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April 2023

Discussion Paper No. 2307

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

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Abstract

This brief study constructs a simple Overlapping Generations Model incorporating endogenous fertility and automation capital, which can be used as a replacement for labor inputs. Furthermore, this study introduces a robot tax on automation capital. In the long run, robot tax promotes not only fertility, but also per capita income.

JEL classification: E10, H50, J11

Keywords: Automation capital, Robot tax, Endogenous fertility, Income growth

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1. Introduction

The decline in fertility[3] has puzzled many developed countries for a long run. Hence, governments in developed countries have been facing serious economic issues in promoting both per capita income growth and population growth. According to Fanti and Gori (2009), there exists a trade-off between population growth and per capita income growth in the standard neoclassical growth literature. Their study demonstrated that child taxes lead raise not only population growth, but also per capita income growth using an overlapping generations model.

Thus, this article focuses on building automation capital based on Fanti and Gori (2009). Prettner (2019) stated that many production steps have already been replaced by machines in the automobile industry. According to Gasteiger and Prettner (2022), many policy makers have been interested in the potential impacts of automation in recent years. Following Prettner (2019), Gasteiger and Prettner (2022) as well as Zhang, Palivos and Liu (2022), this study considers automation capital as a perfect substitute for labor inputs. Further, we introduce a robot tax levied on automation capital similar to those proposed in Gasteiger and Prettner (2022) as well as Zhang, Palivos and Liu (2022). The results show that robot tax promotes population growth as well as per capita income growth.

The remainder of this article is organized as follows. Section 2 describes our proposed model. Finally, Section 3 concludes the study.

2. Model

2-1. Households

We employ a standard overlapping generations model with fertility choice. Identical households experience two periods: young and old. They derive utility from consumption during these two periods and the number of children. During the young period, they endow one unit of labor and supply it inelastically to the labor market. The assumption is that there is full employment. When households are young, they divide their wage income into consumption, savings, and child care. When households become old, they retire and consume their savings.

Following van Groezen, Leers and Meijdam (2003) as well as Fanti and Gori (2009, 2012), the utility function is as follows:

$$\log c_t + \beta \log d_{t+1} + \log n_t. \tag{1}$$

[3] According to Becker and Barro (1988) as well as Barro and Becker (1989), a higher child care cost in developed countries causes a decline in fertility.

Where c_t and d_{t+1} denote the consumption during young and old periods, respectively, n_t is the number of children, and $\beta < 1$ is the discount factor. We denote N_t as the population size born in the period t. The population growth is given by $N_{t+1} = n_t N_t$. The budget constraints are given as follows:

$$c_t + s_t + (\varepsilon w_t + \theta)n_t = w_t,\ 0 < \varepsilon < 1,\ \theta > 0 \tag{2}$$

$$d_{t+1} = R_{t+1}s_t. \tag{3}$$

where s_t is the savings, w_t is the wage, $\varepsilon w_t + \theta$ is the child care cost, and R_{t+1} is the gross interest rate. Following Fanti and Gori (2012), we assume two types of child care costs. In Equation (2), εw_t is the child care cost depending on the working income, while θ is the constant child care cost.

The optimal allocations are given as follows:

$$\frac{s_t}{n_t} = \beta(\varepsilon w_t + \theta), \tag{4}$$

$$n_t = \frac{w_t}{(2+\beta)(\varepsilon w_t + \theta)}. \tag{5}$$

An increase in wage improves fertility from Equation (5).

2-2. Firms

Under a competitive market, identical firms provide final goods. Following Prettner (2019), Gasteiger and Prettner (2022), as well as Zhang, Palivos, and Liu (2022), the factors of production include traditional capital, automation capital, and labor inputs. It is important to note that automation capital is a perfect substitute for labor inputs. The aggregate production function is expressed as:

$$Y_t = AK_t^{\alpha}(N_t + P_t)^{1-\alpha},\ A > 0,\ 0 < \alpha < 1. \tag{6}$$

where Y_t is the total output, K_t is the aggregate traditional capital, N_t is the total labor input and P_t is the aggregate automation capital. We assume full depreciation. The total revenue is calculated as follows:

$$AK_t^{\alpha}(N_t + P_t)^{1-\alpha} - w_t N_t - R_t^k K_t - (1+\tau)R_t^p P_t. \tag{7}$$

where R_t^k and R_t^p are the gross rental prices of traditional capital and automation capital, respectively, and τ is the robot tax as in Gasteiger and Prettner (2022) as well as Zhang, Palivos, and Liu (2022). The factor demands are given as follows:

$$w_t = A(1-\alpha)\left(\frac{k_t}{1+p_t}\right)^{\alpha}, \tag{8}$$

$$R_t^k = A\alpha\left(\frac{1+p_t}{k_t}\right)^{1-\alpha}, \tag{9}$$

$$R_t^p = \frac{A(1-\alpha)}{1+\tau}\left(\frac{k_t}{1+p_t}\right)^{\alpha}. \tag{10}$$

where $k_t \equiv K_t/N_t$ and $p_t \equiv P_t/N_t$ represent per capita traditional capital and automation capital, respectively. We denote $y_t \equiv Y_t/N_t$ as per capita output and y_t is described as $y_t = Ak_t^{\alpha}(1+p_t)^{1-\alpha}$. If we omit automation capital, that is $p_t = 0$, the per capita output boils down to the standard form, that is $y_t = Ak_t^{\alpha}$.

According to Gasteiger and Prettner (2022) as well as Zhang, Palivos, and Liu (2022), a no-arbitrage condition exists between traditional capital and automation capital:

$$R_t^k = R_t^p. \tag{11}$$

Based on Equations (9)-(11), we obtain:

$$k_t = \frac{\alpha(1+\tau)[1+p_t]}{1-\alpha}. \tag{12}$$

2-3. Governments

The government imposes a robot tax on automation capital in order to finance government expenditures under a balanced budget. The government is subject to the following budgetary constraint.

$$\tau R_t^p P_t = G_t. \tag{13}$$

where G_t is the government expenditure. Following Uhlig and Yanagawa (1996), we assume that G_t does not contribute to productivity and welfare. In other words, G_t is a waste of government resources.

2-4. Equilibrium

In equilibrium, $R_t^k = R_t^p = R_t$ holds. When we substitute Equation (12) into equation (9), the equilibrium gross interest rate is given by $R = \alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha-1}$. We assume large enough A to ensure $R > 1$. The dynamics[4] of this economy can be described as follows:

$$k_{t+1} + p_{t+1} = \frac{s_t}{n_t}. \tag{14}$$

Based on Equations (4), (12), and (14), we obtain the following long-run per capita automation capital:

$$p = \frac{\beta(1-\alpha)[\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha} + \theta] - \alpha(1+\tau)}{1+\alpha\tau}. \tag{15}$$

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[4] See Appendix for the derivation of equation (14)

If $\beta(1-\alpha)[\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}+\theta] \le \alpha(1+\tau)$ holds, then $p=0$ holds. We assume that $\beta(1-\alpha)[\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}+\theta] > \alpha(1+\tau)$ throughout the rest of the article to ensure the interior solution regarding automation capital. From equation (15), we obtain:

$$\frac{dp}{d\tau} = \frac{\alpha(1-\alpha)}{(1+\alpha\tau)^2} \times \left\{\beta\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}\left[\frac{1+\alpha\tau}{1+\tau}-1\right]-(1+\beta\theta)\right\} < 0. \tag{16}$$

Since $\frac{1+\alpha\tau}{1+\tau}-1<0$ uniquely holds in this equation, $\frac{dp}{d\tau}<0$ uniquely satisfies. By substituting Equation (15) into Equation (12), the long-run per capita traditional capital[5] can be calculated by:

$$k = \frac{\alpha(1+\tau)[1+\beta\theta+\beta\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}]}{1+\alpha\tau}. \tag{17}$$

From this equation, we obtain:

$$\frac{dk}{d\tau} = \frac{\alpha(1-\alpha)}{(1+\alpha\tau)^2} \times \{\beta\varepsilon\alpha^{\alpha}(1-\alpha)^{-\alpha}A(1+\tau)^{\alpha}(1+\alpha^2\tau)+1+\beta\theta\} > 0. \tag{18}$$

A higher robot tax leads to a shift from automation capital to traditional capital as indicated by Gasteiger and Prettner (2022). Thus, an increase in robot tax reduces long-run per capita automation capital though increases in long-run per capita traditional capital.

To investigate how robot tax affects fertility in the long run, we derive the equilibrium wage. By substituting Equation (12) into Equation (8), we obtain the following constant wage:

$$w = \alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}. \tag{19}$$

When this equation is differentiated with respect to τ, we obtain:

$$\frac{dw}{d\tau} = \alpha^{1+\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha-1} > 0. \tag{20}$$

A higher robot tax increases per capita traditional capital, despite a reduction in automation capital. These two effects raise wages. Next, we focus on fertility choices in the long run. From Equations (5) and (19), the long-run fertility under generic form of τ is given by:

[5] If $p=0$ holds, the long-run per capita traditional capital is implicitly described as $k = \beta[\varepsilon(1-\alpha)Ak^{\alpha}+\theta]$.

$$n = n[w(\tau)]. \tag{21}$$

When this equation is differentiated with respect to τ, we obtain:

$$\frac{dn}{d\tau} = \underbrace{\overbrace{\frac{dn}{dw}}^{+}\overbrace{\frac{dw}{d\tau}}^{+}}_{+}. \tag{22}$$

A higher robot tax increases wages, which leads to improved fertility in the long run. When we substitute Equation (19) into Equation (5), we obtain:

$$\begin{aligned} n &= \frac{w}{(2+\beta)(\varepsilon w + \theta)}, \\ &= \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}}{(2+\beta)[\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}+\theta]}. \end{aligned} \tag{23}$$

From Equation (23), we obtain:

$$\frac{dn}{d\tau} = \frac{\alpha^{1+\alpha}(1-\alpha)^{1-\alpha}A\theta(1+\tau)^{\alpha-1}}{(2+\beta)[\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}+\theta]^{2}} > 0. \tag{24}$$

Therefore, we derive the following proposition.

Proposition 1

A rise in robot tax improves fertility

Now, we focus on the impact of robot tax on the long-run per capita output. Based on Equations (6), (12), and (14), the long-run per capita output under generic form of τ is described as follows:

$$y = y[k(\tau), p(\tau)]. \tag{25}$$

We recall that a rise in the robot tax promotes per capita capital accumulation, while per capita automation capital reduces, as shown by Equations (16) and (18). From equation (25), we obtain:

$$\frac{dy}{d\tau} = \underbrace{\overbrace{\frac{\partial y}{\partial k}}^{+}\overbrace{\frac{\partial k}{\partial \tau}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial y}{\partial p}}^{+}\overbrace{\frac{\partial p}{\partial \tau}}^{-}}_{-}. \tag{26}$$

A higher robot tax has the following effects on the long-run per capita outputs. First, an increase in the robot tax promotes traditional capital accumulation, which in turn increases the long-run per capita outputs. Second, an increase in the robot tax reduces automation capital, thereby reducing the long-run per capita outputs. The first and second terms in the right-hand side of Equation (26) represent the first and second effects, respectively. If the first effect dominates the second effect, income growth arises with a higher robot tax. From equations (6), (12), and (15), we obtain the following long-run per capita outputs:

$$y = Ak^{\alpha}(1+p)^{1-\alpha}$$
$$= \frac{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\tau)^{\alpha}[\beta\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}+1+\beta\theta]}{(1+\alpha\tau)}. \quad (27)$$

After differentiating Equation (27) with respect to τ, we derive:

$$\frac{dy}{d\tau} = \frac{\alpha^{1+\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}}{(1+\alpha\tau)^2} \times$$
$$\left\{\beta\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}\left[\frac{2(1+\alpha\tau)}{1+\tau}-1\right]+(1+\beta\theta)\left[\frac{1+\alpha\tau}{1+\tau}-1\right]\right\} \quad (28)$$

Whether long-run per capita outputs increase with a higher robot tax depends on these parameters. If the sign of the large brackets of Equation (28) is positive, an increase in robot tax is likely to promote long-run income growth. Finally, we have the following proposition.

Proposition 2

If $\beta\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1+\tau)^{\alpha}\left[\frac{2(1+\alpha\tau)}{1+\tau}-1\right]+(1+\beta\theta)\left[\frac{1+\alpha\tau}{1+\tau}-1\right]>0$ holds, a higher robot tax promotes per capita income growth.

Table 1 presents a numerical example. The present study sets the following parameters: $\alpha = 0.3, \beta = 0.5, \varepsilon = 0.1$ $\theta = 1$ and $A = 14$.

[Table 1]

3. Conclusion

Low fertility has been observed in many developed countries. Several developed countries face the serious economic issue of promoting population growth and per capita income growth. The current study constructs a simple overlapping generations model incorporating endogenous fertility and automation capital. Our results demonstrate that a robot tax is effective in promoting not only population growth but also income growth.

Appendix

The clearing condition for the goods market is given as follows:

$$Y_t = C_t + K_{t+1} + P_{t+1} + G_t. \tag{A.1}$$

where C_t is the aggregate consumption at period t. From Equations (8), (9), and (10), we have:

$$Y_t = w_t N_t + R_t K_t + (1+\tau) R_t P_t \tag{A.2}$$

As indicated above, $R_t^k = R_t^p = R_t$ satisfies the equilibrium. Using Equations (2), (3), (A.1), and (A.2), we obtain:

$$\begin{aligned} & w_t N_t + R_t K_t + (1+\tau) R_t P_t \\ & \qquad = (w_t - s_t) N_t + R_t s_{t-1} N_{t-1} + K_{t+1} + P_{t+1} + G_t \end{aligned} \tag{A.3}$$

From Equations (12) and (A.3), we obtain:

$$R_t (K_t + P_t - s_{t-1} N_{t-1}) = K_{t+1} + P_{t+1} - s_t N_t \tag{A.4}$$

To satisfy equation (A.4) for a period t, Equation (14) should hold.

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Table 1

τ	0	0.1	0.2	0.3	0.4	0.5
p	0.316	0.285	0.256	0.227	0.200	0.174
n	1.727	1.755	1.781	1.805	1.827	1.848
y	14.289	14.359	14.400	14.417	14.416	14.399

$\alpha = 0.3, \beta = 0.5, \varepsilon = 0.1$ $\theta = 1$ and $A = 14$.