Minimum Wage, Economic Growth and Preference for Consumption

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Abstract

We build a simple overlapping generations model with minimum wage. Many eariler papers do not enough consider household's preference in the context of minimum wage and ecoomic growth under a dynamic framework Therefore our study focus on the household's preference for consumption. Results show whether an increase in minimum wage promotes economic growth or not depends on the preference for consumption.

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1 Intoroduction

It is said that minimum wage contribute to avoid poverty and to keep the minimu standard of living. But it is also said that minimu wage has negative effect on macro economic performance, for example employment and GDP. There are many previous papers discuss economic growth and minimum wage. Cahuc and Michel(1996) use a overlapping generations model with human capital accumulation and discuss econmic growth and welfare increase or not with minimum wage. Ravn and Sorensen(1999) consider the realationship economic growth and minimum wage with skill formation. Askenazy(2003) introduce R & D sector and open economy with Ramsey model, and consider long-run effect by minimum wage. Barany(2016) also constructs a two sector endgenous growth model with R & D sector. Irmen and Wigger(2006) consider two-country with overlapping genrations model. Flaschel and Greiner(2011) develop a growth model with minimum wage and dual labor market.

Many ealier papers consider two sectors, for example skilled/unskilled sector, production sector/R & D sector, and domestic/foreign sector. Fanti and Gori(2011) use a simple one sector over lapping generations model with minimum wage and show the conditions that minimum wage promotes economic growth. Following Fanti and Gori(2011), we aloso use a simple one sector overlapping generations model with minimum wage and show the positive reraltion between economic growth and minimum wage. But there are two major differences compared to Fanti and Gori(2011). Firstly, we use more general utility function. Fanti and Gori(2011) use log utility function, therefore the savings and growth rate depends on only wage income. On the other hands, we use more general form than log utility function to foucus on household's preference for consumption and the effect by interest rate. The preference for consumption is not enough considered in eariler papers. Secondly, we omit unemployment benefit. In Fanti and Gori(2011), unemployment benefit has a crusial role to their result. But we explain an increase in minimum wage can promote economic growth with household's preference.

The reminder of our paper is composed as follows. The second chapter build a model. The thirds chapter concludes our model.

2 Model

We use a simple one sector overlapping generations model. In this economy, there exists two agents, households and firms. Households experience two periods and they work in young period. The population size, N_t, is unity.

2.1 Firms

The firms produce final goods with labor and capital stock. The basic framework of production is similar to Fanti and Gori(2011). The production technology of firm $i = 1...I$ is described by

$$Y_{i,t} = K_{i,t}^{\alpha}(A_{i,t}L_{i,t})^{1-\alpha} \tag{1}$$

where $K_{i,t}$, $A_{i,t}$,$L_{i,t}$ are respectively the capital,the productivity which defined $A_{i,t} \equiv \lambda^{\frac{1}{1-\alpha}} \frac{K_t}{N_t}$ $(\lambda > 0)$, and the labor inputs. $\alpha \in (0, 1)$ is a constant parameter. In this economy, all firms are identical. Therefore we denote $K_{i,t} = K_t$, $A_{i,t} = A_t$ and $L_{i,t} = L_t$. We assume households have one unit time and the population size is unity, therefore if the unemployment rate is denoted u_t, L_t is described by $L_t = (1 - u_t)N_t$ where N_t is the population size. In our paper, u_t is not the number of unemployment but the unemployment time. This asumption is adopted in Fanti and Gori(2010). Assuming fully depriciation, the factor demand is

$$w_{m,t} = (1-\alpha)K_t^{\alpha}A_t^{1-\alpha}L_t^{-\alpha} \tag{2}$$
$$(1+r_t) = \alpha K_t^{\alpha-1}(A_tL_t)^{1-\alpha} \tag{3}$$

where $w_{m,t}$ is the minimum wage and r_t is the interest rate. In this economy, the minimu wage is prevaild. We assume the rerationship between the minimum wage and the competetive wage denoted $w_{c,t}$ as

$$w_{m,t} = \mu w_{c,t} \tag{4}$$

where $\mu > 1$ is the constant mark up ratio. The competetive wage is described by

$$w_{c,t} = (1-\alpha)K_t^{\alpha}A_t^{1-\alpha} \tag{5}$$

Using (2), (4) and (5), the unemployment rate is described by

$$u_t = 1 - \mu^{-\frac{1}{\alpha}} \tag{6}$$

From (6) the unemployment rate decreases with an increase μ.

2.2 Households

Households live two periods and have one unit of time with each period. When they are young, they supply one unit of time to the labor market. The utility function of households is defined by

$$U_t = \frac{c_{y,t}^{1-\theta} - 1}{1-\theta} + \beta\frac{c_{o,t+1}^{1-\sigma} - 1}{1-\sigma} \qquad \theta > 0, \;\; \sigma > 0 \tag{7}$$

where $c_{y,t}$ is the consumption in young period, $c_{o,t}$ is the consumption in old period, β is the discount factor and θ, σ are the positive parameter. If $\theta \to 1$ and $\sigma \to 1$, the utility function changes to log utility form. Therfore the utility function in our model is more general form rather than log utility form. The budget constraints of households are described by

$$c_{y,t} + s_t = (1-u_t)w_{m,t} \tag{8}$$
$$c_{o,t+1} = (1+r_{t+1})s_t \tag{9}$$

where s_t is the saving and $1-u_t$ is the working time. The optimal saving is

$$[(1-u_t)w_{m,t} - s_t]^{-\theta} = \beta(1+r_{t+1})^{1-\sigma}s_t^{-\sigma} \tag{10}$$

2.3 Dynamics

Nextly we described the dynamics in this economy. The capital stock in period $t+1$ depends on the saving at t period. The rerationship between K_{t+1} and s_t is described by

$$K_{t+1} = s_t \tag{11}$$

We assume the unemployment rate as the fraction of time, therfore the capital stock at $t+1$ period is equal to the saving at t period and this assumption contributes to slove the model analytically.

The wage income and interst rate in equiribrium are

$$(1-u_t)w_{m,t} = (1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}}K_t \tag{12}$$
$$1 + r_t = \alpha\lambda\mu^{\frac{\alpha-1}{\alpha}} \tag{13}$$

Using (11), (12) and (13), the dynamics of this economy is

$$K_{t+1} + \beta^{-\frac{1}{\theta}}(\alpha\lambda\mu^{\frac{\alpha-1}{\alpha}})^{\frac{\sigma-1}{\theta}} K_{t+1}^{\frac{\sigma}{\theta}} = (1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} K_t \tag{14}$$

In the steady state $K_{t+1} = K_t$ is satisfied. Denote K^* is the steady state capital stock, the stady state capital stock is

$$K^* = \Big[\frac{(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} - 1}{\beta^{-\frac{1}{\theta}}(\alpha\lambda\mu^{\frac{\alpha-1}{\alpha}})^{\frac{\sigma-1}{\theta}}}\Big]^{\frac{1}{\frac{\sigma}{\theta}-1}} \tag{15}$$

If $(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} > 1$ is assumed, there exists a uniqe steady state. In equation (15), $\alpha\lambda\mu^{\frac{\alpha-1}{\alpha}}$ denotes interest rate. If the utility function is given by log form, the savings and the steady state capital stock does not depend on interest rate because the substitution effect and the income effect are canceled out. But our model assumes utility function as (7), interest rate also affects savings and steady state capital stock. Nextly we discuss the stability in the steady state. If $dK_{t+1}/dK_t < 1$ is satisfied in the steady state, the dynamics is loccaly stable. The capital accumulation is denoted (14), therefore dK_{t+1}/dK_t is shown as follows

$$\frac{dK_{t+1}}{dK_t} = \frac{(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}}}{1 + \frac{\sigma}{\theta}\beta^{-\frac{1}{\theta}}(\alpha\lambda\mu^{\frac{\alpha-1}{\alpha}})^{\frac{\sigma-1}{\theta}} K_{t+1}^{\frac{\sigma}{\theta}-1}} \tag{16}$$

In the steady state, $K_{t+1} = K_t = K^*$ is hold. Using (14) the following equation is established.

$$\beta^{-\frac{1}{\theta}}(\alpha\lambda\mu^{\frac{\alpha-1}{\alpha}})^{\frac{\sigma-1}{\theta}} K^{*\frac{\sigma}{\theta}-1} = (1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} - 1 \tag{17}$$

Substitute (17) to (16), the locally stable condition is described as follows

$$\sigma > \theta \tag{18}$$

This condition is intuitive. If [1]$\sigma > \theta$ is hold, households prefer to the consumption in young period rather than the consumption in old period with capital accumulation. Therefore the capital stock converge to K^*. From (15) and (18), the following proposition is established.

Proposition.1
If $(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} > 1$ and $\sigma > \theta$ are hold, there exists a locally stable steady state

Finally we discuss the relation between minimum wage and economic growth. From (15), K^* is denoted

$$\begin{aligned} K^* &= \Big(\frac{\beta^{\frac{1}{\theta}}}{(\alpha\lambda)^{\frac{\sigma-1}{\theta}}}\Big)^{\frac{1}{\frac{\sigma}{\theta}-1}} \phi^{\frac{1}{\frac{\sigma}{\theta}-1}} \\ \phi &\equiv \frac{(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} - 1}{\mu^{\frac{\alpha-1}{\alpha}\frac{\sigma-1}{\theta}}} \end{aligned}$$

We assume $(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} > 1$ and $\sigma > \theta$ to exist the locally stable steady state. The derivative K^* with respect to μ gives

$$\frac{dK^*}{d\mu} = \Big(\frac{\beta^{\frac{1}{\theta}}}{(\alpha\lambda)^{\frac{\sigma-1}{\theta}}}\Big)^{\frac{1}{\frac{\sigma}{\theta}-1}} \frac{1}{\frac{\sigma}{\theta}-1} \phi^{\frac{1}{\frac{\sigma}{\theta}-1}-1} \frac{d\phi}{d\mu}$$

[1]See Watanabe and Yasuoka(2009)

The sign of $dK^*/d\mu$ is depend on the sign of $d\phi/d\mu$. If $d\phi/d\mu > 0$ is satisfied then $dK^*/d\mu > 0$ is hold(vice versa). The condition $d\phi/d\mu > 0$ is hold is described as follows

$$\frac{\sigma - 1}{\theta}\Big(1 - \frac{1}{(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}}}\Big) > 1 \tag{19}$$

Therefore the following proposition is establlished.

Proposition.2
If $\frac{\sigma-1}{\theta}\Big(1 - \frac{1}{(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}}}\Big) > 1$ is hold, an increase in minimum wage increases the steady state capital stock. Therefore, economic gowth is promoted until the eoconomy reaches at a new steady state.

The sign of the brackets in (19) is always positive becasue we assume $(1-\alpha)\lambda\mu^{\frac{\alpha-1}{\alpha}} > 1$ to exist the steady state capital stock. Therefore prop.2 is hold or not depends on θ and σ. The intuition of this propositin is described as follows. An increase in μ always decreases wage income and interest rate. A decrease in wage income always leads to decrease savings and the steady state capital stock. In our model the utility function is given by (7), savings depend on not only wage income but also interest rate because the substitution effect and the income effect are not canceled out. If savings increases enough with a decrease in interest rate, the economic growth is promoted.

3 Conclusion

It is said that minimum wage contribute to avoid poverty and to keep the minimu standard of living. But it is also said that minimu wage has negative effect on macro economic performance, for example employment and GDP. Many earlier papers which discuss economic growth and minimum wage do not enough consider the household's preference for consumption. Therefore we extend Fanti and Gori(2011) and provide the complementary model. Our study consider the preference for consumption with more general utility function and the interest rate affects the captital stock in the steady state. Results show whether an increase in minimum wage promotes economic growth or not depends on the preference for consumption.

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