Monopolistic Competition and Nominal Stickiness in Generalized New Keynesian Model

Rui WANG

November 25,2019

Discussion Paper No.1913

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

Monopolistic Competition and Nominal Stickiness in Generalized New Keynesian Model

Rui WANG*

Department of Economics, Kanto Gakuen University†

November 25, 2019

Abstract

In this paper, we extend the standard New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model to allow both staggered price setting and staggered wage setting and derive a generalized version of New Keynesian model to study how these distortions affect the steady state and dynamics of model given different annual target inflation rates. The main finding is that the imperfection of labor market has more distortionary power on aggregate output and aggregate welfare given positive target inflation rate. Sensitivity analysis of structural parameters in the context of static steady state provides us a macroeconomic structural model-based explanation for the stylized fact that many central banks set the target inflation rate within a range from 1% to 2%. Also, given positive target inflation rate, the dynamic responses of macroeconomic variables on exogenous shocks are more sensitive to the change of structural parameters related to labor market. By comparing the dynamics generated under different sets of calibration, we find that the structure of labor market with high degree of monopolistic competition, low wage stickiness and low wage indexation is more desirable in an economy with positive target inflation rate.

Key Word: Price Stickiness, Wage Stickiness, Generalized New Keynesian Model, Distortion

JEL classification: E43, E44, E52, E58

1 Introduction

Monopolistic competition and nominal stickiness are two basic microeconomic-foundations in the standard New Keynesian model, which is the workhorse in the modern monetary macroeconomics. Monopolistic competition follows the mechanism proposed by Dixit and Stiglitz (1977). Differentiated intermediate-goods are not perfect substitutes for each other and intermediate-good firms are in monopolistic competition market, which means intermediate-good firms have some monopolistic power to add a markup on the marginal cost of product. The competition degree of monopolistic market can be represented by a specific structural parameter, *elasticity of substitution*. Nominal stickiness can be introduced by the Calvo (1983) staggered pricing mechanism which is the most popular and widely-used way to introduce nominal stickiness into the New Keynesian model. The Calvo (1983) price setting mechanism is simple and easy to track and interpret. In each period, an intermediate-good firm has a random change to optimize its price with a fixed probability. Those intermediate-good firms that can't re-optimize price setting just index the price to the last period inflation rate. The role of price indexation is to fit and generate the high persistence in the dynamic response of inflation on exogenous shock, which is usually observed in empirical analysis. Given these basic features, the standard New Keynesian model[1] has the property of *super-neutrality*, which means that at steady state with zero inflation, aggregate output is totally decided by real factors such as endogenous economic structure represented by structural parameters and exogenous shock process. However, if we allow steady state

*Assistant professor at Faculty of Economics, Department of Economics, Kanto Gakuen University, Japan. Research fellow at Graduate School of Economics, Kobe University, Japan. E-mail: kizuna.wr@gmail.com

†*200 Fujiagu-cho, Ōta, Gunma Prefecture, 373-0034, Japan.*

[1]For the standard literature about the New Keynesian model and its application in monetary policy analysis, please refer to Woodford (2003), Galí (1999, 2000, 2015), Miao (2014) and Walsh (2017).

with non-zero inflation rate, *super-neutrality* will not hold anymore. New Keynesian model with non-zero inflation steady state, in another terminology, *trend inflation*, is also known as Generalized New Keynesian model[2]. Note that non-zero steady state inflation or trend inflation in the New Keynesian model can also be explained as the target inflation rate anchored by the central bank in the practice of monetary policy. This is why we should be concerned with this issue. Most of existing literatures on generalized New Keynesian model only focus on monopolistic intermediate-good market and assume that labor market is completely competitive. In this paper, following Erceg, Henderson, and Levin (2000), we introduce monopolistic labor market, wage stickiness and wage indexation into the generalized New Keynesian model proposed by Ascari and Sbordone (2014). Under this setting, we have 6 key structural parameters that dominate these nominal and real distortions in the model.

ε_p	monopolistic competition in intermediate-good market
ε_w	monopolistic competition in labor market
θ_p	nominal stickiness in staggered price setting
θ_w	nominal stickiness in staggered wage setting
ϱ_p	backward-looking indexation in staggered price setting
ϱ_w	backward-looking indexation in staggered wage setting

Table 1: Economic Explanation of Structural Parameters

As listed in Table 1, these structural parameters have explicit economic interpretation. For the value of parameters related to price (wage) setting, $\theta_p(\theta_w)$ and $\varrho_p(\varrho_w)$, these parameters can be identified and estimated in the empirical estimation of DSGE models such as Smets and Wouters (2003, 2007) and Christiano et al. (2005). These parameters can also be estimated from micro-level data. For ε_p and ε_w, these parameters are generally calibrated to fit the markup rate. As we will show in the later[3], if target inflation rate is zero, no matter what the values of these structural parameters are, they don't effect price dispersion or wage dispersion, *super-neutrality* does hold in this case. However, if target inflation rate is not zero, aggregate output and aggregate welfare will be directly affected through the price dispersion and wage dispersion. Based on this reason, in this paper, we investigate these how these structural parameters affect the steady state and dynamics of generalized New Keynesian model.

We find that in our generalized version of New Keynesian model, in the context of steady state, increasing of elasticity of substitution in intermediate-good market (ε_p) always leads to increasing of aggregate output given every target inflation rate from 0% to 8%. But increasing of elasticity of substitution in intermediate-good market (ε_p) only leads to positive aggregate welfare gain given moderate target inflation rate from 0% to 4%. Besides, high elasticity of substitution in labor market (ε_w) only leads to high aggregate output given target inflation from 0% to 2%. Also, increasing of elasticity of substitution in labor market (ε_w) only leads to positive aggregate welfare gain given strict 0% target inflation rate. These findings may imply us a macroeconomic structural model-based explanation for the stylized fact that the most central banks set their target inflation rate within a range from 1% to 2%. From the sensitivity analysis of structural parameters in the context of dynamic impulse response, we find that impulse response functions are more sensitive to the change of structural parameters of labor market than the counterparts of intermediate-good market. Labor market with high degree of monopolistic competition (low ε_w), low wage stickiness (low θ_w) and low wage indexation (low ϱ_w) is desirable in the economy with positive target inflation rate. Labor market with low degree of monopolistic competition, in another word, high degree of competition (high ε_w) is not desirable in this situation.

This remaining paper is organized as follows. In Section 2, we firstly derive out generalized New Keynesian model. Then we linearize the equilibrium conditions around the steady state associated with non-zero target inflation rate. In Section 3, we check how these nominal and real distortions dominated by the key structural parameters affect the deterministic steady state of nonlinear model by changing the key structural parameters given different positive target inflation rates. Section 4 checks the sensitivity of key parameters in the context of dynamic impulse response. Finally, Section 5 summarizes this paper, implies policy implications and prospects the future research.

[2]Ascari and Sbordone (2014) provides a survey about empirical results of trend inflation. They also show a baseline generalized New Keynesian model.

[3]Please refer to Appendix.

2 Generalized New Keynesian Model

2.1 Household and Labor Supply

The period utility function $U(C_{h,t}, N_{h,t}) = \frac{C_{h,t}^{1-\sigma}}{1-\sigma} - \chi e^{\varsigma_t} \frac{N_{h,t}^{1+\varphi}}{1+\varphi}$ of the representative household with indexation $h \in (0,1)$ is assumed to be separable in consumption $C_{h,t}$ and labor $N_{h,t}$, where ς_t is an exogenous labor supply shock. σ and φ represent the inverse of inter-temporal elasticity of substitution and the inverse of Frisch elasticity of labor supply, respectively. Households supply differentiated kinds of labor to competitive labor "packer" which combines heterogeneous types of labor into a homogenous composite labor good that it then leases to intermediate-good producers at prevailing nominal wage rate W_t. The imperfect substitutability between differentiated types of labor gives households some monopolistic power in setting their own wage. The period budget constraint is given by $C_{h,t} + \frac{B_{h,t}}{P_t(1+i_t)} = \frac{W_{h,t}}{P_t} N_{h,t} + D_{h,t} + \frac{B_{h,t-1}}{P_t}$ where i_t is nominal interest rate, $B_{h,t}$ is one-period risk-free bond holdings, $W_{h,t}$ is the nominal wage rate paid to household h that has specialized kind of labor h, and $D_{h,t}$ is distributed dividends. The representative household maximizes the expected sum of discounted inter-temporal utility $\sum_{t=0}^{\infty} \beta^t U(C_{h,t}, N_{h,t})$ subject to period budget constraint. β is subjective discount factor. According to Erceg, Henderson, and Levin (2000), if there exist state contingent claims that insure households against idiosyncratic wage risk, and if preferences are separable in consumption and leisure, all households will be identical in their optimal choice of consumption and bond-holdings, and will only differ in the wage they charge and labor supply. Based on this reason, we can omit the indexation h in the first-order conditions of consumption $C_{h,t}$ and bond-holdings $B_{h,t}$. In our model, there doesn't exist government and capital accumulation, final output Y_t is all used as consumption C_t.

$$Y_t = C_t \tag{1}$$

Euler equation derived from utility maximization can be written as

$$\frac{1}{Y_t^{\sigma}} = \beta \mathbb{E}_t \left[\left(\frac{1+i_t}{\pi_{t+1}} \right) \left(\frac{1}{Y_{t+1}^{\sigma}} \right) \right] \tag{2}$$

where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is defined as gross inflation rate from period t to period $t+1$. Competitive labor packer uses CES technology $N_t = \left(\int_0^1 N_{h,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dh \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$ to pack differentiated kinds of heterogeneous labor into homogenous labor which is used by intermediate-good producers as input. $\varepsilon_w > 1$ is the elasticity of substitution among differentiated kinds of labor with indexation h. The profit maximization problem of the competitive labor packer is $\max_{\{N_{h,t}\}} \left(W_t N_t - \int_0^1 W_{h,t} N_{h,t} dh \right)$ from which we can solve the demand of specialized kind of labor h as $N_{h,t} = \left(\frac{W_{h,t}}{W_t} \right)^{-\varepsilon_w} N_t$. Substituting the demand equation of $N_{h,t}$ into the definition of N_t leads to the aggregate level of nominal wage W_t as $W_t = \left(\int_0^1 W_{h,t}^{1-\varepsilon_w} dh \right)^{\frac{1}{1-\varepsilon_w}}$. Optimal nominal wage set by the representative household that supplies $N_{h,t}$ is denoted as $W_{h,t}^*$. The wage setting here follows the assumption of Calvo (1983) and Erceg, Henderson, and Levin (2000). There is a probability θ_w that a household will be stuck with the current wage in each period. Those households can't optimize their wage just index the nominal wage to previous period gross inflation rate as $W_{h,t+j} = \pi_{t+j-1}^{\varrho_w} W_{h,t+j-1}$. $\varrho_w \in [0,1]$ indicates the degree of backward-looking indexation. Real wage rate w_t is defined as $w_t = \frac{W_t}{P_t}$. Wage optimization problem of the representative household h can be written as $\max_{\{W_{h,t}^*\}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta\theta_w)^j \left[\frac{C_{t+j}^{-\sigma} N_{h,t+j} W_{h,t+j}}{P_{t+j}} - \chi e^{\varsigma_{t+j}} \frac{N_{h,t+j}^{1+\varphi}}{1+\varphi} \right]$ which means the representative household maximizes the expected sum of discounted real labor income $\frac{W_{h,t+j}}{P_{t+j}} N_{h,t+j}$ measured by marginal utility of consumption $C_{t+j}^{-\sigma}$ by choosing optimal real wage $w_{h,t}^*$ at period t subject to the labor demand $N_{h,t+j}$, given the disutility of labor supply $\chi e^{\varsigma_{t+j}} \frac{N_{h,t+j}^{1+\varphi}}{1+\varphi}$. The first-order condition of wage optimization is

$$\left(w_{h,t}^* \right)^{1+\varepsilon_w \varphi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\psi_{w,t}}{\varphi_{w,t}} \tag{3}$$

where $w_{h,t}^*$ is optimal real wage set by household at period t. $\psi_{w,t}$ and $\varphi_{w,t}$ are two auxiliary variables with recursive structure which are defined to rewrite the first-order condition of wage optimization.

$$\psi_{w,t} = \chi e^{\varsigma_t} \left(w_t^{\varepsilon_w} N_t \right)^{1+\varphi} + \beta \theta_w \pi_t^{-\varrho_w \varepsilon_w (1+\varphi)} \mathbb{E}_t \left[\pi_{t+1}^{\varepsilon_w (1+\varphi)} \psi_{w,t+1} \right] \tag{4}$$

$$\varphi_{w,t} = C_t^{-\sigma} w_t^{\varepsilon_w} N_t + \beta \theta_w \pi_t^{\varrho_w (1-\varepsilon_w)} \mathbb{E}_t \left[\pi_{t+1}^{\varepsilon_w - 1} \varphi_{w,t+1} \right] \tag{5}$$

According to Law of Large Numbers (LLN), aggregate level of wage W_t can be written as $W_t = \left(\int_0^1 W_{h,t}^{1-\varepsilon_w} dh \right)^{\frac{1}{1-\varepsilon_w}} = \left[\theta_w \left(\pi_{t-1}^{\varrho_w} W_{t-1} \right)^{1-\varepsilon_w} + (1-\theta_w) \left(W_{h,t}^* \right)^{1-\varepsilon_w} \right]^{\frac{1}{1-\varepsilon_w}}$ from which we can solve the optimal real wage $w_{h,t}^*$ by writing this equation into real term.

$$1 = \theta_w \left(\frac{\pi_{t-1}^{\varrho_w}}{\pi_t} \right)^{1-\varepsilon_w} \left(\frac{w_{t-1}}{w_t} \right)^{1-\varepsilon_w} + (1-\theta_w) \left(\frac{w_{h,t}^*}{w_t} \right)^{1-\varepsilon_w} \tag{6}$$

Define nominal wage dispersion as $s_{w,t} = \int_0^1 \left(\frac{W_{h,t}}{W_t} \right)^{-\varepsilon_w} dh$ and write $s_{w,t}$ into recursive formulation.

$$s_{w,t} = (1-\theta_w) \left(\frac{w_{h,t}^*}{w_t} \right)^{-\varepsilon_w} + \theta_w \left(\frac{w_{t-1} \pi_{t-1}^{\varrho_w}}{w_t \pi_t} \right)^{-\varepsilon_w} s_{w,t-1} \tag{7}$$

2.2 Intermediate-good Producer and Final-good Producer

Final-good producers aggregate intermediate-goods $Y_{i,t}$ to final-good Y_t with CES production technology $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ where $\varepsilon_p > 1$ indicates the elasticity of substitution among differentiated intermediate-goods. Final-good producer maximizes the profit $P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$ subject to this production technology. The profit maximization of final-good producer is $\max_{\{Y_{i,t}\}} \left(P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \right)$ from which we can solve the optimal input of each intermediate-good $Y_{i,t}$ as $Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon_p} Y_t$. Substituting this demand equation of $Y_{i,t}$ into production function of Y_t leads to definition of general price level $P_t = \left(\int_0^1 P_{i,t}^{1-\varepsilon_p} di \right)^{\frac{1}{1-\varepsilon_p}}$. The production technology of intermediate-good producer is $Y_{i,t} = A_t N_{i,t}^{1-\alpha}$ where A_t is total factor productivity that is identical for all intermediate-good producers. Real total cost $TC_{i,t}^r$ and real marginal cost $MC_{i,t}^r$ are given as $TC_{i,t}^r = \frac{W_t}{P_t} N_{i,t} = \frac{W_t}{P_t} \left(\frac{Y_{i,t}}{A_t} \right)^{\frac{1}{1-\alpha}}$ and $MC_{i,t}^r = \frac{\partial TC_{i,t}^r}{\partial Y_{i,t}} = \frac{A_t^{\frac{1}{\alpha-1}}}{1-\alpha} \frac{W_t}{P_t} Y_{i,t}^{\frac{\alpha}{1-\alpha}}$ respectively. Substituting demand equation of $Y_{i,t}$ into $MC_{i,t}^r$ leads to $MC_{i,t}^r = \frac{A_t^{\frac{1}{\alpha-1}}}{1-\alpha} \frac{W_t}{P_t} \left[\left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon_p} Y_t \right]^{\frac{\alpha}{1-\alpha}}$ which means the different intermediate-good producers charging different prices would produce different levels of output and have different marginal costs if $\alpha \neq 0$. If $\alpha = 0$, the model economy has the property of constant return to scale and all intermediate-good producers have the same real marginal cost. The pricing mechanism of intermediate-good producer is based on Calvo (1983). In each period, there is a fixed probability $1 - \theta_p$ that an intermediate-good producer can optimize its nominal price and set it to optimal level $P_{i,t}^*$. With probability θ_p, instead, the intermediate-good producer that can't optimize its price setting just indexes its price to the precious period gross inflation rate like $P_{i,t} = \pi_{t-1}^{\varrho_p} P_{i,t-1}$ where $\varrho_p \in [0,1]$ indicates the degree of backward-looking indexation. Intermediate-good producer optimizes its price to maximize the expected sum of discounted profit in real term as $\max_{\{P_{i,t}^*\}} \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \theta_p^j \left(\frac{P_{i,t+j}}{P_{t+j}} Y_{i,t+j} - TC_{i,t+j}^r \right)$ where $\mathcal{D}_{t,t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_t}$ is the stochastic discount factor and λ_{t+j} denotes the marginal utility of consumption $C_{t+j}^{-\sigma}$. The first-order condition of price optimization is

$$\left(p_{i,t}^* \right)^{1+\frac{\alpha \varepsilon_p}{1-\alpha}} = \frac{\varepsilon_p}{(1-\alpha)(\varepsilon_p - 1)} \frac{\psi_{p,t}}{\varphi_{p,t}} \tag{8}$$

where we rewrite $p_{i,t}^*$ as $p_{i,t}^* = P_{i,t}^*/P_t$ and define two auxiliary variables $\psi_{p,t}$ and $\varphi_{p,t}$ to simplify the first-order condition. $\psi_{p,t}$ and $\varphi_{p,t}$ can be rewritten into recursive form.

$$\psi_{p,t} = w_t Y_t^{\frac{1}{1-\alpha}-\sigma} A_t^{-\frac{1}{1-\alpha}} + \beta\theta_p \pi_t^{-\frac{\varrho_p \varepsilon_p}{1-\alpha}} \mathbb{E}_t \left[\pi_{t+1}^{\frac{\varepsilon_p}{1-\alpha}} \psi_{t+1} \right] \tag{9}$$

$$\varphi_{p,t} = Y_t^{1-\sigma} + \beta\theta_p \pi_t^{\varrho_p(1-\varepsilon_p)} \mathbb{E}_t \left[\pi_{t+1}^{\varepsilon_p - 1} \varphi_{p,t+1} \right] \tag{10}$$

According to Law of Large Numbers (LLN), $1-\theta_p$ proportion of all intermediate-good producers can optimize price and θ_p proportion of all intermediate-good producers just index the price to last period gross inflation rate. The aggregate price level P_t evolves as following equation.

$$P_t = \left(\int_0^1 P_{i,t}^{1-\varepsilon_p} di \right)^{\frac{1}{1-\varepsilon_p}} = \left[\theta_p \left(\pi_{t-1}^{\varrho_p} P_{t-1} \right)^{1-\varepsilon_p} + (1-\theta_p) \left(P_{i,t}^* \right)^{1-\varepsilon_p} \right]^{\frac{1}{1-\varepsilon_p}}$$

Rewrite this equation by dividing P_t and solve the optimal relative price $p_{i,t}^*$.

$$p_{i,t}^* = \left[\frac{1-\theta_p \pi_{t-1}^{\varrho_p(1-\varepsilon_p)} \pi_t^{\varepsilon_p - 1}}{1-\theta_p} \right]^{\frac{1}{1-\varepsilon_p}} \tag{11}$$

Price dispersion $s_{p,t}$ can be written into recursive form under the assumption of the Calvo (1983).

$$s_{p,t} = (1-\theta_p) \left(p_{i,t}^* \right)^{-\frac{\varepsilon_p}{1-\alpha}} + \theta_p \pi_t^{\frac{\varepsilon_p}{1-\alpha}} \pi_{t-1}^{-\frac{\varrho_p \varepsilon_p}{1-\alpha}} s_{p,t-1} \tag{12}$$

2.3 Aggregate Output and Potential Output

Denote $N_{(h,i),t}$ as the type of labor h packed by labor packer that provides homogenous labor to intermediate-good producer i. Aggregate labor demand N_t is defined as $N_t = \int_0^1 \left(\int_0^1 N_{(h,i),t} dh \right) di$.

$$N_t = \int_0^1 \left[N_{i,t} \left(\int_0^1 \frac{N_{(h,i),t}}{N_{i,t}} dh \right) \right] di = s_{w,t} s_{p,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \tag{13}$$

We use f to denote the variables under flexible price and wage. Y_t^f can be solved by setting $\theta_p = \theta_w = 0$ in equilibrium conditions. When price and wage are both flexible, every household and intermediate-good producer will make identical optimal decisions for price and wage setting, so price dispersion and wage dispersion will disappear, which means $s_{w,t}^f = s_{p,t}^f = 1$.

$$p_{i,t}^{f*} = 1 = \frac{(1-\alpha)(\varepsilon_p - 1)}{\varepsilon_p} \frac{\psi_{p,t}^f}{\varphi_{p,t}^f}, \; \left(w_{h,t}^{f*} \right)^{1+\varepsilon_w \varphi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\psi_{w,t}^f}{\varphi_{w,t}^f}$$

$$w_{h,t}^{f*} = w_t^f, \; Y_t^f = C_t^f, \; N_t^f = \left(\frac{Y_t^f}{A_t} \right)^{\frac{1}{1-\alpha}}$$

Combining these equilibrium conditions leads to Y_t^f.

$$Y_t^f = \left[\frac{(1-\alpha)(\varepsilon_p - 1)(\varepsilon_w - 1)}{\chi \varepsilon_p \varepsilon_w} \right]^{\frac{1-\alpha}{\varphi+\alpha+\sigma(1-\alpha)}} \left(\frac{A_t^{\frac{1+\varphi}{1-\alpha}}}{e^{\varsigma_t}} \right)^{\frac{1-\alpha}{\varphi+\alpha+\sigma(1-\alpha)}} \tag{14}$$

From this result, we can find that potential output Y_t^f is totally decided by real factors and not affected by target inflation rate.

2.4 Monetary Policy

The central bank follows a standard Taylor rule with weight φ_{π_p} on deviation of inflation from its target π, weight φ_{π_w} on wage inflation and weight φ_y on output deviation from potential output level Y_t^f. v_t is a monetary policy shock.

$$\left(\frac{1+i_t}{1+i}\right) = \left(\frac{\pi_t}{\pi}\right)^{\varphi_{\pi_p}} \left(\frac{w_t\pi_t}{w_{t-1}\pi}\right)^{\varphi_{\pi_w}} \left(\frac{Y_t}{Y_t^f}\right)^{\varphi_y} e^{v_t} \tag{15}$$

Output gap can be defined as the difference between output under decentralized equilibrium with sticky price and sticky wage and potential output under decentralized equilibrium with flexible price and flexible wage.

$$x_t = \frac{Y_t}{Y_t^f} \tag{16}$$

2.5 Aggregate Welfare Function

Aggregate welfare $\mathbb{W}_t$ can be defined as the aggregation of each individual household's welfare $\mathbb{W}_{h,t}$ as $\mathbb{W}_t = \int_0^1 \mathbb{W}_{h,t} dh$, where $\mathbb{W}_{h,t}$ is given as $\mathbb{W}_{h,t} = \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{h,t}^{1-\sigma}}{1-\sigma} - \chi e^{\varsigma_t} \frac{N_{h,t}^{1+\varphi}}{1+\varphi}\right)$.

$$\mathbb{W}_t = \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi e^{\varsigma_t}}{1+\varphi}\int_0^1 N_{h,t}^{1+\varphi} dh\right) = \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi e^{\varsigma_t}}{1+\varphi}\int_0^1 \left(\left(\frac{W_{h,t}}{W_t}\right)^{-\varepsilon_w} N_t\right)^{1+\varphi} dh\right]$$

Rewrite $\mathbb{W}_t$ in recursive form and define $\int_0^1 \left(\frac{W_{h,t}}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dh$ as $\Delta_{w,t}$.

$$\begin{aligned}\mathbb{W}_t &= \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi e^{\varsigma_t} N_t^{1+\varphi}}{1+\varphi}\int_0^1 \left(\frac{W_{h,t}}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dh + \beta \mathbb{E}_t \mathbb{W}_{t+1} \\ &= \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi e^{\varsigma_t} N_t^{1+\varphi}}{1+\varphi}\Delta_{w,t} + \beta \mathbb{E}_t \mathbb{W}_{t+1}\end{aligned} \tag{17}$$

Similar to the wage dispersion $s_{w,t}$, we rewrite $\Delta_{w,t}$ in recursive formulation.

$$\Delta_{w,t} = (1-\theta_w)\left(\frac{w_{h,t}^*}{w_t}\right)^{-\varepsilon_w(1+\varphi)} + \theta_w \left(\frac{w_{t-1}\pi_{t-1}^{\varrho_w}}{w_t\pi_t}\right)^{-\varepsilon_w(1+\varphi)} \Delta_{w,t-1} \tag{18}$$

We have 18 equilibrium conditions from equation (1) to equation (18) with 18 endogenous variables

$$\left\{C_t, Y_t, i_t, \pi_t, w_t, N_t, p_{i,t}^*, w_{i,t}^*, \psi_{p,t}, \varphi_{p,t}, \psi_{w,t}.\varphi_{w,t}, s_{p,t}, s_{w,t}, Y_t^f, x_t, \Delta_{w,t}, \mathbb{W}_t\right\}$$

and 3 shock process variables $\{A_t, \varsigma_t, v_t\}$.

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t},\ \varepsilon_{A,t} \sim \mathcal{N}(0, \sigma_A^2)$$

$$\varsigma_t = \rho_\varsigma \varsigma_{t-1} + \varepsilon_{\varsigma,t},\ \varepsilon_{\varsigma,t} \sim \mathcal{N}(0, \sigma_\varsigma^2)$$

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t},\ \varepsilon_{v,t} \sim \mathcal{N}(0, \sigma_v^2)$$

Linear approximating all nonlinear equilibrium conditions around the steady state associated with target inflation rate leads to linear form of generalized New Keynesian model[4]. By setting zero annual target inflation rate as $\pi = 1$, after some algebraic manipulations, we can reduce our model to a more compact form[5].

[4]Please refer to Appendix.

[5]Please refer to the New Keynesian model with price stickiness and wage stickiness in Chapter 6 in Galí (2015)

3 Analysis of Deterministic Steady State

Following standard literature of New Keynesian model such as Woodford (2003), Galí (2015), Walsh (2017) and Miao (2014), we calibrate the structural parameters given in Table 2 as benchmark case. $\sigma = 1$ means the representative household has the log utility. $\alpha = 0$ means the model economy has the property of constant return to scale. $\varepsilon_p = 6$ means the intermediate-good producer add 20% markup rate[6] on marginal cost. Similarly, household adds 28% markup rate[7] on marginal rate of substitution. θ_p and θ_w are both calibrated to be 0.75, which means the price and wage can be reset every 4 quarters. ϱ_p and ϱ_w are calibrated to be zero, implying that there doesn't exist backward-looking indexation in price setting and wage setting. Once price or wage has been optimized to its optimal value, it will not change until next change of re-optimization. We calibrate the aggregate output and potential aggregate output to be 1 under zero target inflation rate. We can calibrate χ with reverse engineering[8] from the steady state of potential aggregate output Y^f. According to Galí (2015), $\varphi_{\pi_p} + \varphi_{\pi_w} > 1$ must hold to ensure the existence of a unique stationary equilibrium. Without loss of generality, we set $\varphi_{\pi_p} = \varphi_{\pi_w} = 1$, which means that the nominal interest rate i_t responses to price inflation and wage inflation with same weight.

σ	β	φ	α	ε_w	ϱ_w	θ_w	ε_p	ϱ_p	θ_p	φ_{π_p}	φ_{π_w}	φ_y	π	χ	ρ_A	ρ_ς	ρ_v
1	0.99	1	0	4.5	0	0.75	6	0	0.75	1	1	0.125	1	5.8333	0.9	0.9	0.5

Table 2: Benchmark Calibration of Structural Parameters

For the analysis of deterministic steady state, all exogenous shock processes disappear, we can solve the steady state of endogenous variables given in Table 3.

C	Y	i	π	w	N	p_i^*	w_h^*	ψ_p	φ_p	ψ_w	φ_w	s_p	s_w	Y^f	x	Δ_w	$\mathbb{W}$
0.3333	0.3333	0.0101	1	0.8333	0.3333	1	0.8333	3.2362	3.8835	0.4878	1.7096	1	1	1	1	1	−142.2686

Table 3: Benchmark Steady State at Zero Target Inflation Rate

Figure 1 shows the steady state of main model variables given positive annual target inflation rate[9] from 0% to 8%. At the steady state associated with zero target inflation rate, each variable attains its benchmark value given in Table 3. With the increasing of target inflation rate π, optimal relative price p_i^* increases from 1 to 1.075, about 7.5% increasing. Optimal real wage w_h^* shows different behavior compared with optimal price p_i^*. Optimal real wage w_h^* decreases slightly from 0.833 to 0.818, about 1.86% decreasing. The increasing of target inflation rate doesn't have much impact on the optimal decision of household and intermediate-good producer. Price dispersion s_p also increases with the increasing of target inflation rate, but very slightly, 8% annual target inflation rate corresponds with 2.5% increasing in price dispersion compared with zero target inflation rate. The situation for wage dispersion s_w is quite different. Wage dispersion s_w increases from 1 to 1.37 with the increasing of annual target inflation rate from 0% to 8%. 37% increasing of wage dispersion is the main distortion that erodes the steady state aggregate output Y and aggregate labor input N. Given the benchmark calibration of other structural parameters and the 3% annual target inflation rate, Y is 0.294 and N is 0.328, which means 11.9% loss and 1.68% loss compared with their corresponding steady states associated with zero target inflation rate.

In New Keynesian model with price stickiness and wage stickiness, there are two type, four sources of distortions. First, monopolistic competition in intermediate-good market and labor market can generate low employment and low output. This first type of two sources of distortions can be corrected by providing employment subsidy to firms. This subsidy can be financed by non-distortionary lump-sum taxes levied from households. The second type of distortion comes from the staggered price setting

[6] $p_i^* = \frac{\varepsilon_p}{\varepsilon_p - 1} MC^r$

[7] $w_h^* = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\chi N^\varphi}{C^{-\sigma}}$

[8] $\chi = \frac{(\varepsilon_p - 1)(\varepsilon_w - 1)}{\varepsilon_p \varepsilon_w}$

[9] For example, 4% annual target inflation rate means that in the model, steady state of quarterly inflation target rate π is given as $\pi = (1.04)^{\frac{1}{4}} \approx 1\%$.

and staggered wage setting. This second type of two sources of distortions can't be corrected simultaneously because there exists only one policy instrument in New Keynesian model. We summarize the structural parameters that dominate these distortions in Table 1. We also check how the degree of backward-looking indexation affect the model[10]. We change the values of these key structural parameters to check how these distortions affect the steady state given positive target inflation rate.

3.1 Steady State of Price Dispersion

Given the non-zero target inflation rate π and the assumption that the model economy has the property of constant scale to scale ($\alpha = 0$), price dispersion is given as $s_p = \frac{1-\theta_p}{1-\theta_p \pi^{\varepsilon_p(1-\varrho_p)}} \left[\frac{1-\theta_p \pi^{(\varepsilon_p - 1)(1-\varrho_p)}}{1-\theta_p}\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$ which means there are 3 structural parameters, ε_p, θ_p and ϱ_p, can affect the price dispersion s_p at steady state. ε_p represents the real distortion of monopolistic competition in intermediate-good market. θ_p represents the nominal distortion of price stickiness in staggered price setting. ϱ_p represents the backward-looking indexation of non-optimizing firms in the Calvo (1983) framework of staggered price setting. Figure 2 shows how the increasing of these 3 structural parameters change the price dispersion s_p at the steady state. For example, the panel (a) of Figure 2 shows that, given the same elasticity of substitution ε_p, price dispersion s_p increases with the increasing of target inflation rate π. Given the same target inflation rate π, higher elasticity of substitution ε_p leads to higher price dispersion s_p. Higher elasticity of substitution means the less monopolistic power of firm. The intuition of this result is that the firm with high monopolistic power is not willing to change its pricing frequently, hence less price adjustment and less price dispersion. Similar pattern can be confirmed in the panel (c) of Figure 2. Given the same target inflation rate π, higher price stickiness θ_p causes more lack of synchronization in price adjustment, which means high price dispersion. Also, given the same θ_p, higher target inflation rate leads to larger price dispersion because of the forward-looking nature of firm. Finally, non-optimizing intermediate-good firms index their prices to the last period gross inflation rate. $\varrho_p = 1$ means total indexation and $\varrho_p = 0$ means no indexation. Given the same positive target inflation rate π, less indexation which means a smaller ϱ_p leads to larger price dispersion. Price dispersion s_p changes with the change in key parameters, but still in very small range. For example, given $\theta_p = 0.8$ and 8% target annual inflation rate, $s_p = 1.053$, only 5.3% increasing from the benchmark case ($s_p = 1$).

3.2 Steady State of Wage Dispersion

Similar to the analysis of price dispersion, wage dispersion is given as $s_w = \frac{1-\theta_w}{1-\theta_w \pi^{\varepsilon_w(1-\varrho_w)}}$ which means that there are 3 structural parameters, ε_w, θ_w and ϱ_w, can affect the wage dispersion at steady state. ε_w represents the real distortion of monopolistic competition in labor market. θ_w represents the nominal distortion of wage stickiness. ϱ_w represents the backward-looking indexation of non-optimizing households in the Calvo (1983) framework of staggered wage setting. Figure 2 shows the change of wage dispersion s_w. The change in key structural parameters leads to significant change of wage dispersion, given the same target inflation rate. At 2% annual target inflation rate, s_w is 1.072 given $\varepsilon_w = 4.5$ and s_w is 1.118 given $\varepsilon_w = 7$. The increasing of ε_w from 4.5 to 7 results in almost 4.3% increasing of s_w, which means the less monopolistic power of household, the larger wage dispersion. The change of θ_w and ϱ_w also results in significant change of wage dispersion, which can be confirmed from Figure 2. For example, given 2% annual target inflation rate, $s_w = 1.031$ with $\theta_w = 0.5$ and $s_w = 1.137$ with $\theta_w = 0.8$ mean almost 10.3% increasing of wage dispersion s_w. Higher wage stickiness means larger wage dispersion. Note that in the panel (a) of Figure 2, the range of price dispersion s_p is from 1 to 1.033, but in the panel (b) of Figure 2, the range of wage dispersion s_w is from 1 to 1.762. These findings imply that the structural parameters of labor market affect the wage dispersion in a more aggressive way compared to the way that the structural parameters of intermediate-good market affect the price dispersion.

3.3 Steady State of Aggregate Output and Aggregate Welfare

Following the way in previous section that we used to check the relationship between structural parameters and price (wage) dispersion given different target inflation rate, we check the steady state of

[10]It is generally known that the backward-looking indexation is an ad-hoc assumption to generate the inertia of inflation, but this assumption has less empirical evidence in microeconomic level of household and wage behavior. In this paper, we don't have much discussion about this assumption and just show the numerical results.

aggregate output given different combinations of target inflation rate and structural parameters. By combing the equilibrium conditions at steady state, we can solve the aggregate output Y as the function of A and structural parameters[11].

$$Y = A^{\frac{1+\varphi}{\varphi+\alpha+\sigma(1-\alpha)}} \left[\begin{array}{l} \frac{(1-\alpha)\left(\varepsilon_p-1\right)\left(\varepsilon_w-1\right)}{\chi\varepsilon_p\varepsilon_w} \times \left(\frac{1-\theta_p\pi^{\left(\varepsilon_p-1\right)\left(1-\varrho_p\right)}}{1-\theta_p}\right)^{\frac{1-\alpha+\varepsilon_p(\alpha+\varphi)}{(1-\varepsilon_p)(1-\alpha)}} \\ \times \left[\left(\frac{1-\theta_p\pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}}{1-\theta_p}\right)\left(\frac{1-\theta_w\pi^{\varepsilon_w(1-\varrho_w)}}{1-\theta_w}\right)\right]^{\varphi} \\ \times \left(\frac{1-\beta\theta_p\pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}}{1-\beta\theta_p\pi^{\left(\varepsilon_p-1\right)\left(1-\varrho_p\right)}}\right) \times \left(\frac{1-\beta\theta_w\pi^{\varepsilon_w(1+\varphi)(1-\varrho_w)}}{1-\beta\theta_w\pi^{(\varepsilon_w-1)(1-\varrho_w)}}\right) \end{array} \right]^{\frac{1-\alpha}{\varphi+\alpha+\sigma(1-\alpha)}}$$

Given the benchmark calibration of structural parameters $(\alpha, \sigma, \varphi, \chi)$, we change the value of other structural parameters to investigate how monopolistic competition and nominal stickiness in intermediate-good market and labor market affect the output at steady state under different target inflation rate. Aggregate welfare can be checked in a similar way.

$$\Delta_w = \frac{1-\theta_w}{1-\theta_w\pi^{\varepsilon_w(1-\varrho_w)(1+\varphi)}}, \quad \mathbb{W} = \frac{1}{1-\beta}\left(\frac{C^{1-\sigma}}{1-\sigma} - \Delta_w\frac{\chi N^{1+\varphi}}{1+\varphi}\right)$$

At steady state, only if target inflation rate is zero ($\pi = 1$) or wage stickiness disappears ($\theta_w = 0$), Δ_w will be unit. If either of these two conditions doesn't hold, Δ_w will be larger than unit and generate the loss of aggregate welfare[12]. Figure 3 and Figure 4 show the sensitivity of aggregate output Y and aggregate welfare $\mathbb{W}$ to the change of key structural parameters given each target inflation rate. We find that steady state aggregate output Y is more sensitive to the change of structural parameters that dominates the imperfection of labor market, especially at the high target inflation rate. Also, given zero target inflation rate, when ε_w increases from 4 to 7, Y has 6.9% increasing from 0.3273 to 0.3499. $\mathbb{W}$ also increases from -142.9 to -140.7. But given 2% target inflation rate, Y doesn't have change much but $\mathbb{W}$ decreases from -153.8 to -161.9.

Flexible Price and Flexible Wage	$\theta_p = 0$ and $\theta_w = 0$
Sticky Price and Sticky Wage	$\theta_p = 0.75$ and $\theta_w = 0.75$
Flexible Price and Sticky Wage	$\theta_p = 0$ and $\theta_w = 0.75$
Sticky Price and Flexible Wage	$\theta_p = 0.75$ and $\theta_w = 0$

Table 4: Model Variation

By setting the value of θ_p and θ_w, we can turn-off/-on the stickiness in the model as shown in Table 4. Figure 5 shows that how the target inflation rate affects the aggregate output Y and aggregate welfare W given other parameters calibrated as benchmark case. Given $\theta_p = \theta_w = 0$, the model economy has both flexible wage and flexible price, so the target inflation rate doesn't have real effect on output and welfare. By comparing other 3 variations of model economy, we can find the distortion caused by wage stickiness is more severe than the counterpart caused by price stickiness, especially given a high target inflation rate.

4 Analysis of Dynamic Impulse Response

As shown in Appendix, the coefficients of linearized model contain target inflation rate π. For example, given zero target inflation rate ($\pi = 1$), equation (5) of price dispersion $\widehat{s}_{p,t}$ will reduce to $\widehat{s}_{p,t} = \theta_p\widehat{s}_{p,t-1}$ and equation (10) will reduce to $\widehat{s}_{w,t} = \theta_w\widehat{s}_{w,t-1}$. This means $\widehat{s}_{p,t} = \widehat{s}_{w,t} = 0$ and the price dispersion and wage dispersion will not affect the model dynamics at least at first-order approximation. But if target

[11]The steady state of technology A is normalized to be $A = 1$.

[12]Note that in the computation of steady state welfare, $\sigma = 1$ reduces $\frac{C^{1-\sigma}}{1-\sigma}$ to $\log C$.

inflation rate is positive, $\widehat{s}_{p,t}$ and $\widehat{s}_{w,t}$ will directly affect the model dynamics. Considering this point, we run a sensitivity analysis of structural parameters as showed in Table 5.

	Benchmark	High ε_p	High ε_w	Low θ_p	Low θ_w	High ϱ_p	High ϱ_w
ε_p	6	9	6	6	6	6	6
ε_w	4.5	4.5	7.5	4.5	4.5	4.5	4.5
θ_p	0.75	0.75	0.75	0.5	0.75	0.75	0.75
θ_w	0.75	0.75	0.75	0.75	0.5	0.75	0.75
ϱ_p	0	0	0	0	0	0.9	0
ϱ_w	0	0	0	0	0	0	0.9

Table 5: Structural Parameter Sensitivity

	Productivity Shock $\varepsilon_{A,t}$	Monetary Policy Shock $\varepsilon_{v,t}$	Labor Supply Shock $\varepsilon_{\varsigma,t}$
ε_p and ε_w	Figure 6	Figure 9	Figure 12
θ_p and θ_w	Figure 7	Figure 10	Figure 13
ϱ_p and ϱ_w	Figure 8	Figure 11	Figure 14

Table 6: Sensitivity Analysis of Structural Parameters in Dynamic Impulse Response

We run two sets of sensitivity analysis, given 0% target inflation rate and 2% target inflation rate and compare the dynamic response of 3 kinds of exogenous shocks. All results are presented from Figure 6 to Figure 14. We pick up some interesting findings from these results.

4.1 Productivity Shock $\varepsilon_{A,t}$

From Figure 6, we can find that high ε_w can significantly strength the response of output $\widehat{Y}_t$ and the response of inflation $\widehat{\pi}_t$ at 2% target inflation rate. From Figure 7, we can find that given 0% target inflation rate, the change of θ_p doesn't result in significant change in the dynamic impulse response of aggregate output $\widehat{Y}_t$ to productivity shock $\varepsilon_{A,t}$, but low wage stickiness ($\theta_w = 0.5$) can weaken the response. The situation is different given 2% target inflation rate. Low price stickiness ($\theta_p = 0.5$) can strengthen the impulse response. Figure 8 shows the sensitivity analysis of ϱ_p and ϱ_w. The impulse response functions of $\widehat{Y}_t$ and $\widehat{\pi}_t$ are less sensitive to the change of target inflation rate. Also, given same target inflation rate, high wage indexation ($\varrho_w = 0.9$) can generate the strongest impulse responses compared to the case of benchmark calibration.

4.2 Monetary Policy Shock $\varepsilon_{v,t}$

The panel (a) and panel (b) of Figure 11 are worth noting. High wage indexation ($\varrho_w = 0.9$) neutralizes the effect of monetary policy shock given 2% target inflation rate. This finding may imply a policy implication that the flexible and rapid adjustment of wage contract is preferable in an economy with non-zero target inflation rate. The changes incurred by other four structural parameters are less significant, which can be confirmed from Figure 9. Comparing the panel (c) and panel (d) of Figure 10, we can find that given the same θ_w, the change of θ_p can generate obvious change of impulse response in the case of 2% target inflation rate, but no obvious change in the case of 0% target inflation rate.

4.3 Labor Supply Shock $\varepsilon_{\varsigma,t}$

Comparing the panel (a) and panel (b) of Figure 12, we can find that high ε_w can strength the response of output $\widehat{Y}_t$ in the case of 2% target inflation rate. But in the case of 0% target inflation rate, the 3 sets of impulse response function are almost same given 3 different sets of calibration. The panel (b) of Figure 14 is also worth noting. High wage indexation ($\varrho_w = 0.9$) neutralizes the effect of labor supply shock, but not in the case of 0% target inflation rate.

Summarizing all these findings, we conclude that given 2% target inflation rate, impulse response functions are more sensitive to the change of structural parameters, especially to those structural parameters of labor market.

5 Conclusion and Policy Implication

Finally, we conclude our paper to summarize the main findings. Standard New Keynesian model with price stickiness generally assumes the steady state associated with zero target inflation rate. We extend the standard New Keynesian model to allow both price stickiness and wage stickiness, and allow the steady state associated with positive target inflation rate. These two modifications change the static properties of the model. Aggregate output and aggregate welfare responses more aggressively to the change in the key structural parameters that dominates the imperfection of labor market. From the sensitivity analysis of structural parameters in the context of static steady state, we can find that if we allow a non-zero target inflation rate, given the reasonable range of key structural parameters, $1\% \sim 2\%$ is the reasonable range of annual target inflation rate, which is consistent with the most of central banks. From the sensitivity analysis of structural parameters in the context of dynamic impulse response, we can find that the change of structural parameters related to labor market can generate significant changes in the responses of macroeconomic variables on exogenous shocks compared those counterparts of structural parameters related intermediate-good market. Given positive target inflation rate, high degree of monopolistic competition (low ε_w), low wage stickiness (low θ_w) and low wage indexation (low ϱ_w) are most desirable compared to other calibrations, at least in the context of this paper.

Some questions are remained for the future research. For example, we need to identify the determinacy and indeterminacy regions of the structural parameters in monetary policy rule. Optimal monetary policy should be evaluated by a rigorous welfare function which is derived by second-order approximating utility function. Previous research only treated the similar question with the generalized New Keynesian model that doesn't consider the imperfection of labor market. We will discuss this question in the future research.

References

[1] Dixit, A. K., & Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. The American Economic Review, 67(3), 297-308.

[2] Ascari, G., & Sbordone, A. M. (2014). The Macroeconomics of Trend Inflation. Journal of Economic Literature, 52(3), 679-739.

[3] Erceg, C. J., Henderson, D. W., & Levin, A. T. (2000). Optimal Monetary Policy with Staggered Wage and Price Contracts. Journal of Monetary Economics, 46(2), 281-313.

[4] Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of A Shock to Monetary Policy. Journal of Political Economy, 113(1), 1-45.

[5] Calvo, G. A. (1983). Staggered Prices in A Utility-maximizing Framework. Journal of Monetary Economics, 12(3), 383-398.

[6] Clarida, R., Gali, J., & Gertler, M. (1999). The Science of Monetary Policy: A New Keynesian Perspective. Journal of Economic Literature, 37(4), 1661-1707.

[7] Galí, J. (2015). Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and its Applications. Princeton University Press.

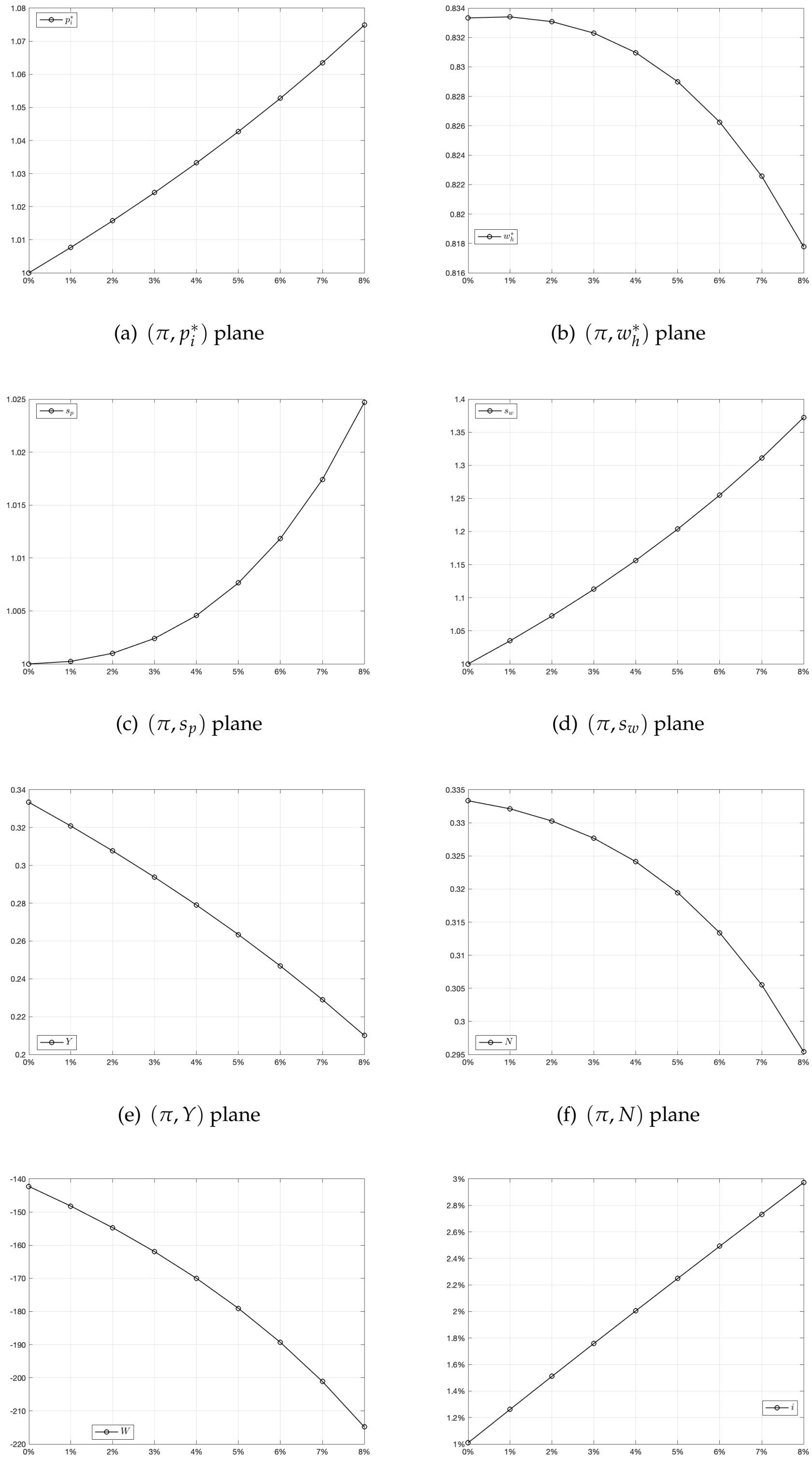
[8] Clarida, R., Galí, J., & Gertler, M. (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. The Quarterly Journal of Economics, 115(1), 147-180.

[9] Walsh, C. E. (2017). Monetary Theory and Policy. MIT Press.

[10] Miao, J. (2014). Economic Dynamics in Discrete Time. MIT Press.

[11] Smets, F., & Wouters, R. (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. Journal of the European Economic Association, 1(5), 1123-1175.

[12] Smets, F., & Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, 97(3), 586-606.



(a) (π, p_i^*) plane

(b) (π, w_h^*) plane

(c) (π, s_p) plane

(d) (π, s_w) plane

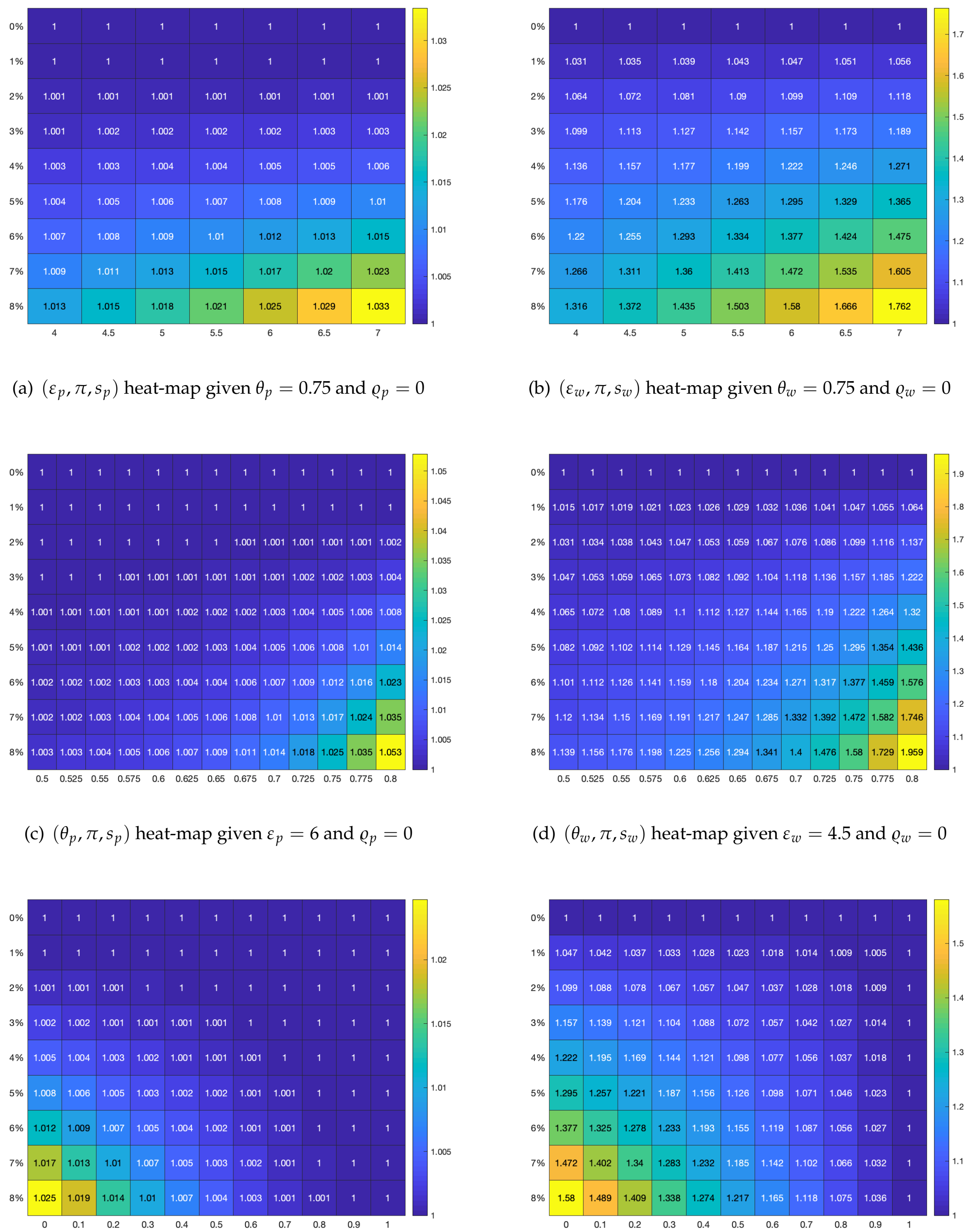
(e) (π, Y) plane

(f) (π, N) plane

(g) $(\pi, \mathbb{W})$ plane

(h) (π, i) plane

Figure 1: Steady State under Different Annual Target Inflation Rates with Benchmark Calibration



(a) $(\varepsilon_p, \pi, s_p)$ heat-map given $\theta_p = 0.75$ and $\varrho_p = 0$

(b) $(\varepsilon_w, \pi, s_w)$ heat-map given $\theta_w = 0.75$ and $\varrho_w = 0$

(c) (θ_p, π, s_p) heat-map given $\varepsilon_p = 6$ and $\varrho_p = 0$

(d) (θ_w, π, s_w) heat-map given $\varepsilon_w = 4.5$ and $\varrho_w = 0$

(e) (ϱ_p, π, s_p) heat-map given $\varepsilon_p = 6$ and $\theta_p = 0.75$

(f) (ϱ_w, π, s_w) heat-map given $\varepsilon_w = 4.5$ and $\theta_w = 0.75$

Figure 2: Steady State of Price Dispersion s_p and Wage Dispersion s_w

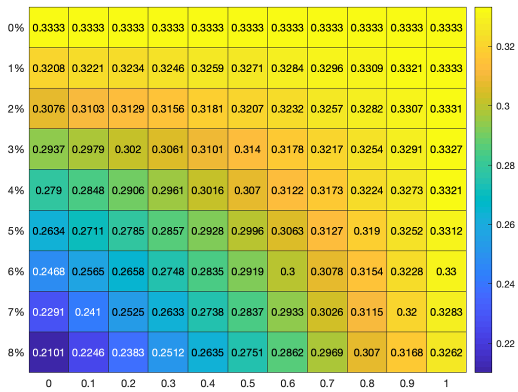
(a) (ε_p, π, Y) surface

(b) (ϵ_w, π, Y) surface

(c) (θ_p, π, Y) surface

(d) (θ_w, π, Y) surface

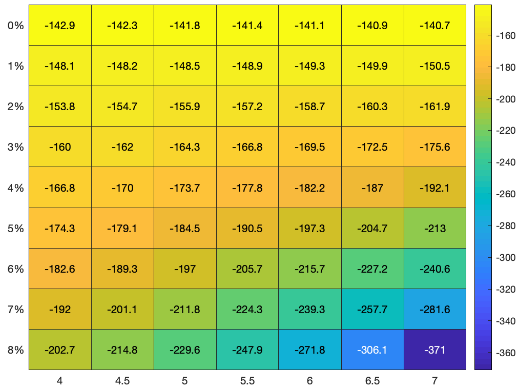
(e) (ϱ_p, π, Y) surface



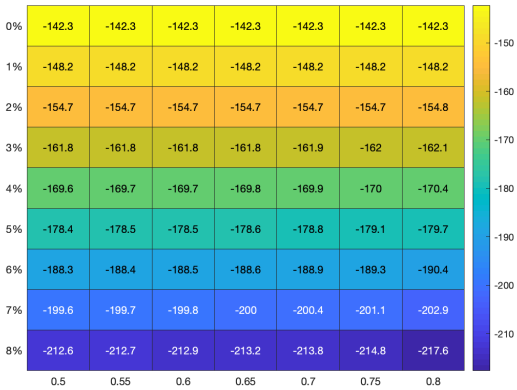
(f) (ϱ_w, π, Y) surface

Figure 3: Steady State of Aggregate Output Y

(a) (ε_p, π, W) surface

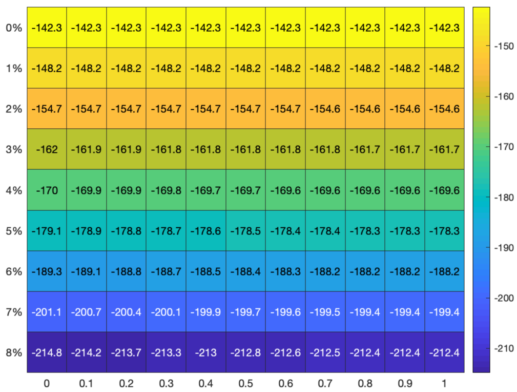


(b) $(\epsilon_w, \pi, \mathbb{W})$ surface

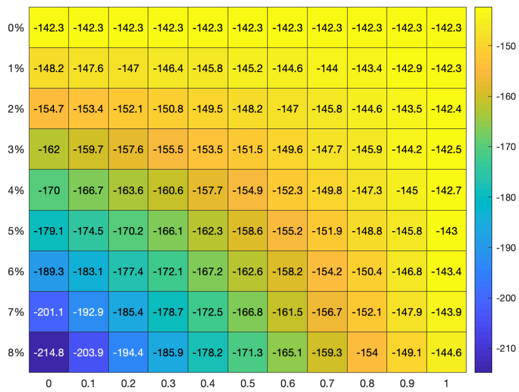


(c) $(\theta_p, \pi, \mathbb{W})$ surface

(d) $(\theta_w, \pi, \mathbb{W})$ surface

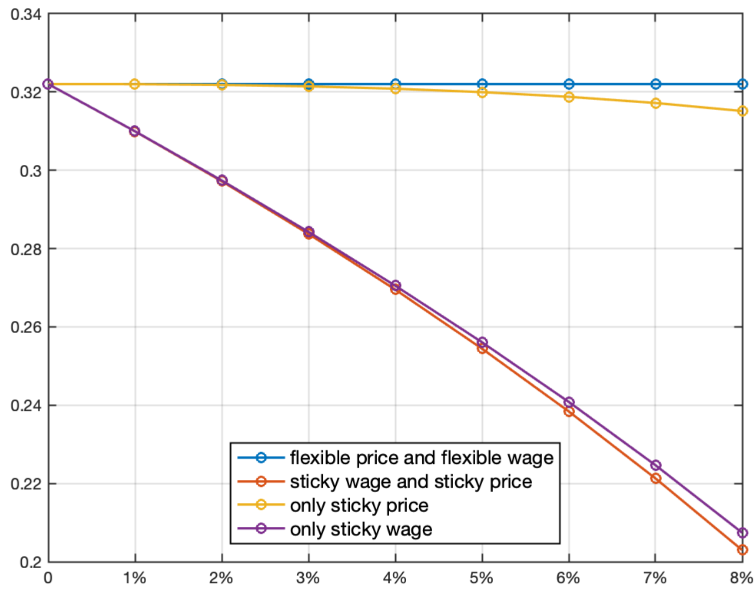


(e) $(\varrho_p, \pi, \mathbb{W})$ surface

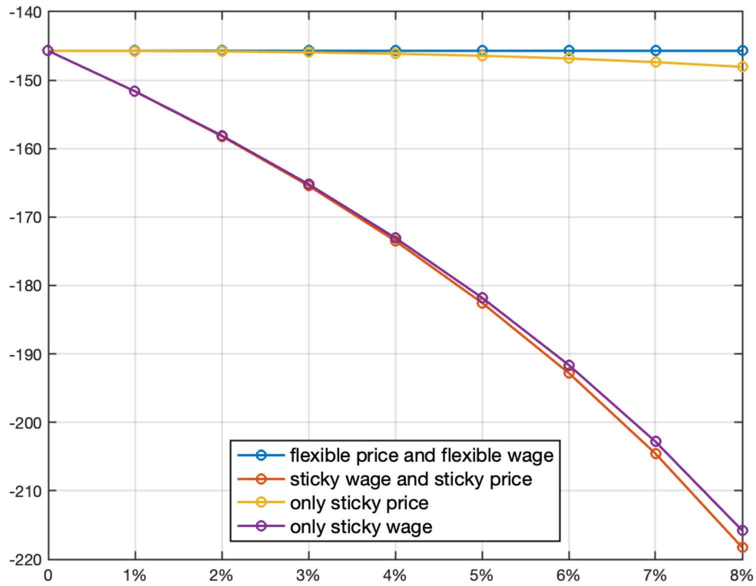


(f) $(\varrho_w, \pi, \mathbb{W})$ surface

Figure 4: Steady State of Aggregate Welfare $\mathbb{W}$

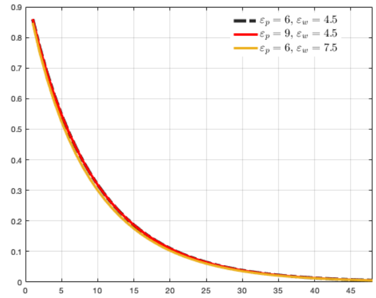


(a) (π, Y) plane

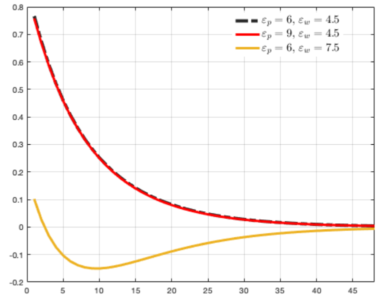


(b) $(\pi, \mathbb{W})$ plane

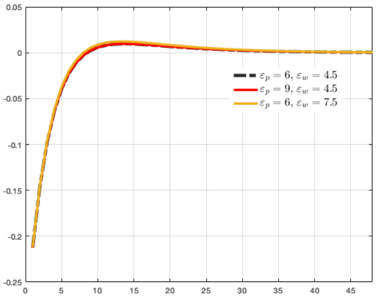
Figure 5: Steady State of Aggregate Output Y and Aggregate Welfare $\mathbb{W}$



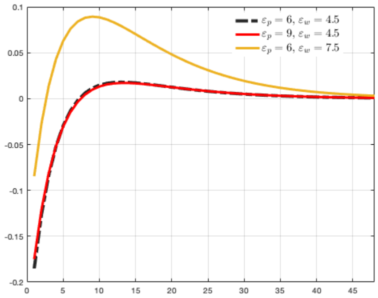
(a) $\varepsilon_{A,t} \to \widehat{Y}_t$ (0%)



(b) $\varepsilon_{A,t} \to \widehat{Y}_t$ (2%)

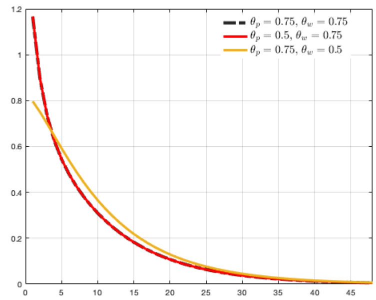


(c) $\varepsilon_{A,t} \to \widehat{\pi}_t$ (0%)

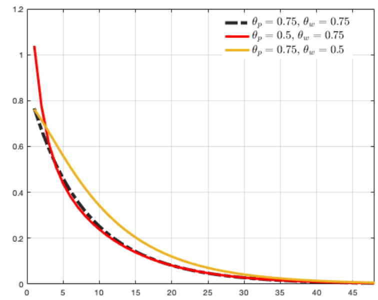


(d) $\varepsilon_{A,t} \to \widehat{\pi}_t$ (2%)

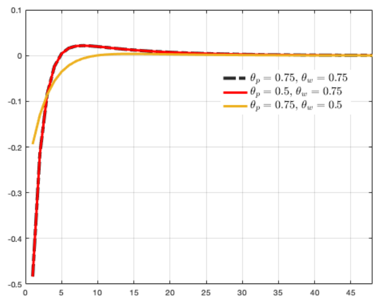
Figure 6: Sensitivity Analysis of ε_p and ε_w in Productivity Shock $\varepsilon_{A,t}$



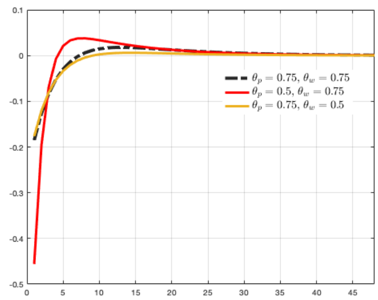
(a) $\varepsilon_{A,t} \to \widehat{Y}_t$ (0%)



(b) $\varepsilon_{A,t} \to \widehat{Y}_t$ (2%)

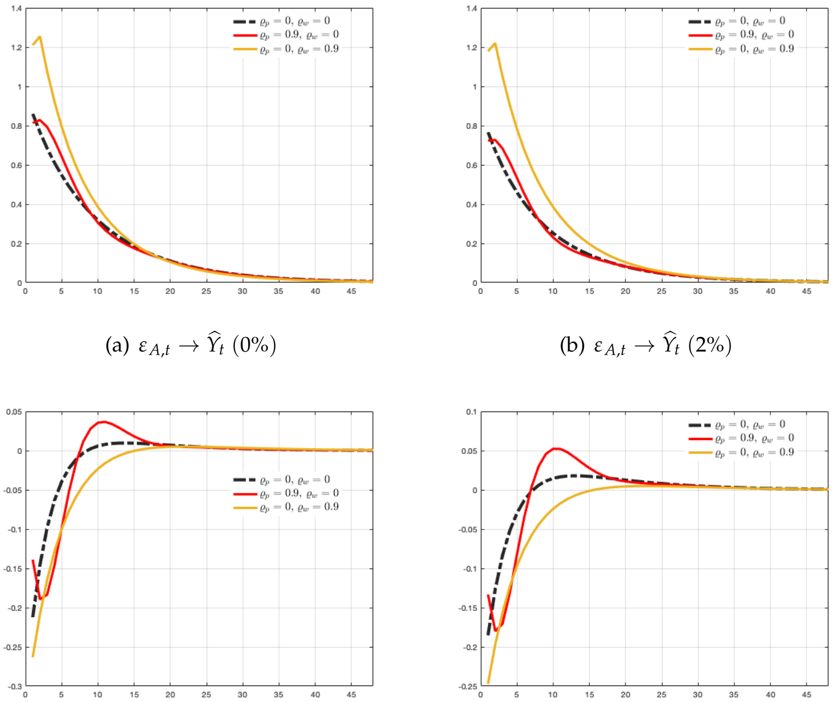


(c) $\varepsilon_{A,t} \to \widehat{\pi}_t$ (0%)



(d) $\varepsilon_{A,t} \to \widehat{\pi}_t$ (2%)

Figure 7: Sensitivity Analysis of θ_p and θ_w in Productivity Shock $\varepsilon_{A,t}$



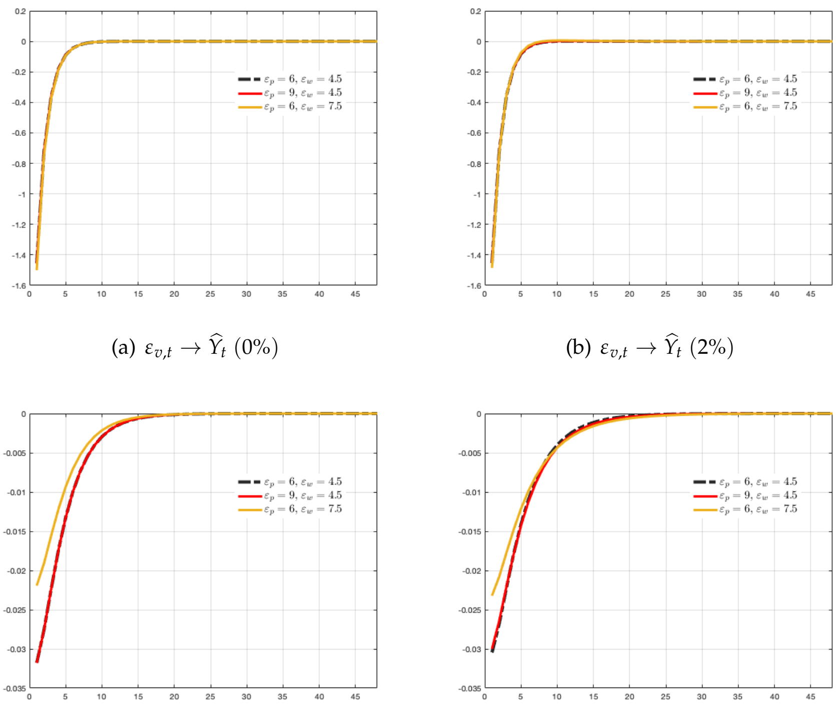
(a) $\varepsilon_{A,t} \rightarrow \widehat{Y}_t$ (0%)

(b) $\varepsilon_{A,t} \rightarrow \widehat{Y}_t$ (2%)

(c) $\varepsilon_{A,t} \rightarrow \widehat{\pi}_t$ (0%)

(d) $\varepsilon_{A,t} \rightarrow \widehat{\pi}_t$ (2%)

Figure 8: Sensitivity Analysis of ϱ_p and ϱ_w in Productivity Shock $\varepsilon_{A,t}$



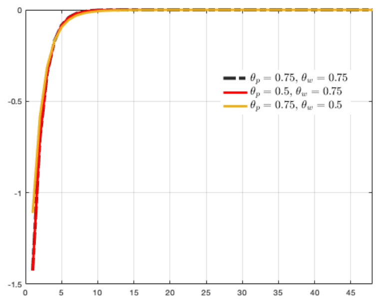
(a) $\varepsilon_{v,t} \rightarrow \widehat{Y}_t$ (0%)

(b) $\varepsilon_{v,t} \rightarrow \widehat{Y}_t$ (2%)

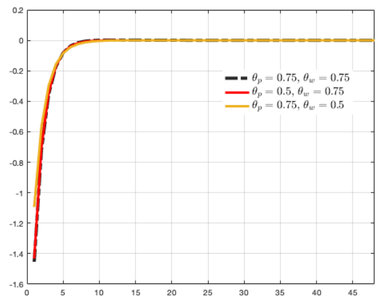
(c) $\varepsilon_{v,t} \rightarrow \widehat{\pi}_t$ (0%)

(d) $\varepsilon_{v,t} \rightarrow \widehat{\pi}_t$ (2%)

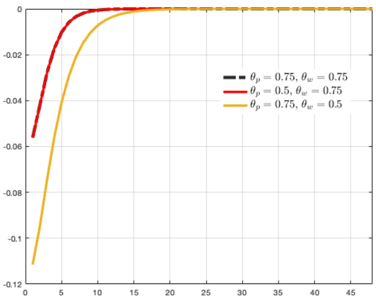
Figure 9: Sensitivity Analysis of ε_p and ε_w in Monetary Policy Shock $\varepsilon_{v,t}$



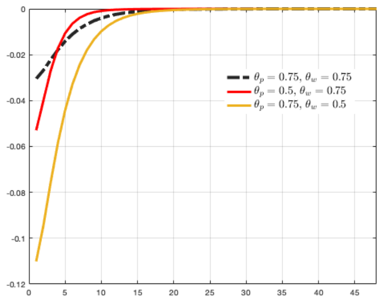
(a) $\varepsilon_{v,t} \rightarrow \widehat{Y}_t$ (0%)



(b) $\varepsilon_{v,t} \rightarrow \widehat{Y}_t$ (2%)

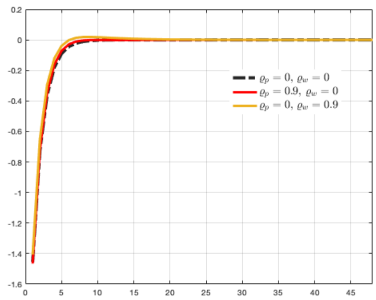


(c) $\varepsilon_{v,t} \rightarrow \widehat{\pi}_t$ (0%)

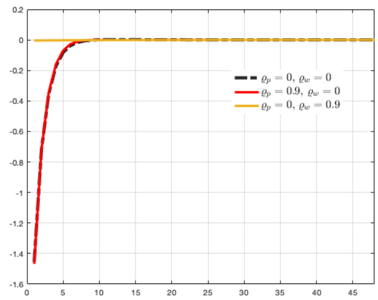


(d) $\varepsilon_{v,t} \rightarrow \widehat{\pi}_t$ (2%)

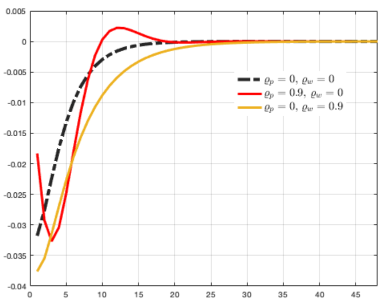
Figure 10: Sensitivity Analysis of θ_p and θ_w in Monetary Policy Shock $\varepsilon_{v,t}$



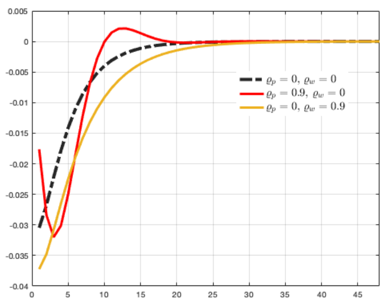
(a) $\varepsilon_{v,t} \rightarrow \widehat{Y}_t$ (0%)



(b) $\varepsilon_{v,t} \rightarrow \widehat{Y}_t$ (2%)

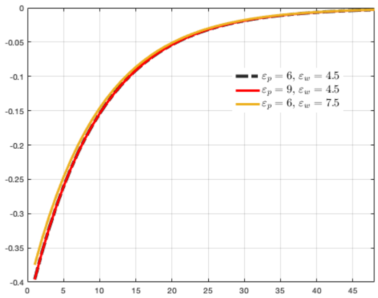


(c) $\varepsilon_{v,t} \rightarrow \widehat{\pi}_t$ (0%)

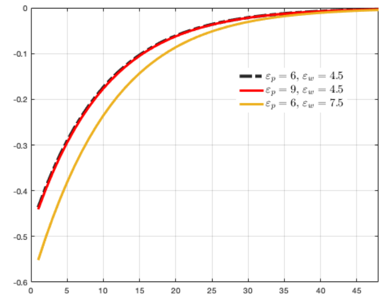


(d) $\varepsilon_{v,t} \rightarrow \widehat{\pi}_t$ (2%)

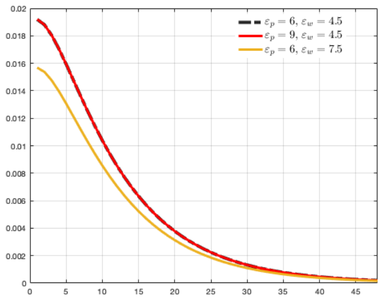
Figure 11: Sensitivity Analysis of ϱ_p and ϱ_w in Monetary Policy Shock $\varepsilon_{v,t}$



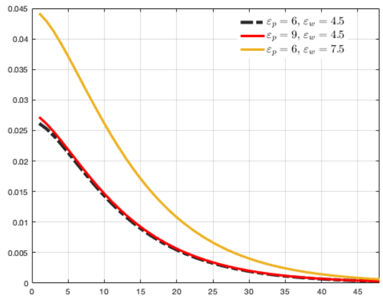
(a) $\varepsilon_{\varsigma,t} \rightarrow \widehat{Y}_t$ (0%)



(b) $\varepsilon_{\varsigma,t} \rightarrow \widehat{Y}_t$ (2%)

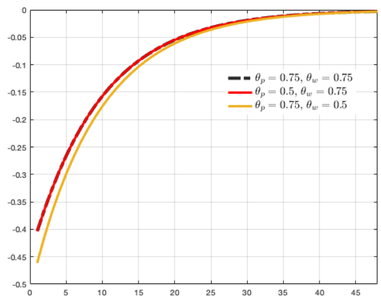


(c) $\varepsilon_{\varsigma,t} \rightarrow \widehat{\pi}_t$ (0%)

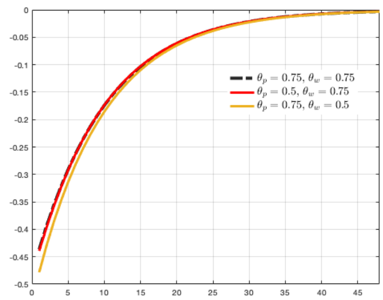


(d) $\varepsilon_{\varsigma,t} \rightarrow \widehat{\pi}_t$ (2%)

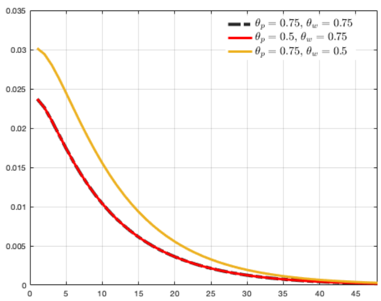
Figure 12: Sensitivity Analysis of ε_p and ε_w in Labor Supply Shock $\varepsilon_{\varsigma,t}$



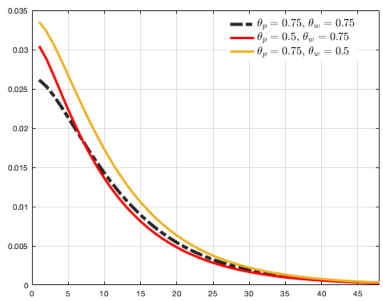
(a) $\varepsilon_{\varsigma,t} \rightarrow \widehat{Y}_t$ (0%)



(b) $\varepsilon_{\varsigma,t} \rightarrow \widehat{Y}_t$ (2%)

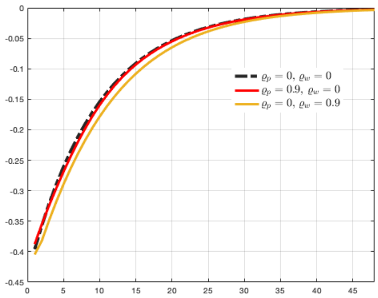


(c) $\varepsilon_{\varsigma,t} \rightarrow \widehat{\pi}_t$ (0%)

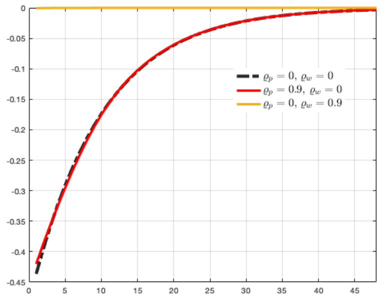


(d) $\varepsilon_{\varsigma,t} \rightarrow \widehat{\pi}_t$ (2%)

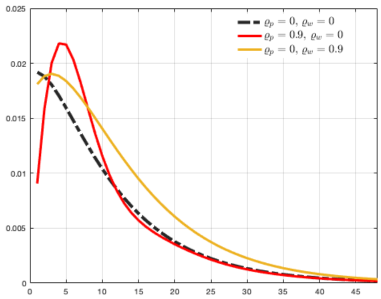
Figure 13: Sensitivity Analysis of θ_p and θ_w in Labor Supply Shock $\varepsilon_{\varsigma,t}$



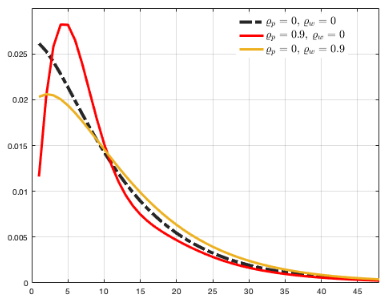
(a) $\varepsilon_{\varsigma,t} \to \widehat{Y}_t$ (0%)



(b) $\varepsilon_{\varsigma,t} \to \widehat{Y}_t$ (2%)



(c) $\varepsilon_{\varsigma,t} \to \widehat{\pi}_t$ (0%)



(d) $\varepsilon_{\varsigma,t} \to \widehat{\pi}_t$ (2%)

Figure 14: Sensitivity Analysis of ϱ_p and ϱ_w in Labor Supply Shock $\varepsilon_{\varsigma,t}$

Appendix

Deterministic Steady State

$$Y = C$$

$$\pi = \beta(1+i)$$

$$s_w = \frac{1-\theta_w}{1-\theta_w \pi^{\varepsilon_w(1-\varrho_w)}}$$

$$\psi_w = \frac{\chi\left(w^{\varepsilon_w} N\right)^{1+\varphi}}{1-\beta\theta_w \pi^{\varepsilon_w(1+\varphi)(1-\varrho_w)}}$$

$$\varphi_w = \frac{C^{-\sigma} w^{\varepsilon_w} N}{1-\beta\theta_w \pi^{(\varepsilon_w-1)(1-\varrho_w)}}$$

$$w_h^* = w = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\chi N^{\varphi}}{C^{-\sigma}} \frac{1-\beta\theta_w \pi^{(\varepsilon_w-1)(1-\varrho_w)}}{1-\beta\theta_w \pi^{\varepsilon_w(1+\varphi)(1-\varrho_w)}}$$

$$p_i^* = \left[\frac{1-\theta_p \pi^{(\varepsilon_p-1)(1-\varrho_p)}}{1-\theta_p}\right]^{\frac{1}{1-\varepsilon_p}}$$

$$(p_i^*)^{1+\frac{\varepsilon_p \alpha}{1-\alpha}} = \frac{\varepsilon_p}{(\varepsilon_p-1)(1-\alpha)} \frac{\psi_p}{\varphi_p}$$

$$\psi_p = \frac{w A^{-\frac{1}{1-\alpha}} Y^{\frac{1}{1-\alpha}-\sigma}}{1-\theta_p \beta \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}}$$

$$\varphi_p = \frac{Y^{1-\sigma}}{1-\theta_p \beta \pi^{(\varepsilon_p-1)(1-\varrho_p)}}$$

$$N = s_w s_p \left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}$$

$$MC_i^r = MC^r = \frac{A^{\frac{1}{\alpha-1}}}{1-\alpha} w Y^{\frac{\alpha}{1-\alpha}}$$

$$s_p = \frac{1-\theta_p}{1-\theta_p \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}} \left(p_i^*\right)^{-\frac{\varepsilon_p}{1-\alpha}} = \frac{1-\theta_p}{1-\theta_p \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}} \left[\frac{1-\theta_p \pi^{(\varepsilon_p-1)(1-\varrho_p)}}{1-\theta_p}\right]^{\frac{\varepsilon_p}{(\varepsilon_p-1)(1-\alpha)}}$$

$$\left(p_i^*\right)^{1+\frac{\alpha\varepsilon_p}{1-\alpha}} = \frac{\varepsilon_p}{(\varepsilon_p-1)(1-\alpha)} \left[\frac{1-\theta_p \beta \pi^{(\varepsilon_p-1)(1-\varrho_p)}}{1-\theta_p \beta \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}}\right] w A^{-\frac{1}{1-\alpha}} Y^{\frac{\alpha}{1-\alpha}}$$

$$\Rightarrow \left(p_i^*\right)^{1+\frac{\alpha\varepsilon_p}{1-\alpha}} = \frac{\varepsilon_p}{\varepsilon_p-1} \left[\frac{1-\theta_p \beta \pi^{(\varepsilon_p-1)(1-\varrho_p)}}{1-\theta_p \beta \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}}\right] MC^r$$

$$Y^f = \left[\frac{(1-\alpha)(\varepsilon_p-1)(\varepsilon_w-1)}{\chi \varepsilon_p \varepsilon_w}\right]^{\frac{1-\alpha}{\varphi+\alpha+\sigma(1-\alpha)}}$$

$$x = \frac{Y}{Y^f}$$

The Linearized Model

$$\widehat{Y}_t = \widehat{C}_t \tag{1}$$

$$\widehat{Y}_t = \mathbb{E}_t \widehat{Y}_{t+1} - \frac{1}{\sigma}\left(\widehat{i}_t - \mathbb{E}_t \widehat{\pi}_{t+1}\right) \tag{2}$$

$$\widehat{N}_t = \widehat{s}_{p,t} + \widehat{s}_{w,t} + \frac{1}{1-\alpha}\left(\widehat{Y}_t - \widehat{A}_t\right) \tag{3}$$

$$\widehat{i}_t = \varphi_{\pi_p} \widehat{\pi}_t + \varphi_{\pi_w}\left(\widehat{w}_t - \widehat{w}_{t-1} + \widehat{\pi}_t\right) + \varphi_y\left(\widehat{Y}_t - \widehat{Y}_t^f\right) + v_t \tag{4}$$

$$\widehat{s}_{p,t} = \left[-\frac{\varepsilon_p}{1-\alpha}\left(1-\theta_p \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}\right)\right] \widehat{p}_{i,t}^* + \left(\theta_p \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}\right)\left[\frac{\varepsilon_p}{1-\alpha}\left(\widehat{\pi}_t - \varrho_p \widehat{\pi}_{t-1}\right) + \widehat{s}_{p,t-1}\right] \tag{5}$$

$$\left(1 + \frac{\alpha\varepsilon_p}{1-\alpha}\right)\widehat{p}_{i,t}^* = \widehat{\psi}_{p,t} - \widehat{\varphi}_{p,t} \tag{6}$$

$$\begin{aligned} \widehat{\psi}_{p,t} = &\left(1-\theta_p \beta \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}\right)\left[\widehat{w}_t - \frac{1}{1-\alpha}\widehat{A}_t + \left(\frac{1}{1-\alpha} - \sigma\right)\widehat{Y}_t\right] \\ &+ \theta_p \beta \pi^{\frac{\varepsilon_p(1-\varrho_p)}{1-\alpha}}\left[\frac{\varepsilon_p}{1-\alpha}\left(\mathbb{E}_t \widehat{\pi}_{t+1} - \varrho_p \widehat{\pi}_t\right) + \mathbb{E}_t \widehat{\psi}_{p,t+1}\right] \end{aligned} \tag{7}$$

$$\begin{aligned} \widehat{\varphi}_{p,t} = &\left(1-\theta_p \beta \pi^{(\varepsilon_p-1)(1-\varrho_p)}\right)(1-\sigma)\widehat{Y}_t \\ &+ \theta_p \beta \pi^{(\varepsilon_p-1)(1-\varrho_p)}\left[(\varepsilon_p-1)\left(\mathbb{E}_t \widehat{\pi}_{t+1} - \varrho_p \widehat{\pi}_t\right) + \mathbb{E}_t \widehat{\varphi}_{p,t+1}\right] \end{aligned} \tag{8}$$

$$\widehat{p}_{i,t}^* = \frac{\theta_p \pi^{(\varepsilon_p-1)(1-\varrho_p)}}{1-\theta_p \pi^{(\varepsilon_p-1)(1-\varrho_p)}}\left(\widehat{\pi}_t - \varrho_p \widehat{\pi}_{t-1}\right) \tag{9}$$

$$\begin{aligned} \widehat{s}_{w,t} = &\ \varepsilon_w\left(1-\theta_w \pi^{\varepsilon_w(1-\varrho_w)}\right)\left(\widehat{w}_t - \widehat{w}_{h,t}^*\right) \\ &+ \theta_w \varepsilon_w \pi^{\varepsilon_w(1-\varrho_w)}\left(\widehat{w}_t - \widehat{w}_{t-1} + \widehat{\pi}_t - \varrho_w \widehat{\pi}_{t-1}\right) + \theta_w \pi^{\varepsilon_w(1-\varrho_w)} \widehat{s}_{w,t-1} \end{aligned} \tag{10}$$

$$(1+\varepsilon_w\varphi)\widehat{w}^*_{h,t} = \widehat{\psi}_{w,t} - \widehat{\varphi}_{w,t} \tag{11}$$

$$\begin{aligned}\widehat{\psi}_{w,t} = &\left(1-\beta\theta_w\pi^{\varepsilon_w(1+\varphi)(1-\varrho_w)}\right)\left[(1+\varphi)\left(\varepsilon_w\widehat{w}_t+\widehat{N}_t\right)+\varsigma_t\right] \\ &+\beta\theta_w\pi^{\varepsilon_w(1+\varphi)(1-\varrho_w)}\left[\varepsilon_w(1+\varphi)\left(\mathbb{E}_t\widehat{\pi}_{t+1}-\varrho_w\widehat{\pi}_t\right)+\mathbb{E}_t\widehat{\psi}_{w,t+1}\right]\end{aligned} \tag{12}$$

$$\begin{aligned}\widehat{\varphi}_{w,t} = &\left(1-\beta\theta_w\pi^{(\varepsilon_w-1)(1-\varrho_w)}\right)\left(-\sigma\widehat{C}_t+\varepsilon_w\widehat{w}_t+\widehat{N}_t\right) \\ &+\beta\theta_w\pi^{(\varepsilon_w-1)(1-\varrho_w)}\left[(\varepsilon_w-1)\left(\mathbb{E}_t\widehat{\pi}_{t+1}-\varrho_w\widehat{\pi}_t\right)+\mathbb{E}_t\widehat{\varphi}_{w,t+1}\right]\end{aligned} \tag{13}$$

$$\widehat{w}_t = \theta_w\pi^{(\varepsilon_w-1)(1-\varrho_w)}\left(\widehat{w}_{t-1}+\varrho_w\widehat{\pi}_{t-1}-\widehat{\pi}_t\right)+(1-\theta_w)\,\widehat{w}^*_{h,t} \tag{14}$$

$$\widehat{Y}^f_t = \frac{1+\varphi}{\varphi+\alpha+\sigma(1-\alpha)}\widehat{A}_t - \frac{1-\alpha}{\varphi+\alpha+\sigma(1-\alpha)}\varsigma_t \tag{15}$$

$$\widehat{x}_t = \widehat{Y}_t - \widehat{Y}^f_t \tag{16}$$

$$\widehat{\mathbb{W}}_t = \frac{C^{1-\sigma}}{\mathbb{W}}\widehat{C}_t - \frac{\chi N^{1+\varphi}\Delta_w}{\mathbb{W}}\widehat{N}_t - \frac{\chi N^{1+\varphi}\Delta_w}{(1+\varphi)\mathbb{W}}\left(\widehat{\Delta}_{w,t}+\varsigma_t\right)+\beta\mathbb{E}_t\widehat{\mathbb{W}}_{t+1} \tag{17}$$

$$\begin{aligned}\widehat{\Delta}_{w,t} = &\frac{\varepsilon_w(1-\theta_w)(1+\varphi)}{\Delta_w}\left(\widehat{w}_t-\widehat{w}^*_{h,t}\right) \\ &+\theta_w\pi^{\varepsilon_w(1+\varphi)(1-\varrho_w)}\left[\varepsilon_w(1+\varphi)\left(\widehat{w}_t+\widehat{\pi}_t-\widehat{w}_{t-1}-\varrho_w\widehat{\pi}_{t-1}\right)+\widehat{\Delta}_{w,t-1}\right]\end{aligned} \tag{18}$$

18 equilibrium conditions consist of 18 endogenous variables

$$\left\{\widehat{C}_t, \widehat{Y}_t, \widehat{i}_t, \widehat{\pi}_t, \widehat{w}_t, \widehat{N}_t, \widehat{p}^*_{i,t}, \widehat{w}^*_{h,t}, \widehat{\psi}_{p,t}, \widehat{\varphi}_{p,t}, \widehat{\psi}_{w,t}, \widehat{\varphi}_{w,t}, \widehat{s}_{p,t}, \widehat{s}_{w,t}, \widehat{Y}^f_t, \widehat{x}_t, \widehat{\Delta}_{w,t}, \widehat{\mathbb{W}}_t\right\}$$

and 3 shock process variables $\left\{\widehat{A}_t, \varsigma_t, v_t\right\}$.

$$\widehat{A}_t = \rho_A\widehat{A}_{t-1}+\varepsilon_{A,t}$$

$$\varsigma_t = \rho_\varsigma\varsigma_{t-1}+\varepsilon_{\varsigma,t}$$

$$v_t = \rho_v v_{t-1}+\varepsilon_{v,t}$$