

**Asset bubbles, labor market frictions,
and R&D-based growth**

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Abstract

Employing an overlapping generations model of research and development (R&D)-based growth with labor market frictions, this paper examines how employment changes induced by labor market frictions influence asset bubbles and long-run economic growth. Asset bubbles can (cannot) exist when the employment rate is high (low), which leads to higher (lower) economic growth through labor market efficiency. We also explore the steady state and transitional dynamics of bubbles, economic growth, and employment. Furthermore, we show that policy or parameter changes that have a negative influence on the labor market can lead to a bubble burst.

Keywords: overlapping generations, asset bubbles, labor market friction, employment rate, R&D

JEL Classification Number: J64, O41, O42

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1. Introduction

As identified by Aliber and Kindleberger (2015), economic bubbles have occurred throughout history, often with major impacts on the economies of concerned countries. Examples include the current recession in the United States and other countries, the Japanese experience in the late 1980s and 1990s, and the 1929 economic crash. These bubbles featured spectacular booms, followed by dramatic crashes. In 1990, stock prices collapsed, and Japan's deepest and longest depression began; the average growth rate for the decade was 1.7%, and the year 1998 recorded a negative growth. Unemployment rose from 2.1% in 1990 to 4.7 % in 1999 (Kaihara, 2008). Similarly, bubbles have been frequently observed when economic activity is booming and the economic growth rate is high (Martin and Ventura, 2012; Farmer and Schelnast, 2013). Additionally, empirical studies show that asset bubbles are accompanied by a reduction in the unemployment rate (Phelps, 1999; Fitoussi et al., 2000).

In this paper, we analyze the interaction between bubbles, unemployment, and the long-run economic growth rate. Because technological progress via research and development (R&D) innovation has been identified as the primary driving force of modern economic growth (e.g., Romer 1990), we are particularly interested in the effects of these interactions and transitional dynamics on R&D-based innovation.

In the literature on asset bubbles and economic growth, Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) examine the conditions necessary for bubbles to exist in an overlapping generations economy. In their studies, when a bubble arises, it diverts savings from capital accumulation and retards economic growth. For an alternative approach that focuses on financial market imperfections, Hirano and Yanagawa (2010), Martin and Ventura (2012), and Kunieda and Shibata (2016) show that asset bubbles can enhance or impair growth depending on the restrictiveness of the collateral constraint. However, these do not consider the possibility of unemployment.

There are theories stating that equilibrium unemployment occurs as a result of friction in the labor market. In an economy with labor market frictions, the wage rate is endogenously determined by agents' negotiation. Bean and Pissarides (1993) introduce labor market search frictions in a standard overlapping generations model, where the wage is negotiated by a vacant firm and a worker,¹ and analyze the relationship between economic growth and unemployment. On the other hand, Corneo and Marquardt (2000) consider a monopolistic trade union in an

¹ See Pissarides (2000) for an introduction to search friction models.

endogenous growth model, where the wages and employment rate are set by the unions.² However, they do not analyze the connection between asset bubbles and unemployment.

A study close to ours is Hashimoto and Im (2016), who use a continuous-time overlapping generations model (Weil, 1989) with labor market frictions and consider the relationship between bubbles and unemployment in an endogenous growth framework (AK model) through a learning-by-doing technological capital externality. However, they use an ad-hoc setting in the determination of wage rates, and the analysis focuses on steady state only. In order to fill this gap, our paper follows standard labor market frictions, where the wage rate is endogenously determined by Nash-bargaining negotiation between a vacant firm and a worker.

A crucial point of departure of our model from existing related studies is that the accumulation of physical capital, technological progress via learning by doing or knowledge spillovers during production are considered as fundamental growth drivers, whereas our study introduces R&D sector, which play a crucial role in modern technological development.³ As such, we are able to analyze the effects of the interactions of bubbles and unemployment on R&D-based innovation. In this framework, we construct a simple and tractable overlapping generations model of R&D-based growth with labor market frictions, and we can explore not only the steady state, but also transitional dynamics of bubbles, economic growth, and employment.⁴

In our model, where unemployment stems from labor market frictions, labor market efficiency is reflected in the interest rate. Subsequently, because asset returns are related to the interest rate, the existence of bubbles depends on labor market conditions. As such, we find the equilibrium employment rate to be a key factor in the existence of bubbles; when it is over a certain level and the interest rate is high, asset bubbles can exist. When the conditions are

² See Aghion and Howitt (1994), Eriksson (1997), Caballero and Hammour (1996), and Haruyama and Leith (2010) for other models of the relationship between growth and unemployment, which address labor market frictions.

³ Olivier (2000) and Tanaka (2011) are exceptional pioneering works that examine the effects of asset bubbles on a variety of products by using a static monopolistic competition model. A key feature of those studies is that they do not consider the influences of unemployment.

⁴ Miao et al. (2016) and Kocherlakota (2011) present studies similar to our own. Miao et al. (2016) investigate the relationship between unemployment and stock market bubbles in an economy with labor and financial market frictions. Kocherlakota (2011) assumes that output is determined by household demand, and, as such, he does not consider the firm's behavior and capital stock accumulation in an economy with matching frictions and bubbles. However, these studies do not consider endogenous economic growth.

satisfied for bubbles to exist, we say that the economy is in a “bubble regime;” conversely, when it is not possible for them to exist, the economy is in a “non-bubble regime.” In a bubble regime, there are multiple equilibria, such that a steady state can exist either with bubbles or without. We show that bubbles divert savings from R&D resources and lower output growth rate, a common finding in the literature. On the other hand, when we compare bubble to non-bubble regimes, we find that the output growth rate is always higher under the former compared to the latter.

Our model allows us to examine the effects of labor market policy or parameter changes on bubbles, economic growth, and employment. For example, we find that, because a rise in search cost decreases the number of firms with vacant positions, it has a negative impact on employment (a standard conclusion among models with search friction). Therefore, if search costs are increased in an economy with bubbles, then the employment rate should decrease and the labor market should become more inefficient, which would lower the interest rate and, consequently, shift the economy from a bubble to a non-bubble regime. As a result, employment rate and economic growth decrease is associated with bubble bursting. In this case, there is a positive relationship between employment and economic growth.⁵

The remainder of this paper is organized as follows. Section 2 outlines the features of the chosen model. Section 3 discusses labor market structure. Section 4 describes the traditional dynamics and the steady-state equilibrium with and without bubbles, and compares the effects of policy and parameter changes under the two regimes on bubbles, economic growth, and employment. The final section summarizes our findings and concludes the paper.

2. Model

This section develops an overlapping generations model with labor market frictions. A new generation is born in each period $t = 0, 1, \dots$, and lives for three periods: young, adult, and old age. Each generation has constant population size (L). The economy consists of three sectors: a final goods, intermediate goods, and R&D sector. Labor market is open for the final goods sector only. In accordance with Rivera-Batiz and Romer (1991) and Barro and Sala-i-Martin (2004, Chapter 6), we regard final goods as the production factor in both the intermediate goods and R&D sectors. R&D firms create blueprints for intermediate goods and conduct the

⁵ In fact, numerous empirical studies show a positive relationship between the employment rate and economic growth (Ball and Moffitt, 2001; Muscatelli and Tirelli, 2001; Staiger et al., 2001; Tripier, 2006; Pissarides and Vallanti, 2007).

market launches of these goods. Conversely, each intermediate good is produced by a single monopoly firm. Each final good is produced by competitive firms, which are successful in matching with labor, with a variety of imperfectly substitutable intermediate goods as inputs.

2.1. Final goods sector

In the final goods sector, many identical firms produce final goods with the same production technology. A firm needs one worker and intermediate goods to produce final goods. In the labor market, there are young households and firms with vacant positions to find each other. When the firm is successful in matching with one worker, it then operates final goods production with inputs of intermediate goods.

Considering the behavior of the operating production firm, firm i produces final goods $y_{i,t}$ at time t with the following production technology:

$$y_{i,t} = \int_0^{N_t} (x_{i,t}(j))^\alpha dj, \quad 0 < \alpha < 1, \quad (1)$$

where $x_{i,t}(j)$ and N_t are the input of intermediate goods for product variety j and the number of varieties available at period t , respectively. Subsequently, the operating profits, which are the remainders of output to be allotted between firm i and its worker, are given by

$$\pi_{i,t}^Y \equiv y_{i,t} - \int_0^{N_t} p_t(j)x_{i,t}(j)dj, \quad (2)$$

where $p_t(j)$ represents the price of intermediate good j . Because the factors market is competitive, from the profit maximization problem, we can get the firm i 's demand function for intermediate goods as

$$x_{i,t}(j) = x_t(j) = \left(\frac{\alpha}{p_t(j)} \right)^{\frac{1}{1-\alpha}}. \quad (3)$$

The firm-specific index in the final goods sector can be dropped because of the symmetry in production technology; $y_{i,t} = y_t$ and $\pi_{i,t}^Y = \pi_t^Y$.

2.2. Intermediate goods sector

Each intermediate good j is produced by monopolistically competitive firms, which hold a blueprint for this intermediate good. One unit of final goods is required to produce one unit of an intermediate good, and the operating profit of each intermediate goods producer, $\pi_t^X(j)$,

is expressed as follows: $\pi_t^X(j) = (p_t(j) - 1)X_t(j)$, where $X_t(j)$ represents the supply of intermediate good j . Under monopolistic competition, each firm maximizes its profits, given a demand curve for its brand. Because final good firms need one worker to produce goods, the number of active firms producing final goods in time t equals the total number of workers, $\sigma_t L$, where σ_t represents the employment rate. Subsequently, the aggregate demand for product variety j is defined as $X_t(j) \equiv \int_0^{\sigma_t L} x_{i,t}(j) di = x_t(j)\sigma_t L$. Using this definition and equation (3), we can obtain the demand curve for intermediate good j :

$$X_t(j) = \left(\frac{\alpha}{p_t(j)} \right)^{\frac{1}{1-\alpha}} \sigma_t L. \quad (4)$$

Then, the optimization problem for intermediate good is that firm j establishes a price equal to a constant markup over unit cost:

$$p_t(j) = p_t = \frac{1}{\alpha}. \quad (5)$$

Thus, the firm-specific index in the intermediate goods sector can be dropped, and profits may therefore be expressed as follows:

$$\pi_t^X = (1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \sigma_t L. \quad (6)$$

2.3. R&D sector

The development of R&D technology requires final goods as its input. Denoting η units of the final goods between periods t and $t + 1$, competitive R&D firms can invent one unit of $N_{t+1} - N_t$ new blueprints, and sell these blueprints to intermediate goods firms at their market values of D_t . Therefore, output is expressed as follows:

$$N_{t+1} - N_t = \frac{1}{\eta} I_t^R, \quad (7)$$

where I_t^R represents R&D inputs. Under the assumption of free entry in the R&D sector, the expected gain of $D_t I_t^R / \eta$ from R&D must not exceed the cost of I_t^R for a finite size of R&D activities at equilibrium. We assume that the R&D cost is given by $\eta = \bar{\eta} L$, which expresses the dilution effect that removes the scale one, as in Laincz and Peretto (2006) and Peretto and Connolly (2007). Thus, we have the following condition:

$$D_t = \bar{\eta}L. \quad (8)$$

We subsequently consider no-arbitrage conditions. The market value of intermediate goods firms, D_t (i.e., the market value of blueprints), is related to the risk-free interest rate, r_t . Shareholders of intermediate goods firms that purchased these shares during period t obtain dividends of π_{t+1}^x during period $t + 1$, and can sell these shares to the subsequent generation at a value of D_{t+1} . In the financial market, total returns from holding the stock of a particular intermediate firm must be equal to the returns on the risk-free asset, $(1 + r_{t+1})D_t$, which implies the following no-arbitrage condition: for all t , the return on one unit of stock must be equal to the interest rate:

$$1 + r_{t+1} = \frac{\pi_{t+1}^x + D_{t+1}}{D_t}. \quad (9)$$

Then, substituting (6) and (8) into (9), we have the interest rate as follows:

$$r_{t+1} = r(\sigma_{t+1}); \quad r(\sigma_{t+1}) \equiv \frac{\pi_{t+1}^x}{D_t} = \frac{1}{\bar{\eta}}(1 - \alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\sigma_{t+1}. \quad (10)$$

2.4. Agents

The first, second, and third periods of agents' lifetimes are referred to as young, adult, and old, respectively. The cohort born in period $t-1$ become active labor in period t . Therefore, we call this cohort generation t . Note that the superscript ι denotes an agent's employment status: $\iota = e$ if employed and $\iota = u$ if unemployed, which is an outcome of job search. An individual derives utility from consumption in old age c_{t+1}^ι , then the lifetime utility of individuals in generation t is expressed as $U_t^\iota = c_{t+1}^\iota$.

During the first period, individuals are endowed with one unit of labor. If match with a firm is successful in the first period (young), the agent can work and receive wage income, w_t , in the second period (adult). Otherwise, the adult receives unemployment benefits from the government, z_t . Individuals transfer lump-sum tax, τ_t , to the government and save the after-tax income. The allocation of savings is devoted to the interest-bearing and bubbly assets. Following Tirole (1985), we consider bubbly assets. Bubbles are intrinsically useless, that is, the fundamental value of the bubbles is zero. The adult will buy bubbly assets only if he/she will be able to resell them at a positive price to the next generation. In the third period,

individuals are retired and spend their savings on old-age consumption. Therefore, the budget constraint for generation t is expressed as follows:

$$s_t^l + p_t^B m_t^l = \omega_t^l - \tau_t, \quad \omega_t^e = w_t \text{ for employed, } \omega_t^u = z_t \text{ for unemployed,}$$

$$c_{t+1}^l = (1 + r_{t+1})s_t^l + p_{t+1}^B m_t^l,$$

where s_t^l is the interest-bearing asset holdings, m_t^l is the demand for bubble assets, and p_t^B is the price of bubble assets at time t in real terms. In order to hold bubbles in equilibrium, the price of bubbles must satisfy the arbitrage condition $p_{t+1}^B / p_t^B = 1 + r_{t+1}$, that is, return of bubbles equals the interest rate. As such, an agent's lifetime utility is given by

$$U_t^l = (1 + r_{t+1})(\omega_t^l - \tau_t). \quad (11)$$

2.5. Government

Government finances unemployment benefits using lump-sum tax on households. From the condition of a balanced government budget, we have

$$\tau_t L = z_t (1 - \sigma_t) L.$$

The left-hand side denotes aggregate tax revenue and the right-hand side represents the payment for unemployment benefits. Unemployment benefits are paid to unemployed workers, following such a policy that $z_t = \bar{z} w_t$, where $\bar{z} \in [0,1)$. In other words, the benefit payment to each unemployment worker is proportional to but below the wage rate in the current period.

3. Labor market

3.1. Matching mechanism

As discussed in the previous section, young agents and employers search each other in the labor market. The matching mechanism follows from the standard model of unemployment. Because young agents and firms face matching frictions in the current economy, unemployment occurs at equilibrium, although each agent is born endowed with one unit of labor inelastically supplied.

Now, consider matching mechanism in this economy. By denoting the number of successful matches as F , this process can be given by the matching function $F(L, v_{t-1})$, where L

and v_{t-1} represent the number of young agents and the number of firms with vacancies, respectively. Following the standard assumptions, the matching function is to be concave, homogeneous of degree one, increasing in both of its arguments, and $0 \leq F(L, v_{t-1}) \leq \min[L, v_{t-1}]$. The tightness of the labor market is expressed by $\theta_{t-1} \equiv v_{t-1} / L$, then the probability that a firm with vacancy matches with a young agent is given by $F(L, v_{t-1}) / v_{t-1} = F(1/\theta_{t-1}, 1) \equiv q(\theta_{t-1})$. Note that the probability $q(\theta)$ holds the following properties; $q(\theta) \in [0, 1]$, $q'(\theta) < 0$, $\lim_{\theta \rightarrow 0} q(\theta) = 1$, and $\lim_{\theta \rightarrow \infty} q(\theta) = 0$.⁶

If the search is successful in time $t-1$, employment is realized in the subsequent period (time t). Using the definition of employment rate σ_t , because the realized number of employment is equal to the number of successful matches, it follows that $F(L, v_{t-1}) = \sigma_t L$, which is rewritten as

$$\sigma_t = \theta_{t-1} q(\theta_{t-1}). \quad (12)$$

This shows the relationship between the employment rate and the tightness of the labor market, from which we obtain $d\sigma_t / d\theta_{t-1} > 0$ because $\partial[\theta_{t-1} q(\theta_{t-1})] / \partial \theta_{t-1} = \partial F(1, \theta_{t-1}) / \partial \theta_{t-1} > 0$.⁷ Therefore, (12) provides a positive relationship between the employment rate and the tightness of the labor market, which is the so-called Beveridge curve. Therefore, when the labor market tightness, θ , approaches zero (infinity), the employment rate σ becomes zero (unity).

If a match is successful, the firm can produce final goods and earn operating profits. The probability that a firm will be matched with a worker in period t is given by $q(\theta_{t-1})$. Thus, $1 - q(\theta_{t-1})$ is the probability that a firm with vacancies cannot be matched to a worker. Let V_t and J_t be the values of a vacant job and an occupied job in period t , respectively. As such, the value of a vacant job is as follows:

$$V_{t-1} = -k_{t-1} + \frac{1}{1+r_t} [q(\theta_{t-1})J_t + (1 - q(\theta_{t-1}))V_t], \quad (13)$$

⁶ See den Haan et al. (2000) and Petrongolo and Pissarides (2001) for a discussion on matching functions.

⁷ Using $\theta_{t-1} \equiv v_{t-1} / L$ and (12), it follows that the number of final goods production firms in time t which are successful in matching in time $t-1$ are $v_{t-1} q(\theta_{t-1}) = \sigma_t L$.

where k_{t-1} denotes the search cost.⁸ The second term on the right-hand side represents the expected current value of a successful or an unsuccessful match. The value of an occupied job is given by

$$J_t = \pi_t^Y - w_t. \quad (14)$$

Since the period of employment is one period (adult age), the value of an occupied job is one period profit (implying the full separation rate in one period).

We assume that the final product firms enter the market freely. Then, from the free entry condition, the value of a vacant job is $V_t = 0$ for all t . Consequently, from (13), the value of an occupied job becomes $J_t = (1 + r_t)k_{t-1} / q(\theta_{t-1})$, and substituting it into (14) yields

$$\pi_t^Y - w_t = \frac{(1 + r_t)k_{t-1}}{q(\theta_{t-1})}. \quad (15)$$

3.2. Nash bargaining

The remainder of output after payments towards intermediate goods is allotted to a firm and its worker. We assume that the wage rate is negotiated and determined by Nash bargaining. The household surplus and the firm surplus are given by $U_t^e - U_t^u$ and $J_t - V_t$, respectively. Using (11) and (14) with free entry condition, $V_t = 0$, the shares of each are determined by maximizing the following Nash product with respect to wage:

$$w_t = \arg \max (U_t^e - U_t^u)^\beta (J_t - V_t)^{1-\beta} = \arg \max ((1 + r_{t+1})(w_t - z_t))^\beta (\pi_t^Y - w_t)^{1-\beta},$$

where $\beta \in (0, 1)$ denotes the worker's bargaining power. Then, the wage rate is given by

$$w_t = (1 - \beta)z_t + \beta\pi_t^Y. \quad (16)$$

Using (1), (2), (3), and (5), the output and operating profit of final goods are given by

$$y_t = \alpha^{\frac{2\alpha}{1-\alpha}} N_t, \quad \pi_t^Y = (1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}} N_t. \quad (17)$$

Then, using (17) and the unemployment benefit policy, $z_t = \bar{z}w_t$, the wage rate is given by the following equation:

$$w_t = \Omega(\bar{z}, \beta)(1 - \alpha)\alpha^{\frac{2\alpha}{1-\alpha}} N_t, \quad (18)$$

⁸ The search cost would cover recruitment activities such as job interviews and the evaluation of reference letters, which are done using the firm's operating resources.

where $\Omega(\bar{z}, \beta) \equiv \beta / (1 - (1 - \beta)\bar{z}) \in (0, 1)$ represents the worker's output share of π_t^Y . Then the following conditions hold: $\partial\Omega/\partial\bar{z} > 0$ and $\partial\Omega/\partial\beta > 0$. This means that the larger outside option that the worker faces leads to a greater share of π_t^Y , and the larger Nash bargaining power enables the worker to obtain a greater share of π_t^Y .

Furthermore, in accordance with Mortensen and Pissarides (1999), Pissarides and Vallanti (2007), and Miyamoto and Takahashi (2011), we assume the form of search cost is as follows: $k_t = \bar{k}y_t$; that is, the search cost is proportionate to the scale of production, $\bar{k} \in (0, 1)$.⁹ Then, substituting $k_t = \bar{k}y_t$, (17), and (18) into (15) yields

$$(1 - \alpha)(1 - \Omega(\bar{z}, \beta)) = \frac{(1 + r_t)\bar{k}}{(1 + g_{t-1})q(\theta_{t-1})}, \quad (19)$$

where $g_{t-1} \equiv (N_t - N_{t-1}) / N_{t-1}$ is growth rate of the variety. Equation (19) is referred to as the job creation condition (Pissarides, 2000), and shows that at higher $\Omega(\bar{z}, \beta)$ or \bar{k} , θ_{t-1} is lower. Additionally, the growth rate of the firm's effective rate of discount, $(1 + g_{t-1}) / (1 + r_t)$, has a positive effect on job creation (higher θ_{t-1}), which is the so-called "capitalization effect" (Aghion and Howitt, 1994). The following section examines the equilibrium growth and employment rates.

4. Equilibrium

4.1. Equilibrium dynamics

Consider the equilibrium dynamics of an economy. First, we derive the growth rate of production variety. The final goods market equilibrium condition is given by

$$Y_t = C_t + N_t X_t + I_t^R + v_t k_t, \quad (20)$$

where Y_t and C_t are the aggregate final goods and the aggregate consumption, respectively; $Y_t \equiv y_t \sigma_t L$ and $C_t \equiv c_t^e \sigma_{t-1} L + c_t^u (1 - \sigma_{t-1}) L$. We can obtain the following asset market equilibrium condition (the derivation is provided in Appendix A):

$$S_t = D_t N_{t+1} + v_t k_t. \quad (21)$$

⁹ This assumption is required to ensure a balanced growth path, in which the search cost follows the pace of economic growth.

We find that the interest-bearing assets consist of the patent of varieties, $D_t N_{t+1}$, and the total search cost of the matching process for the final goods production $v_t k_t$.¹⁰ On the other hand, from the budget constraint of households, the aggregate holdings of the interest-bearing assets $S_t (\equiv s_t^e \sigma_t L + s_t^u (1 - \sigma_t) L)$ can be given by

$$S_t = w_t \sigma_t L - B_t, \quad (22)$$

where $B_t \equiv p_t^B [m_t^e \sigma_t L + m_t^u (1 - \sigma_t) L]$ represents aggregate demand of asset bubbles. Then, denoting the growth rate of varieties as $g_t \equiv (N_{t+1} - N_t)/N_t$, we obtain the following equation (the derivation is provided in Appendix B):

$$1 + g_t = \frac{\frac{1}{\bar{\eta}} \left[\frac{\Omega}{\alpha} \bar{\eta} r(\sigma_t) - b_t \right]}{\left[1 + \frac{1 - \Omega}{\alpha} \frac{r(\sigma_{t+1})}{1 + r(\sigma_{t+1})} \right]}, \quad (23)$$

where $b_t \equiv B_t / LN_t$ is defined as the normalized bubbles. We find that the growth rate depends on the variables of employment rate and bubbles; $g_t = g(\sigma_{t+1}, \sigma_t, b_t)$.

Subsequently, using (10), (19), and (23) with $\theta_t = \theta(\sigma_{t+1})$ from (12), we obtain the dynamics of the employment rate:

$$\sigma_{t+1} = \sigma(\sigma_t, b_t) \Leftrightarrow \frac{1 + r(\sigma_{t+1})}{1 + g(\sigma_{t+1}, \sigma_t, b_t)} = \frac{1}{\Delta} q(\theta(\sigma_{t+1})), \quad (24)$$

where $\Delta \equiv \bar{k} / (1 - \alpha)(1 - \Omega(\beta, \bar{z}))$ can be interpreted as the cost parameter of firm entry, because parameter Δ increases in both search cost, \bar{k} , and worker's profit share $\Omega(\beta, \bar{z})$. Using (12), the probability q is described as a function of σ ; $q'(\sigma) < 0$, $q(0) = 1$ and $q(1) = 0$. As such, we obtain the following properties: $\partial \sigma_{t+1} / \partial \sigma_t > 0$ and $\partial \sigma_{t+1} / \partial b_t < 0$ (the derivation is provided in Appendix C).

¹⁰ Using (9), holding the patent of varieties can be rewritten as the current value of the return: $D_t N_{t+1} = \frac{\pi_{t+1}^X + D_{t+1}}{1 + r_{t+1}} N_{t+1}$. Additionally, using (12) and (15) with $\theta_t \equiv v_t / L$, the

total search cost can be rewritten as the current value of the return: $v_t k_t = \frac{\pi_{t+1}^Y - w_{t+1}}{1 + r_{t+1}} \sigma_{t+1} L$.

Therefore, we find that the interest-bearing asset is devoted to the investments towards the expected profits of the final and intermediate goods sectors.

Let $B_t = p_t^B M$ be the real value of the bubble at time t , where M is the total nominal supply of bubbles; then, the equilibrium condition is given by $M = m_t^e \sigma_t L + m_t^u (1 - \sigma_t) L$. By the arbitrage condition, we have the dynamics of bubbles: $B_{t+1} = (1 + r_{t+1}) B_t$. Using $b_t \equiv p_t^B M / LN_t$, the dynamics of the normalized bubbles can be obtained as follows:

$$b_{t+1} = b(\sigma_t, b_t) \Leftrightarrow b_{t+1} = \frac{1 + r(\sigma_{t+1})}{1 + g(\sigma_{t+1}, \sigma_t, b_t)} b_t \quad (25)$$

using (24). The equilibrium of this economy is completely described by these equations: (24) and (25) in σ_t and b_t .

The phase diagram can be drawn on the (σ_t, b_t) plane. We refer to the locus of plane (σ_t, b_t) representing $\sigma_{t+1} = \sigma_t$ as the σ locus, and that representing $b_{t+1} = b_t$ as the b locus. Using (23), from (24) and (25), the σ and b loci are represented as equal parts of (26) and (27), respectively;

$$\sigma_{t+1} \geq \sigma_t : b_t \leq \Gamma(\sigma_t) \equiv \bar{\eta} \left[\frac{\Omega}{\alpha} r(\sigma_t) - \frac{\Delta}{q(\sigma_t)} \left(1 + r(\sigma_t) + \frac{1 - \Omega}{\alpha} r(\sigma_t) \right) \right], \quad (26)$$

$$b_{t+1} \geq b_t : \begin{cases} b_t \geq \Phi(\sigma_t) \Leftrightarrow b_t \geq \bar{\eta} \left[\frac{\Omega}{\alpha} r(\sigma_t) - \left(1 + r(\sigma_{t+1}) + \frac{1 - \Omega}{\alpha} r(\sigma_{t+1}) \right) \right], \\ b_t = 0. \end{cases} \quad (27)$$

where $\sigma_{t+1} = \sigma(\sigma_t, b_t)$ from (24). The phase diagram is as shown in Figure 1. This shows that the slope of the σ locus represents the inverse-U shape, in which $\Gamma(0) < 0$, $\Gamma(1) < 0$ and $\Gamma''(\sigma_t) < 0$ are satisfied. The b locus has two lines; one is represented by the horizontal line of $b = 0$, and the other is represented by an increase curve $\Phi'(\sigma_t) > 0$ for $b > 0$.¹¹ As such, we can show that there are two phases: “non-bubble regime” and “bubble regime” as shown in Figures 1 (i) and (ii), respectively. In the “non-bubble regime,” there is a unique saddle path to point E^{NB} , since the path to point E is the source. In this regime, bubbles cannot occur. In the “bubble regime,” however, there are two equilibria, E^N and E^B , since the path to point E is

¹¹ See Appendix D for the slope of σ and b loci.

the source. The equilibrium path to E^N is the sink, while the equilibrium path to E^B is saddle-point stable.¹² In this regime, bubble equilibrium can occur at point E^B .

4.2. Bubbles, employment and growth

In this section, we derive the condition under which bubbles exist in a steady state. At bubble equilibrium, using equations (25) and (10), the growth rate with a positive bubble, g^B , can be expressed by

$$g^B = r(\sigma) \equiv \frac{1}{\bar{\eta}}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\sigma, \quad (28)$$

the growth rate depending only on the interest rate. By substituting (28), (12), and (19) into (23) and rearranging them, we can get the value of the equilibrium bubble:

$$b = \bar{\eta} \left[r(\sigma) \left(\frac{\Omega}{\alpha} - \frac{1-\Omega}{\alpha} - 1 \right) - 1 \right]. \quad (29)$$

We assume the parameter condition $\Omega > (1+\alpha)/2$ for the possibility of bubble equilibrium.¹³ From (29) and (10), we obtain the condition of employment rate for a bubble regime:

$$\sigma > \hat{\sigma} \equiv \frac{\bar{\eta}}{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \left(\frac{\Omega}{\alpha} - \frac{1-\Omega}{\alpha} - 1 \right)}. \quad (30)$$

Then, the bubble regime where positive bubbles can exist ($b > 0$) holds for $\sigma > \hat{\sigma}$, while the non-bubble regime where bubble cannot exist ($b = 0$) holds for $\sigma \leq \hat{\sigma}$. This implies that the equilibrium employment rate plays an important role in the existence of bubbles.

In the non-bubble economy, from (23) and $b = 0$, the growth rate without bubble g^N can be rewritten as

$$1 + g^N(\sigma) = \frac{\Omega}{\frac{\alpha}{r(\sigma)} + \frac{1-\Omega}{1+r(\sigma)}}. \quad (31)$$

¹² See Appendix E for the local stability analysis of these equilibrium paths in each regime.

¹³ This assumption is imposed to allow bubble equilibrium to occur; otherwise, the possibility of bubbles is intrinsically avoided. Note that this assumption is the possibility of a non-bubble equilibrium not being eliminated. In fact, under the following proposition, we obtain non-bubble equilibrium.

The relationship between the growth rate without bubble and the one with bubble is described in Figure 2. As such, the economy will be in a bubble regime (non-bubble regime) when the equilibrium employment rate is higher (lower) than the threshold level.

Now, we derive the equilibrium of employment rate. In the steady state, (24) gives the level of the employment rate with (31) in non-bubble equilibrium, and with (28) in bubble equilibrium.

$$\frac{1+r(\sigma)}{1+g^N(\sigma)} = \frac{1}{\Delta} q(\sigma) \quad \text{for non-bubble equilibrium,} \quad (32)$$

$$1 = \frac{1}{\Delta} q(\sigma) \quad \text{for bubble equilibrium,} \quad (33)$$

where $\Delta \equiv \bar{k}/(1-\alpha)(1-\Omega(\beta, \bar{z}))$. The relationship between the employment rate with and without bubble is described in Figures 3 (a) and (b). Equations (32) and (33) determine the equilibrium of employment rate. Therefore, we obtain the following proposition.

Proposition 1: *If the equilibrium employment rate is over a threshold level $\hat{\sigma}$ in (30), then bubbles can exist at equilibrium; if not, bubbles cannot exist.*

Moreover, we summarize the determinants of employment rate in the following lemma.

Lemma 1: *An increase in the search cost (\bar{k}) decreases the employment rate. An increase in the R&D cost ($\bar{\eta}$) has a negative effect on the employment rate in the non-bubble equilibrium, while it has no effect on the employment rate in the bubble equilibrium. Increases in unemployment benefit rate (\bar{z}) and the bargaining power of the worker (β) decrease the employment rate in the bubble equilibrium, while, under the non-bubble economy, they increase the employment rate for $\Omega(\beta, \bar{z}) < \Omega^*$ and decrease it for $\Omega(\beta, \bar{z}) > \Omega^*$.*

Proof. See Appendix F.

An increase in \bar{k} is captured by the down shift of right-hand side of (32) and (33), which leads to a negative effect on the employment rate. This is because an increase in search cost decreases the entry of firms with vacancies, which decreases the labor market tightness

and causes the employment rate to fall. An increase in $\bar{\eta}$ decreases the left hand side of (32), and, then, the employment rate decreases in non-bubble equilibrium. The effect of R&D cost increases the relative interest rate to growth rate, which, in turn, decreases the expected current value of profit. As such, the entry of firms with vacancies decreases and the employment rate falls. Since under the bubble regime the growth rate always equals the interest rate, the R&D cost has no impact on the determinant of employment. Analogous to the case of \bar{k} , an increase in $\Omega(\beta, \bar{z})$, which increases by β or \bar{z} , decreases the right hand side of (32) and (33), making the employment rate decrease. Additionally to this effect, an increase in $\Omega(\beta, \bar{z})$ has a positive effect on the growth rate through an increase in household income, which shifts the left hand side of (32) downward, thereby increasing the employment rate under the non-bubble economy. As shown in Appendix F, in the low (high) range of $\Omega(\beta, \bar{z})$, unemployment benefit rate (\bar{z}) and the bargaining power of the worker (β) have a positive (negative) effect on the employment rate, as the positive effect (income effect) dominates the negative effect (entry cost effect).

4.3. Dynamics of boom and bubble collapse

The economy will be in a non-bubble or bubble regime when the equilibrium employment rate is lower or higher, respectively, than the threshold level. As pointed out by Aliber and Kindleberger (2015), the term “bubble” foreshadows the end of an economic bubble. If the cause of bubble bursting stems from the realization of a sunspot, then the bubble equilibrium shifts to the non-bubble equilibrium under the bubble regime, which leads to higher economic growth. Therefore, the bubble burst caused by a sunspot creates a crowding-out effect, a negative relationship between bubbles and growth (Grossman and Yanagawa, 1993). In our framework, we follow the approach of the bubble boom-bust by Brunnermeier and Oehmke (2015), who point out that an initial boom in asset bubbles is often triggered by fundamentals.¹⁴ Our model shows that labor market frictions can lead to a bubbly steady state (i.e., a bubble regime). Therefore, our framework focuses on the boom-bust of asset bubbles caused by changes in fundamental variables, such as labor market conditions or R&D production technologies.

As such, we can examine the boom and bust of bubbles through regime shifts resulting from changes in parameter conditions. If changes in policies or parameters cause a decrease in

¹⁴ See Farhi and Tirole (2012) for a discussion on the boom and bust properties of bubbles; two types of causes lead asset bubbles.

the employment rate (Lemma 1; e.g., a rise in search cost or a fall in R&D technology), the economy will shift from a bubble regime to a non-bubble regime. We also find that equilibrium employment E^B in Figure 3 (b) changes to E^{NB} in Figure 3(a). Moreover, a bubble burst is accompanied by a decrease in employment rate and economic growth rate. Then, as shown in Figure 2, the output growth rate is always higher under the bubble regime than under the non-bubble regime, even when bubbles occur.

Furthermore, using comparative dynamics in response to parameter changes, we can analyze the dynamic properties of the boom and bust of a bubble in the phase diagram. We consider a rise in search cost (\bar{k}), which leads to a decreased employment rate in the steady state. From (26) and (27) with (24), the σ locus shifts downward and the b locus shifts upward.¹⁵ Therefore, if changes in search cost occur under bubble equilibrium, bubbles can suddenly burst and both employment and growth rates converge to a lower steady state. Conversely, a decrease in search cost can lead to a bubble boom, where both employment rate and economic growth rate converge to a high steady state. The dynamic behavior of a bubble burst is shown in Figure 4 (a) and bubble boom in Figure 4 (b).

On the effect of production technologies, a fall in R&D productivity (an increase in $\bar{\eta}$) has no effect on the employment rate under bubble steady state equilibrium (Lemma 1), while it decreases the region of the bubble regime (an increase in the threshold level $\hat{\sigma}$ from Proposition 1). Subsequently, from (26) and (27) with (24), both the σ and b loci decrease.¹⁶ Therefore, if a negative shock of R&D productivity occur and the threshold level exceeds the employment rate under bubble equilibrium, bubbles can suddenly burst and employment rate and growth rate converges to a lower steady state under non-bubble equilibrium.

These results are formally stated in the following proposition.

Proposition 2: *If policies or parameters change to cause a decrease (an increase) in the employment rate under bubble economy (under non bubble economy), asset bubbles can burst (boom) immediately, and the employment rate converges to a lower (higher) equilibrium, which, in turn, leads to a lower (higher) growth rate.*

¹⁵ See Appendix G for the mathematical derivation of the changes in σ and b loci: $\partial\Gamma/\partial\bar{k} < 0$ and $\partial\Phi/\partial\bar{k} > 0$.

¹⁶ See Appendix G for the mathematical derivation of the changes in σ and b loci: $\partial\Gamma/\partial\bar{\eta} < 0$ and $\partial\Phi/\partial\bar{\eta} < 0$.

Figure 5 summarizes the dynamic paths of bubbles, employment rate, and economic growth after bubble burst.

5. Conclusion

In this paper, we developed an overlapping generations model with labor market friction and examined the conditions for bubbles. We showed theoretical relationships between bubbles, economic growth, and employment. In contrast to previous studies, we introduced labor market frictions into an endogenous growth model, so that the interest rate depends on labor market conditions. Allowing for unemployment, fluctuations induced by the labor market determine the type of regime that the economy will be under for both non-bubble and bubble equilibria. Based on our finding that bubbles can (cannot) occur when the equilibrium employment rate is high (low), and the interest and economic growth rates are high (low), we conclude that policies that have a positive impact on the labor market (e.g., a decrease in the search cost) can improve employment and place the economy under a bubble regime. This, in turn, will raise both the interest rate and the economic growth rate.

Appendices

Appendix A: The derivation of the equation (21)

From (15) and (1), the output of a firm can be expressed by

$$y_t = w_t + \frac{(1+r_t)k_{t-1}}{q(\theta_{t-1})} + p_t x_t N_t. \quad (\text{A1})$$

Based on equation (A1), and using (12), $\theta_{t-1} \equiv v_{t-1}/L$, $X_t \equiv x_t \sigma_t L$, and the fact that the number of firms in the final goods sector are equal to the number of successful matches $\sigma_t L$, we can obtain the aggregate the output $Y_t \equiv y_t \sigma_t L$ as follows:

$$Y_t \equiv y_t \sigma_t L = w_t \sigma_t L + (1+r_t)k_{t-1} v_{t-1} + p_t X_t N_t. \quad (\text{A2})$$

Therefore, the market-clearing condition (20) for final goods is expressed in the following manner:

$$w_t \sigma_t L + (1+r_t)v_{t-1}k_{t-1} + p_t X_t N_t = C_t + N_t X_t + I_t^R + v_t k_t. \quad (\text{A3})$$

Using $\pi_t^x = (p_t - 1)X_t$, $\eta = \bar{\eta}L$, (7), (8), and (9), we obtain the following expression:

$$S_t + p_t^B M + (1 + r_t) \upsilon_{t-1} k_{t-1} + (p_t - 1) X_t N_t = (1 + r_t) S_{t-1} + p_t^B M + D_t (N_{t+1} - N_t) + \upsilon_t k_t$$

$$\Leftrightarrow S_t - D_t N_{t+1} - \upsilon_t k_t = (1 + r_t) [S_{t-1} - D_{t-1} N_t - \upsilon_{t-1} k_{t-1}].$$

Because initial assets are given by $S_{-1} = D_{-1} N_0 + \upsilon_{-1} k_{-1}$, we obtain (18) $t > 0$ for any period.

Appendix B: The derivation of (23)

Dividing equation (21) by LN_t and substituting equations (8), (18), (22), and $k_t = \bar{k} y_t$ into (21) yields the growth rate of varieties $g_t \equiv (N_{t+1} - N_t) / N_t$ as follows:

$$1 + g_t = \frac{1}{\bar{\eta}} \left[\Omega \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha) \sigma_t - \theta_t \bar{k} \alpha^{\frac{2\alpha}{1-\alpha}} - b_t \right]. \quad (\text{B1})$$

From (19) with (12), the following condition can be obtained:

$$(1 - \alpha)(1 - \Omega) \sigma_{t+1} \frac{1 + g_t}{1 + r_{t+1}} = \theta_t \bar{k}. \quad (\text{B2})$$

After using (B1) and (B2) with (10) to eliminate $\theta_t \bar{k}$, we obtain (23) as follows:

$$\begin{aligned} 1 + g_t &= \frac{1}{\bar{\eta}} \left[\Omega \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha) \sigma_t - (1 - \Omega) \alpha^{\frac{2\alpha}{1-\alpha}} (1 - \alpha) \sigma_{t+1} \frac{1 + g_t}{1 + r_{t+1}} - b_t \right] \\ \Leftrightarrow (1 + g_t) \left[1 + \frac{1 - \Omega}{\alpha} \frac{r(\sigma_{t+1})}{1 + r(\sigma_{t+1})} \right] &= \frac{1}{\bar{\eta}} \left[\frac{\Omega}{\alpha} \bar{\eta} r(\sigma_t) - b_t \right] \\ \Leftrightarrow 1 + g_t &= \frac{\frac{1}{\bar{\eta}} \left[\frac{\Omega}{\alpha} \bar{\eta} r(\sigma_t) - b_t \right]}{\left[1 + \frac{1 - \Omega}{\alpha} \frac{r(\sigma_{t+1})}{1 + r(\sigma_{t+1})} \right]}. \end{aligned}$$

Appendix C: The property of $\sigma(\sigma_t, b_t)$

Using (23), Equation (24) can be rewritten as

$$\begin{aligned} \frac{1 + r(\sigma_{t+1})}{1 + g(\sigma_{t+1}, \sigma_t, b_t)} &= \frac{1}{\Delta} q(\sigma_{t+1}) \Leftrightarrow \\ \bar{\eta} \left\{ 1 + r(\sigma_{t+1}) + \frac{1 - \Omega}{\alpha} r(\sigma_{t+1}) \right\} \frac{1}{q(\sigma_{t+1})} &= \frac{1}{\Delta} \left[\frac{\Omega}{\alpha} \bar{\eta} r(\sigma_t) - b_t \right]. \quad (\text{C1}) \end{aligned}$$

Totally differentiating (C1) leads to

$$[A]d\sigma_{t+1} = \left[\frac{1}{\Delta} \frac{\Omega}{\alpha} \bar{\eta} r' \right] d\sigma_t + \left[\frac{-1}{\Delta} \right] db_t,$$

$$\begin{aligned} \text{where } A &\equiv \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_{t+1}} - \frac{q'(\sigma_{t+1})}{\Delta} \\ &= \bar{\eta} r' \left(1 + \frac{1-\Omega}{\alpha} \right) \frac{1}{q(\sigma_{t+1})} + \bar{\eta} \left\{ 1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha} r(\sigma_{t+1}) \right\} \frac{-q'(\sigma_{t+1})}{q(\sigma_{t+1})^2} > 0, \end{aligned}$$

and $r' = \frac{1}{\bar{\eta}}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$ is positive and constant. Therefore, we have

$$\frac{\partial \sigma_{t+1}}{\partial \sigma_t} = \frac{1}{A} \left[- \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t} \right] = \frac{1}{A} \left[\frac{1}{\Delta} \frac{\Omega}{\alpha} \bar{\eta} r' \right] > 0, \quad (\text{C2})$$

$$\frac{\partial \sigma_{t+1}}{\partial b_t} = \frac{1}{A} \left[- \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial b_t} \right] = \frac{1}{A} \left[\frac{-1}{\Delta} \right] < 0. \quad (\text{C3})$$

Appendix D: The slope of the σ and b loci

First, we consider the slope of the σ locus. From (26), we have this slope is as follows:

$$\frac{db_t}{d\sigma_t} = \Gamma'(\sigma_t) = \bar{\eta} \left[\frac{\Omega}{\alpha} r' - \frac{\Delta}{q(\sigma_t)} \left(r' + \frac{1-\Omega}{\alpha} r' \right) + \frac{q'(\sigma_t)\Delta}{q(\sigma_t)^2} \left(1 + r(\sigma_t) + \frac{1-\Omega}{\alpha} r(\sigma_t) \right) \right].$$

Furthermore, we obtain the second derivatives are as follows:

$$\frac{d^2 b_t}{d\sigma_t^2} = \Gamma''(\sigma_t) = \frac{\bar{\eta}\Delta}{q^3} \left[2q'q \left(r' + \frac{1-\Omega}{\alpha} r' \right) + \left[-2(q')^2 + q''q \right] \left(1 + r + \frac{1-\Omega}{\alpha} r \right) \right].$$

Then, the condition $\Gamma''(\sigma_t) < 0$ holds as long as the probability $q(\cdot)$ is not too convex. We assume the functional form $q(\cdot)$ to satisfy $\Gamma''(\sigma_t) < 0$. In addition to the above properties, using σ locus, $\sigma_{t+1}(\sigma_t, b_t) = \sigma_t$, we have

$$\frac{\partial \sigma_{t+1}}{\partial b_t} db_t = \left[1 - \frac{\partial \sigma_{t+1}}{\partial \sigma_t} \right] d\sigma_t.$$

Therefore, the slope of the σ locus is positive when $\partial\sigma_{t+1}/\partial\sigma_t > 1$, and negative when $\partial\sigma_{t+1}/\partial\sigma_t \in (0,1)$.

Next, we consider the slope of the b locus. Totally differentiating (27) gives

$$\left[1 + \left(1 + \frac{1-\Omega}{\alpha}\right)\bar{\eta}r' \frac{\partial\sigma_{t+1}}{\partial b_t}\right] db_t = \left[\frac{\Omega}{\alpha} - \left(1 + \frac{1-\Omega}{\alpha}\right) \frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right] \bar{\eta}r' d\sigma_t. \quad (D1)$$

Using (C2), (C3), and the condition $q(\sigma_{t+1})/\Delta = 1$ in positive bubble equilibrium ($b_t > 0$), each coefficient of db_t and $d\sigma_t$ is given by

$$\left[1 + \left(1 + \frac{1-\Omega}{\alpha}\right)\bar{\eta}r' \frac{\partial\sigma_{t+1}}{\partial b_t}\right] = \frac{1}{A}\bar{\eta}\left\{1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha}r(\sigma_{t+1})\right\} \frac{-q'(\sigma_{t+1})}{q(\sigma_{t+1})^2} > 0, \quad (D2)$$

$$\left[\frac{\Omega}{\alpha} - \left(1 + \frac{1-\Omega}{\alpha}\right) \frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right] \bar{\eta}r' = \frac{1}{A}\bar{\eta}r'\bar{\eta}\frac{\Omega}{\alpha}\bar{\eta}\left\{1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha}r(\sigma_{t+1})\right\} \frac{-q'(\sigma_{t+1})}{q(\sigma_{t+1})^2} > 0. \quad (D3)$$

Therefore, the slope of the b locus, $b_t = \Phi(\sigma_t)$, is positive; $db_t/d\sigma_t = \Phi'(\sigma_t) > 0$ in $b_t > 0$.

Appendix E: Dynamic stability

Totally differentiating (24) and (25) leads to

$$d\sigma_{t+1} = \frac{1}{A} \left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t} \right] d\sigma_t + \frac{1}{A} \left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t} \right] db_t, \quad (E1)$$

$$db_{t+1} = b \left[\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}} \frac{\partial\sigma_{t+1}}{\partial\sigma_t} + \frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t} \right] d\sigma_t + \left[\frac{1+r_{t+1}}{1+g_t} + b \left[\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}} \frac{\partial\sigma_{t+1}}{\partial b_t} + \frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t} \right] \right] db_t. \quad (E2)$$

Then, the Jacobian matrices of this system are as follows:

$$\begin{bmatrix} \frac{1}{A} \left[-\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t} \right] & \frac{1}{A} \left[-\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial b_t} \right] \\ b \left(\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_{t+1}} \frac{\partial \sigma_{t+1}}{\partial \sigma_t} + \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t} \right) & \frac{1+r_{t+1}}{1+g_t} + b \left(\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_{t+1}} \frac{\partial \sigma_{t+1}}{\partial b_t} + \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial b_t} \right) \end{bmatrix}.$$

First, we consider the stability of equilibrium under non-bubble regime ($b = 0$). The following conditions are satisfied around the steady state (E^{NB}):

$$\frac{\partial \sigma_{t+1}}{\partial \sigma_t} = \frac{1}{A} \left[-\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t} \right] > 0, \quad \frac{\partial \sigma_{t+1}}{\partial b_t} = \frac{1}{A} \left[-\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial b_t} \right] < 0, \quad \frac{\partial b_{t+1}}{\partial \sigma_t} = 0, \quad \frac{\partial b_{t+1}}{\partial b_t} = \frac{1+r}{1+g^N} > 1.$$

Furthermore, around the steady state at the point E^{NB} in Figure 3 (a), we have

$$\frac{\partial \sigma_{t+1}}{\partial \sigma_t} < 1 \Leftrightarrow \frac{-q'(\sigma)}{\Delta} > \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_{t+1}} + \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t}. \quad (\text{E3})$$

We denote the trace and determinant of the Jacobian matrices as T and D , respectively. Additionally, the eigenvalues are denoted as λ_j ($j = 1, 2$), and the characteristic polynomial is expressed as $\xi(\lambda) \equiv \lambda^2 - T\lambda + D$. Under non-bubble regime, $\xi(0) > 0$ and $\xi(1) < 0$ are obtained. It is well known (Azariadis, 1993; Chapter 6) that the steady state is a saddle if the relations $\xi(0) > 0$ and $\xi(1) < 0$ hold simultaneously. Therefore, the steady state (E^{NB}) under non-bubble regime is stable and a saddle.

Next, we consider the equilibrium under bubble regime, non-bubble equilibrium (E^N) and bubble equilibrium (E^B). Around the steady state under the bubble regime with non-bubble equilibrium (E^N), the following conditions are satisfied:

$$\frac{\partial \sigma_{t+1}}{\partial \sigma_t} = \frac{1}{A} \left[-\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t} \right] < 1, \quad \frac{\partial \sigma_{t+1}}{\partial b_t} = \frac{1}{A} \left[-\frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial b_t} \right] < 0, \quad \frac{\partial b_{t+1}}{\partial \sigma_t} = 0, \quad \frac{\partial b_{t+1}}{\partial b_t} = \frac{1+r}{1+g^N} < 1,$$

using (E3). As such, $\xi(0) > 0$, $\xi(1) > 0$, and $T < 2$ are obtained. Therefore, the steady state (E^N) under the bubble regime is a sink.

Around the steady state under the bubble regime with bubble equilibrium (E^B), the following conditions are satisfied:

$$\frac{\partial \sigma_{t+1}}{\partial \sigma_t} = \frac{1}{\hat{A}} \frac{\partial g_t}{\partial \sigma_t} > 0, \quad \frac{\partial \sigma_{t+1}}{\partial b_t} = \frac{1}{\hat{A}} \frac{\partial g_t}{\partial b_t} < 0,$$

$$\frac{\partial b_{t+1}}{\partial \sigma_t} = \frac{1}{1+g} b \left[\left(\frac{\partial r_{t+1}}{\partial \sigma_{t+1}} - \frac{\partial g_t}{\partial \sigma_{t+1}} \right) \frac{\partial \sigma_{t+1}}{\partial \sigma_t} - \frac{\partial g_t}{\partial \sigma_t} \right],$$

$$\frac{\partial b_{t+1}}{\partial b_t} = 1 + \frac{1}{1+g} b \left[\left(\frac{\partial r_{t+1}}{\partial \sigma_{t+1}} - \frac{\partial g_t}{\partial \sigma_{t+1}} \right) \frac{\partial \sigma_{t+1}}{\partial b_t} - \frac{\partial g_t}{\partial b_t} \right],$$

where $\hat{A} \equiv -\frac{q'(\sigma)}{q'(\sigma)}(1+g) + \frac{\partial r_{t+1}}{\partial \sigma_{t+1}} - \frac{\partial g_t}{\partial \sigma_{t+1}} > 0$. Then, we obtain $\xi(1) \equiv 1 - T + D$ and

$\xi(-1) \equiv 1 + T + D$ as:

$$\xi(-1) = -\frac{b}{1+g} \left[H \frac{\partial \sigma_{t+1}}{\partial b_t} - \left(1 - \frac{\partial \sigma_{t+1}}{\partial \sigma_t} \right) \frac{\partial g_t}{\partial b_t} \right] > 0,$$

$$\xi(1) = 2 + 2 \frac{\partial \sigma_{t+1}}{\partial \sigma_t} + \frac{b}{1+g} \left[H \frac{\partial \sigma_{t+1}}{\partial b_t} - \left(1 - \frac{\partial \sigma_{t+1}}{\partial \sigma_t} \right) \frac{\partial g_t}{\partial b_t} \right] < 0,$$

$$\text{where } H \equiv \frac{\partial r_{t+1}}{\partial \sigma_{t+1}} - \frac{\partial g_t}{\partial \sigma_{t+1}} - \frac{\partial g_t}{\partial \sigma_t} = \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_{t+1}} + \frac{\partial \left(\frac{1+r_{t+1}}{1+g_t} \right)}{\partial \sigma_t} < 0.$$

Therefore, the steady state with bubbles (E^B) under bubble regime is stable and a saddle, since $\xi(-1) > 0$ and $\xi(1) < 0$ hold simultaneously.

Appendix F: The proof of Lemma 1

Under bubble regime, totally differentiating (33) gives

$$-q'(\sigma)d\sigma = -\frac{\partial \Delta}{\partial \bar{k}} d\bar{k} - \frac{\partial \Delta}{\partial \Omega} \frac{\partial \Omega}{\partial \beta} d\beta - \frac{\partial \Delta}{\partial \Omega} \frac{\partial \Omega}{\partial \bar{z}} d\bar{z},$$

where $\Delta \equiv \bar{k}/(1-\alpha)(1-\Omega)$ and $\Omega \equiv \beta/(1-(1-\beta)\bar{z})$. Thus, we obtain

$$\bar{k} \uparrow, \beta \uparrow, \bar{z} \uparrow \Rightarrow \sigma^B \downarrow.$$

Under non-bubble economy, substituting (31) into (32) leads to

$$\left(\frac{1+r}{1+g^N} = \right) \frac{\alpha}{\Omega} \frac{1+r(\sigma)}{r(\sigma)} + \frac{1-\Omega}{\Omega} = \frac{1}{\Delta} q(\sigma). \quad (\text{F1})$$

Totally differentiating (F1) gives

$$\bar{A} d\sigma = \frac{-1}{\Delta^2} \frac{\partial \Delta}{\partial \bar{k}} q(\sigma) d\bar{k} + [\Xi] \left(\frac{\partial \Omega}{\partial \beta} d\beta + \frac{\partial \Omega}{\partial \bar{z}} d\bar{z} \right) - \frac{\partial \left(\frac{1+r}{1+g^N} \right)}{\partial \bar{\eta}} d\bar{\eta},$$

$$\text{where } \bar{A} \equiv \left[\frac{-q'(\sigma)}{\Delta} + \frac{\partial \left(\frac{1+r}{1+g^N} \right)}{\partial \sigma} \right] > 0, \quad \frac{\partial \left(\frac{1+r}{1+g^N} \right)}{\partial \bar{\eta}} > 0,$$

$$\begin{aligned} \Xi &\equiv \left[-\frac{\partial \left(\frac{1+r}{1+g^N} \right)}{\partial \Omega} + \frac{-1}{\Delta^2} \frac{\partial \Delta}{\partial \Omega} q(\sigma) \right] = \frac{1}{\Omega^2} \left(\alpha \frac{1+r}{r} + 1 \right) - \frac{1}{1-\Omega} \left(\frac{\alpha}{\Omega} \frac{1+r}{r} + \frac{1-\Omega}{\Omega} \right) \\ &= \frac{1}{\Omega^2} \left[\left(\alpha \frac{1+r}{r} + 1 \right) - \frac{\Omega}{1-\Omega} \left(\alpha \frac{1+r}{r} + 1 - \Omega \right) \right]. \end{aligned}$$

We can easily confirm that $\Xi > 0$ when Ω approaches to 0, and $\Xi < 0$ when Ω approaches to 1.

Then, the threshold value of $\Omega = \Omega^*$ to satisfy $\Xi = 0$ is

$$\Omega^* = \psi - \sqrt{\psi^2 - \psi}, \quad \text{where } \psi \equiv \alpha \frac{1+r}{r} + 1.$$

Thus, we obtain

$$\bar{k} \uparrow, \bar{\eta} \uparrow \Rightarrow \sigma^N \downarrow,$$

$$\beta \uparrow, \bar{z} \uparrow \Rightarrow \sigma^N \uparrow (\downarrow) \text{ if } \Omega(\beta, \bar{z}) < (>) \Omega^*.$$

Appendix G: Comparative dynamics: \bar{k} and $\bar{\eta}$

From (26), we obtain the following conditions:

$$\frac{\partial \Gamma(\sigma_t)}{\partial \bar{k}} = -\frac{\bar{\eta}}{q(\sigma_t)} \left(1 + r(\sigma_t) + \frac{1-\Omega}{\alpha} r(\sigma_t) \right) \frac{\partial \Delta}{\partial \bar{k}} < 0.$$

$$\frac{\partial \Gamma(\sigma_t)}{\partial \bar{\eta}} = -\frac{\Delta}{q(\sigma_t)} < 0.$$

Using (D1), totally differentiating (27) gives

$$\frac{\partial \Phi(\sigma_t)}{\partial \bar{k}} = \frac{-1}{\Theta} \bar{\eta} r' \left(1 + \frac{1-\Omega}{\alpha} \right) \frac{\partial \sigma_{t+1}}{\partial \bar{k}}, \quad (\text{G1})$$

$$\frac{\partial \Phi(\sigma_t)}{\partial \bar{\eta}} = \frac{-1}{\Theta} \left[1 + \bar{\eta} r' \left(1 + \frac{1-\Omega}{\alpha} \right) \frac{\partial \sigma_{t+1}}{\partial \bar{\eta}} \right], \quad (\text{G2})$$

where $\Theta \equiv \left[1 + \left(1 + \frac{1-\Omega}{\alpha} \right) \bar{\eta} r' \frac{\partial \sigma_{t+1}}{\partial b_t} \right] > 0$ from (D2).

Totally differentiating (C1) leads to the following conditions:

$$\frac{\partial \sigma_{t+1}}{\partial \bar{k}} = \frac{1}{A} \frac{-1}{\Delta^2} \frac{\partial \Delta}{\partial \bar{k}} \left(\frac{\Omega}{\alpha} \bar{\eta} r(\sigma_t) - b_t \right) < 0, \quad (\text{G3})$$

$$\frac{\partial \sigma_{t+1}}{\partial \bar{\eta}} = \frac{1}{A} \frac{-1}{q(\sigma_{t+1})} < 0. \quad (\text{G4})$$

Using (G3) and (G4), we obtain the sign of (G1) and (G2) as follows:

$$\frac{\partial \Phi(\sigma_t)}{\partial \bar{k}} = \frac{1}{\Theta} \bar{\eta} r' \left(1 + \frac{1-\Omega}{\alpha} \right) \frac{1}{A} \frac{1}{\Delta^2} \frac{\partial \Delta}{\partial \bar{k}} \left(\frac{\Omega}{\alpha} \bar{\eta} r(\sigma_t) - b_t \right) > 0,$$

$$\frac{\partial \Phi(\sigma_t)}{\partial \bar{\eta}} = \frac{-1}{\Theta} \frac{1}{A} \bar{\eta} \left(1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha} r(\sigma_{t+1}) \right) \frac{-q'(\sigma_{t+1})}{(q(\sigma_{t+1}))^2} < 0.$$

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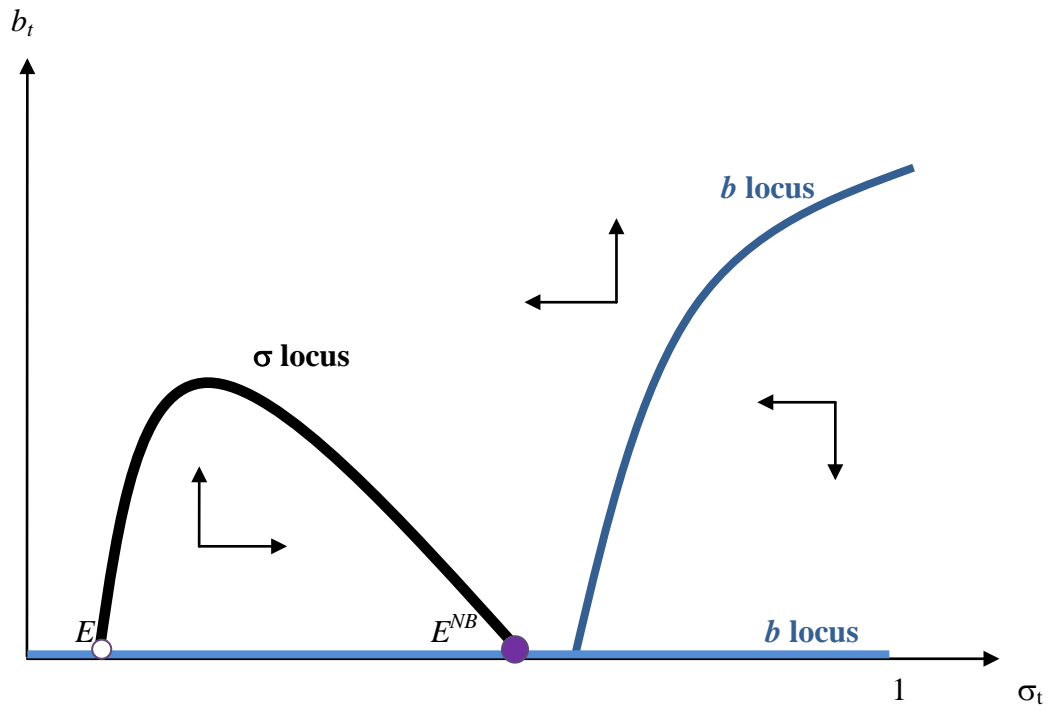


Figure 1. Phase diagram: (i) non-bubble regime

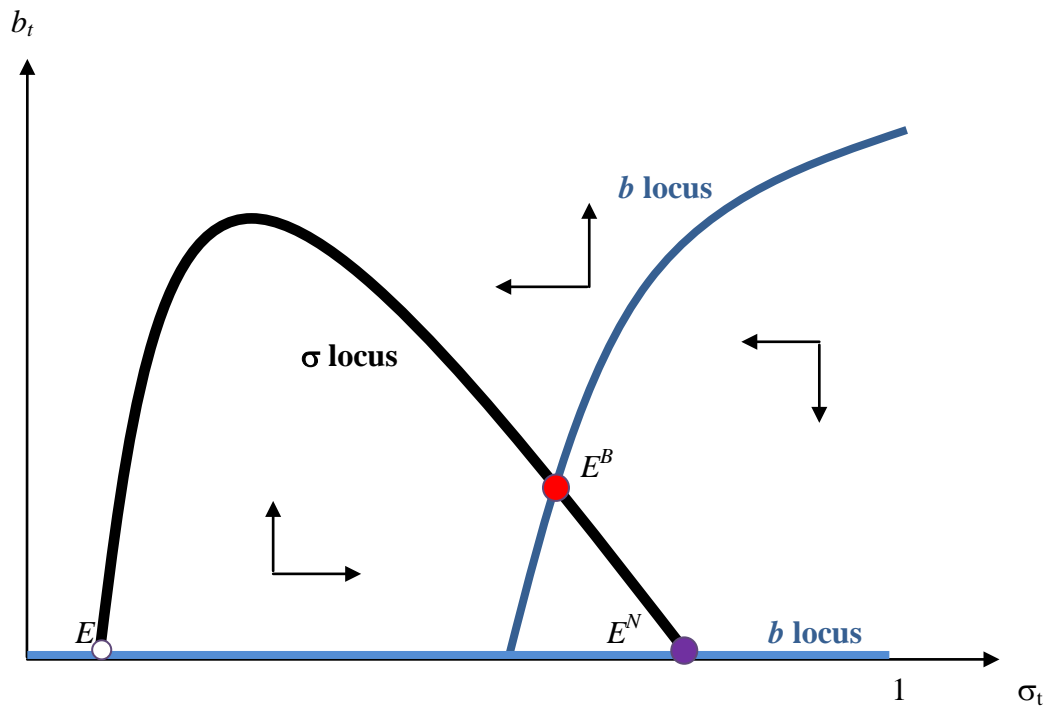


Figure 1. Phase diagram: (ii) bubble regime

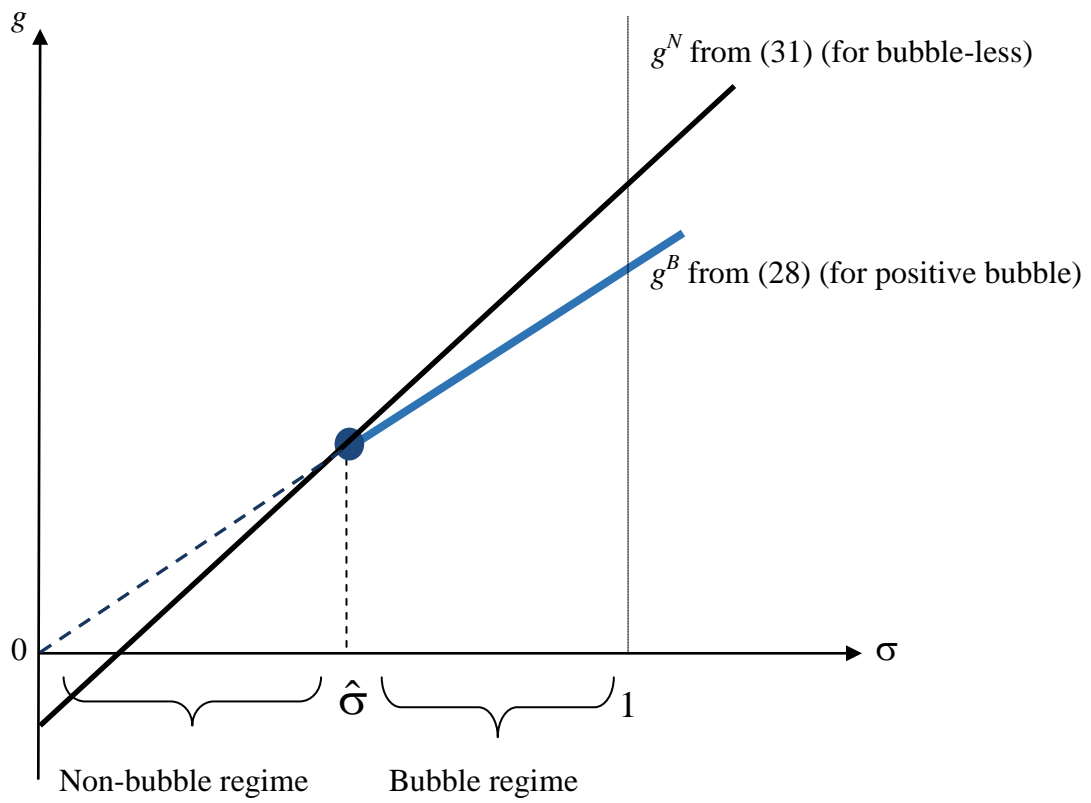


Figure 2. Employment and growth rate.

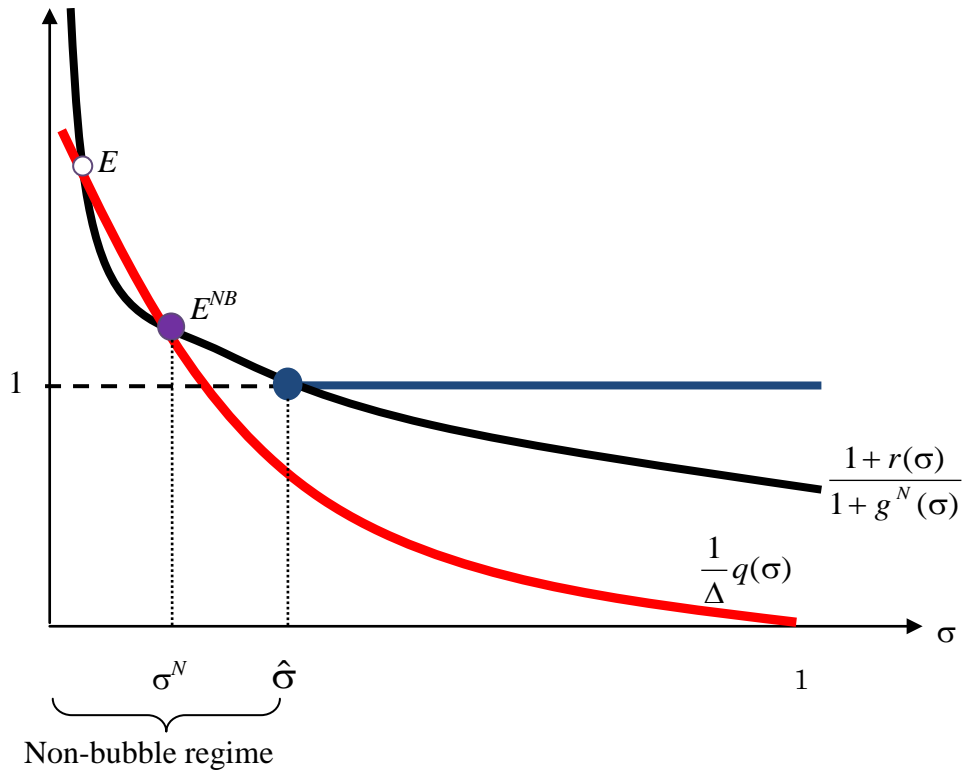


Figure 3 (a): The equilibrium of employment rate under non-bubble regime

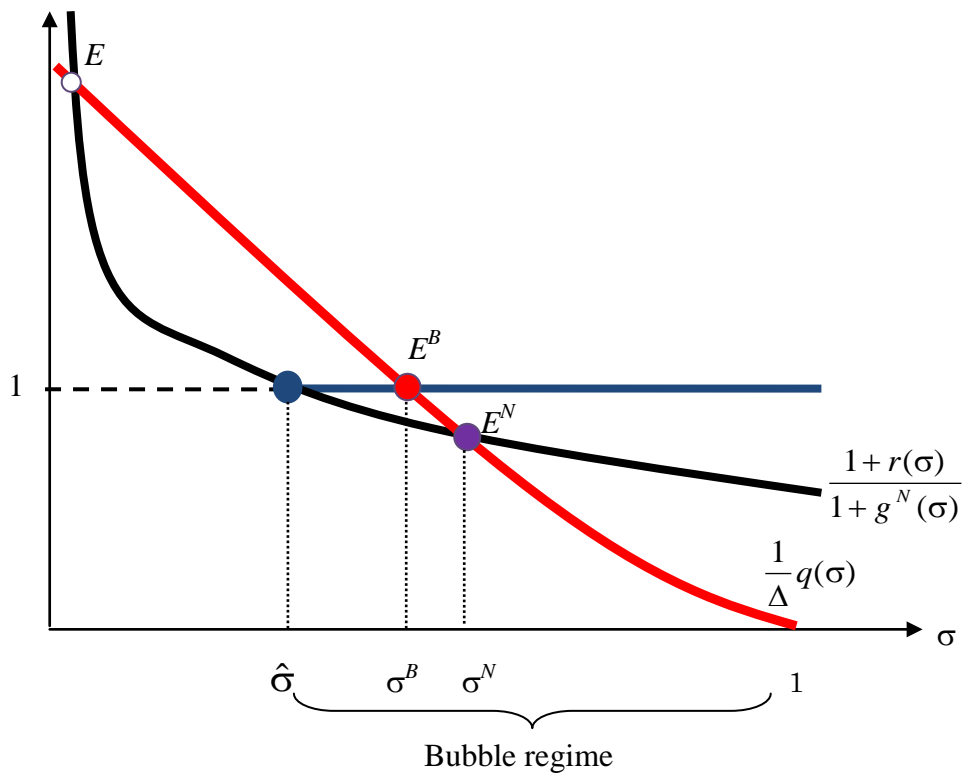


Figure 3 (b): The equilibrium of employment rates under bubble regime

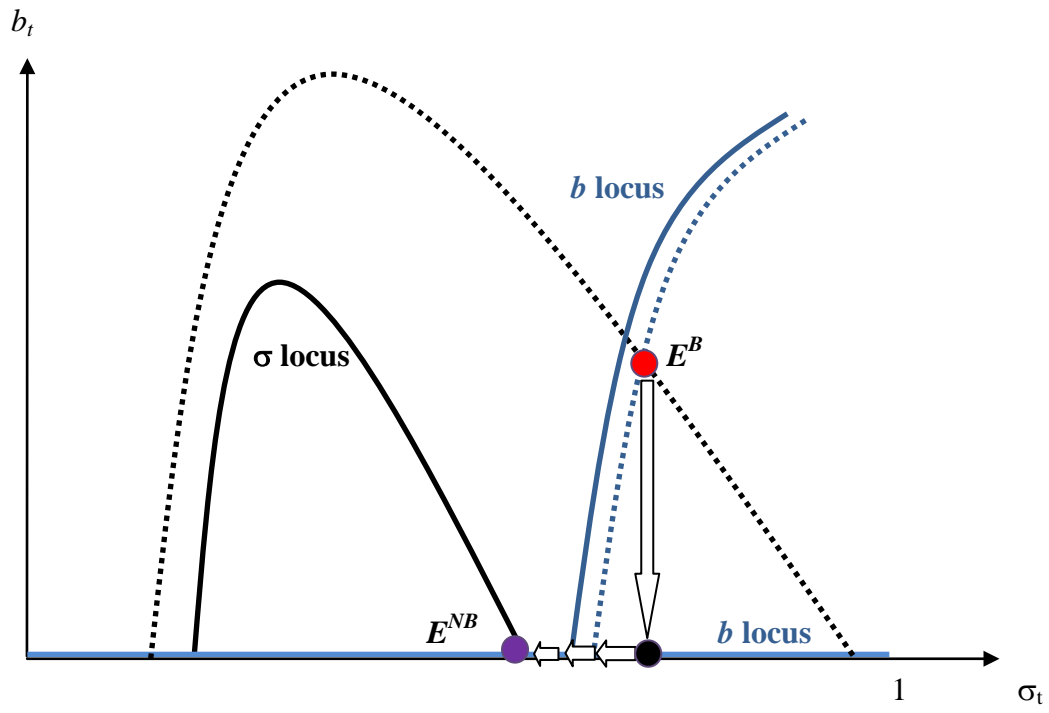


Figure 4 (a). The pattern of bubble bust

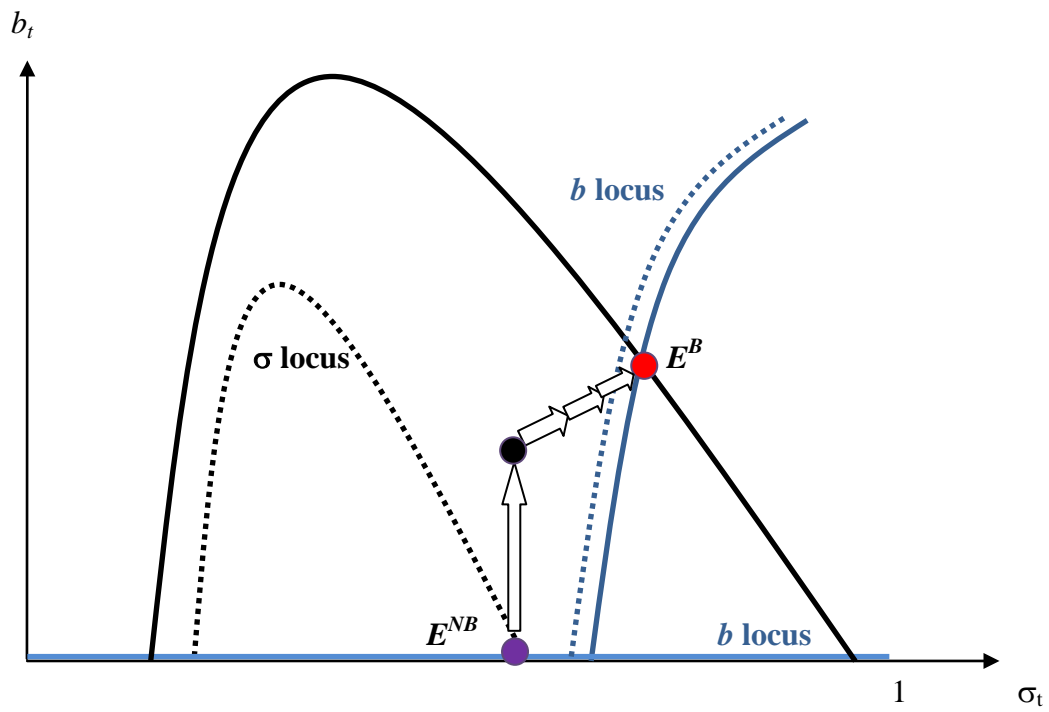


Figure 4 (b). The pattern of bubble boom

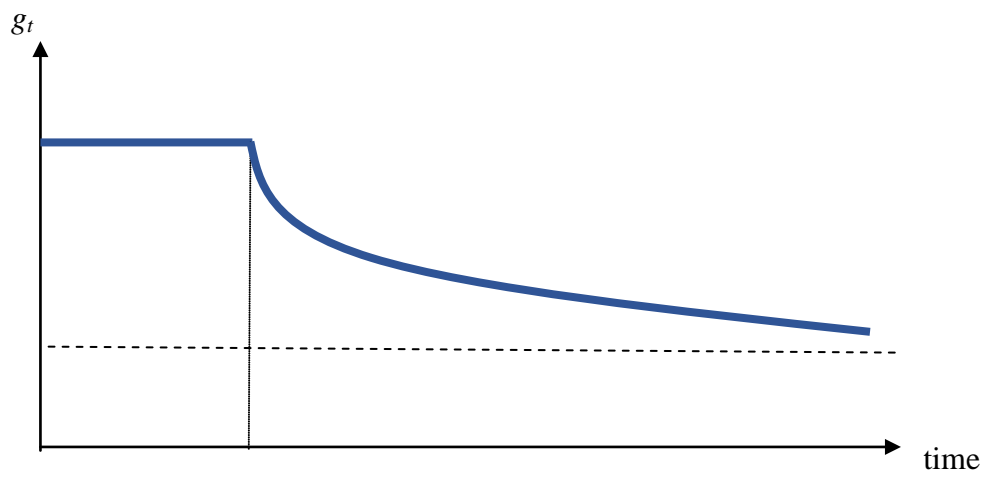
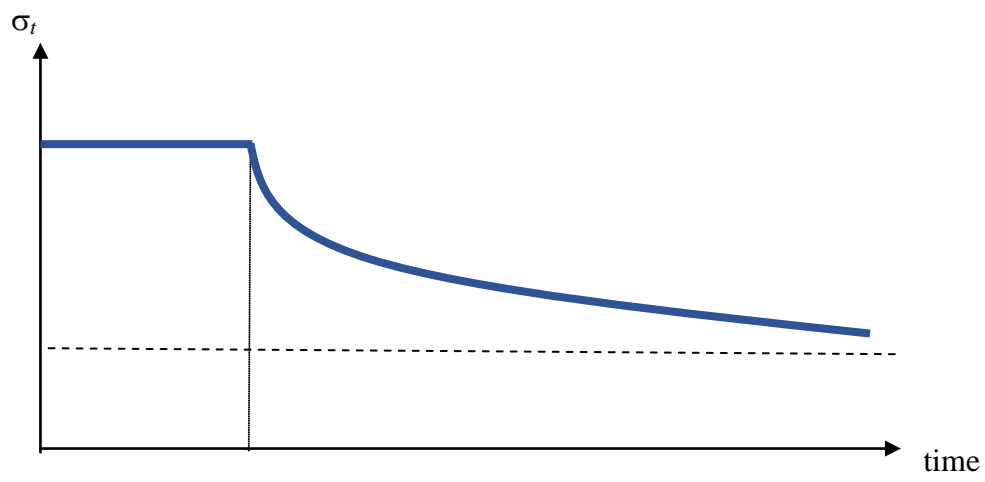
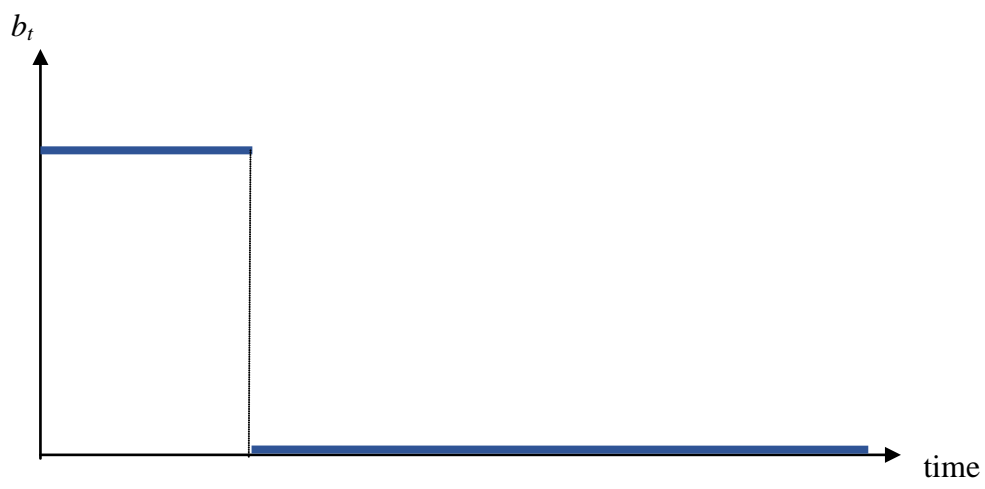


Figure 5. Dynamic paths after bubble bust