# Endogenous choice of price or quantity contract with upstream advertising

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**January 2023**

**Discussion Paper No. 2301**

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

# Endogenous choice of price or quantity contract with upstream advertising*

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January 17, 2023

**Abstract**

We investigate a supply chain comprising a manufacturer engaged in advertising and two retailers who compete with differentiated products. We examine the endogenous choice between competing on quantity or price for the retailers. Our analysis reveals that, depending on the level of product substitutability, the range of possible outcomes is varied and includes Cournot, Bertrand, and Cournot-Bertrand under informative advertising. This result contradicts the established understanding that firms tend to engage in Cournot competition as their dominant strategy. Furthermore, we find that under persuasive advertising, Cournot or Bertrand outcomes may be optimal, but Cournot-Bertrand never arises as an equilibrium.

**JEL codes**: D43, L13, M21.

**Keywords**: endogenous competition mode, advertising, vertical relationship.

*The third author gratefully acknowledges the financial support from the Japan Society for the Promotion of Science, KAKENHI Grant Numbers JP22H00043 and JP20K01678. The usual disclaimer applies.

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# 1 Introduction

We analyze the endogenous choice between quantity or price competition faced by retailers in a vertical market with an upstream manufacturer engaged in advertising and two downstream retailers competing with differentiated products. Singh and Vives (1984) analyzed the endogenous choice in the absence of advertising by the upstream manufacturer, using Cournot, Bertrand, and Cournot-Bertrand static duopoly models within the framework of Dixit (1979). They demonstrated that when products are substitutes, Cournot competition is the dominant strategy for firms. In this study, we challenge this conventional wisdom by considering the influence of advertising by the upstream manufacturer.

We consider a four-stage game played by a manufacturer and two retailers offering differentiated products.[1] In the first (pre-play) stage, the retailers simultaneously choose the type of market contract (quantity or price). In the second stage, the manufacturer chooses the level of advertising, which affects the size of the retailers' markets. In the third stage, the manufacturer decides the wholesale price. In the fourth stage, the retailers compete in accordance with the market contract choices made in the first stage.

We find that in the case of informative advertising, retailers' market contract choices depend on the degree of product substitutability. Specifically, when the degree of product substitutability is sufficiently high, both retailers choose to offer a quantity contract (Cournot) in equilibrium, consistent with the result in Singh and Vives (1984). However, when the product substitutability is sufficiently low, both retailers may choose to offer a price contract (Bertrand), in contrast with the result in Singh and Vives (1984). Additionally, when the product substitutability is at an intermediate level, there may be two asymmetric subgame perfect Nash equilibria (SPNE) in the first stage, with one retailer offering a price contract and the other offering a quantity contract (Cournot-Bertrand). The retailers' choices are strategic substitutes, which never occurs in the model provided by Singh and Vives (1984).

[1] The literature on advertising (Gross and Shapiro, 1984; Soberman, 2004; Zhang et al., 2012; Zhang et al., 2020) allows firms to sell differentiated products.

The intuition underlying this result is as follows: It is well-known that the market is the most competitive under Bertrand competition, relatively less competitive under the Cournot-Bertrand market, and the least competitive under Cournot competition. The manufacturer tends to advertise more in a more competitive retail market to increase output, which benefits retailers by increasing demand. Simultaneously, a more competitive market adversely affects retailers' profits. When the degree of product substitutability is low, the scale of advertising is also small. In this case, the retailer has an incentive to intensify competition to encourage advertising investment. If product substitutability is sufficiently low, both retailers choose a price contract. As the degree of product substitutability increases, the incentive to select a price contract decreases. At some level of product substitutability, if one retailer chooses a price contract, the other retailer loses the incentive to choose a price contract because the manufacturer's advertising investment increases discontinuously. Consequently, for an intermediate level of product substitutability, the retailers choose asymmetric contracts. Finally, in the case of high product substitutability, no retailer chooses a price contract because the level of manufacturer advertising is sufficient.

Additionally, we find that in the case of persuasive advertising, a Bertrand outcome occurs in equilibrium when the advertising investment is sufficiently efficient or the degree of product substitutability is sufficiently low. When the degree of product substitutability is low, both retailers may choose a price contract, which is similar to the case of informative advertising. Additionally, when advertising investment is efficient, the beneficial advertising effect is sufficiently strong to persuade one retailer to choose a quantity contract, leading to a Cournot outcome. However, when advertising investment is inefficient or the degree of product substitutability is intermediate, a Cournot-Bertrand outcome never occurs in equilibrium. Instead, either a Cournot or a Bertrand outcome may be obtained.

Our study is related to previous research that has examined the well-known Cournot advantage property of endogenous choice between quantity or price competition in various contexts (Zanchettin, 2006; Arya et al., 2008; Tremblay et al., 2009; Basak and Wang, 2016). These studies have demonstrated that the subgame perfect equilibrium could be

Bertrand or Cournot-Bertrand. However, all of these studies considered models without upstream manufacturer advertising. Our work is closely related to that of Hu and Mizuno (2021), who investigated retailers' second-mover advantage property of Bertrand competition by introducing advertising by the upstream manufacturer. They found that simultaneous pricing may occur in equilibrium when product substitutability is sufficiently low. Our study complements their work by analyzing the endogenous choice between quantity or price competition faced by retailers by considering Cournot, Bertrand, and Cournot-Bertrand in the presence of both informative and persuasive advertising by the upstream manufacturer. In Hu and Mizuno (2021), the efficiency of advertising by the manufacturer does not play a significant role; however, in our model, advertising efficiency plays an important role in the determination of retailers' market contract choices under persuasive advertising.

Additionally, we find that the assumption that the retailers choose the type of contract before informative advertising is crucial for our main results. Furthermore, we demonstrate that the result of Singh and Vives (1984) still holds in a vertical structure with an upstream manufacturer, but it does not hold when informative advertising by the manufacturer is introduced. This highlights the importance of advertising by the manufacturer for retailers' market contract decisions.

The remainder of the paper is organized as follows: In Section 2, we describe the basic model. In Section 3, we analyze retailers' market contract choices in the case of informative advertising. In Section 4, we present the game in reversed timing and analyze the case of persuasive advertising. Finally, in Section 5, we present our conclusion.

# 2 Model

We consider a market with a manufacturer and two retailers, referred to as retailer $i$ and retailer $j$ ($i, j = 1, 2$ and $i \neq j$). The retailers purchase products at a wholesale price $w$ and sell it to consumers. The products sold by the retailers are differentiated (e.g. each retailer sells a product with a differentiated service). The retailers choose either their price or output, and the price and output of retailer $i$ are denoted by $p_i$ and $Q_i$, respectively. The profit of retailer $i$ is then given by $\pi_i \equiv (p_i - w)Q_i$.

The manufacturer can engage in informative advertising. We denote the level of informative advertising by $\theta$, which represents the fraction of consumers who receive the advertisement. Consumers who view the advertisement can buy either product, while those who do not view it buy none. We assume that if $\theta$ consumers receive the advertisement, the manufacturer must incur an advertising cost of $k\theta^2$, where $k$ is a positive and constant parameter and $\theta \in [0, 1]$.[2] The manufacturer produces the product at a constant marginal cost of $c$. Consequently, the profit of the manufacturer is given by $\Pi_M \equiv (w - c)(Q_i + Q_j) - k\theta^2$.

There is a unit mass of consumers, each with a symmetric utility function: $u(q_i, q_j, m) \equiv a(q_i + q_j) - b(q_i^2 + 2\gamma q_i q_j + q_j^2)/2 + m$, where $q_i$ and $q_j$ are the consumption levels for products $i$ and $j$, respectively; $m$ is the quantity of a numeraire good; $\gamma \in (0, 1)$ is the measure of product substitutability; and $a$ and $b$ are positive parameters. This utility function yields the following demand function: $q_i = [a(1-\gamma) - p_i + \gamma p_j]/[b(1-\gamma^2)]$. As we assume that consumers must receive the advertisement to buy the products, the demand faced by the retailers depends on $\theta$ (indicating the fraction of consumers viewing the advertisement)and equals $Q_i(p_i, p_j) \equiv \theta q_i$ and $Q_j(p_i, p_j) \equiv \theta q_j$.[3] To guarantee an interior solution, we assume $k > (a-c)^2/[4b(2+\gamma-\gamma^2)]$. This assumption is obtained from the condition that the equilibrium level of informative advertising is less than 1. Additionally, this assumption is a sufficient condition for concavity of the manufacturer's profit.

We define consumer and total surpluses as follows: $CS \equiv \theta[a(q_i + q_j) - b(q_i^2 + 2\gamma q_i q_j + q_j^2)/2 - p_i q_i - p_j q_j]$, $TS \equiv CS + \Pi_M + \pi_i + \pi_j$.

The timing of this game is as follows: In the first (pre-play) stage, the retailers can choose the type of market contract (quantity or price). In the second stage, the manufacturer chooses the level of informative advertising $\theta$. In the third stage, the manufacturer decides

[2] We assume a quadratic advertising cost. Although some previous studies have employed linear costs and concave benefits from advertising (Nakata, 2011; Shy, 1995), quadratic advertising cost is a standard assumption in the literature (e.g., Simbanegavi, 2009; Soberman, 2004; Zhang et al., 2012; Zhang et al., 2020; Hu and Mizuno, 2021).

[3] By defining a utility function of the representative consumer as $u(Q_i, Q_j, m) = a(Q_i + Q_j) - b(Q_i^2 + 2\gamma Q_i Q_j + Q_j^2)/(2\theta) + m$, we can derive these demand functions. This demand function and set-up under informative advertising are similar to that in Zhang et al. (2020) and Hu and Mizuno (2021).

the wholesale price $w$. In the fourth stage, the retailers compete according to the market contract chosen in the first stage. We solve this model using backward induction.

# 3 Analysis

## 3.1 Fourth stage: retail competition

When both retailers choose the price contract in the first stage, in the fourth stage, they choose $p_i$ to maximize $\pi_i = (p_i - w)\theta[a(1-\gamma) - p_i + \gamma p_j]/[b(1-\gamma^2)]$. Solving the first-order conditions, we obtain the prices that the retailers choose, and then the quantities as follows:

$$p_i^B(w) = \frac{a(1-\gamma)+w}{2-\gamma}, \quad Q_i^B(w,\theta) = \frac{\theta(a-w)}{b(2-\gamma)(\gamma+1)}, \tag{1}$$

where the superscript $B$ denotes Bertrand competition.

Next, we consider the case in which both retailers choose quantities. Solving $Q_i(p_i, p_j)$ and $Q_j(p_i, p_j)$ for $p_i$ and $p_j$, we obtain the inverse demand function $p_i(Q_i, Q_j) = a - b(Q_i + \gamma Q_j)/\theta$. Substituting the inverse demand function into the profit functions of retailers and solving the first-order conditions, we obtain the quantities chosen by the retailers and then the prices as follows:

$$Q_i^C(w,\theta) = \frac{(a-w)\theta}{b(2+\gamma)}, \quad p_i^C(w) = \frac{a+w(1+\gamma)}{2+\gamma}, \tag{2}$$

where the superscript $C$ denotes Cournot competition.

Finally, we examine the asymmetric case where one retailer chooses a quantity and the other chooses a price. Without loss of generality, we assume that retailer $i$ chooses quantity $Q_i$, and retailer $j$ chooses price $p_j$. Solving $Q_i(p_i, p_j) = \theta[a(1-\gamma) - p_i + \gamma p_j]/[b(1-\gamma^2)]$ for $p_i$ and $Q_j$, we obtain the following demand systems in strategic variables $Q_i$ and $p_j$: $p_i(Q_i, p_j) = a(1-\gamma) + p_j\gamma - bQ_i(1-\gamma^2)/\theta$, and $Q_j(Q_i, p_j) = (a-p_j)\theta/b - Q_i\gamma$, respectively. Using the above demand systems and solving the first-order conditions, we obtain the following results:

$$Q_i(w,\theta) = Q^{CB}(w,\theta) = \frac{(a-w)\theta(2-\gamma)}{b(4-3\gamma^2)}, \tag{3}$$

$$p_i(w) = p^{CB}(w) = \frac{a(2-\gamma-2\gamma^2+\gamma^3)+w(2+\gamma-\gamma^2-\gamma^3)}{4-3\gamma^2}, \tag{4}$$

$$Q_j(w,\theta) = Q^{BC}(w,\theta) = \frac{(a-w)\theta(2-\gamma-\gamma^2)}{b(4-3\gamma^2)}, \tag{5}$$

$$p_j(w) = p^{BC}(w) = \frac{a(2-\gamma-\gamma^2)+w(2+\gamma-2\gamma^2)}{4-3\gamma^2}, \tag{6}$$

where the superscript $CB$ ($BC$) denotes that these results are obtained when one retailer competes in quantity (price) while the rival competes in price (quantity).

## 3.2 Third and second stages: manufacturer decisions

In the third stage, the manufacturer chooses the wholesale price. In the Bertrand case, maximizing $\Pi_M = (w-c)[Q_i^B(w,\theta)+Q_j^B(w,\theta)] - k\theta^2$ for $w$ yields $w=(a+c)/2$. Similarly, in the Cournot case, maximizing $\Pi_M = (w-c)[Q_i^C(w,\theta)+Q_j^C(w,\theta)] - k\theta^2$ leads to $w=(a+c)/2$. In the asymmetric Cournot-Bertrand case, on maximizing $\Pi_M = (w-c)[Q^{CB}(w,\theta)+Q^{BC}(w,\theta)] - k\theta^2$, we obtain $w=(a+c)/2$. Therefore, we obtain the following lemma.

**Lemma 1** *The manufacturer chooses*

$$w = \frac{a+c}{2},$$

*and the equilibrium wholesale price is independent of the type of market contract and level of informative advertising.*

Lemma 1 has a simple intuition. The manufacturer faces a linear demand, and the type of market contract and level of informative advertising affect the size of demand for the manufacturer, but do not affect the choke price. Therefore, the wholesale price that maximizes the manufacturer's profit takes the same value for all types of market contracts and levels of advertising.

In the second stage, the manufacturer decides the level of informative advertising. In the Bertrand case, we substitute (1) and Lemma 1 into $\Pi_M$ and solve the first-order condition for $\theta$ to obtain the following:

$$\theta^B = \frac{(a-c)^2}{4kb(2-\gamma)(1+\gamma)}. \tag{7}$$

Next, in the Cournot case, we substitute (2) and Lemma 1 into $\Pi_M$ and solve the first-order condition for $\theta$, to obtain the following level of advertising:

$$\theta^C = \frac{(a-c)^2}{4kb(2+\gamma)}. \tag{8}$$

Similarly, in the asymmetric Cournot-Bertrand case, we substitute (3), (5), and Lemma 1 into $\Pi_M$, and solve the first-order condition for $\theta$ to obtain the following:

$$\theta^{BC} = \theta^{CB} = \frac{(a-c)^2(4-2\gamma-\gamma^2)}{8kb(4-3\gamma^2)}. \tag{9}$$

Comparing the advertising levels in the three cases above, we observe that $\theta^B - \theta^{BC} = (a-c)^2\gamma^2(2-\gamma-\gamma^2)/[8kb(2+\gamma-\gamma^2)(4-3\gamma^2)] > 0$ and $\theta^{BC} - \theta^C = (a-c)^2\gamma^2(2-\gamma)/[8kb(2+\gamma)(4-3\gamma^2)] > 0$, which leads to the following result.[4]

**Lemma 2** *The manufacturer chooses the level of informative advertising as* $\theta^B = (a-c)^2/[4kb(2-\gamma)(1+\gamma)]$ *in the Bertrand case;* $\theta^C = (a-c)^2/[4kb(2+\gamma)]$ *in the Cournot case; and* $\theta^{BC} = (a-c)^2(4-2\gamma-\gamma^2)/[8kb(4-3\gamma^2)]$ *in the asymmetric Cournot-Bertrand case, with* $\theta^B > \theta^{BC} > \theta^C$.

The above results can be interpreted as follows: Following previous research (Singh and Vives, 1984), we know that the market is the most competitive under price competition, relatively less competitive under asymmetric competition, and the least competitive under quantity competition. Additionally, advertising expenditure increases with the intensity of competition because the manufacturer faces greater demand in a more competitive market. Therefore, we can infer that $\theta^B > \theta^{BC} (\equiv \theta^{CB}) > \theta^C$, suggesting that the manufacturer

[4] Owing to $0 \leq \gamma < 1$, the inequality is satisfied.

advertises the most in the symmetric Bertrand competition market, relatively less in the asymmetric Cournot-Bertrand market, and the least in the symmetric Cournot competition market.

Here, we present the profits of the retailers and manufacturer as well as the consumer surplus and total surplus in each case. Using (1)-(9) and Lemma 1, we obtain the following outcomes. First, in the Bertrand case, we obtain the following results:

$$\pi^B = \frac{(a-c)^4(1-\gamma)}{16b^2k(1+\gamma)^2(2-\gamma)^3}, \quad \Pi_M^B = \frac{(a-c)^4}{16b^2k(2-\gamma)^2(1+\gamma)^2},$$
$$CS^B = \frac{(a-c)^4}{16b^2k(2-\gamma)^3(1+\gamma)^2}, \quad TS^B = \frac{(a-c)^4(5-3\gamma)}{16b^2k(2-\gamma)^3(1+\gamma)^2}.$$

Next, in the Cournot case, we have the following outcomes:

$$\pi^C = \frac{(a-c)^4}{16b^2k(2+\gamma)^3}, \quad \Pi_M^C = \frac{(a-c)^4}{16b^2k(2+\gamma)^2},$$
$$CS^C = \frac{(a-c)^4(1+\gamma)}{16b^2k(2+\gamma)^3}, \quad TS^C = \frac{(a-c)^4(5+2\gamma)}{16b^2k(2+\gamma)^3}.$$

Finally, in the asymmetric case, we obtain the following outcomes:

$$\pi^{CB} = \frac{(a-c)^4(2-\gamma)^2(1-\gamma^2)[4-\gamma(2+\gamma)]}{32b^2k(4-3\gamma^2)^3}, \quad \pi^{BC} = \frac{(a-c)^4(2-\gamma-\gamma^2)^2[4-\gamma(2+\gamma)]}{32b^2k(4-3\gamma^2)^3},$$
$$\Pi_M^{BC} = \frac{(a-c)^4[4-\gamma(2+\gamma)]^2}{64b^2k(4-3\gamma^2)^2}, \quad CS^{BC} = \frac{(a-c)^4(2-\gamma^2)[4-\gamma(2+\gamma)]}{64b^2k(4-3\gamma^2)^2},$$
$$TS^{BC} = \frac{(a-c)^4[4-\gamma(2+\gamma)][5-\gamma(3+\gamma)]}{32b^2k(4-3\gamma^2)^2}.$$

## 3.3 First stage: endogenous market contracts

Now, we can analyze the choice of quantity versus price in the first stage. Per the outcomes in each type of market contract, the payoff matrix in the first stage is presented in Table 1.

Table 1: Payoff matrix in the endogenous competition choice

| | | Retailer $j$ | |
|---|---|---|---|
| | | Quantity | Price |
| Retailer $i$ | Quantity | $\pi^C, \quad \pi^C$ | $\pi^{CB}, \quad \pi^{BC}$ |
| | Price | $\pi^{BC}, \quad \pi^{CB}$ | $\pi^B, \quad \pi^B$ |

The retailers simultaneously and independently choose the type of market contract. To identify the equilibrium contract type, we compare the retail profits between $\pi^B$, $\pi^C$, $\pi^{CB}$, and $\pi^{BC}$, and obtain the following inequalities.

$$\begin{cases} \pi^B \leq \pi^{CB}, \quad \pi^{BC} < \pi^C & \text{if } \; 0.392 \leq \gamma < 1, \\ \pi^B < \pi^{CB}, \quad \pi^C \leq \pi^{BC} & \text{if } \; 0.378 \leq \gamma < 0.392, \\ \pi^{CB} < \pi^B, \quad \pi^C < \pi^{BC} & \text{if } \; 0 < \gamma < 0.378. \end{cases} \tag{10}$$

From the above result, we have the following proposition.

**Proposition 1** *In the case of informative advertising by the manufacturer, the following can be observed: (i) When the degree of product substitutability is relatively high, i.e.,* $0.392 \leq \gamma < 1$*, SPNE will result in both retailers choosing to offer quantity contracts (CC). (ii) When the degree of product substitutability is not too high, i.e.,* $0.378 \leq \gamma < 0.392$*, there are two asymmetric SPNE at the first stage, with one retailer offering a price contract and the other offering a quantity contract (BC and CB). (iii) When the degree of product substitutability is relatively low, i.e.,* $0 < \gamma < 0.378$*, SPNE will result in both retailers choosing to offer price contracts in the first stage (BB).*

The intuition behind this proposition is as follows: There are two opposing effects on the retailers' decisions. The first is competition effect, which suggests that the adverse impact on retailers' profits intensifies as the market becomes increasingly competitive. It is understood that the market is the most competitive under price competition, relatively less competitive under asymmetric market conditions, and the least competitive under quantity competition. Therefore, in the absence of informative advertising by the manufacturer, retailers are likely to choose quantity competition, which is conventional wisdom (Singh and Vives, 1984).

However, in our model, there is also a second effect—the advertising effect. From Lemma 2, the manufacturer advertises most heavily in Bertrand competition market, relatively less in Cournot-Bertrand market, and the least in Cournot competition market. Therefore, there will be more latent consumers who are converted to actual buyers in a Bertrand competition market than in an asymmetric market or a Cournot competition market. More advertising leads to greater demand for the retailers, which benefits them. As the degree of substitutability falls, competition effect becomes relatively unimportant and is dominated by the advertising effect. Consequently, when the degree of product substitutability is sufficiently low, both retailers choose prices. On the contrary, when the degree of product substitutability is sufficiently high, competition effect becomes important and dominates advertising effect, incentivizing the two retailers to choose quantities. Finally, when the degree of product substitutability is not too high, as in the asymmetric Cournot-Bertrand competitive market, which is less competitive than a Bertrand market but more competitive than a Cournot competition market, the strategic substitutes outcomes ($BC$ and $CB$) occur.

Next, we compare the manufacturer's profits, consumer surpluses, and total surpluses under different retailers' market contract regimes. Owing to $1 > \gamma > 0$, we can easily derive that $\Pi_M^B > \Pi_M^{BC} > \Pi_M^C$, $CS^B > CS^{BC} > CS^C$, and $TS^B > TS^{BC} > TS^C$. Therefore, we obtain the following proposition.

**Proposition 2** *The profit of the manufacturer, consumer surplus, and total surplus are the highest under the Bertrand case, second highest under the Cournot-Bertrand case, and the lowest under the Cournot case.*

This result is highly intuitive. Bertrand competition intensifies competition and increases informative advertising. Therefore, when both retailers choose prices, the problem of double marginalization becomes less important, and many consumers can buy the products. Consequently, the manufacturer's profit, consumer surplus, and total surplus are the highest under Bertrand competition. By contrast, Cournot competition is the least competitive and offers the least informative advertising. The asymmetric case lies between Bertrand competition and Cournot competition in terms of the intensity of competition, which leads to the above result.

It is a well-known result that while Cournot competition is the optimal choice for firms, consumer and total surpluses are invariably the largest under Bertrand competition. However, in our model, Bertrand competition could occur in equilibrium and simultaneously benefit both consumer and total surpluses.

# 4 Discussion

## 4.1 Timing of advertising

In the previous section, we assumed that retailers choose the market contract before informative advertising. In this subsection, we examine whether the main result of Proposition 1 still holds if the timing of choosing the market contract is changed.

Hence, we consider the following two types of timings: (i) In the first stage, the manufacturer chooses the level of advertising; in the second stage, the manufacturer sets its wholesale price; in the third stage, the retailers choose the market contract (quantity or price); and in the fourth stage, following the market contracts determined in the third stage, the retailers compete with each other. (ii) In the first stage, the manufacturer chooses the level of advertising; in the second stage, the retailers choose the market contract; in the third stage, the manufacturer sets the wholesale price; and in the fourth stage, the retailers compete on the variable chosen in the second stage.

First, we consider case (i). Outcomes in the fourth stage are the same as those obtained in the previous section, (1)–(6). Then, the profits of retailers under each case are as follows:

$$\pi^{BT} = \frac{(a-w)^2(1-\gamma)\theta}{b(2-\gamma)^2(1+\gamma)}, \quad \pi^{CT} = \frac{(a-w)^2\theta}{b(2+\gamma)^2},$$
$$\pi^{CBT} = \frac{(a-w)^2(2-\gamma)^2(1-\gamma^2)\theta}{b(4-3\gamma^2)^2}, \quad \pi^{BCT} = \frac{(a-w)^2(2-\gamma-\gamma^2)^2\theta}{b(4-3\gamma^2)^2}.$$

where the superscript $BT$, $CT$, and $CBT$ ($BCT$) denote—all with reversed timing of the game—Bertrand competition, Cournot competition, and the asymmetric case obtained when the retailer competes in quantity (price) while the rival competes in price (quantity), respectively. Comparing the above profits, we obtain $\pi^{CT} > \pi^{CBT} > \pi^{BT} > \pi^{BCT}$. Hence, in any subgame, the retailers prefer choosing a quantity contract, which results in a Cournot game ($CC$). This result is the same as the well-known result of Singh and Vives (1984). Our main result is that the case wherein both retailers choose price ($BB$), and the asymmetric case wherein one retailer chooses price and the other chooses quantity ($BC$ and $CB$) never occur in equilibrium.

Next, we consider case (ii). In the third stage, the manufacturer chooses a wholesale price. From Lemma 1, the manufacturer chooses $w = (a + c)/2$ in any market contract. Additionally, with case (i), we already revealed that for any wholesale price $w$, the retailers prefer choosing a quantity contract. Consequently, both retailers choose quantities ($CC$). Importantly, this result proves that the result of Singh and Vives (1984) still holds in a vertical structure with an upstream manufacturer setting the wholesale price, but it does not hold when informative advertising by the manufacturer is introduced.

Summarizing the results in the two cases, we find that the assumption that the retailers choose the market contract before informative advertising is crucial for our main results.

## 4.2 Persuasive advertising

While in the previous section we considered informative advertising, in this subsection, we discuss persuasive advertising, which increases the willingness to buy or marginal utility for consumers.

**Model** We assume that the utility function of the representative consumer is $u = (a + \theta)(Q_i + Q_j) - b(Q_i^2 + 2\gamma Q_i Q_j + Q_j^2)/2 + m$, where $\theta$ denotes the level of persuasive advertising.[5] Then, the demand for each product is $Q_i = [(a+\theta)(1-\gamma) - p_i + p_j\gamma]/[b(1-\gamma^2)]$. The profits of the manufacturer and retailer $i$ are $\Pi_M = (w - c)(Q_i + Q_j) - k\theta^2$ and $\pi_i = (p_i - w)Q_i$,

[5] The utility function under persuasive advertising is similar to Zhang et al. (2020), and Hu and Mizuno (2021).

respectively. The other settings are the same as those in the previous section. For concavity of the manufacturer's profit, we assume $bk \equiv z > (1 + a - c)/[2(2 + \gamma - \gamma^2)] \equiv z^{SOC}$.

**Calculating equilibrium** In the fourth stage, when both firms choose prices, on substituting the demand function into the retailers' profit function and maximizing it with respect to the price, we obtain the following price and quantity:

$$p_i^{Bpa}(w, \theta) = \frac{(1-\gamma)(a+\theta)+w}{2-\gamma}, \quad Q_i^{Bpa}(w, \theta) = \frac{a-w+\theta}{b(2+\gamma-\gamma^2)},$$

where the superscript $Bpa$ denotes Bertrand competition with persuasive advertising.

Next, we consider the case when both retailers choose quantities. Solving $Q_i = [(a + \theta)(1-\gamma) - p_i + p_j\gamma]/[b(1-\gamma^2)]$ for $p_i$ and $p_j$, we can obtain the inverse demand function $p_i(Q_i, Q_j) = a+\theta-b(Q_i+\gamma Q_j)$. Using this inverse demand function, we obtain the following quantities and prices in this subgame.

$$Q_i^{Cpa}(w, \theta) = \frac{a-w+\theta}{b(2+\gamma)}, \quad p_i^{Cpa}(w, \theta) = \frac{a+\theta+w(1+\gamma)}{2+\gamma},$$

where the superscript $Cpa$ denotes Cournot competition with persuasive advertising.

Finally, we analyze the asymmetric case wherein one retailer chooses quantity and the other chooses price. Without loss of generality, we assume that retailer $i$ chooses quantity $Q_i$, and retailer $j$ chooses price $p_j$. Solving the demand function $Q_i = [(a + \theta)(1-\gamma) - p_i + p_j\gamma]/[b(1-\gamma^2)]$ for $p_i$ and $Q_j$, we obtain the demand systems in strategic variables $Q_i$ and $p_j$: $p_i(Q_i, p_j) = (a+\theta)(1-\gamma) + p_j\gamma - bQ_i(1-\gamma^2)$, and $Q_j(Q_i, p_j) = (a+\theta-p_j - bQ_i\gamma)/b$, respectively. Using the above demand systems and solving the first-order conditions

$\partial \pi_i(Q_i, p_j)/\partial Q_i = 0$ and $\partial \pi_j(Q_i, p_j)/\partial p_j = 0$, we obtain the following quantities and prices:

$$
\begin{aligned}
Q_i(w,\theta) = Q^{CBpa}(w,\theta) &= \frac{(a-w+\theta)(2-\gamma)}{b(4-3\gamma^2)}, \\
p_i(w,\theta) = p^{CBpa}(w,\theta) &= \frac{(a+\theta)(2-\gamma-2\gamma^2+\gamma^3)+w(2+\gamma-\gamma^2-\gamma^3)}{4-3\gamma^2}, \\
Q_j(w,\theta) = Q^{BCpa}(w,\theta) &= \frac{(a-w+\theta)(2-\gamma-\gamma^2)}{b(4-3\gamma^2)}, \\
p_j(w,\theta) = p^{BCpa}(w,\theta) &= \frac{(a+\theta)(2-\gamma-\gamma^2)+w(2+\gamma-2\gamma^2)}{4-3\gamma^2}
\end{aligned}
$$

where the superscript $CBpa$ ($BCpa$) indicates that this result is obtained when a retailer competes in quantity (price) while its rival competes in price (quantity) with persuasive advertising.

In the third stage, the manufacturer chooses its wholesale price. Regardless of which market contract regime the retailers choose, the manufacturer chooses the same wholesale price: $w = (a+\theta+c)/2$.

In the second stage, for the case wherein both retailers choose prices, the manufacturer chooses the following:

$$\theta^{Bpa} = \frac{a-c}{2z(2+\gamma-\gamma^2)-1}.$$

Note that $z \equiv bk$. Then, for the case wherein both retailers choose quantities, we derive the following level of advertising:

$$\theta^{Cpa} = \frac{a-c}{2z(2+\gamma)-1}.$$

For the asymmetric case wherein one retailer chooses quantity and the other chooses price, we obtain the following:

$$\theta^{BCpa} = \theta^{CBpa} = \frac{(a-c)(4-2\gamma-\gamma^2)}{4z(4-3\gamma^2)+2\gamma+\gamma^2-4}.$$

Comparing the advertising levels in the three cases above, we observe the following:

$$\theta^{Bpa} - \theta^{BCpa} = \frac{2z(a-c)\gamma^2(2-\gamma-\gamma^2)}{[1-2z(2-\gamma)(1+\gamma)][4-\gamma(2+\gamma)-4z(4-3\gamma^2)]} > 0,$$
$$\theta^{BCpa} - \theta^{Cpa} = \frac{2z(a-c)\gamma^2(2-\gamma)}{[1-2z(2+\gamma)][4-\gamma(2+\gamma)-4z(4-3\gamma^2)]} > 0.$$

Notably, the assumption $z > z^{SOC}$ ensures that the denominators are positive. Hence, we obtain the following lemma with an intuition similar to Lemma 2.

**Lemma 3** *The level of persuasive adverting is ranked as follows:* $\theta^{Bpa} > \theta^{BCpa} = \theta^{CBpa} > \theta^{Cpa}$.

Here, substituting the subgame outcomes into the profits of retailers, we obtain the retailer's profits as follows:

$$\pi^{Bpa} = \frac{\xi(1-\gamma^2)}{[1-2z(2+\gamma-\gamma^2)]^2}, \quad \pi^{Cpa} = \frac{\xi}{1-2z(2+\gamma)]^2},$$
$$\pi^{CBpa} = \frac{4\xi(2-\gamma)^2(1-\gamma^2)}{[4-2\gamma-\gamma^2+4z(4-3\gamma^2)]^2}, \quad \pi^{BCpa} = \frac{4\xi(2-\gamma-\gamma^2)^2}{[4-2\gamma-\gamma^2+4z(4-3\gamma^2)]^2},$$

where $\xi \equiv (a-c)^2bk^2 > 0$.

Now, we consider the first stage. Comparing the retailers' profits in the above cases, we present the ranking for the same.

$$\begin{cases} \pi^{CBpa} < \pi^{Bpa}, \quad \pi^{Cpa} < \pi^{BCpa} & \text{if} \quad z^{SOC} < z < 1/(4\gamma), \\ \pi^{Bpa} < \pi^{CBpa}, \quad \pi^{BCpa} \leq \pi^{Cpa} & \text{if} \quad 1/(4\gamma) \leq z. \end{cases}$$

This result directly leads to the following proposition:

**Proposition 3** *If a manufacturer engages in persuasive advertising, when the advertising investment is sufficiently efficient (small $k$) or the degree of product substitutability is sufficiently low (small $\gamma$)—that is, $z \equiv bk < 1/(4\gamma)$, the retailers offer a price contract (BB); otherwise, they offer a quantity contract (CC). The asymmetric regime wherein one retailer chooses price and the other chooses quantity never occurs.*

The intuition is as follows: When $k$ is small, the advertising investment is efficient, which strengthens the advertising effect. When this advertising effect is sufficiently large because of efficient advertising, it may dominate the competition effect, which influences retailers to choose a more competitive market contract ($BB$). Meanwhile, if the degree of product substitutability is sufficiently low, similar to Proposition 1, competition effect becomes less important and may be dominated by advertisement effect; this also leads both firms to offer a price contract ($BB$). Similarly, when investment is inefficient with large $k$ or the degree of product substitutability is sufficiently large, retailers offer a quantity contract ($CC$). The incentives to offer a price contract (or quantity contract) differ depending on the type of contract chosen by the rival retailer. Hence, there are two thresholds for offering a price contract, and in this study, those thresholds are equal. Consequently, each firm chooses a contract type based on its dominant strategy.

# 5 Conclusions

We consider a supply chain with a manufacturer engaged in advertising and two retailers producing differentiated products. We analyze the endogenous choice between quantity or price competition for retailers. We find that depending on the degree of product substitutability, the set of possible outcomes is rich and includes Cournot, Bertrand, and Cournot-Bertrand under informative advertising, which contrasts with the well-known result that the dominant strategy is for firms to compete à la Cournot. Moreover, we demonstrate that under both informative and persuasive advertising, the manufacturer, consumer, and total surpluses are the largest when retailers offer a price contract. Additionally, under persuasive advertising, either Cournot or Bertrand outcomes may occur in equilibrium, but Cournot-Bertrand never occurs.

It is worthwhile to consider wholesale price discrimination and also advertising differentiation, wherein the manufacturer offers different levels of advertising to the two retailers. We leave this investigation for future research.

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