

# Asymmetric forecasting and commitment policy in a robust control problem

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## Abstract

This paper provides a piece of results regarding asymmetric forecasting and commitment monetary policy with a robust control algorithm. Previous studies provide no clarification of the connection between asymmetric preference and robust commitment policy. Three results emerge from general equilibrium modeling with asymmetric preference: (i) the condition for system stability implies an average inflation bias with respect to asymmetry (ii) the effect of asymmetry can be mitigated if policy makers relinquish a concern for robustness, and (iii) commitment policy may be superior to discretionary policy under widely used calibration sets, regardless of asymmetry.

**Keywords:** asymmetric forecasting, commitment monetary policy, robust control

**JEL classification:** E50, E52, E58

## 1 Introduction

Recent macroeconomic scholarship has dramatically advanced in understanding economic expectations. Research has examined not only standard rational expectations paradigms but also more general forms of agent expectations. Amid the recent fruitful environment

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for macroeconomics, this paper distinguishes how asymmetric forecasting (asymmetric preference or asymmetry) affects a robustly optimized commitment policy. To the best of my knowledge, this study is the first to implement a general equilibrium analysis of asymmetric preference under state-contingent policy.

Asymmetric forecasting is one of the general forms of expectations and has been established by several authors, including distinguished contributions by Nobay and Peel (2003), Ruge-Murcia (2003), and Surico (2007). Asymmetric preference is useful in identifying a bias in macroeconomic expectations. The standard rational expectations require that agents assign a fixed-symmetric weight in their assessment of risk. In practice, however, it is natural to suppose that agents assigns a different weighting when assessing future values (and risk) for inflation and/or output.

Previous studies of asymmetric preference have an advantage to formulate the agent's expectations realistically, but traditional methods involving asymmetry offer no general equilibrium analysis (system analysis). Prior to this paper, inquiries have lagged developments in mainstream macroeconomics by entirely depending on a Euler equation of the model and in being based on outcomes of a partial equilibrium analysis. This study develops a theory of asymmetric preference via general equilibrium modeling to revise the macroeconomic study of asymmetric preference. It sheds light on relationships between asymmetric preference and commitment monetary policy with a robust control algorithm.

This paper proceeds as follows. Section 2 explains the developments of asymmetric preference and motivations of this paper. Section 3 provides analytical and numerical results of the model. Section 4 concludes.

## **2 Literature review and Motivation**

Asymmetric preference conventionally has been used to identify distortions in the objectives of monetary authority and criticize their economic consequences in the context

of Barro and Gordon's (1983) average inflation bias. Nobay and Peel (2003) introduced a linear loss function into monetary policy analysis, in which the shape of the objective function is disciplined by asymmetric preference. Existence of asymmetric preference generates a bias in the forecasts of inflation and/or output. On the basis of the theory, Ruge-Murcia (2003) provided empirical evidence for asymmetry in the output (unemployment) of Federal Reserve. Surico (2007) tailored asymmetry a New Keynesian theory and indicated the existence of expansionary-asymmetric preference for output in the pre-Volcker period, which in turn triggered an average inflation bias.<sup>1</sup> Riboni and Ruge-Murcia (2010) considered the voting protocol of the monetary policy committee and mentioned member-specific preferences as an asymmetry. The estimated interest rate rule suggested that most monetary policies in developed countries do not follow a consensus voting protocol. Again, all scholarly contributions are based on the concept of a partial equilibrium; presently, no contribution exists in the context of a general equilibrium perspective.

Recently, studies on asymmetric preference have been rapidly developed by econometric researchers—that is, the elicitation of agent expectations with general assumptions rather than those used in the standard rational expectations approach. Among the vast literature, studies closely related to this paper are Capistrán and Timmermann (2009) and Branch (2013).<sup>2</sup> Capistrán and Timmermann (2009) pursued a determinant form of inflation expectations using the Survey of Professional Forecasters and reported that asymmetry exists in the professionals' forecasts. They also observed a correlation between asymmetry and cross-sectional dispersion of forecasts and suggested irrationality in expectations. Branch (2013) sought to explain the “too low for too long” phenomenon of federal funds rates by referring to the central bank's use of a “nowcasting” Taylor

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<sup>1</sup>He reported that the asymmetry (and inflation bias) disappeared after the Volcker era.

<sup>2</sup>See Pesaran and Weale (2006) and Manski (2004) for surveys of the fields. Engelberg et al. (2009) also reported asymmetric tendencies in the historical evolution of market expectations with the Survey of Professional Forecasters, although they simultaneously expressed skepticism about whether asymmetric preference exists.

rule with asymmetric preference in assessing current period expectations for inflation and output gap. He supported the existence of asymmetry and stressed the importance of considering asymmetry in policy rules for understanding the recent behavior of U.S. monetary policy.

The next advances in understanding asymmetry is likely to emerge from these recent studies, so I draw heavily from Branch (2013), who modeled asymmetric forecasting  $\hat{E}$  for  $z_{t+1}$  in period  $t$  as

$$\hat{E}_t z_{t+1} = E_t z_{t+1} + \varphi_z \sigma_{z,t}, \quad (1)$$

where  $E_t z_{t+1}$  is the level of standard rational expectations,  $\varphi_z$  denotes the asymmetric preference of forecaster, and  $\sigma_{z,t}$  is uncertainty induced by asymmetry. If forecasters' estimates are severely asymmetrical, the deviation from symmetric (rational) expectations is enlarged. The positive sign for asymmetry indicates that they prefer negative deviations of  $z_{t+1}$  from its target level to positive deviations, and *vice versa*.<sup>3</sup>

Based on the relationship in (1), the New Keynesian Phillips curve with asymmetric expectations takes the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \beta \varphi_\pi \sigma_{\pi,t} + u_t, \quad (2)$$

where  $\varphi_\pi$  is asymmetry in inflation forecasting and  $\sigma_{\pi,t}$  is its distortion. Note that I assume that firms offering asymmetric forecasts breed subsequent uncertainty in the basic New Keynesian model, in which policymakers attempt to control for uncertainty robustly. This assumption allows for specifying the policy objective as a quadratic form:

$$Loss_t = \left(\frac{1}{2}\right) E_t \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda(x_t - x^*)^2 - \theta \sigma_{\pi,t}^2]. \quad (3)$$

To complete this approach, I also benefit from the robust control theory developed

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<sup>3</sup>See Nobay and Peel (2003) for details of intuition for asymmetric preference with a linex loss function.

by Hansen and Sargent (2008). They suppose that a robust policy suspects this model and worry about misspecifications. To mitigate failures brought by model uncertainty, policy makers engage in a mental zero-sum game: a robust policy minimizes social loss, given the fictitious malevolent agent’s strategy of maximizing uncertainty in the economy. Applications of this algorithm in monetary policy include Giordani and Söderlind (2004) for the most basic New Keynesian model and Leitemo and Söderström (2008) for a small open economy. Numerical results suggest that the robust policy reacts more severely to a cost-push shock than in the case of rational expectations because it becomes uncertainty averse. Walsh (2004) analyzes robust control with a New Keynesian model and indicates an “equivalent result” in the sense that concern for robustness does not affect optimization conditions with respect to inflation and output. In contrast to these findings, Tillmann (2009) insists the weakened response of robust policy when a cost channel exists.<sup>4</sup>

Per the policy objective in (3), parameter  $\theta$  denotes the policy’s disregard for robustness. An infinite value of  $\theta$  implies that policy has no concern with robustness. The robust policy in this study first elicits the market’s expectations, and then optimizes the policy objective with model distortions specified as asymmetry-induced uncertainty in the market’s expectations: policy makers worry about asymmetric distortions in agent expectations and try to robustify the model against it. It is natural to assume this sequence in the robust control problem since the monetary authority elicits actual market expectations from survey data before making a policy decision conditioned by economic uncertainty. Moreover, Capistrán and Timmermann (2009) and Branch (2013) empirically establish asymmetric expectations in the market.<sup>5</sup> Their findings support the approach in this paper.

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<sup>4</sup>He interpreted the tendency as the “Brainard principle” of the robust policy.

<sup>5</sup>Although Branch (2013) assumed policy has asymmetry, his assumption is essentially the same as that taken in this paper. He elicited actual asymmetry from survey data of market participants. This paper models the policy decision process in more detail with respect to the policy eliciting market expectations.

### 3 Model

This section fully exposes the model and its properties. It reveals the model's analytical solutions and presents numerical simulations based on them.

#### 3.1 Analytical solution

This subsection distinguishes the relationship between commitment policy and asymmetry and then derives social losses under both commitment and discretionary policies.

##### 3.1.1 Characterizing commitment policy with asymmetry

The robust policy solves the Lagrangian of the form:

$$\mathcal{L}_t = \left(\frac{1}{2}\right) E_t \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda(x_t - x^*)^2 - \theta \sigma_{\pi,t}^2 + 2s_t(\beta E_t \pi_{t+1} + \kappa x_t + \beta \varphi_{\pi} \sigma_{\pi,t} - \pi_t)],$$

where  $(\beta, \kappa, \lambda, \theta, x^*) \in \mathbb{R}^{5+}$  without the loss of accuracy. These parameters are generally positive in optimal monetary policy analyses.

Optimal conditions for state-contingent policy are obtained as

$$\pi_t = s_t - s_{t-1}, \tag{4}$$

$$x_t = x^* - \frac{\kappa}{\lambda} s_t, \tag{5}$$

$$\sigma_{\pi,t} = \frac{\beta \varphi_{\pi}}{\theta} s_t, \quad \forall t \geq t_0. \tag{6}$$

As usual, policymakers commit to an inertial plan to control the economy. History dependency requires their policy to include both a purely forward-looking component and a past state of the economy.

The distorted Phillips curve leads to the second-order difference equation of the La-

grange multiplier  $s_t$  as

$$\left[ L^{-1} - \frac{1}{\beta}(1 + \beta + g(\varphi_\pi, \theta)) + \frac{1}{\beta}L \right] s_t = -\frac{1}{\beta}(\kappa x^* + u_t), \quad (7)$$

$$g \equiv \frac{\theta \kappa^2 - \lambda \beta^2 \varphi_\pi^2}{\lambda \theta},$$

and the following proposition arises.

**proposition 1** *The parameterization in standard rational expectations solutions are re-stored as*

$$\lim_{\varphi_\pi \rightarrow 0} g(\varphi_\pi, \theta) = \lim_{\theta \rightarrow \infty} g(\varphi_\pi, \theta) = \frac{\lambda}{\kappa^2}.$$

This condition also implies no uncertainty in the model since  $\lim_{\varphi_\pi \rightarrow 0} \sigma_{\pi,t} = \lim_{\theta \rightarrow \infty} \sigma_{\pi,t} = 0$  in optimal condition. The rational expectations approach implies a policy that is unconcerned with robustness, in which firms make symmetric forecasts, or both.

I factorize the difference equation as

$$(L^{-1} - \mu_1)(L^{-1} - \mu_2)Ls_{t+1} = -\frac{1}{\beta}(\kappa x^* + u_t),$$

where

$$\mu_1 + \mu_2 = \frac{1 + \beta + g}{\beta},$$

$$\mu_1 \mu_2 = \frac{1}{\beta},$$

and obtain the solution for the smaller root as

$$\mu_1 = \frac{(1 + \beta + g) - \sqrt{(1 + \beta + g)^2 - 4\beta}}{2\beta}. \quad (8)$$

The solution of the root (8) induces the following proposition.

**proposition 2** *Given  $(\beta, \kappa, \lambda, \theta, x^*) \in \mathbb{R}^{5+}$ , difference equation (7) has two real roots of*

which  $0 < \mu_1 < 1 < \mu_2$  if and only if

$$\varphi_\pi \in I_{\varphi_\pi} = \left( -\sqrt{\frac{\theta\kappa^2}{\lambda\beta^2}}, \sqrt{\frac{\theta\kappa^2}{\lambda\beta^2}} \right).$$

After solving the lag-polynomial, backward recursion produces

$$s_t = \frac{1}{g}\kappa x^* + \mu_1 \sum_{j=0}^{\infty} \mu_1^j u_{t-j}, \quad (9)$$

if the cost-push shock is i.i.d.<sup>6</sup>

The solution of the difference equation directly disciplines the price level dynamics, so that the unconditional expectation of controlled price is

$$Ep_t = \frac{1}{g}\kappa x^*. \quad (10)$$

In equation (10), average inflation bias arises if  $1/g > 0$  with  $\kappa \neq 0$ . Accordingly, I obtain following corollary.

**corollary 2.1** *Proposition 2 implies the average inflation bias w.r.t.  $\varphi_\pi$ . Otherwise,  $\varphi_\pi \in I_{\varphi_\pi}^C$  implies a deflationary-explosive price pass.*

Corollary 2.1 stresses the harmfulness of the asymmetric preference for the outcome of optimal policy. The asymmetry enhances the distortion in the Phillips curve and, at best, worsens long-run inflation. The deflationary scenario will destroy the economy.

### 3.1.2 Loss evaluation

I derive the unconditional expectations for the worst-case loss of the form:

$$ELoss_t = \left(\frac{1}{2}\right) E \left\{ E_t \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda(x_t - x^*)^2 - \theta\sigma_{\pi,t}^2] \right\}, \quad \forall t > t_0. \quad (11)$$

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<sup>6</sup>I restrict the discussion within the i.i.d. shock case since evaluations of the expectation and loss function are especially difficult in the serial correlation case.

The Appendix shows that the unconditional loss under a commitment policy is calculated as

$$ELoss_t^{com} = \frac{1}{2(1-\beta)} \left\{ \frac{1}{g} \kappa^2 x^{*2} + [g + 2(1-\mu_1)] \frac{\mu_1^2}{1-\mu_1^2} \sigma_u^2 \right\}, \quad (12)$$

and the loss under a discretionary policy is

$$ELoss_t^{dis} = \frac{\tilde{\mu}}{2(1-\beta)} \left[ \left( \frac{\kappa x^*}{1-\beta\tilde{\mu}} \right)^2 + \sigma_u^2 \right], \quad (13)$$

where  $\tilde{\mu} \equiv 1/(1+g)$ . I compare these two through numerical simulation in the next section, as they involve a complicated bundle of parameters.

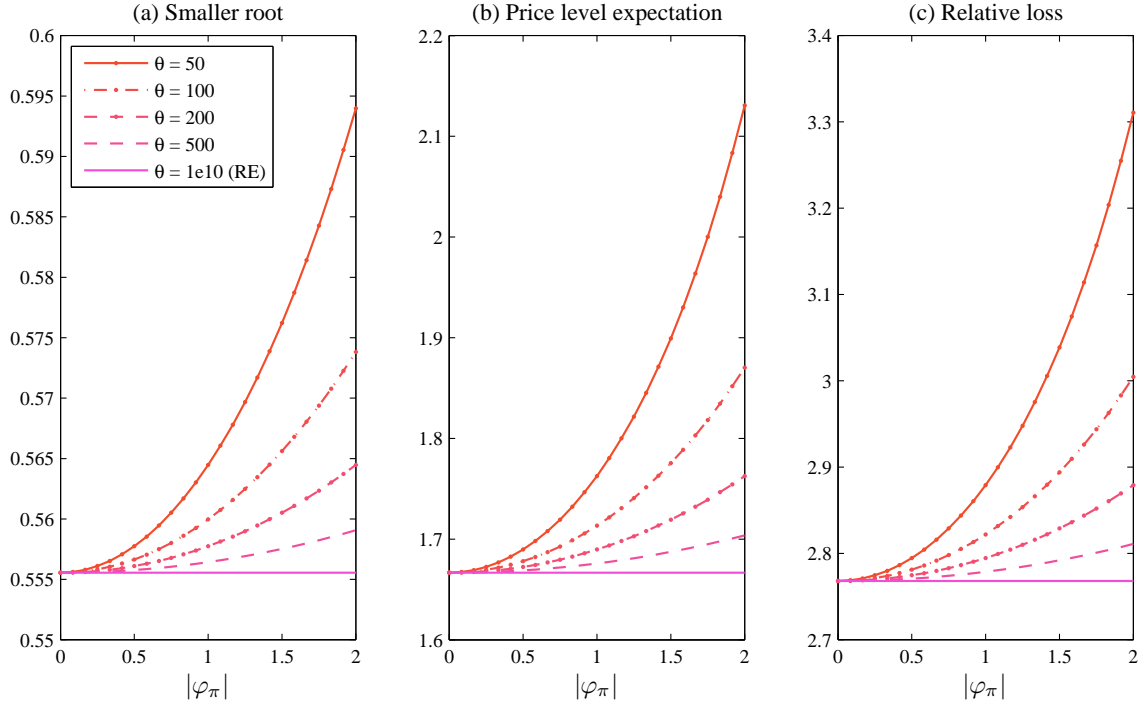


Figure 1: Stable root, Expected price level and Relative loss w.r.t.  $\varphi_\pi$  and  $\theta$

### 3.2 Simulation

Figure 1 displays three results from the numerical simulation. Panel (a) indicates the smaller root of (8). Panel (b) is the expectation for the average price level from (10). Panel (c) indicates the relative unconditional loss calculated as  $ELoss_t^{dis}/ELoss_t^{com}$  in Section 2. In this simulation, I set  $\kappa = 0.30$  as in Clarida, Galí and Gertler (2000),  $\lambda = 0.25$  as in McCallum and Nelson (2000) and  $\beta = 0.99$  as usual. I examine  $\varphi_\pi \in [-2, 2] \subset \cap_\theta I_{\varphi_\pi(\theta)}$  with  $\theta = \{50, 100, 200, 500, 1e10\}$  under the parameterizations.<sup>7</sup>

Panel (a) indicates that the stable roots increase with asymmetry, which expands the effects of a cost-push shock (see (9)). In panel (b), an increase in asymmetry produces increases in expectations for the price level. Under these parameterizations, both the stochastic and deterministic components of the economy increase with asymmetry. Panel (c) stresses the superiority of a commitment policy over a discretionary policy given the parameterizations. The greatest gain under commitment exceeds 300% relative to the

<sup>7</sup>Branch (2013) and Capistràn and Timmermann (2009) reported the interval of  $|\varphi_\pi|$  from about 0 to 2.

discretionary policy. These tendencies are monotone under the calibration. Note that the monotonically increasing effects of asymmetry can be suppressed with monetary policy that relinquishes concern for robustness (has larger  $\theta$ ).

Many researchers have emphasized the superiority of commitment policies over discretionary policies (e.g., Benigno and Woodford, 2005). I support the prevailing view of monetary policy analysis because the existence of asymmetry does not essentially affect the stylized fact in the simulation. The notable differences from asymmetry are its effects on the determinants of system stability and its role in creating the average inflation bias.

## 4 Conclusion

This study examined relationships between asymmetric forecasting and a robust commitment policy and established three results. First, the asymmetry providing stable roots produces an average, long-run inflationary bias in commitment policy. Second, the effect of asymmetry can be mitigated if policymakers disregard robustness. Third, a commitment policy is superior to discretionary policy even if asymmetry exists. This superiority depends on the calibration in this study, but parameters were chosen from a widely used set and are not specific to this study.

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## Appendix: Calculating unconditional loss

### a.) commitment case

The Lagrange multiplier with an i.i.d. cost-push shock takes the AR(1) form of

$$s_t = \mu_1 s_{t-1} + \frac{\mu_1}{1 - \beta\mu_1} \kappa x^* (1 + \mu_1) + \mu_1 u_t.$$

Backward recursion yields

$$s_t = \frac{1}{g} \kappa x^* + \mu_1 \sum_{j=0}^{\infty} \mu_1^j u_{t-j}$$

with  $\mu_1 + \mu_2 = (1/\beta)(1 + \beta + g)$  and  $\mu_1\mu_2 = 1/\beta$ .

Accordingly, the squared sequence of multiplier

$$s_t^2 = \left[ \frac{1}{g} \kappa x^* + \mu_1 (u_t + \mu_1 u_{t-1} + \mu_1^2 u_{t-2} + \dots) \right] \left[ \frac{1}{g} \kappa x^* + \mu_1 (u_t + \mu_1 u_{t-1} + \mu_1^2 u_{t-2} + \dots) \right]$$

takes an unconditional expectation of the form

$$E s_t^2 = \left( \frac{1}{g} \kappa x^* \right)^2 + \frac{\mu_1^2}{1 - \mu_1^2} \sigma_u^2.$$

Moreover, the cross product of  $s_t$  and  $s_{t-1}$  is

$$s_t s_{t-1} = \left[ \frac{1}{g} \kappa x^* + \mu_1 (u_t + \mu_1 u_{t-1} + \mu_1^2 u_{t-2} + \dots) \right] \left[ \frac{1}{g} \kappa x^* + \mu_1 (u_{t-1} + \mu_1 u_{t-2} + \mu_1^2 u_{t-3} + \dots) \right],$$

and, therefore, its expectation is derived as

$$E s_t s_{t-1} = \left( \frac{1}{g} \kappa x^* \right)^2 + \frac{\mu_1^2}{1 - \mu_1^2} \mu_1 \sigma_u^2.$$

Inserting these expectations into the loss function in (8) in Sec. 3.1.2 yields

$$E Loss_t^{com} = \frac{1}{2(1 - \beta)} \left\{ \frac{1}{g} \kappa^2 x^{*2} + [g + 2(1 - \mu_1)] \frac{\mu_1^2}{1 - \mu_1^2} \sigma_u^2 \right\}.$$

## b.) discretionary case

Discretionary robust policy solves a one-period optimization problem subject to a distorted Phillips curve. The first-order necessary conditions are

$$\begin{aligned}\pi_t &= q_t, \\ x_t &= x^* - \frac{\kappa}{\lambda} q_t, \\ \sigma_{\pi,t} &= \frac{\beta \varphi_{\pi}}{\theta} q_t,\end{aligned}$$

where  $q_t$  is the Lagrange multiplier associated with discretionary optimization.

By the distorted Phillips curve, the first-order difference equation

$$(1 - \beta \tilde{\mu} L^{-1}) q_t = \tilde{\mu} (\kappa x^* + u_t)$$

is obtained, and it is solved as

$$q_t = \frac{\tilde{\mu}}{1 - \beta \tilde{\mu}} \kappa x^* + \tilde{\mu} u_t.$$

The second moment of the solution is immediately obtained as

$$E q_t^2 = \left( \frac{\tilde{\mu}}{1 - \beta \tilde{\mu}} \kappa x^* \right)^2 + \tilde{\mu}^2 \sigma_u^2.$$

Consequently, the unconditional loss function under discretionary policy is evaluated as

$$ELoss_t^{dis} = \frac{\tilde{\mu}}{2(1 - \beta)} \left[ \left( \frac{\kappa x^*}{1 - \beta \tilde{\mu}} \right)^2 + \sigma_u^2 \right].$$