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Abstract

We construct a continuous-time overlapping-generations model with labor market friction in order to examine the relationship between bubbles, economic growth, and unemployment. We show that the existence of bubbles is contingent upon the equilibrium unemployment rate. Asset bubbles can (not) exist when unemployment is low (high), which leads to higher (lower) interest rates and economic growth through labor market efficiency. Hence, economic growth under the bubble regime where bubbles can exist is higher than that under the non-bubble regime where bubbles cannot exist. Furthermore, policy or parameter changes that have a positive effect on the labor market shift the economy from a non-bubble regime to a bubble regime.

Keywords: overlapping generations, endogenous growth, labor market friction, unemployment

JEL Classification Number: J64, O41, O42

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1. Introduction

Bubbles have occurred many times in the past and have had major impacts on the macro economy. In particular, bubbles have been frequently observed when economic activity is booming and the growth rate of GDP is high (Martin and Ventura, 2012; Farmer and Schelnast, 2013, ch. 6). Also, empirical studies show that asset bubbles are accompanied by a reduction in the unemployment rate (Phelps, 1999; Fitoussi et al., 2000). Since there are no studies that address the theoretical interaction between bubbles, growth and unemployment, the purpose of this paper is to examine the relationship analytically.

In his seminal study, Tirole (1985) examines the condition of the existence of bubbles on intrinsically useless assets in an overlapping generations model. Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) extend Tirole's model to an endogenous growth framework. They re-examine the conditions that are necessary for bubbles to exist and show the relationships between bubbles and economic growth. In their studies, when a bubble arises in the economy, it diverts savings from capital accumulation and retards economic growth. For an alternative perspective on bubbles, Olivier (2000) considers bubbles not as useless assets, but as assets tied to capital goods. As such, he is able to show that bubbles can increase economic growth. However, these do not consider the possibility of unemployment.

There are theories that state that equilibrium unemployment occurs as a result of friction in the labor market. Diamond (1982), Mortensen (1982), and Pissarides (1985) develop search and matching models of unemployment (which scholars have since applied to a wide variety of fields¹). Eriksson (1997) introduces labor market frictions into the standard dynamic optimizing (Ramsey) model with capital stock externalities through learning by doing in order to ensure long-run economic growth. He then examines the effects of various policies (capital taxes and unemployment benefits) on both economic growth and unemployment.² However, unlike in an overlapping-generations model, rational bubbles

¹ See Pissarides (2000) for an introduction to search friction models.

² See Bean and Pissarides (1993), Aghion and Howitt (1994), Caballero and Hammour (1994), and Haruyama and Leith (2010) for other models of the relationship between growth and unemployment that address labor market frictions.

cannot be generated in an infinitely lived, representative-agent model (Ramsey-type model) (Tirole, 1985; Santos and Woodford, 1997).³

To the best of our knowledge, no studies have used an endogenous growth model to analyze the connection between bubbles and unemployment. In order to fill this void, we develop a theoretical framework with which to examine the conditions necessary for bubbles to exist in an economy with endogenous unemployment, and we explore the relationships between bubbles, economic growth, and unemployment.⁴ In order to do so, we adopt Eriksson's (1997) endogenous growth and labor market friction approach, and merge the Ramsey framework with Weil's (1989) continuous-time overlapping-generations model.

In our model, where unemployment stems from labor market friction, labor market efficiency is reflected in the interest rate because the marginal productivity of capital is influenced by it. Then, because asset returns are related to the interest rate, the existence of bubbles depends on conditions in the labor market. As such, we find that the equilibrium unemployment rate is a key factor in the existence of bubbles; when it is below a certain level and interest rate is high, bubbles asset can exist. When the conditions are satisfied for bubbles to exist, we say that the economy is in a "bubble regime"; conversely, when it is not possible for them to exist, we say that the economy in a "non-bubble regime." In a bubble regime, there are multiple equilibria, such that a steady state can exist either with bubbles or without. We show that bubbles divert savings away from physical capital and lower the output growth rate, which is a common finding in the literature. On the other hand, when we compare bubble regimes to non-bubble regimes, we find that the output growth rate is always higher under the former than the latter.

Additionally, with our model we can examine the effects of labor market policy or parameter changes on bubbles, economic growth, and unemployment. For example, we find that, because unemployment benefits raise the value of unemployment, they have a negative impact on employment (which is a standard conclusion among models with search friction).

³ As Santos and Woodford (1997) point out, it is hard to generate rational bubbles in an infinitely lived agent model without market frictions. For an alternative approach that focuses on the friction in the financial market see Hirano and Yanagawa (2010) and Martin and Ventura (2012); these authors examine the existence of bubbles and show that they can be growth enhancing or growth impairing depending on the restrictiveness of the collateral constraint.

⁴ Miao et al. (2012) and Kocherlakota (2011) present studies that are similar to our own. Miao et al. (2012) investigate the relationship between unemployment and stock market bubbles in an economy with labor market friction and financial market friction. Kocherlakota (2011) assumes that output is determined by household demand, and as such, he does not consider the firm's behavior in an economy with matching frictions and bubbles. However, these studies do not consider economic growth endogenously.

Thus, if unemployment benefits are reduced, then the employment rate should increase and the labor market should become more efficient, which would raise the interest rate and consequently shift the economy from a non-bubble regime to a bubble regime. In this case, there would be a negative relationship between unemployment and economic growth.⁵

The remainder of this paper is organized as follows: Section 2 outlines the features of the model. Section 3 describes the steady-state equilibrium with and without bubbles, and compares the effects of policy and parameter changes under the two regimes on bubbles, economic growth, and unemployment. The final section summarizes our findings and concludes the paper.

2. The Model

There are many infinitely lived dynasties in the economy. At each point in time, a new and identical infinitely lived dynasty appears at rate n . The total population at time 0 is N_0 ; thus, the total population at time t is $N_t = N_0 e^{nt}$. Unless it is necessary, time notation is suppressed for the remainder of the paper.

2.1. Matching

In the labor market, unemployed workers and firms with vacant positions strive to find each other. We include mating frictions in the labor market in order to generate unemployment. By denoting the number of successful matches as f , this process can be described by the matching function

$$f(uN, \upsilon N),$$

where u and υ represent the unemployment rate and the vacancy rate, respectively. As such, uN is the number of unemployed workers and υN is the number of vacant jobs in the economy. Following the standard assumptions,⁶ the matching function is satisfied to be concave, homogeneous of degree one, and increasing in both of its arguments. If the tightness of the labor market is defined as

⁵ In fact, many empirical studies show a negative relationship between unemployment and economic growth (Ball and Moffitt, 2001; Muscatelli and Tirelli, 2001; Staiger et al., 2001; Tripier, 2006; Pissarides and Vallanti, 2007).

⁶ See Petrongolo and Pissarides (2001) for a discussion on matching functions.

$$\theta \equiv \frac{vN}{uN} = \frac{v}{u},$$

then the probability that a firm with vacancies will be matched to an unemployed worker can be given by the following property:

$$\frac{f(uN, vN)}{vN} = f\left(\frac{1}{\theta}, 1\right) \equiv q(\theta), \text{ where } \frac{\partial q(\theta)}{\partial \theta} < 0.$$

2.2. Firms

There are N identical firms in the economy. The number of firms is assumed to equal the number of consumers. The production function of firm j is described by

$$y_j = Ak_j^\alpha (\bar{k}l_j)^{1-\alpha}, \quad 0 < \alpha < 1,$$

where A , k_j , and l_j represent productivity, capital stock, and the number of workers employed by firm j , respectively. \bar{k} represents labor productivity, which is assumed to be driven by spillovers (similar to those proposed by Romer (1986) that emanate from a firm's accumulated investments per worker). In order to ensure the existence of a long-run growth path, it is assumed that \bar{k} takes the form $\bar{k} = \left(\int_0^N k_j dj\right)/N$, which represents the average capital stock.

In order to create matches, firm j must put out job vacancies. If we let v_j be the number of firm j 's vacancies, then $q(\theta)v_j$ workers are hired by firm j in every moment.⁷ Furthermore, firm j fires or loses workers at a rate of δl_j , where δ represents the exogenous separation rate. By summing these two flows, we find that the size of the labor force changes according to the equation

$$\dot{l}_j = q(\theta)v_j - \delta l_j. \quad (1)$$

In accordance with Eriksson (1997), Mortensen and Pissarides (1999), and Pissarides and Vallanti (2007), we assume that the cost to recruit workers for a vacancy is proportional to the wage rate, γw (where w is the wage rate and γ is a cost parameter).⁸ In order to simplify the model, we do not consider the adjustment cost of investment. Consequently, firm j 's profit maximization problem can be written as

⁷ As θ is given for all firms, the probability $q(\theta)$ is the same for all firms.

⁸ This assumption is required in order to ensure that there is a balanced growth path.

$$\max \int_t^\infty (y_j - rk_j - wl_j - \gamma wv_j) e^{-\int_t^i r, dt} di, \text{ subject to (1)}. \quad (2)$$

This inter-temporal maximization problem can be solved using the current-value Hamiltonian function $H = (y_j - rk_j - wl_j - \gamma wv_j) + \chi(q(\theta)v_j - \delta l_j)$, where χ is the current shadow value of labor. Assuming that the market shares are small enough that each firm takes average capital (\bar{k}) and market tightness (θ) as constant, the first-order conditions are $\partial H/\partial k_j = 0$, $\partial H/\partial v_j = 0$, and $\partial H/\partial l_j = r\chi - \dot{\chi}$. Combining these conditions yields:

$$\left(\frac{\partial y_j}{\partial k_j} = \right) \alpha A k_j^{\alpha-1} (\bar{k} l_j)^{1-\alpha} = r, \quad (3)$$

$$\left(\frac{\partial y_j}{\partial l_j} = \right) (1-\alpha) A k_j^\alpha \bar{k}^{1-\alpha} l_j^{-\alpha} = w + \frac{\gamma w}{q(\theta)} \left(r + \delta - \frac{\dot{w}}{w} + \frac{q'(\theta)\dot{\theta}}{q(\theta)} \right). \quad (4)$$

All firms are considered identical because of the symmetry in production technology, and as such, $\bar{k} = k_j = k$ in equilibrium. Also, the vacancy rate is equal to the number of firm j 's vacancies, $v \equiv \left(\int_0^N v_j dj \right) / N = v_j = v$. Thus, the tightness of the labor market is determined by the ratio of the number of vacancies per firm to unemployment rate, $\theta \equiv vN/uN = v/u$. Furthermore, from the definition of the employment rate, we get:

$$1 - u \equiv \left(\int_0^N l_j dj \right) / N = l. \quad (5)$$

As a result, per capita output can be written as

$$y = Ak(1-u)^{1-\alpha}. \quad (6)$$

Following the example of other studies that include search friction (Pissarides, 2000), we assume that wages are negotiated by the worker and the firm after they meet. When a match is made, the firm can put the worker into production and save on the vacancy cost. Hence, the upper bound on the wage is the marginal benefits of labor, which is determined by the marginal product and the marginal value of the saved vacancy cost $(\partial y/\partial l + \theta\gamma w)$.⁹ The lower bound, on the other hand, is a worker's opportunity income, that is, the unemployment benefit, λw (which is proportionate to the wage rate, $\lambda \in (0, 1)$). It is assumed that a

⁹ The upper bound on the wage can be derived as follows: $\partial(y - \gamma wv)/\partial l$ with $v = \theta u = \theta(1 - l)$. This is consistent with the marginal benefit of labor with an internalized unemployment rate.

negotiation between a firm and a worker results in a wage that is somewhere between these two extremes (Eriksson, 1997); such that $w = (1 - \beta)\lambda w + \beta(\partial y/\partial l + \theta\gamma w)$, where $\beta \in (0, 1)$ denotes the worker's bargaining power.¹⁰ Consequently, the wage rate can be expressed as follows:

$$w = \frac{\beta}{1 - (1 - \beta)\lambda - \beta\gamma\theta} \cdot \frac{\partial y}{\partial l}. \quad (7)$$

2.3. Households

We approach households in much the same way as Weil (1989). In our model, there are many infinitely lived households in the economy. Households do not have any assets when they first enter into the economy, though we assume that the first generation has a set of initial assets. We distinguish this first generation of households from those that are born at time 0 (denoted 0^+) by denoting them 0^- . The lifetime utility of a representative household in generation s is

$$\int_t^\infty \ln c(s, i) e^{-\rho(i-t)} di,$$

where $c(s, t)$ represents the consumption of generation s at time t , and ρ is the subjective discount rate. Each household allocates its total assets (z) between physical capital (k) and the bubble asset (m) (the bubble asset is an intrinsically useless paper asset, specifically money). Because of the arbitrage condition, the price of the bubble asset in terms of goods ($1/p$) must satisfy the condition $(\dot{1/p})/(1/p) = -\dot{p}/p = r$, or rather, the return on one unit of the bubble asset must be equal to r . By using this relation, we can obtain the following flow/stock budget constraint in terms of real goods:¹¹

$$\frac{dz(s, t)}{dt} = rz(s, t) + w(1 - u) + \lambda w u + v^I - \tau - c(s, t), \quad (8)$$

$$z(s, t) = k(s, t) + m(s, t),$$

where $z(s, t)$ represents the total asset holdings of generation s at time t , λw is the unemployment benefit, τ is a lump-sum tax, and v^I is the income from vacancies (defined as

¹⁰ This formulation of the wage equation is broadly used in the literature on unemployment that includes search friction (Aghion and Howitt, 1994; Caballero and Hammour, 1996; Eriksson, 1997). See Pissarides (2000) and Hall (2005) for discussions on the determination of the wage equation.

¹¹ See Appendix A for a detailed explanation of the derivation.

$v^I = \left(\int_0^N \gamma^{wv_j} dj \right) / N = \gamma^{wv}$). In order to eliminate any uncertainty regarding employment, we assume that each household has a large number of members. The first-order maximization conditions are given by

$$\frac{dc(s,t)}{dt} = (r - \rho)c(s,t), \quad (9)$$

and the transversality condition is given by $\lim_{t \rightarrow \infty} z(s,t)e^{-\int_s^t r_i di} = 0$. Using Equations (8) and (9), we can obtain the following consumption function:

$$c(s,t) = \rho[z(s,t) + h_t], \quad (10)$$

$$\text{where } h_t \equiv \int_t^\infty \left(w_i(1 - u_i) + \lambda w_i u_i + v_i^I - \tau_i \right) e^{-\int_t^i r_i dt} di.$$

2.4. Government

The government supplies the useless paper asset, B , which is priced in terms of goods at $1/p$. We define the real value of the supply of this asset as $M \equiv B/p$. The government gives this asset to the first generation (0) at time 0 and continues to supply it to each household at constant rate of $\mu (= \dot{B}/B)$. Since the government's real revenue is the sum of the lump-sum tax (τN) and the bubble expansion (μM), and its expenditure is the unemployment benefit ($\lambda w u N$), we can write its budget constraint as

$$\tau N + \mu M = \lambda w u N. \quad (11)$$

2.5. Market clearing conditions

We define the aggregate variable X_t as follows (Weil, 1989):

$$X_t \equiv x(0^{-1}, t)N_0 + \int_0^t x(s,t)ne^{ns}N_0 ds.$$

By combining Equations (8), (9), and (10) with the above definition, we are able to formulate the following economy-wide dynamic equations:¹²

$$\dot{C} = (r - \rho + n)C - npZ, \quad (12)$$

$$\dot{Z} = rZ + \omega N - C, \quad (13)$$

¹² Equation (12) is derived by using Equations (9) and (10), and $z(0, 0) = 0$. Equation (13) is derived by using Equation (8). Equation (14) is derived by using $M = B/p$ with $\dot{B}/B = \mu$ and $-\dot{p}/p = r$.

$$\dot{M} = (\mu + r)M, \quad (14)$$

where $Z = K + M$ and $\omega \equiv w(1 - u) + \gamma wv + \mu M/N$.

The aggregate output is defined as $Y \equiv \int_0^N y_j dj = yN$. As such, by using Equation (6) we can get

$$Y = AK(1 - u)^{1-\alpha}. \quad (15)$$

Also, using Equations (13) and (14), we obtain the goods market equilibrium condition¹³

$$Y = C + \dot{K}. \quad (16)$$

2.6. The equilibrium conditions on a balanced growth path

In this subsection, we derive the conditions on a balanced growth path. Using Equations (1) and (5), in conjunction with the definition $v = \theta u$, we obtain $\dot{u} = -\theta q(\theta)u + \delta(1 - u)$.

Consequently, the following condition is satisfied in the steady state:

$$u = \frac{\delta}{\delta + \theta q(\theta)}, \quad (17)$$

where $\partial u / \partial \delta > 0$ and $\partial u / \partial \theta < 0$.¹⁴ Equation (17) represents the Beveridge curve; this implies that the unemployment rate rises when the separation rate increases or the labor market becomes less tight.

Also, based on Equation (3), and using the fact that $\bar{k} = k_j = k$, the interest rate can be given by

$$r = \alpha A(1 - u)^{1-\alpha}. \quad (18)$$

With this we can easily confirm that there is a negative relationship between the unemployment rate and the interest rate.

We define the growth rate of aggregate output ($Y (= yN)$) as $g \equiv \dot{Y}/Y$. Based on Equations (5), (6), and (7) and the fact that $\partial y / \partial l = (1 - \alpha)Ak l^{-\alpha}$, we find that (in a steady state where u and θ are constant) the wage growth rate equals the per capita output growth rate such that

¹³ See Appendix B for the derivation of (16)

¹⁴ Based on the matching function property provided in subsection 2.1, we obtain $\partial(\theta q(\theta))/\partial \theta > 0$, which implies $\partial u / \partial \theta < 0$.

$$\dot{w}/w = g - n. \quad (19)$$

As such, substituting Equations (7) and (19) into Equation (4) yields:

$$\left[\frac{(1-\beta)(1-\lambda)}{\beta\gamma} - \theta \right] q(\theta) = r + \delta - g + n. \quad (20)$$

We define the variables normalized by aggregate output as $c \equiv C/Y$ and $m \equiv M/Y$. Using the definitions (c, m) and the fact that $Z = K + M$, as well as Equations (12), (14), and (15) and recalling that the steady state levels of c and m are constant when $\dot{C}/C = \dot{K}/K = \dot{Y}/Y = g$ on a balanced growth path, we are able to derive the following equilibrium conditions:

$$(r - \rho + n - g)c - n\rho \left(\frac{1}{A(1-u)^{1-\alpha}} + m \right) = 0, \quad (21)$$

$$(\mu + r - g)m = 0. \quad (22)$$

By dividing Equation (16) by Y and using Equations (15) and (18), we obtain the following expression

$$c = 1 - g \frac{\alpha}{r}. \quad (23)$$

Based on Equations (23) and (18), and the requirement that the consumption-output ratio be positive ($c > 0$), the condition $g < r/\alpha = A(1-u)^{1-\alpha}$ must be satisfied. In the steady state, Equations (17), (18), (20), (21), (22), and (23) give the equilibrium values of c, m, g, r, u , and θ .

If we consider the case in which the population growth rate is zero ($n = 0$), as in a representative-agent (Ramsey) model with one dynasty (Eriksson, 1997), then we can easily confirm that $g = r - \rho$ for $c > 0$ (based on Equation (21)). Consequently, (based on equation (22)) the value of asset bubbles must become zero ($m = 0$). As a result, in a Ramsey-type representative-agent model, bubbles cannot exist in equilibrium.

3. The conditions for, and consequences of bubbles

3.1. The conditions for bubbles

In this section, we derive the conditions under which bubbles may exist in an economy. Under a bubble regime, there are multiple steady states, the positive bubble equilibrium ($m > 0$) and the bubble-less equilibrium ($m = 0$). Using Equation (22), we are able to obtain the aggregate output growth rate (g) with a positive bubble:

$$g = r + \mu. \quad (24)$$

In the positive bubble equilibrium, the output growth rate depends positively on the interest rate and the bubble's expansion rate.

The equilibrium bubble value can be found by substituting Equations (18), (23) and (24) into Equation (21), and then rearranging them:

$$m = \frac{\alpha}{n\rho r} \left[\left(\frac{r}{\alpha} - g \right) (n - \rho - \mu) - n\rho \right] = \frac{(n - \rho - \mu)(1 - \alpha)}{n\rho r} \left[r - \frac{\alpha}{1 - \alpha} \left(\mu + \frac{n\rho}{n - \rho - \mu} \right) \right]. \quad (25)$$

Given unemployment rate, u (which gives the interest rate from Equation (18)), both the aggregate output growth rate and the bubble satisfy Equations (24) and (25). Based on the first equality in Equation (25), a positive value of m must require $n - \rho - \mu > 0$ under the condition $r/\alpha > g$ for $c > 0$ from Equation (23). Also, from the second equality, we see that a bubble regime requires:

$$r > \bar{r} \equiv \frac{\alpha}{1 - \alpha} \left(\mu + \frac{n\rho}{n - \rho - \mu} \right). \quad (26)$$

Then, using $r = \alpha A(1 - u)^{1-\alpha}$ from (18), we obtain the condition of the unemployment rate for a bubble regime:

$$u < \bar{u} \equiv 1 - \left[\frac{1}{(1 - \alpha)A} \left(\mu + \frac{n\rho}{n - \rho - \mu} \right) \right]^{\frac{1}{1-\alpha}}. \quad (27)$$

Thus, we are able to attain the following proposition from Equation (27):

Proposition 1: *If Equation (27) is satisfied in equilibrium, then bubbles can exist in the economy; if not, then bubbles cannot exist.*

If the equilibrium unemployment rate is below a certain threshold (\bar{u}), and the interest rate is greater than \bar{r} , the bubbles asset can exist.

Under full employment ($u = 0$), since the interest rate is given by a production parameter: $r = \alpha A$, the existence of bubbles depends on the condition $\alpha A > \bar{r}$. With unemployment, however, the equilibrium level of unemployment plays a crucial role in the existence of bubbles. The following subsection examines the equilibrium unemployment rate in greater detail.

3.2. The properties of Non-Bubble regime

In this subsection, we consider the steady state equilibrium under a non-bubble regime. Using Equations (18), (21), and (23), we denote the growth rate in a non-bubble regime as g^{NB} , then define it as

$$g^{NB} = \Gamma(r) \Leftrightarrow (g - r - n + \rho) \left(g - \frac{r}{\alpha} \right) - n\rho = 0, \quad (28)$$

where $\Gamma(0) < 0$.¹⁵ In equilibrium $g < r + n - \rho$ must be satisfied because $g < r/\alpha$. Also, by taking the total derivative of Equation (28), we obtain the following property of $\Gamma(r)$:

$$\Gamma'(r) = \frac{\partial g}{\partial r} = \frac{(r/\alpha - g) + (r + n - \rho - g)/\alpha}{(r/\alpha - g) + (r + n - \rho - g)} > 1.$$

Furthermore, (using Equations (24) and (28)) we can confirm that at the threshold level \bar{r} (given in Equation (26)) the growth rate in a non-bubble regime is equivalent to the growth rate in a bubble regime; $g = \bar{r} + \mu = \Gamma(\bar{r})$, as shown in Figure 1.

Next, we consider the equilibrium unemployment rate, which determines the interest rate $r = r(u)$. We know from Equation (17) that θ is given by a function of $\theta = \theta(u; \delta)$; $\partial\theta/\partial u < 0$ and $\partial\theta/\partial\delta > 0$. Then, by substituting Equation (28) and $r = r(u)$ into Equation (20), we find that the equilibrium unemployment rate in a non-bubble regime (u^{NB}) is determined by

$$\Phi_L(u^{NB}; \beta, \lambda, \gamma, \delta) = \Phi_R(u^{NB}; \delta, n), \quad (29)$$

$$\text{where } \Phi_L(u^{NB}; \beta, \lambda, \gamma, \delta) \equiv \left[\frac{(1-\beta)(1-\lambda)}{\beta\gamma} - \theta(u^{NB}; \delta) \right] q(\theta(u^{NB}; \delta)),$$

$$\Phi_R(u^{NB}; \delta, n) \equiv r(u^{NB}) + \delta - g(r(u^{NB})) + n,$$

¹⁵ See Appendix C for the explicit expression for $\Gamma(r)$.

$$\Phi'_L(u) > 0, \text{ and } \Phi'_R(u) > 0.$$

By defining $\Psi(u) \equiv \Phi_L(u) - \Phi_R(u)$, we can obtain the following conditions: $\Psi(0) < 0$ and $\Psi(1) > 0$. Furthermore, if we assume that the equilibrium is unique, then $\Psi'(u) > 0$ ($\Phi'_L(u) > \Phi'_R(u)$) must be satisfied. Figure 2 (a) shows the equilibrium unemployment rate under a non-bubble regime at the point E^{NB} . Additionally, we can confirm that the intersection of $\Phi_R(u)$ and $\delta + n - \mu$ gives the threshold level of \bar{u} in Equation (27), since $g^{NB} = g = \bar{r} + \mu$ at the threshold. From Equations (18) and (28), we obtain the equilibrium interest and growth rates under the non-bubble regime, $r^{NB} = \alpha A(1 - u^{NB})^{1-\alpha}$ and $g^{NB} = \Gamma(r^{NB})$. This relationship is depicted at the point E^{NB} in Figure 3.

With Equation (29), we can examine the properties of the equilibrium level of u^{NB} depending on various parameters, including β , λ , γ , δ , and n . Consequently, we find that the partial derivatives of u^{NB} indicate

$$\begin{aligned} \frac{\partial u^{NB}}{\partial \beta} &= \frac{(1-\lambda)q}{\beta^2 \gamma \Psi'} > 0, & \frac{\partial u^{NB}}{\partial \lambda} &= \frac{(1-\beta)q}{\beta \gamma \Psi'} > 0, & \frac{\partial u^{NB}}{\partial \gamma} &= \frac{(1-\beta)(1-\lambda)q}{\beta \gamma^2 \Psi'} > 0, & (30) \\ \frac{\partial u^{NB}}{\partial \delta} &= \frac{1}{\Psi'} \left[1 + \left\{ q - \frac{\partial q}{\partial \theta} \left[\frac{(1-\beta)(1-\lambda)}{\beta \gamma} - \theta \right] \right\} \frac{\partial \theta}{\partial \delta} \right] > 0, & \frac{\partial u^{NB}}{\partial n} &= \frac{1}{\Psi'} > 0. \end{aligned}$$

Using Equation (30) in conjunction with (18) and (28), we get

$$\text{sign} \frac{\partial g^{NB}}{\partial i} = \text{sign} \frac{\partial r^{NB}}{\partial i} = \text{sign} -\frac{\partial u^{NB}}{\partial i}, \quad (\text{for } i = \beta, \lambda, \gamma, \delta, n). \quad (31)$$

Thus, we obtain the following proposition from Equations (30) and (31).

Proposition 2: *Under a non-bubble regime, the bargaining power of labor (β), unemployment benefits (λ), the vacancy cost (γ), the separation rate (δ), and the population growth rate (n) all serve to increase the unemployment rate, so that the equilibrium interest and output growth rates fall.*

Intuitively, greater bargaining power among workers (β) results in higher wages, this implies that the incentive for job creation would decrease and firms would reduce vacancies. Then, the tightness of the labor market ($\theta = v/u$) would decrease and the steady state unemployment rate would rise. The effects of the vacancy cost (γ) and unemployment benefits (λ) are the same as mentioned above. In addition to the above effect, a higher separation rate (δ) would

also shift the Beveridge curve and raise unemployment. These findings regarding the effects on the unemployment rate are in line with those of the standard search friction model (Pissarides, 2000).

Moreover, we can see from Equation (19) that the wage growth rate decreases as the population growth rate (n) increases. A decrease in the growth rate of wages means that future vacancy costs will be lower; hence, firms would postpone creating them. As such, the labor market would become more restrictive and, in turn, the unemployment rate would rise. As a result of this effect on the unemployment rate, the changes in the parameters $(\beta, \lambda, \gamma, \delta, n)$ that increase the unemployment rate would also have a negative effect on the interest and output growth rates (see Equations (18) and (28)).

3.3. The properties of Bubble regime

In this subsection, we analyze the properties of bubble regime. Under this regime, there are two equilibria: the positive bubble equilibrium (B) and the bubble-less equilibrium (N). We denote the variables in this regime as u_ι^B , r_ι^B , and g_ι^B where $\iota = (B, N)$. The variables in the bubble-less equilibrium are given by u_N^B , r_N^B , and g_N^B and are equivalent to those in the non-bubble regime described in the previous subsection. The bubble-less equilibrium unemployment rate (u_N^B) is given by $\Phi_L(u) = \Phi_R(u)$ and shown at the point E_N^B in Figure 2 (b). Then, from Equations (18) and (28), we obtain the equilibrium interest and growth rates, $r_N^B = \alpha A(1 - u_N^B)^{1-\alpha}$ and $g_N^B = \Gamma(u_N^B)$. This relationship is depicted at the point E_N^B in Figure 3.

Next, we consider the positive bubble equilibrium unemployment rate. When there is a positive bubble, the growth rate can be given by $g_B^B = r + \mu$. By substituting this expression into Equation (20), we can obtain the equilibrium condition for the unemployment rate (u_B^B),

$$\Phi_L(u_B^B; \beta, \lambda, \gamma, \delta) = \delta + n - \mu. \quad (32)$$

As shown at the point E_B^B in Figure 2 (b), the equilibrium unemployment rate with a positive bubble is larger than that without a bubble; that is, $u_B^B > u_N^B$, which implies $r_B^B < r_N^B$. The positive bubble equilibrium interest and growth rates are $r_B^B = \alpha A(1 - u_B^B)^{1-\alpha}$ and $g_B^B = r_B^B + \mu$. This relationship is depicted at the point E_B^B in Figure 3.

By taking the partial derivatives of u_B^B we find

$$\frac{\partial u_B^B}{\partial \beta} = \frac{(1-\lambda)q}{\beta^2 \gamma \Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial \lambda} = \frac{(1-\beta)q}{\beta \gamma \Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial \gamma} = \frac{(1-\beta)(1-\lambda)q}{\beta \gamma^2 \Phi'_L} > 0, \quad (33)$$

$$\frac{\partial u_B^B}{\partial \delta} = \frac{1}{\Phi'_L} \left[1 + \left\{ q - \frac{\partial q}{\partial \theta} \left[\frac{(1-\beta)(1-\lambda)}{\beta \gamma} - \theta \right] \right\} \frac{\partial \theta}{\partial \delta} \right] > 0, \quad \frac{\partial u_B^B}{\partial n} = \frac{1}{\Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial \mu} = \frac{-1}{\Phi'_L} < 0.$$

Then, using Equation (33) in conjunction with (18) and (24), we also find

$$\text{sign} \frac{\partial g_B^B}{\partial i} = \text{sign} \frac{\partial r_B^B}{\partial i} = \text{sign} -\frac{\partial u_B^B}{\partial i} < 0, \quad (\text{for } i = \beta, \lambda, \gamma, \delta, n), \quad (34)$$

$$\text{sign} \frac{\partial r_B^B}{\partial \mu} = \text{sign} -\frac{\partial u_B^B}{\partial \mu} > 0, \quad \frac{\partial g_B^B}{\partial \mu} = \frac{\partial r_B^B}{\partial \mu} + 1 > 0.$$

Consequently, we obtain the following proposition from Equations (33) and (34).

Proposition 3: *In a positive bubble equilibrium, the bargaining power of labor (β), unemployment benefits (λ), vacancy cost (γ), the separation rate (δ), and the population growth rate (n) raise the unemployment rate so that the equilibrium interest and output growth rates fall. The bubble expansion rate (μ), on the other hand, decreases the unemployment rate and increases the equilibrium interest and output growth rates.*

The effects of β , λ , γ , δ , and n on unemployment, the interest rate, and the output growth rate are all analogous to those in a non-bubble regime.

The bubble expansion rate (μ) has two positive effects on the output growth rate: The first occurs directly by means of an increase in asset holdings (asset effect) (Futagami and Shibata, 2000); this effect shifts the $g_B^B = r_B^B + \mu$ curve upward. Furthermore, this increase in economic growth raises wages growth, which in turn increases the number of operating vacancies so that the unemployment rate falls. The second effect stems from the improvement in employment; this induces economic growth through an increase in the interest rate. This effect moves the positive bubble economy along the g_B^B curve at interest rate r_B^B . Figure 4 depicts how the presence of bubbles makes E_B^B (bubble economy) approach E_N^B (bubble-less economy).

3.4. Comparison between regimes

The economy will be in a non-bubble regime (bubble regime) when the equilibrium unemployment rate is higher (lower) than the threshold level. Thus, when policies or parameters are changed in such a way that decreases the equilibrium unemployment rate (e.g., a decrease in unemployment benefits) the economy will shift from a non-bubble regime to a bubble regime. As shown in Figure 3, the output growth rate is always higher under a bubble regime than under a non-bubble regime, even if bubbles occur.

Additionally, under a bubble regime, a steady state equilibrium can be achieved in either the presence or the absence of bubbles (Tirole, 1985). As for equilibrium unemployment, the relationship $u_N^B < u_B^B < \bar{u}$ holds under bubble regimes. Thus, the conditions $r_N^B > r_B^B > \bar{r}$ and $g_N^B > g_B^B$ are satisfied. In regards to the growth rate, we find that bubbles create a crowd-out effect by reducing capital accumulation (Grossman and Yanagawa, 1993; Futagami and Shibata 2000).

The above properties can be formally restated as follows:

Proposition 4: *The growth rate (g^{NB}) in a non-bubble regime ($u > \bar{u}$) is lower than the growth rate (g_v^B) in a bubble regime ($u < \bar{u}$), that is, $g^{NB} < g_v^B$. Under a bubble regime, the growth rate is lower when there are bubbles (g_B^B) than when there are not (g_N^B), that is, $g_B^B < g_N^B$.*

4. Conclusion

In this paper, we developed a continuous-time overlapping-generation model with labor market friction and examined the conditions for bubbles. We showed theoretical relationships between bubbles, economic growth, and unemployment. In contrast to previous studies that only accounted for production technology in the calculation of the interest rate, we introduce labor market friction into our endogenous growth model, so that the interest rate depends on labor market conditions. This, in turn, determines the efficiency of labor in the production. Allowing for unemployment, fluctuations induced by the labor market determine the type of regime that the economy will be under. In particular, the lower the equilibrium unemployment rate is (when the interest rate is relatively high), the more likely it is that there will be a positive bubble equilibrium. Furthermore, labor market efficiency affects capital

accumulation by means of the marginal product of capital, so that economic growth is also deeply contingent upon the employment situation.

Based on our finding that bubbles can (not) occur when the equilibrium unemployment rate is low (high), and the interest and economic growth rates are high (low), we conclude that policies that have a positive impact on the labor market (e.g., a decrease in unemployment benefits) can improve employment and place the economy under a bubble regime. This, in turn, will raise both the interest rate and the economic growth rate.

Appendix

Appendix A: The derivation of the flow budget constraint (8)

We define bubble asset holdings in nominal terms as $b(s, t)$. Then, the flow budget constraint of generation s at time t is

$$[db(s, t)/dt]/p + dk(s, t)/dt = rk(s, t) + w(1 - u) + \lambda wu + v^J - \tau - c(s, t).$$

Since the real value of the bubble asset is given by $m(s, t) = b(s, t)/p$, we obtain $dm(s, t)/dt = [db(s, t)/dt]/p + rm(s, t)$ by using the arbitrage condition $-(\dot{p}/p) = r$. Substituting this into the above-equation yields

$$dm(s, t)/dt - rm(s, t) + dk(s, t)/dt = rk(s, t) + w(1 - u) + \lambda wu + v^J - \tau - c(s, t).$$

Therefore, using $z(s, t) = k(s, t) + m(s, t)$, we can obtain the flow budget constraint (8).

Appendix B: The derivation of the equilibrium condition in the goods market (16)

From Equations (13) and (14) and $Z = K + M$, we have

$$\dot{K} = rK + [w(1 - u) + \gamma wv]N - C. \quad (\text{B1})$$

Because there is perfect competition in the market for production factors firms receive zero profits. By using this information in conjunction with Equations (2) and (5), we find

$$Y = rK + [w(1 - u) + \gamma wv]N, \quad (\text{B2})$$

implying that total output is distributed among the capital income, the wage income, and the vacancy income. Substituting (B2) into (B1) yields the goods market equilibrium condition (16).

Appendix C: The explicit expression for $\Gamma(r)$

Using Equation (28), the quadratic equation for the growth rate (g) can be solved for the two following solutions:

$$g = \left[(r + n - \rho + r/\alpha) \pm \sqrt{(r + n - \rho + r/\alpha)^2 - 4((r + n - \rho)r/\alpha - n\rho)} \right] / 2. \text{ However, since it is}$$

required that $g < r/\alpha$ in order for c to take positive value (from Equation (23)), only one solution may be used:

$$g = \Gamma(r) \equiv \frac{1}{2} \left[(r - \rho + n + r/\alpha) - \sqrt{(r - \rho + n + r/\alpha)^2 - 4((r - \rho + n)r/\alpha - n\rho)} \right],$$

where $\Gamma(0) \equiv \left[(n - \rho) - \sqrt{(n - \rho)^2 + 4n\rho} \right] / 2 < 0.$

References

- Aghion, Philippe and Peter Howitt (1994): “Growth and unemployment”, *Review of Economic Studies*, 61 (3), pp. 477–494.
- Ball, Laurence and Robert Moffitt (2001): “Productivity growth and the philips curve”, In: Krueger, A. B., Solow, R. (Eds.), *The Roaring Nineties: Can Full Employment Be Sustained?*, Russell Sage Foundation, New York, pp. 61-90.
- Bean, Charles and Christopher A. Pissarides (1993): “Unemployment, consumption and growth”, *European Economic Review*, 37 (4), pp. 837–854.
- Caballero, Ricardo J. and Mohamad L. Hammour (1996): “On the timing and efficiency of creative destruction”, *Quarterly Journal of Economics*, 111 (3), pp. 805–852.
- Diamond, Peter A. (1982): “Aggregate demand management in search equilibrium”, *Journal of Political Economy*, 90 (5), pp. 881-894.
- Eriksson, Clas (1997): “Is there a trade-off between employment and growth?”, *Oxford Economic Papers*, 49 (1), pp. 77–88.
- Grossman, Gene M. and Yanagawa Noriyuki (1993): “Asset bubbles and endogenous growth”, *Journal of Monetary Economics*, 31 (1), pp. 3–19.
- Farmer, Karl and Matthias Schelnast (2013): *Growth and International Trade: An Introduction to the Overlapping Generations Approach*, Springer.
- Fitoussi, Jean-Paul, David Jestaz, Edmund S. Phelps, and Gylfi Zoega (2000): “Roots of the Recent Recoveries: Labor Reforms or Private Sector Forces?”, *Brookings Papers on Economic Activity*, vol. 31 (1), pp. 237-311.
- Futagami, Koichi and Akihisa Shibata (2000): “Growth effects of bubbles in an endogenous growth model”, *The Japanese Economic Review*, 51 (2), pp. 221–235.
- Hall, Robert, E. (2005): “Employment fluctuations with equilibrium wage stickiness”, *American Economics Review*, 95 (1), pp. 50-65.
- Haruyama, Tetsugen and Campbell Leith (2010): “Unemployment and the productivity slowdown: An efficiency wage perspective”, *The Japanese Economics Review*, 61 (3), pp. 301-319.
- Hirano, Tomohiro and Noriyuki Yanagawa (2010): “Asset bubbles, endogenous growth, and financial frictions”, CARF F-Series CARF-F-223, Center for Advanced Research in Finance, Faculty of Economics, The University of Tokyo.
- King, Ian and Don Ferguson (1993): “Dynamic inefficiency, endogenous growth, and ponzi games”, *Journal of Monetary Economics*, 32, pp. 79-104.
- Kocherlakota, Narayana R. (2011): “Bubbles and unemployment”, Federal Reserve Bank of Minneapolis, discussion paper.
- Martin, Alberto and Jaime Ventura (2012): “Economic growth with bubbles”, *American Economic Review*, 102 (6), pp. 3033-3058.

- Miao, Jianjun, Pengfei Wang and Lifang Xu (2012): “Stock market bubbles and unemployment”, Boston University, Working Paper Series WP2012-011.
- Mortensen, Dale T. (1982): “Property rights and efficiency in mating, racing, and related games”, *American Economic Review*, 72 (5), pp. 968-979.
- Mortensen, Dale T. and Christopher A. Pissarides (1999): “Job reallocation and employment fluctuations”, In: Woodford, M., Taylor, J. B. (Eds.), *Handbook of Macroeconomics*, Vol. 1. Elsevier Science, Amsterdam, pp. 1171-1227.
- Muscattelli, Anton V. and Patrizio Tirelli (2001): “Unemployment and growth: some empirical evidence from structural time series models”, *Applied Economics*, 33 (8), pp. 1083–1088.
- Olivier, Jacques (2000): “Growth-enhancing bubbles”, *International Economic Review*, 41 (1), pp. 133–151.
- Petrongolo, Barbara and Christopher A. Pissarides (2001): “Looking into the black box: A survey of the matching function”, *Journal of Economic Literature*, 39 (2), pp. 390–431.
- Phelps, Edmund S. (1999): “Behind This Structural Boom: The Role of Asset Valuations”, *American Economic Review*, 89 (2), pp. 63-68.
- Pissarides, Christopher A. (1985): “Short-run equilibrium dynamics of unemployment vacancies, and real wages”, *American Economic Review*, 75 (4), pp. 676-690.
- Pissarides, Christopher A. (2000): *Equilibrium Unemployment Theory 2nd edition*, MIT Press, Cambridge, MA.
- Pissarides, Christopher A. and Giovanna Vallanti (2007): “The impact of TFP growth on steady-state unemployment”, *International Economic Review*, 48 (2), pp. 607-640.
- Romer, Paul M. (1986): “Increasing returns and long-run growth”, *Journal of Political Economy*, 94 (5), pp. 1002–1037.
- Santos, Manuel S. and Michael Woodford (1997): “Rational asset pricing bubbles”, *Econometrica*, 65 (1), pp. 19–58.
- Staiger, Douglas, James H. Stock, and Mark W. Watson (2001): “Prices, wages and the U.S. NARIU in the 1990s”, In: Krueger, A. B., Solow, R. (Eds.), *The Roaring Nineties: Can Full Employment Be Sustained?*, Russell Sage Foundation, New York, pp. 3-64.
- Tirole, Jean (1985): “Asset bubbles and overlapping generations”, *Econometrica*, 53 (6), pp. 1499–1528.
- Tripier, Fabien (2006): “Sticky prices, fair wages, and the co-movements of unemployment and labor productivity growth”, *Journal of Economic Dynamics and Control*, 30 (12), pp. 2749–2774.
- Weil, Philippe (1989): “Overlapping families of infinitely lived agents”, *Journal of Public Economics*, 38 (2), pp. 183–198.

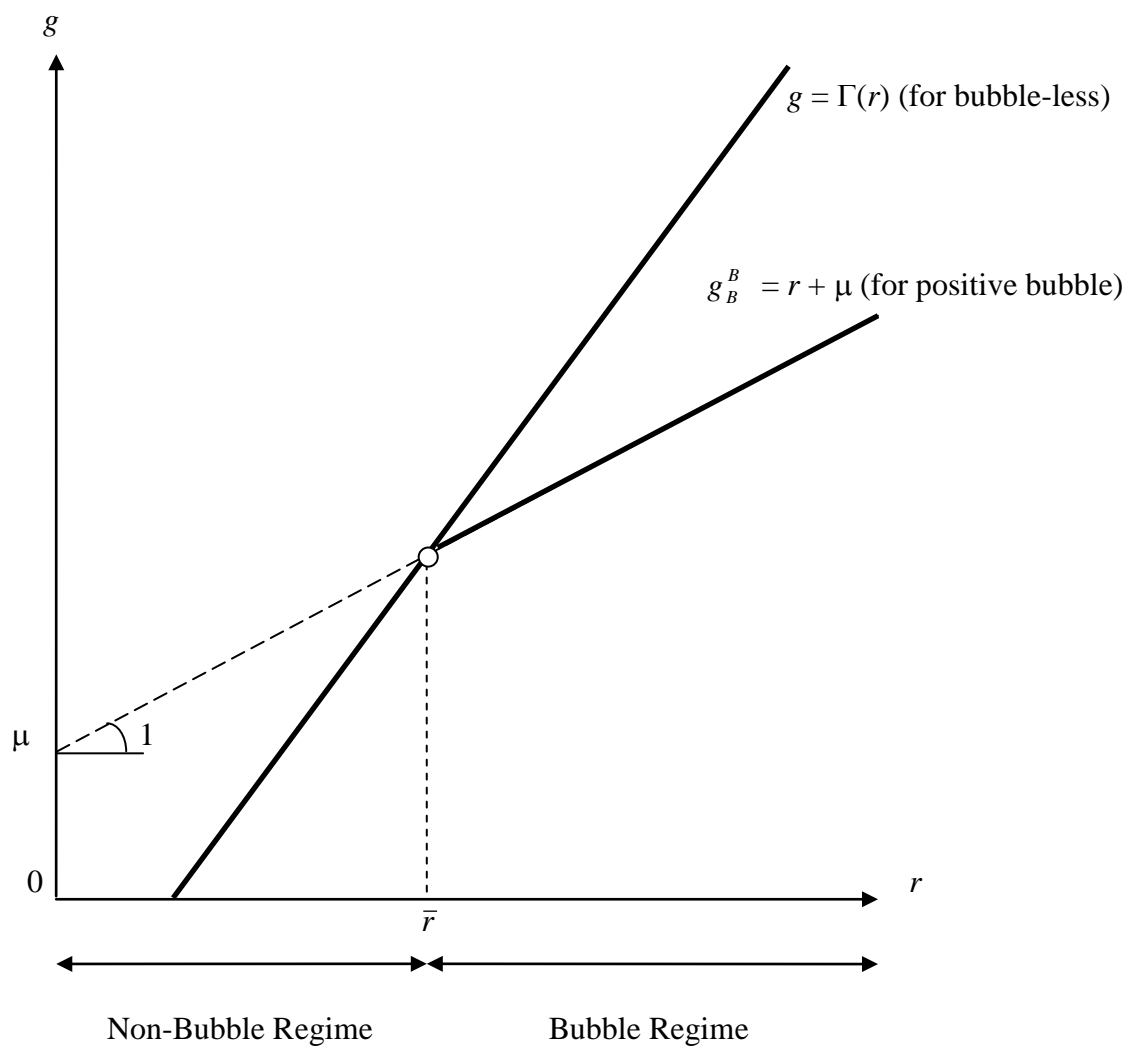


Figure 1. The threshold between a non-bubble regime and a bubble regime

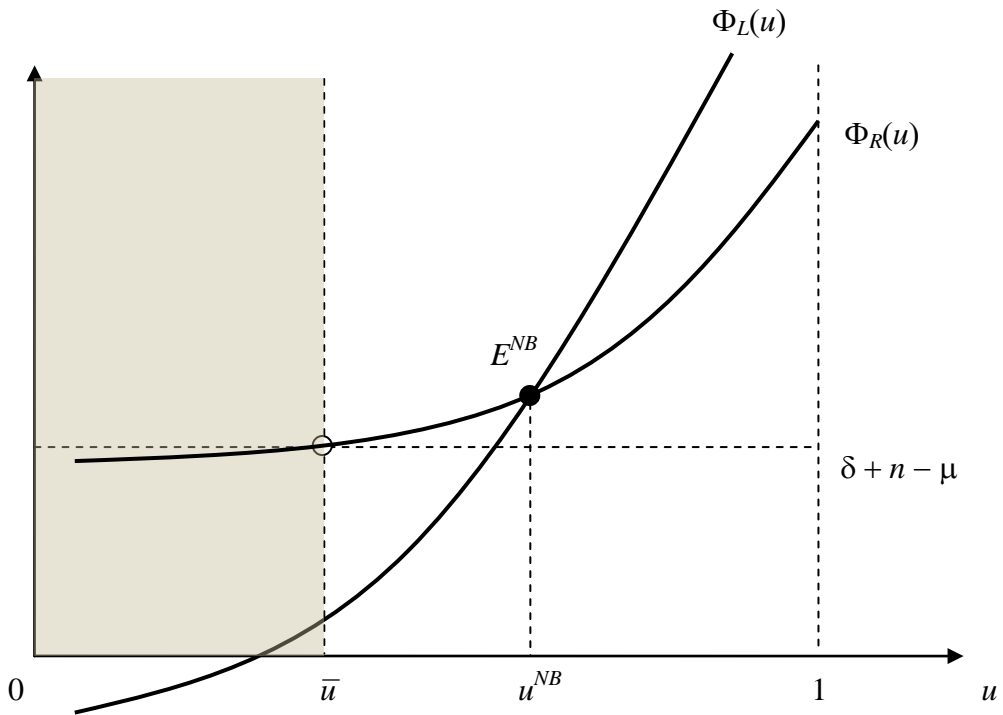


Figure 2 (a). The steady state unemployment equilibrium under a non-bubble regime ($u > \bar{u}$)

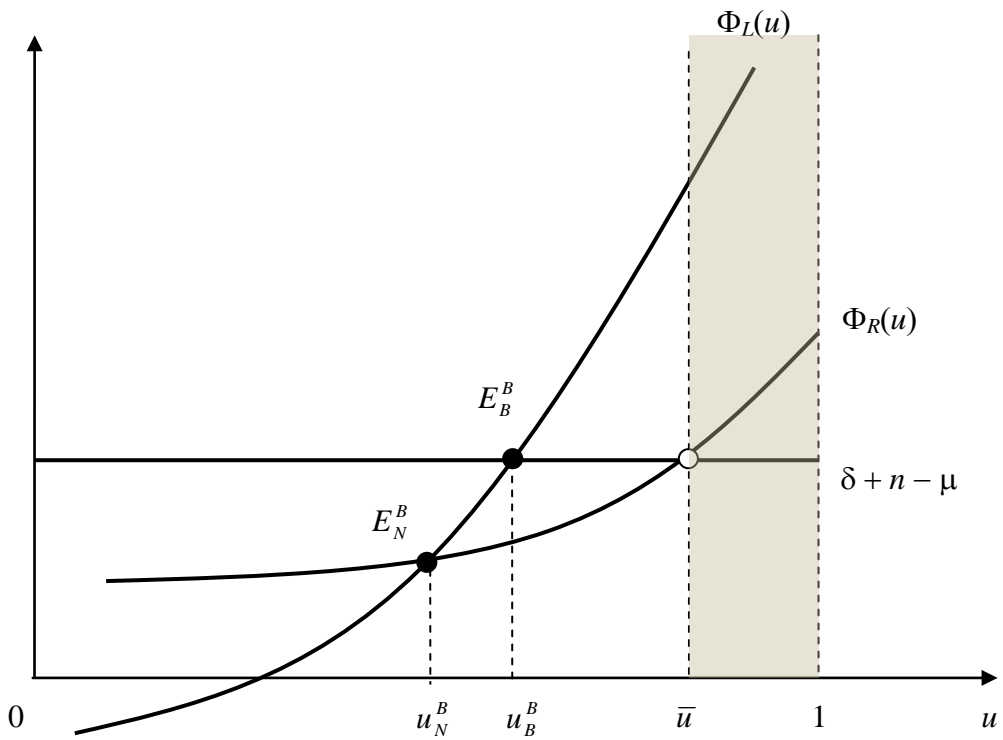


Figure 2 (b). The steady state unemployment equilibrium under a bubble regime ($u < \bar{u}$)

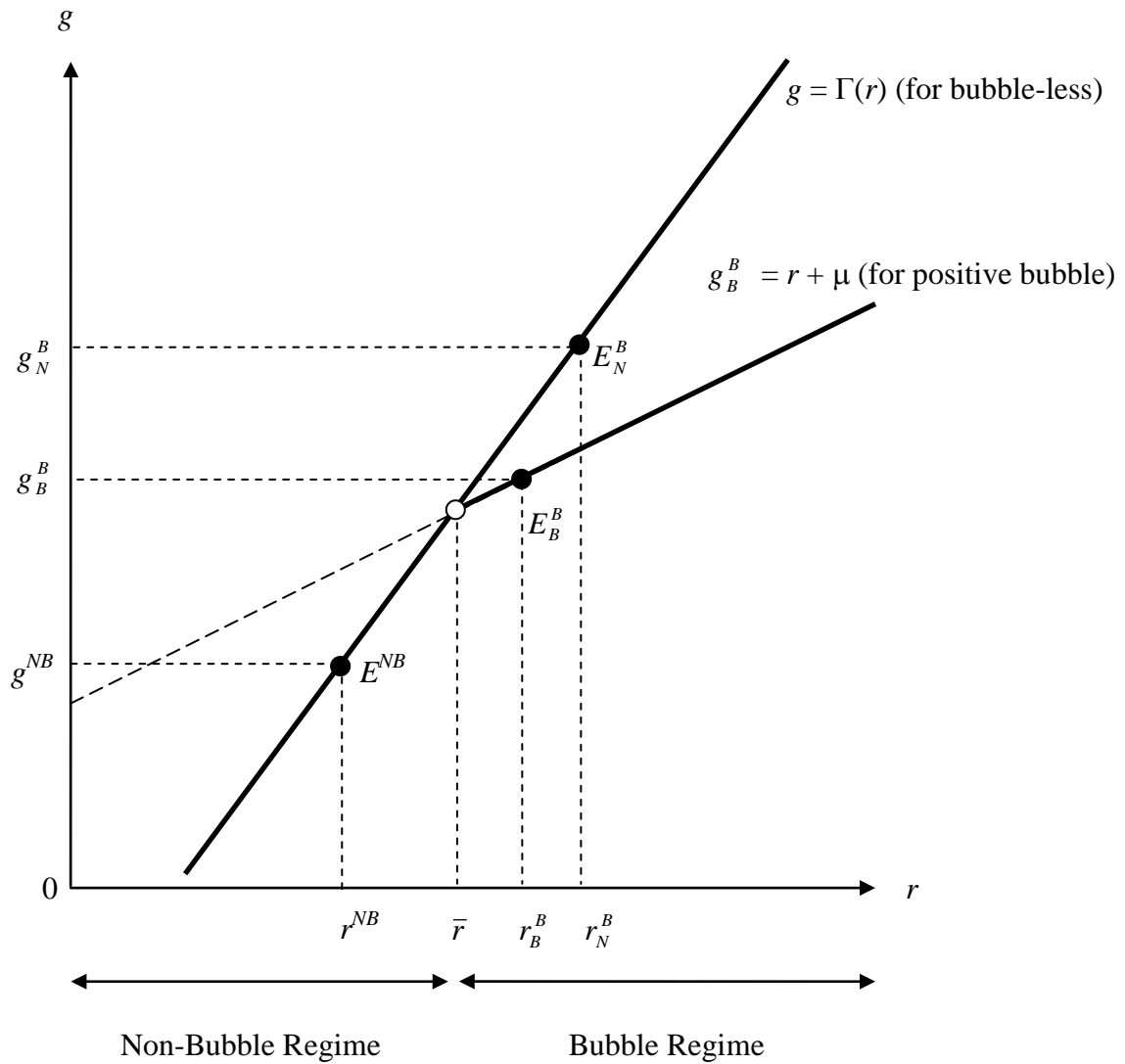


Figure 3. Comparison between regimes

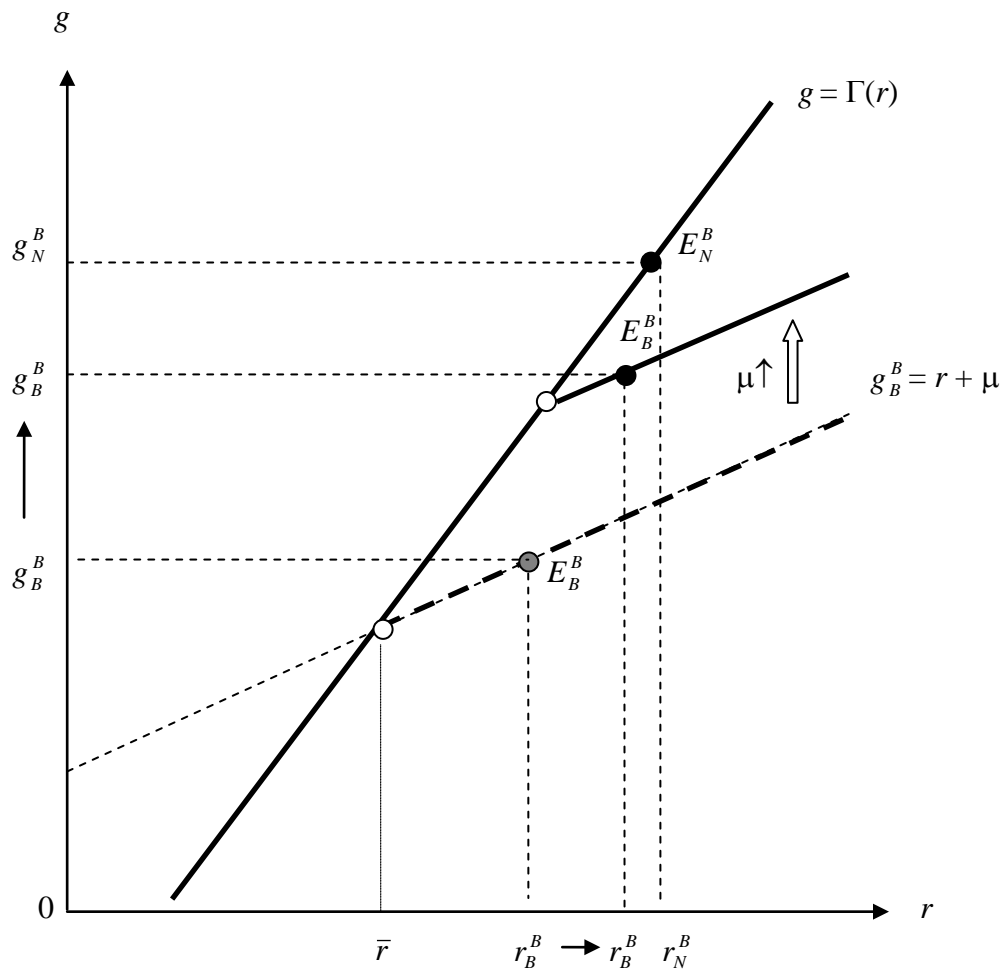


Figure 4. The effect of bubble growth (μ)