that before the break year the estimates of β_2 and β_3 had turned to be insignificant and then returned to be significant after the break year.

Finally, I might infer that the estimation values of the beta can be summarized as shown as Table 5 below.

⁴ <http://www.gdc.ne/pwt>

Table 5: Summary

	1960–1969	1970–1979	1980–1989	1990–1999
$\widehat{\beta}_2$	0.50	0.54	0.42	0.32–0.33

It is clear that the savings retention coefficient beta declines over time. This result is consistent with results reported by Georgopoulos and Hejazi (2005), and by But and Morley (2017). It follows from the fact that the recent economic openness and financial market integration increase capital mobility and reduce financial friction.

In Case 3, I will not only estimate the beta for the overall observation period: 2000–2014, but also estimate it for two sub-periods: 2000–2008 and 2009–2014. The main reason I use 2008 as the break point is that it marks the 2008 financial crisis.

-Figure 1 -

According to Figure 1, where the diagram of total foreign claims⁵ of BIS reporting banks (32 countries, 246 banks) is depicted, the total foreign claims surged suddenly to 34 M USD in the early 2000s. Subsequently, it plummeted to the 26 M USD level after 2008.

- Figure 2-

Total world direct investment shown in Figure 2 also exhibits a similar process to that of foreign claims in Figure 1. Our results from the 2000s are reported in Table 4. Estimation results of Columns (1) and (2) in Table 4 indicate that the estimates of beta become not significant during 2000–2008. In other words, the saving–investment relation temporarily disappeared during this period. It also implies that the financial friction disappeared too. I will identify that the period when the saving–investment relation disappeared as coinciding with a worldwide financial bubble period: 2000–2008. Consequently, Column (3) in Table 4 exhibits that after 2008, the saving–investment relation returned: the estimated beta was 0.51, which is a similar value to that of the 1960s and 1970s. After the crisis, financial frictions also returned, as reported by But and Morley (2017).

4. Conclusion

I set up a representative firm’s investment model to avoid specification errors. Based on the perfect-foresight saddle-path dynamics of investment under convex adjustment cost,

⁵ Note that Foreign claims = Cross-border claims + Local claims of foreign affiliates in foreign currency + Local claims of foreign affiliates in local currency.

I have derived the dynamic regression model. It is worth to note that the error term will include various shocks and effect of financial frictions. Then, using the dynamic panel method, I estimated the savings retention coefficient during 1960–2014. Replication of the FH estimation yields 0.52, which is consistent with the calibration result of 0.52 reported by Bai and Zhang (2010) from their specific examination of the financial friction for solving FHP. Furthermore, I demonstrate that beta fell from 0.54 to 0.33 in the 1980s and 1990s and the saving–investment relation temporarily disappeared from 2000 through 2008. Then the higher beta value of 0.51 returns after 2008. These empirical results support the theoretical solution of the FHP presented by Bai and Zhang (2010).

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Mathematical Appendix

In order to derive the estimation equation, I start making the following assumptions.

Assumption A1.

$C(I)$ is C^2 on the interval $(0, \bar{I})$ and strictly convex, where \bar{I} is the upper bound of investment.

Moreover, $\lim_{I \rightarrow \bar{I}} C(I) = \lim_{I \rightarrow \bar{I}} C'(I) = \infty$ and $C'(0) = 0$.

Assumption A2.

$R(k)$ is C^2 defined on $\mathfrak{K} \equiv \left(0, \frac{\bar{I}}{\delta}\right) = (0, \bar{k})$, and strictly concave with respect to k .⁶

To solve the problem, I rewrite the problem by using the following objective function as defined below.

Definition A1.

$$V(k_{t-1}, k_t) \equiv R(k_t) - I_t - C(I_t) = R(k_t) - (k_t - (1 - \delta)k_{t-1}) - C(k_t - (1 - \delta)k_{t-1}) \quad (t \geq 1)$$

Then using the newly defined objective function, the original problem is rewritten as

⁶ \bar{k} is the maximum reproducible capital when \bar{I} is given.

follows.

$$\begin{aligned} & \max_{\{k_t\}} \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} V(k_t, k_{t+1}) \\ & s.t. (k_t, k_{t+1}) \in \mathfrak{T} \times \mathfrak{T} \text{ and } k_0. \end{aligned}$$

Concerning with the rewritten optimal problem, the following Euler equations can be derived quickly.

$$\frac{\partial V(k_{t-1}, k_t)}{\partial k_t} + \left(\frac{1}{1+r} \right) \frac{\partial V(k_t, k_{t+1})}{\partial k_t} = 0 \quad (t \geq 1) \quad (\text{A1})$$

From the Euler equation (A1), the optimal steady state is defined as presented below.

Definition A2. The optimal steady state k^* is a solution of the following equation.

$$\frac{\partial V(k^*, k^*)}{\partial k_t} + \left(\frac{1}{1+r} \right) \frac{\partial V(k^*, k^*)}{\partial k_t} = 0 \quad (t \geq 1)$$

Because of Assumption 1, there exists an upper-bound $\bar{k} = \frac{\bar{I}}{\delta} > k_t > 0$ ($t \geq 0$). It follows that $\lim_{t \rightarrow \infty} (1+r)^{-t} k_t < \lim_{t \rightarrow \infty} (1+r)^{-t} \bar{k} = 0$. Therefore, the transversality condition should hold.

Consequently, any optimal path is expected to satisfy this condition.

Next, to study the local stability around the steady state k^* , let us linearize the Euler equations around k^* . Then after some calculations, I obtain

$$\begin{aligned} & \left(\frac{1}{1+r} \right) \left(\frac{\partial V^2(k_t, k_{t+1})}{\partial k_t \partial k_{t+1}} \right) \Big|_{(k^*, k^*)} Z_{t+1} + \left\{ \left(\frac{1}{1+r} \right) \frac{\partial V^2(k_t, k_{t+1})}{\partial k_t^2} \Big|_{(k^*, k^*)} + \frac{\partial V^2(k_{t-1}, k_t)}{\partial k_t^2} \Big|_{(k^*, k^*)} \right\} Z_t \\ & + \left[\frac{\partial V^2(k_{t-1}, k_t)}{\partial k_t \partial k_{t-1}} \right] Z_{t-1} = 0, \end{aligned} \quad (\text{A2})$$

where $Z_t = k_t - k^*$,

Using Definition A1 and calculating each second order partial derivative evaluated at k^* , Eq. (A2) can be rewritten as follows.

$$(1-\delta)C^{**} Z_{t+1} + \left\{ (1+r) [R^{**} - C^{**}] - C^{**} (1-\delta)^2 \right\} Z_t + (1+r)(1-\delta)C^{**} Z_{t-1} = 0,$$

where the symbol $*$ denotes that the functions are evaluated in the optimal steady state k^* . Then further simplification will yield the following second-order characteristic

equation $f(\lambda) = 0$ related to the Euler equation.

$$f(\lambda) \equiv \lambda^2 + \left\{ \left(\frac{1+r}{1-\delta} \right) \left[\frac{C^{**} - R^{**}}{C^{**}} \right] - (1-\delta) \right\} \lambda + (1+r) = 0. \quad (\text{A3})$$

Then I can prove the following important proposition. I will derive the estimation equation based on it. I leave all proof in Appendix.

Proposition. If $f(1) < 0$ and $f(0) > 0$ hold, then the steady state k^* is saddle-point stable.

Proof. Note that $f(0) = (1+r) > 0$ already holds. Therefore, I need show that $f(1) < 0$ will also hold. Eq. (A3) quickly provides the following relation:

$$\begin{aligned} f(1) &= 1 + \left\{ \left(\frac{1+r}{1-\delta} \right) \left(\frac{R^{**}}{C^{**}} - 1 \right) - (1-\delta) \right\} + (1+r) \\ &= (1+r+\delta) + \left(\frac{1+r}{1-\delta} \right) \left(\frac{R^{**}}{C^{**}} - 1 \right) = \left\{ (1+r+\delta) - \left(\frac{1+r}{1-\delta} \right) \right\} + \left(\frac{1+r}{1-\delta} \right) \left(\frac{R^{**}}{C^{**}} \right) \\ &= \frac{-\delta(r+\delta)}{1-\delta} + \left(\frac{1+r}{1-\delta} \right) \left(\frac{R^{**}}{C^{**}} \right) < 0 \quad (\text{from the fact that } R^{**} < 0 \text{ and } C^{**} > 0). \end{aligned}$$

Therefore, the needed property is established. ■

The proposition implies that there are two positive roots and one of two roots has its absolute value less than one.

As claimed before, the transversality conditions are held. Therefore saddle-point stability implies that the optimal path should be on stable manifolds. For our estimation of the savings retention coefficient, I use the saddle-point stability property of the optimal accumulation path, instead of estimating the firm's investment function directly.

Let λ_1 be the root satisfying $0 < \lambda_1 < 1$. Then the optimal path is expected to satisfy the following difference equations:

$$(k_t - k^*) = \lambda_1 (k_{t-1} - k^*) \text{ for all } t \geq 1. \quad (\text{A4})$$

From Eq. (A4), subtraction to eliminate k^* gives

$$\begin{cases} (k_t - k_{t-1}) = \lambda_1 (k_{t-1} - k_{t-2}) \\ (k_{t-1} - k_{t-2}) = \lambda_1 (k_{t-2} - k_{t-3}). \end{cases}$$

From the capital accumulation equations, it follows that

$$\begin{cases} I_t - I_{t-1} = (k_t - k_{t-1}) - (1 - \delta)(k_{t-1} - k_{t-2}), \\ I_{t-1} - I_{t-2} = (k_{t-1} - k_{t-2}) - (1 - \delta)(k_{t-2} - k_{t-3}). \end{cases}$$

Combining both relations will readily give $I_t - I_{t-1} = \lambda_1(I_{t-1} - I_{t-2})$. Finally, I obtain the following relation along the saddle-point path as the optimal investment behavior:

$$I_t = (1 + \lambda_1)I_{t-1} - \lambda_1 I_{t-2}. \quad (\text{A5})$$

Next, I must rewrite the equation (A5) obtained above into the GDP share terms originally used by Feldstein and Horioka (1980). To do so, I make the following assumption.

Assumption A3. The rate of average GDP growth g is constant and $0 < g < 1$.

Dividing both sides by $Y_t = GDP_t$ gives

$$\frac{I_t}{Y_t} = (1 + \lambda_1) \left(\frac{Y_{t-1}}{Y_t} \right) \left(\frac{I_{t-1}}{Y_{t-1}} \right) - \lambda_1 \left(\frac{Y_{t-2}}{Y_t} \right) \left(\frac{I_{t-2}}{Y_{t-2}} \right).$$

Due to Assumption A3, the following theoretical model will be finally obtained:

$$\frac{I_t}{Y_t} = \frac{(1 + \lambda_1)}{(1 + g)} \left(\frac{I_{t-1}}{Y_{t-1}} \right) - \frac{\lambda_1}{(1 + g)^2} \left(\frac{I_{t-2}}{Y_t} \right) = \frac{(1 + \lambda_1)}{(1 + g)} \tilde{I}_{t-1} - \frac{\lambda_1}{(1 + g)^2} \tilde{I}_{t-2},$$

and finally

$$\tilde{I}_{it} = \beta_0 \tilde{I}_{it-1} + \beta_1 \tilde{I}_{it-2},$$

where $\beta_0 = \frac{(1 + \lambda_1)}{(1 + g)}$ and $\beta_1 = -\frac{\lambda_1}{(1 + g)^2}$.

Since $0 < \lambda_1 < 1$ holds, it follows that $\beta_0 > 0$ and $-1 < \beta_1 < 0$ should hold.

Table 1: Replication of the FHP

Dependent variable: nominal national investment share of GDP (\tilde{I}_{it})				
Sample period: 1960–1974	(1)	(2)	(3)	(4)
Specification	One-step difference GMM (collapsed instrument)	Two-step difference GMM (collapsed instrument)	One-step system GMM (collapsed instrument)	Two-step system GMM (collapsed instrument)
Regressor				
\tilde{I}_{it-1}	0.34360 (0.2203)	0.15888 (0.2730)	0.94495(0.1304)***	0.93204(0.1215)***
\tilde{I}_{it-2}	- 0.18563 (-0.1856) ***	- 0.12046 (0.6025)*	- 0.17236 (0.0826)**	- 0.14632 (0.1075)
\tilde{S}_{it}	0.66894 (0.1058)***	0.76820 (0.1316)***	0.51702 (0.1076)***	0.63176 (0.1553)***
\tilde{S}_{it-1}	-0.01540 (-0.0154)	- 0.0050 (0.1439)	- 0.42195 (0.1418)***	- 0.52326 (0.1767)***
Const.			0.03875 (0.0250)	0.03088 (0.0243)
# of groups (countries)	28	28	28	28
# of instruments	16	16	18	18
A-B test for AR(1)	0.174	0.625	0.001	0.007
A-B test for AR(2)	0.102	0.039	0.171	0.161
Hansen J-test	0.215	0.215	0.147	0.147
Diff-in-Hansen test			0.479	0.479
Wald test: $\beta_2 = -\beta_3$	0.000	0.000	0.4407	0.3996

Notes

Significant at * 10%, ** 5%, *** 1%. () are standard errors. *P*-values are reported for Arellano–Bond tests, Hansen J-test, Difference-in –Hansen test and Wald test.

Table 2: Sample by 10 years

Dependent variable: nominal national investment share of GDP (\tilde{I}_{it})				
Sample period:	(1) 1960–1969	(2) 1970–1979	(3) 1980–1989	(4) 1990–1999
Specification	One-step system GMM (maximum lag = 2)	One-step difference GMM (collapsed instruments)	One-step difference GMM (maximum lag = 2)	One-step system GMM (maximum lag = 2)
Regressors				
\tilde{I}_{it-1}	0.97956 (0.1200)***	0.50447 (0.0946)***	0.97424 (0.1447)***	1.0311 (0.0666)***
\tilde{I}_{it-2}	- 0.14896 (0.0735)**	- 0.13925 (0.0429)***	- 0.11557 (0.0828)	- 0.14686 (0.6800)**
\tilde{S}_{it}	0.49905 (0.1610)***	0.56865 (0.1479)**	0.55954 (0.1355)***	0.31699 (0.1215)**
\tilde{S}_{it-1}	- 0.43101(0.1895)***	- 0.02761 (0.1111)	- 0.24231 (0.1451)	- 0.28881 (0.1318)**
Const.	0.02755 (0.0187)			0.01926 (0.0082)
# of groups (countries)	28	28	28	28
# of instruments	26	21	23	34
A-B test for AR(1)	0.002	0.000	0.002	0.001
A-B test for AR(2)	0.365	0.306	0.504	0.721
Hansen J-test	0.312	0.211	0.190	0.559
Diff-in-Hansen test	0.421			0.992
Wald test: $\beta_2 = -\beta_3$	0.4521	0.0000	0.0113	0.2703

Note

Significant at * 10%, ** 5%, *** 1%. () are standard errors. *P*-values are reported for Arellano-Bond tests, Hansen J-test, Difference-in –Hansen test and Wald test.

Table 3: Sample by 10 years with Two-step System GMM

Dependent variable: nominal national investment share of GDP (I_{it})				
Sample period:	(1) 1960–1969	(2) 1970–1979	(3) 1980–1989	(4) 1990–1999
Specification	Two-step system GMM (maximum lag = 2)	Two-step system GMM (collapsed instruments)	Two-step system GMM (maximum lag = 1)	Two-step system GMM (maximum lag = 1)
Regressors				
\tilde{I}_{it-1}	0.98967 (0.0979)***	0.68673 (0.1424)***	0.97518 (0.0604)***	1.04328 (0.0695)***
\tilde{I}_{it-2}	- 0.14559 (0.0802)*	- 0.15147 (0.0887)*	- 0.02993 (0.0819)	- 0.13789 (0.0732)*
\tilde{S}_{it}	0.50419 (0.1686)***	0.53558 (0.1583)***	0.42619 (0.1216)***	0.33307 (0.1143)***
\tilde{S}_{it-1}	- 0.44019 (0.1899)***	- 0.34202 (0.1310) **	- 0.34943 (0.1207)***	- 0.30925 (0.1217)**
Const.	0.02441 (0.0175)	0.07520 (0.0270)***	- 0.00569 (0.0145)	0.01546 (0.0100)
# of groups (countries)	28	28	28	28
# of instruments	26	23	24	24
A-B test for AR(1)	0.008	0.001	0.006	0.013
A-B test for AR(2)	0.425	0.316	0.375	0.772
Hansen J-test	0.312	0.233	0.148	0.118
Diff-in-Hansen test	0.421	0.203	0.507	0.266
Wald test: $\beta_2 = -\beta_3$	0.0903	0.2368	0.0206	0.2995

Note

Significant at * 10%, ** 5%, *** 1%. () are standard errors. *P*-values are reported for Arellano-Bond tests, Hansen J-test, Difference-in –Hansen test and Wald test.

Table 4: Break sample in 2000s

Dependent variable: nominal national investment share of GDP (I_{it})

Sample period:	(1) 2000–2008	(2) 2000–2008	(3) 2009–2014	(4) 2000–2014
Specification	One-step difference GMM (maximum lag = 1)	Two-step difference GMM (maximum lag = 1)	One-step system GMM (maximum lag = 2)	Two-step difference GMM (maximum lag = 1)
Regressors				
\tilde{I}_{it-1}	1.05583 (0.1164)***	1.16306 (0.1413)***	0.96651 (0.0838)***	0.87132 (0.1506)***
\tilde{I}_{it-2}	- 0.55433 (0.1719)***	- 0.69848 (0.2223)***	- 0.04373 (0.1001)	- 0.34514 (0.1265)***
\tilde{S}_{it}	0.20021(0.1652)	0.26918 (0.1538)*	0.51255 (0.1306)***	0.51774 (0.1617)***
\tilde{S}_{it-1}	-0.14750 (0.1317)	- 0.16673 (0.1696)	- 0.48142 (0.1374)***	- 0.28662 (0.1781)
Const.			0.00592 (0.0135)	
# of groups (countries)	28	28	28	28
# of instruments	12	12	22	18
A-B test for AR(1)	0.006	0.011	0.000	0.014
A-B test for AR(2)	0.311	0.320	0.099	0.189
Hansen J- test	0.103	0.103	0.170	0.102
Diff-in-Hansen test			0.821	
Wald test: $\beta_2 = -\beta_3$	0.7141	0.5116	0.1872	0.1426

Note

Significant at * 10%, ** 5%, *** 1%. () are standard errors. *P*-values are reported for Arellano-Bond tests, Hansen J-test, Difference-in –Hansen and Wald test.

test

Foreign claims of BIS reporting banks in USD mn

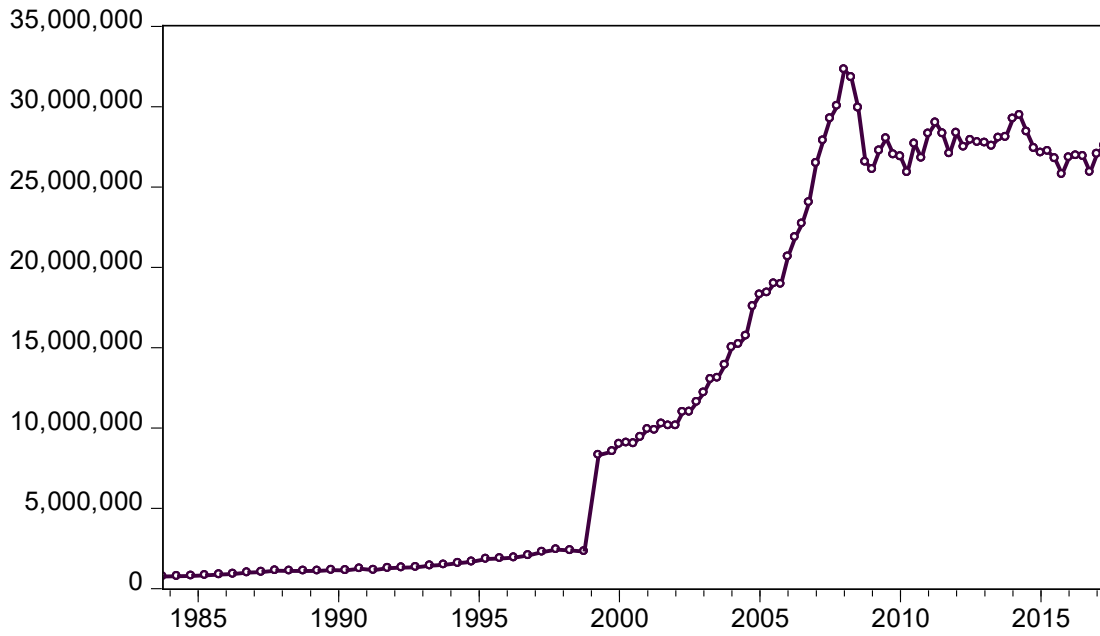


Figure 1

World Direct Investment in USD mn

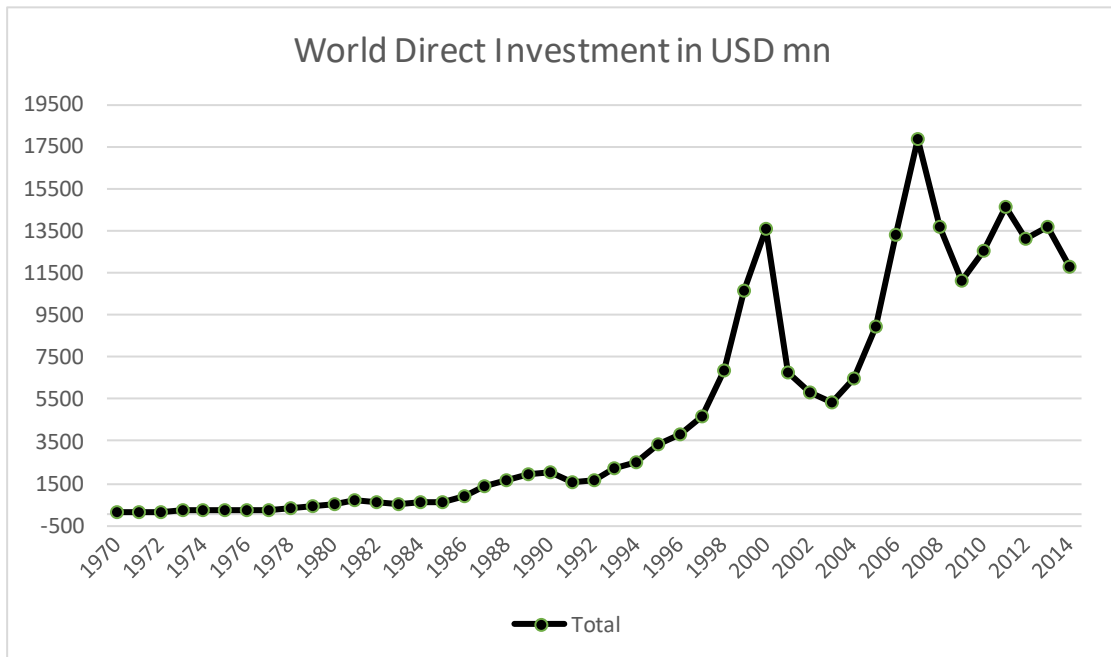


Figure 2