# A fractal analysis of US industrial sector stocks

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**Abstract**

This paper found that most of US industrial sectors are fractal, and therefore have a long autocorrelated dependence.

**Keywords:** Fractal geometry, Hurst exponent, random walk
**JEL classification:** C18, E39, F31

## 1 Introduction

The fractal geometry has a concern with a long autocorrelated dependence of time-series by a power law distributions. In this methodology, we can consider the wider degree of autocorrelations than the unitroot process (random walk). In this regard, the fractal geometry is a method to detect generalized autocorrelations more than the unitroot process.

The generalization of the random walk model with the fractals suggests that the efficient market hypothesis does not valid anymore if the autocorrelation is higher than the one suggested in the unitroot case, because an independence among sequence of past shocks on asset prices is destroyed by the higher order autocorrelations of the fractals.

We believe that the fractal geometry provides new perspectives on the economics, however, the formal application of the fractal geometry to the economics are not enough except for Ikeda (2016), which found that the whole US stock market is fractal rather than the random walk with Hurst's R/S analysis.[1]

This paper aims to provide a formal fractal analysis of the US industrial sector stock prices. To the best of our knowledge, there is no investigation to provide a fractal analysis for the stock prices by industrial sectors. We distinguish the long-run dependence for the US industrial sector stocks.

This paper proceeds as follows. Section 2 introduces a fractal Brownian motion and R/S analysis to determine the long dependence of stock prices. Section 3 presents empirical results. Section 4 concludes.

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[1]Also, see Booth et al. (1982) for the case of exchange rates of developed European countries.

## 2 Methodology

This section introduces a fractional Brownian motion which provides mathematical backgrounds of the fractal geometry, and then the Hurst's R/S analysis to determine the generalized autocorrelation of stock prices.

### A fractional Brownian motion

A fractional Brownian motion (fBm) of the fractal geometry is a key concept to generalize the ordinary Brownian motion (random walk) in the unit root analysis. To reveal the relationship between the two, we denote the difference of the fractional Brownian motion $B_H(t)$ for any two times $t$ and $t_0$ as $|B_H(t) - B_H(t_0)|$. Therefore, the second moment of the difference can be calculated as:

$$\langle |B_H(t) - B_H(t_0)|^2 \rangle \propto |t - t_0|^{2H}, \tag{1}$$

where $\langle\rangle$ is an ensemble average and $H$ is the Hurst exponent which takes a value 1/2 for the random walk case.[2] Accordingly, it is relevant that the variance of the ordinary Brownian motion is the special case of the fBm's variance equation.

Also, note that the variance of the fBm goes to infinity with time increasing, and the rate is faster than the one of the random walk in the case $1/2 < H < 1$.

### The Hurst's R/S analysis

The method, so-called R/S analysis, has been proposed by Hurst (1955) in order to determine the long-run persistence in time-series. We describe the increments of time series in logarithms as $x(t) \in \big(x(1), ..., x(T)\big)$, and calculate the ensemble average over any window-size $\tau$ as:

$$\langle x_t|\tau \rangle = \frac{1}{\tau} \sum_{\tau(l-1)+1}^{\tau l} x(t), \tag{2}$$

for $l = 1 : \text{ceil}(T/\tau)$.

Then, we calculate the accumulated sum of dispersion of $x(t)$ from the average as follows:

$$x(t,\tau) = \sum_{t=1}^{T} \big(x(t) - \langle x(t)|\tau \rangle\big). \tag{3}$$

By the subtraction of the local averages, constant trends in the sample are excluded.

In addition, we calculate $R(\tau)$ of the max-min range over the window-size $\tau$ and $S(\tau)$ of the standard deviations in each segment as:

$$R(\tau) = \max_{\tau(l-1)+1 \leq t \leq \tau l} x(t,\tau) - \min_{\tau(l-1)+1 \leq t \leq \tau l} x(t,\tau), \tag{4}$$

[2]The variance equation of the random walk takes well-known formulation as:

$$\langle |B_H(t) - B_H(t_0)|^2 \rangle \propto |t - t_0|.$$

and

$$S(\tau) = \sqrt{\frac{1}{\tau} \sum_{\tau(l-1)+1}^{\tau l} \big(x(t) - \langle x(t) \rangle\big)^2}. \tag{5}$$

Clearly, the range $R$ is an increasing function for $\tau$.

Finally, the R/S statistics is defined as a fraction of (4) and (5), which obeys the power law distribution as:

$$R(\tau)/S(\tau) \propto \tau^H. \tag{6}$$

where, $H$ is the Hurst exponent as already introduced in equation (1).

Using Hurst's $H$, we can categorize the autocorrelation nature of the time-series into three cases. First, the case $0 < H < 1/2$ corresponds to *antipersistence* or *short correlation*, where the time-series increasing today must decrease tomorrow. Second, the case $H = 1/2$ implies the random walk as frequently used assumption in financial time-series analysis, which implies the shock terms of the series are not interdependent over time-differences. Third, $1/2 < H < 1$ denotes *long dependence* where the past shocks do not disappear and do have an eternal effect on the series. In the long dependence, increasing trends of the variable can turn suddenly to decreasing trends.[3]

Accordingly, it is apparent that the fractal geometry develops the concept of autocorrelation in traditional time-series analyses.

# 3 Empirical results

This section exhibits the statistical Hurst exponents for the 15 industrial sector stock indexes. To this end, we utilize the sector stock indexes categorized by Dow Jones.[4][5]

Here, we implement 5555 times bootstrapping to construct confidence intervals for estimated $H$. This is because the variance for $H$ goes to infinity in the case $1/2 < H < 1$. The nonparametric bootstrapping is appropriate in such situation.

## 3.1 Benchmark results

Table 1 presents the statistical Hurst exponents for the industrial sector stocks. The long autocorrelated dependence is significant in 7/15 industries: Auto and Parts, Consumer Goods, Consumer Services, Media, Real Estate, Technology, Telecommunications. The lower confidence intervals for the Hurst exponents of these stocks take values more than 0.50.[6]

Figure 1 confirms these regressions. In almost all figures, the fitted lines replicated by 5555 times bootstrapping contain the realized R/S values, which grantees the appropriateness for our regressions. However, there are only two exceptions: Farming and

[3]See Feder (1988, Chap. 9) for this classifications.

[4]All data are weekly-frequency and obtained from Thomson Reuters Datastream.

[5]All results in this paper are very similar to daily observations.

[6]The result of Real Estate is consistent with Ikeda (2016), who suggested the fractality of the US stock market.

Fishing and Oil and Gas. These realized values violate our bootstrapped lines, which implies the large structural changes for $H$. This problem deeply corresponds to a *crossovers phenomenon*, discussed in next section.

### 3.2 Time-dependent crossovers for the Hurst exponents

Following Peng et al. (1994), time-series has *crossovers* in which estimated $H$ changes largely.[7][8] This section examines whether the time-dependent crossovers appears on the stock price indexes by sequential estimations.

Figure 2 suggests the crossovers for all sectors. The results are amazing in the sense that the most of the sectors (12/15 sectors) has long-run dependence. This fact suggests that the crossovers are critical, also in economic time-series.

## 4 Conclusion

This paper found that the most of the US industrial sectors has a long-run dependence when we consider the crossovers for the statistical Hurst exponent. The exact specification for the crossovers by multiracial is challenges of the future.

## References


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[5] Peng, C.K., Buldyrev, S.V., Havlin, S., Simons, M., Stanley, H.E., Goldberger, A.L., (1994) "Mosaic organization of DNA nucleotides." *Physical Review E*, 49, 1685–1689.


[7] They stressed that the crossovers should be considered in DNA sequences.

[8] The crossovers suggest changes of the shape of the power law distributions for the variable.

# Figures and Tables

Table 1: Hurst exponents by sector

| No. | Name | Hurst exponent | Lower | Upper |
|---|---|---|---|---|
| 1 | The Americas Auto and Parts | 0.64570 | 0.54456 | 0.75352 |
| 2 | The Americas Basic Resources | 0.50475 | 0.46536 | 0.60480 |
| 3 | The Americas Consumer Goods | 0.60409 | 0.53906 | 0.65110 |
| 4 | The Americas Consumer Services | 0.62969 | 0.58543 | 0.71856 |
| 5 | The Americas Farming and Fishing | 0.49060 | 0.39037 | 0.53957 |
| 6 | The Americas Financial Services | 0.56600 | 0.47879 | 0.63285 |
| 7 | The Americas Food and Beverages | 0.56925 | 0.48216 | 0.60622 |
| 8 | The Americas Media | 0.68863 | 0.55370 | 0.83186 |
| 9 | The Americas Oil and Gas | 0.49178 | 0.41657 | 0.50922 |
| 10 | The Americas Real Estate | 0.57230 | 0.53531 | 0.67868 |
| 11 | The Americas Retail | 0.57423 | 0.47975 | 0.59372 |
| 12 | The Americas Technology | 0.68715 | 0.57431 | 0.75518 |
| 13 | The Americas Telecommunications | 0.65490 | 0.52544 | 0.74653 |
| 14 | The Americas Travel and Leisure | 0.52539 | 0.41885 | 0.58782 |
| 15 | The Americas Utilities | 0.52157 | 0.45394 | 0.62740 |

*Note:* 'Lower' and 'Upper' denote bootstrapped 95% confidence intervals.

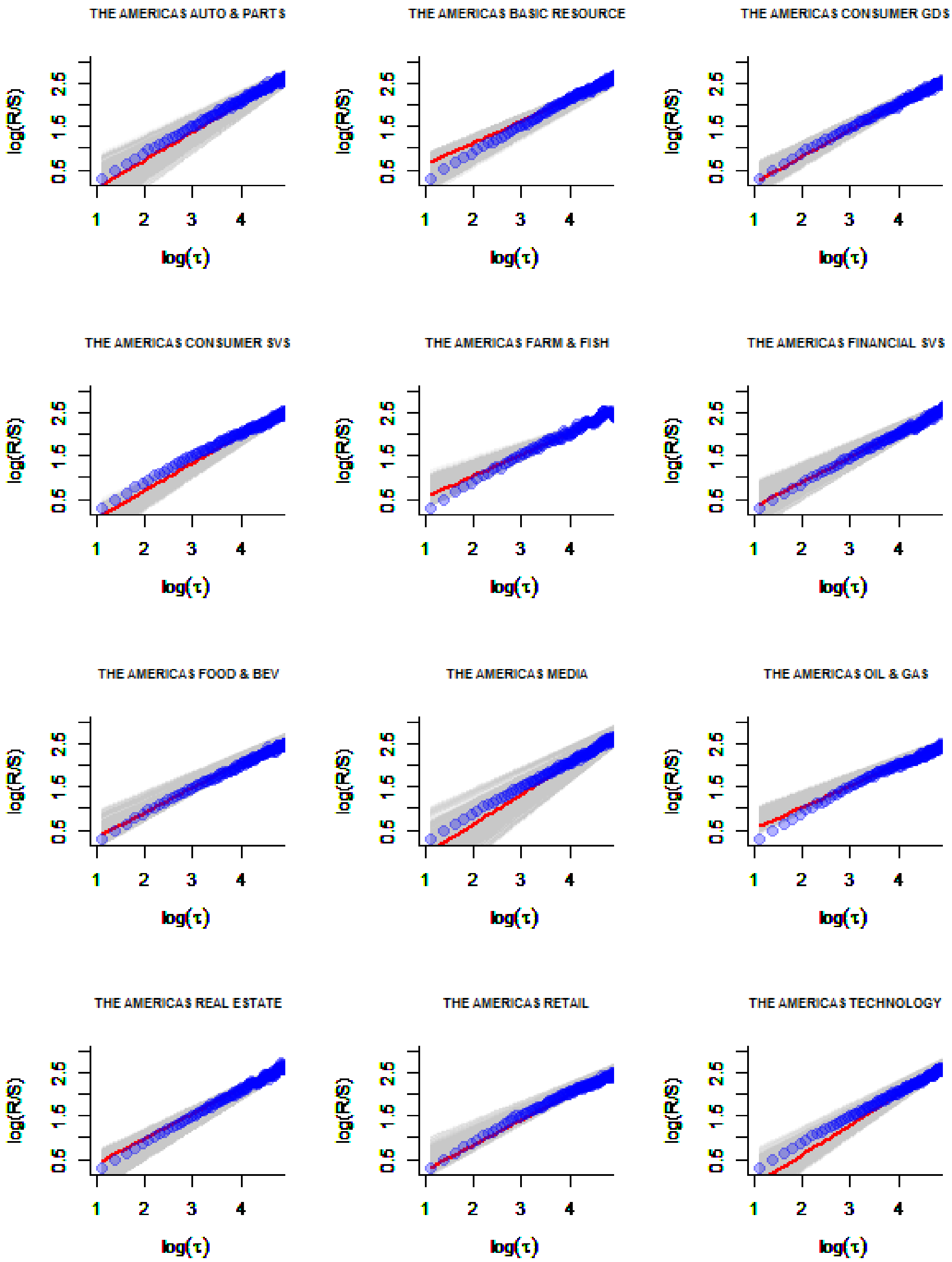
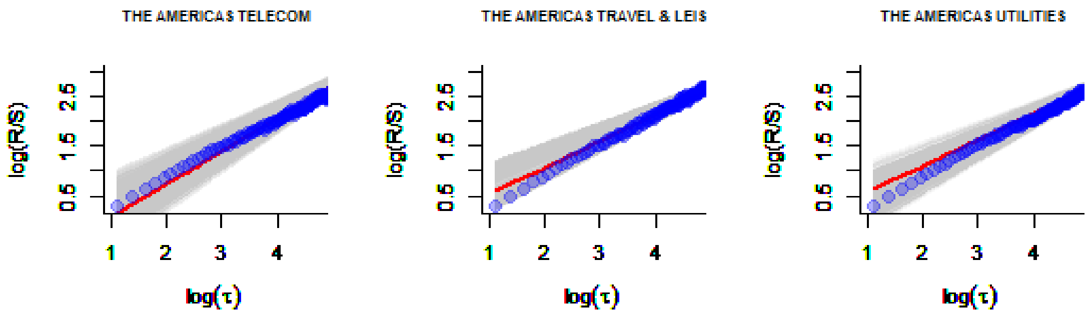



Figure 1: Bootstrapped regressions

*Note:* Gray lines are generated by regressions using 5555 bootstrapping.




*Note:* Gray lines are generated by regressions using 5555 bootstrapping.

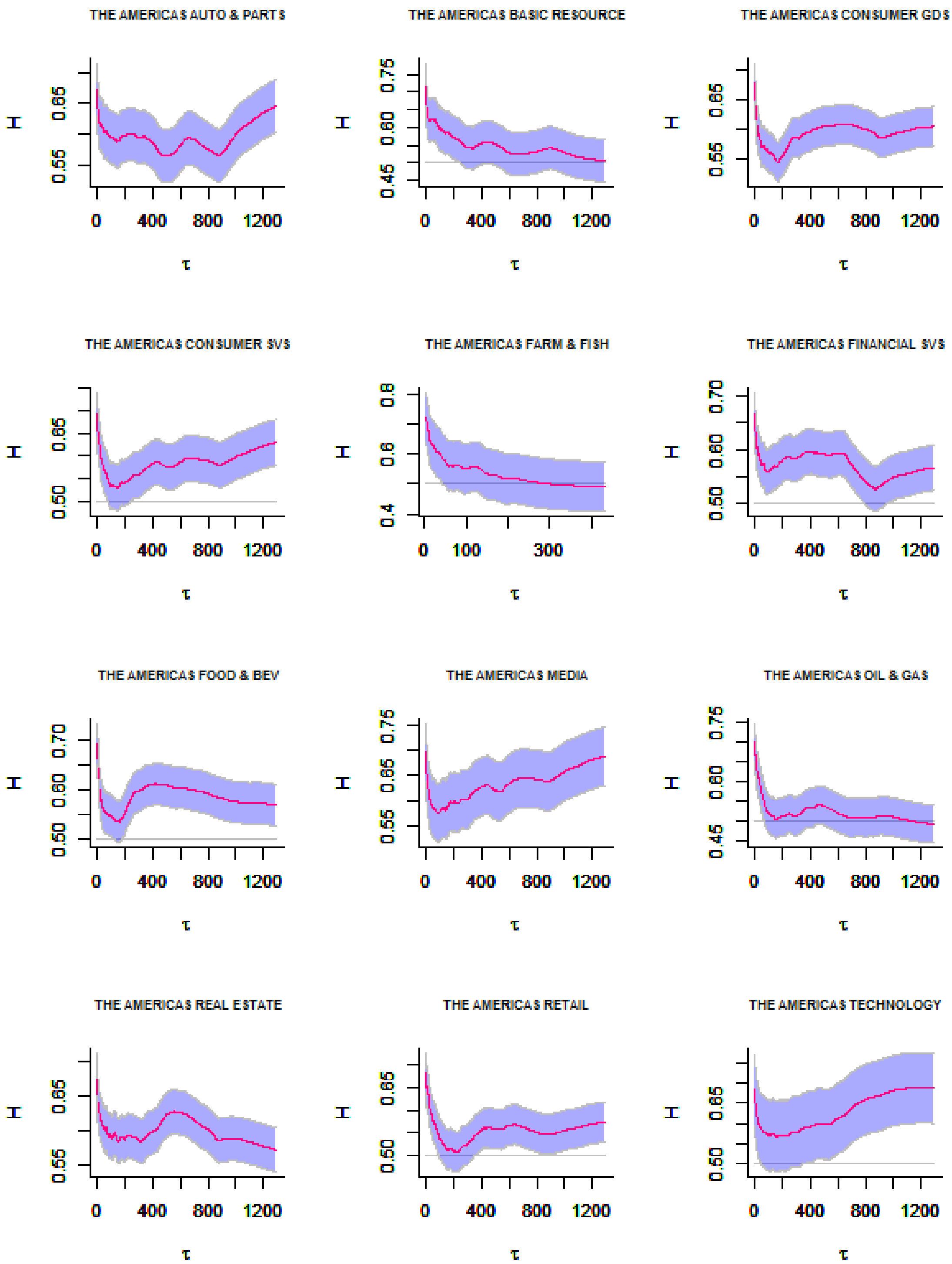
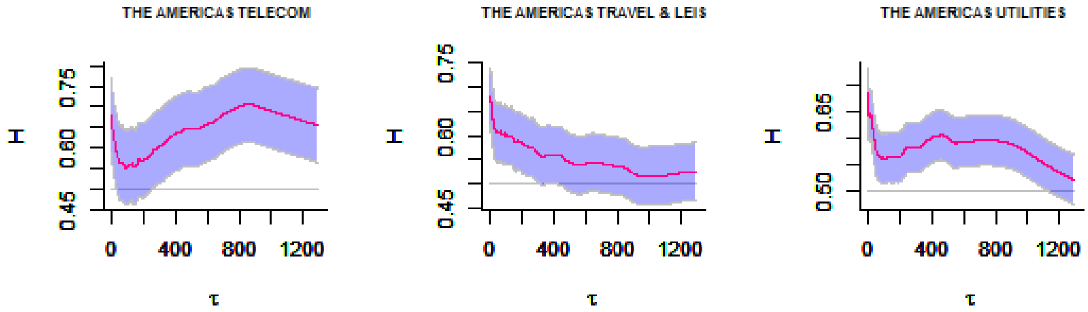



Figure 2: Sequential estimation of $H$

*Note:* Shaded area denotes 95% confidence interval for $H$.




*Note:* Shaded area denotes 95% confidence interval for $H$.