（前期第Ⅱ期・社会人）

令和５年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

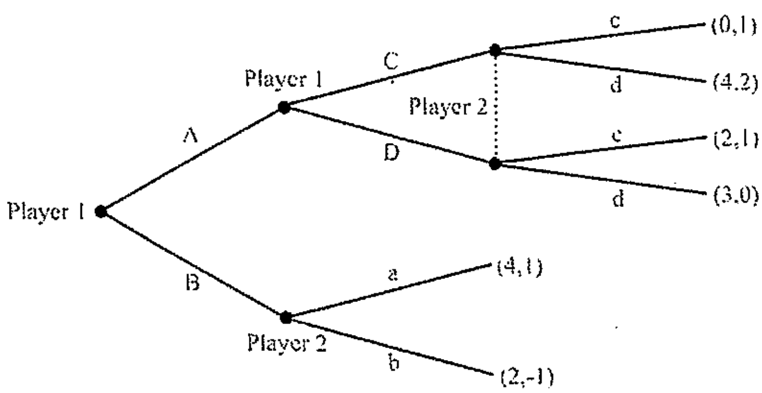
経済理論

・第１問〜第４問のすべてに日本語か英語で答えなさい。
・各問の解答は，それぞれ別の解答用紙に記入しなさい。

第１問　二人の消費者と二つの財x, yからなる交換経済を考える。各消費者の効用関数は$u_1(x_1, y_1) = x_1 y_1^2$，$u_2(x_2, y_2) = \min\{x_2, y_2\}$，初期賦存は$\omega_1 = (1,2)$, $\omega_2 = (2,3)$とする。

（１）各財の価格を所与として各消費者の最適化問題を記述せよ。(15 点)
（２）この市場におけるワルラス均衡の定義を記述せよ。(15 点)
（３）この市場のワルラス均衡を求めなさい。(20 点)

第２問　（１）純粋戦略ナッシュ均衡の数学的定義を記述せよ。ただし使用した数学記号の意味を全て説明すること。(10 点)
（２）純粋戦略ナッシュ均衡が存在するとして，それが実現すると考えられる状況を可能な限り（最大四つ）記述せよ。(5 点×4)
（３）下記ゲームの部分ゲーム完全均衡ではない純粋戦略ナッシュ均衡を全て求めなさい。ただし利得は（Player 1, 2）の順に記述してあり，点線は二つのノードが同じ情報集合に含まれることを意味する。(20 点)



第３問　以下の用語を全て使用し，マクロ経済モデルにおける「一般均衡」とは何かを説明しなさい。その際，概念図を用いながら説明すること。なお，各用語について，最初に使用した箇所に下線を引くこと。(50 点)

使用する用語：
家計，企業，政府，財市場，生産要素市場，金融市場，価格，賃金

第４問　金利の期間構造における期待理論を考える（金利は全て年率)。

（１）　今月の１ヶ月物金利は１％で，来月以降は同金利が１年以上２％で維持されると期待されている。このとき，ターム・プレミアムを無視すると，現在の１年物金利の水準はおよそ何%になるか。(10 点)

（２）　１ヶ月物金利が２％，２ヶ月物金利は３％とする。２ヶ月物のタームプレミアムを１％とすると，１ヶ月後の１ヶ月物金利は何%だと予想されるか。(15 点)

（３）　t時点におけるmヶ月物金利を i_t^m とする。このとき，i_t^3 を１ヶ月物金利で表しなさい（ただしtは離散時間を表す)。(15 点)

（４）　日本銀行の金融政策では，「10 年物国債金利がゼロ％程度で推移するよう」金融調節を行っている。期間構造の期待理論の観点から，この政策は短期金利の推移にどのような含意があるのかを議論しなさい。(10 点)

（前期第Ⅱ期・社会人）

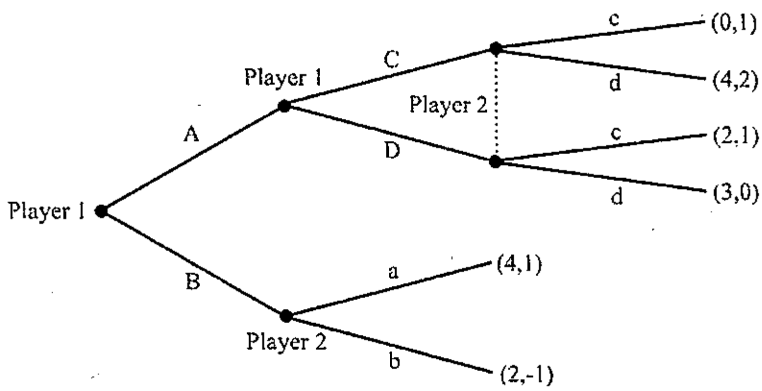
令和５年度　神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

Economic Theory

・Answer all of the following four questions in either English or Japanese.
・Use a separate answer sheet for each question.

1. Consider an exchange economy with two consumers and two goods x, y. The utility function of each consumer is $u_1(x_1, y_1) = x_1 y_1^2$ and $u_2(x_2, y_2) = \min\{x_2, y_2\}$, respectively, while the initial endowment for each consumer is $\omega_1 = (1,2)$ and $\omega_2 = (2,3)$.
   (1) Given a price for each good, describe a maximization problem for each consumer. (15 points)
   (2) Write down the definition of a Walrasian equilibrium in this economy. (15 points)
   (3) Derive a Walrasian equilibrium in this economy. (20 points)

2. (1) Write down the mathematical definition of a pure-strategy Nash equilibrium (whenever you use mathematical symbols, you must describe their meanings). (10 points)
   (2) Provided that a pure-strategy Nash equilibrium exists, describe as many situations as possible (up to four) when such an equilibrium would occur. (5 points×4)
   (3) In the following game (where for each payoff profile, player 1's payoff is specified first, and the dotted line means that the two nodes are in the same information set), derive all the pure-strategy Nash equilibria that are *not* subgame perfect. (20 points)



3. Using all of the following keywords, explain what "general equilibrium" means in macroeconomic models. Use a schematic figure in the description. Underline the keywords where they are used for the first time. (50 points)

Keywords:
Household, firm, government, goods market, factor market, financial market, price, wage.

4. Consider the expectations theory of the term structure of interest rates (e.g., interest rates are annualized).

(1) This month's one-month interest rate is 1% and is expected to be at 2% for at least one year from the next month onward. If term premiums are ignored, what is the current level of the one-year interest rate? (10 points)

(2) Assume that the 1-month interest rate is 2% and the 2-month interest rate is 3%. If the term premium for the 2-month rate is 1%, what is the 1-month rate expected to be after 1 month? (15 points)

(3) Let i_t^m denote the m-month interest rate. Describe i_t^3 in terms of one-month interest rates (t denotes a discrete time period). (15 points)

(4) The Bank of Japan conducts monetary policy "so that 10-year JGB (i.e., Japanese government bonds) yields will remain at around zero percent." From the standpoint of the expectations theory of the term structure, discuss what implications this policy has for the evolution of short-term interest rates. (10 points)

(前期第II期・社会人)

令和5年度 神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

統計学

- 第1問〜第3問のすべてに日本語か英語で答えなさい。
- 各問の解答は，それぞれ別の解答用紙に記入しなさい。
- 本研究科で貸与する電卓のみ使用を認めます。
- 必要に応じて，添付の統計分布表を利用しなさい。

第1問 2つの変数 (X,Y) のデータが次のように与えられているとする。

x_i	3	4	3	5	5
y_i	4	5	5	6	10

以下の問いに答えなさい。ただし，小数が出る場合は，小数第2位まで求めなさい。

(1) x_i の平均 $\bar{x}$ と標本不偏分散 s_x^2 を求めなさい。(10点)

(2) x_i が正規母集団 $\mathrm{N}(\mu_x,\sigma_x^2)$ から得られた無作為標本である時，μ_x の信頼係数 0.95 の信頼区間を求めなさい。(10点)

(3) x_i が正規母集団 $\mathrm{N}(\mu_x,\sigma_x^2)$ から得られた無作為標本である時，σ_x^2 の信頼係数 0.95 の信頼区間を求めなさい。(10点)

(4) x_i, y_i $(i=1,2,\ldots,5)$ が互いに独立な正規母集団 $\mathrm{N}(\mu_x,4)$, $\mathrm{N}(\mu_y,9)$ から得られた無作為標本であるとする。ただし，それぞれの母集団の分散は既知であるとする。帰無仮説 $\mathrm{H}_0:\mu_x=\mu_y$ を対立仮説 $\mathrm{H}_1:\mu_x\neq\mu_y$ に対して有意水準 0.10 で検定する方法を説明しなさい。(20点)

第2問 離散確率変数 X の確率関数が

$$f(x)=\begin{cases} p^{1-x}(1-p)^x & x=0,1 \\ 0 & \text{その他} \end{cases}$$

で与えられるとする。ただし p は $0<p<1$ の定数である。以下の問いに答えなさい。

(1) X の平均と分散を求めなさい。(20点)

(2) $X_1,X_2,\cdots,X_n$ はこの分布から独立に得られた大きさ n の標本であるとする。p の最尤推定量 $\widehat{p}$ を求めなさい。(20点)

(3) (2) で求めた最尤推定量 $\widehat{p}$ の平均と分散を求めなさい。(20点)

第３問　次の回帰モデルを考える。

$$y_i = \alpha + \beta x_i + u_i, \qquad i = 1, \ldots, n.$$

ここで, y は被説明変数, x は説明変数, u は誤差項, α と β は推定すべきパラメータ, n はサンプルサイズである。被説明変数 y と説明変数 x の標本平均をそれぞれ次式で定義する。

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

(1) β の最小二乗推定量 $\hat{\beta}$ を求めなさい。(15 点)

(2) α の最小二乗推定量 $\hat{\alpha}$ を求めなさい。 なお, $\hat{\alpha}$ の式は $\hat{\beta}$ に依存する形で表記してよい。(15 点)

これ以降, 説明変数 x は次式を満たすダミー変数であると仮定する。

$$x_i = \begin{cases} 1 & (i = 1, 2, \ldots, n_1 \text{ のとき}) \\ 0 & (i = n_1 + 1, n_1 + 2, \ldots, n \text{ のとき}) \end{cases}$$

ただし, $1 \leq n_1 < n$ かつ $1 \leq n_0 < n$ であると仮定する。ここで, $n_0 = n - n_1$ である。

(3) $\sum_{i=1}^{n}(x_i - \bar{x})^2$ の値を求めなさい。(15 点)

(4) 説明変数 x が 1 をとるグループと 0 をとるグループそれぞれについて, 被説明変数 y の標本平均を次式で定義する。

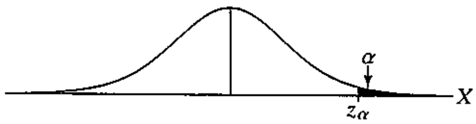
$$\bar{y}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i, \qquad \bar{y}_0 = \frac{1}{n_0}\sum_{i=n_1+1}^{n} y_i.$$

$\bar{y}_1$ と $\bar{y}_0$ を用いて, $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$ の値を求めなさい。(15 点)

(5) $\hat{\beta}$ の値を求めなさい。(15 点)

(6) $\hat{\alpha}$ の値を求めなさい。(15 点)

正規分布表： $X \sim \mathrm{N}(0, 1)$

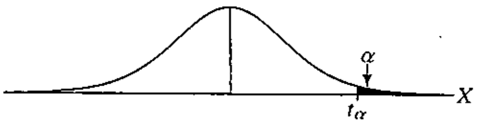


$$\alpha = \mathrm{P}(X > z_\alpha) = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)\mathrm{d}x$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4091	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2644	.2611	.2579	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

α	.10	.05	.025	.010	.005	.001	.0005	.0001	.00001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	4.2649

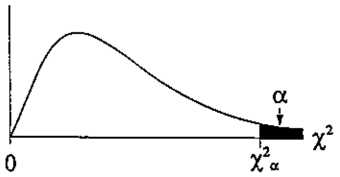
t 分布表： $X \sim t(k)$



$$\alpha = P(X > t_\alpha) = \int_{t_\alpha}^{\infty} f(x)\mathrm{d}x$$

α / k (自由度)	.10	.05	.025	.010	.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8187
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
120	1.2886	1.6577	1.9799	2.3578	2.6174
∞	1.2816	1.6449	1.9600	2.3263	2.5758

カイ２乗分布表： $\chi^2(m)$



$$\alpha = P(\chi^2 > \chi^2_\alpha) = \int_{\chi^2_\alpha}^{\infty} f(\chi^2)\, d\chi^2$$

α m (自由度)	.995	.99	.975	.95	.90	.10	.05	.025	.010	.005
1	.0000393	.000157	.000982	.00393	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.06	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43	104.21
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17

令和 5 年度 神戸大学大学院経済学研究科
博士課程前期課程入学試験問題

Statistics

- Answer all of the following questions either in English or in Japanese.
- Answer each question on a separate sheet.
- Applicants are authorized to use a calculator lent by our Graduate School.
- Use the statistical tables if necessary.

1. Assume that the data of the variables (X, Y) are given as follows.

x_i	3	4	3	5	5
y_i	4	5	5	6	10

Answer the following questions. If the answers are decimals, calculate down to 2 decimal places.

(1) Find the mean $\bar{x}$ and the sample unbiased variance s_x^2 of x_i. (10 points)

(2) Assume that x_is are random samples from a normal population $\mathrm{N}(\mu_x, \sigma_x^2)$. Find the 0.95 confidence interval of μ_x. (10 points)

(3) Assume that x_is are random samples from a normal population $\mathrm{N}(\mu_x, \sigma_x^2)$. Find the 0.95 confidence interval of σ_x^2. (10 points)

(4) Assume that x_i and y_i $(i = 1, 2, \ldots 5)$ are random samples from mutually independent normal populations with known variances $\mathrm{N}(\mu_x, 4)$ and $\mathrm{N}(\mu_y, 9)$. Explain the way to test the null hypothesis $\mathrm{H}_0 : \mu_x = \mu_y$ against the alternative $\mathrm{H}_1 : \mu_x \neq \mu_y$ at the 0.10 significance level. (20 points)

2. Assume that the probability function of a discrete random variable X is given by

$$f(x) = \begin{cases} p^{1-x}(1-p)^x & x = 0, 1 \\ 0 & \text{otherwise,} \end{cases}$$

where p is a constant such that $0 < p < 1$. Answer the following questions.

(1) Find the mean and the variance of X. (20 points)

(2) Let $\{X_1, X_2, \cdots, X_n\}$ be a sample of size n independently drawn from this distribution. Find the maximum likelihood estimator $\widehat{p}$ for p. (20 points)

(3) Find the mean and the variance of maximum likelihood estimator $\widehat{p}$ derived in (2). (20 points)

3. Consider the following regression model:

$$y_i = \alpha + \beta x_i + u_i, \qquad i = 1, \ldots, n,$$

where y is the regressand, x is the regressor, u is the error term, α and β are the parameters to estimate, and n is the sample size. Define the sample means of y and x as follows:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

(1) Compute $\hat{\beta}$, the ordinary least squares estimator for β. (15 points)

(2) Compute $\hat{\alpha}$, the ordinary least squares estimator for α. The expression of $\hat{\alpha}$ can depend on $\hat{\beta}$. (15 points)

In what follows, assume that x is a dummy variable which satisfies the following:

$$x_i = \begin{cases} 1 & \text{for } i = 1, 2, \ldots, n_1, \\ 0 & \text{for } i = n_1 + 1, n_1 + 2, \ldots, n. \end{cases}$$

Assume that $1 \leq n_1 < n$ and $1 \leq n_0 < n$, where $n_0 = n - n_1$.

(3) Compute $\sum_{i=1}^{n}(x_i - \bar{x})^2$. (15 points)

(4) Define the sample means of y for the group of individuals with $x = 1$ and for the group of individuals with $x = 0$ as:

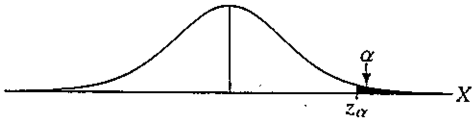
$$\bar{y}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} y_i, \qquad \bar{y}_0 = \frac{1}{n_0}\sum_{i=n_1+1}^{n} y_i.$$

Compute $\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$, using $\bar{y}_1$ and $\bar{y}_0$. (15 points)

(5) Compute $\hat{\beta}$. (15 points)

(6) Compute $\hat{\alpha}$. (15 points)

Normal distribution: $X \sim N(0,1)$

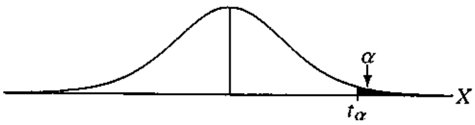


$$\alpha = P(X > z_\alpha) = \int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)dx$$

z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4841	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4091	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2644	.2611	.2579	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2207	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1563	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1094	.1075	.1057	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002

α	.10	.05	.025	.010	.005	.001	.0005	.0001	.00001
z_α	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905	3.7190	4.2649

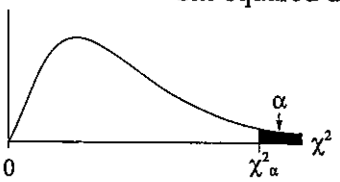
t distribution: $X \sim t(k)$



$$\alpha = P(X > t_\alpha) = \int_{t_\alpha}^{\infty} f(x)dx$$

k (Degrees of Freedom) \ α	.10	.05	.025	.010	.005
1	3.0777	6.3138	12.7062	31.8205	63.6567
2	1.8856	2.9200	4.3027	6.9646	9.9248
3	1.6377	2.3534	3.1824	4.5407	5.8409
4	1.5332	2.1318	2.7764	3.7469	4.6041
5	1.4759	2.0150	2.5706	3.3649	4.0321
6	1.4398	1.9432	2.4469	3.1427	3.7074
7	1.4149	1.8946	2.3646	2.9980	3.4995
8	1.3968	1.8595	2.3060	2.8965	3.3554
9	1.3830	1.8331	2.2622	2.8214	3.2498
10	1.3722	1.8125	2.2281	2.7638	3.1693
11	1.3634	1.7959	2.2010	2.7181	3.1058
12	1.3562	1.7823	2.1788	2.6810	3.0545
13	1.3502	1.7709	2.1604	2.6503	3.0123
14	1.3450	1.7613	2.1448	2.6245	2.9768
15	1.3406	1.7531	2.1314	2.6025	2.9467
16	1.3368	1.7459	2.1199	2.5835	2.9208
17	1.3334	1.7396	2.1098	2.5669	2.8982
18	1.3304	1.7341	2.1009	2.5524	2.8784
19	1.3277	1.7291	2.0930	2.5395	2.8609
20	1.3253	1.7247	2.0860	2.5280	2.8453
21	1.3232	1.7207	2.0796	2.5176	2.8314
22	1.3212	1.7171	2.0739	2.5083	2.8187
23	1.3195	1.7139	2.0687	2.4999	2.8073
24	1.3178	1.7109	2.0639	2.4922	2.7969
25	1.3163	1.7081	2.0595	2.4851	2.7874
26	1.3150	1.7056	2.0555	2.4786	2.7787
27	1.3137	1.7033	2.0518	2.4727	2.7707
28	1.3125	1.7011	2.0484	2.4671	2.7633
29	1.3114	1.6991	2.0452	2.4620	2.7564
30	1.3104	1.6973	2.0423	2.4573	2.7500
40	1.3031	1.6839	2.0211	2.4233	2.7045
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
120	1.2886	1.6577	1.9799	2.3578	2.6174
∞	1.2816	1.6449	1.9600	2.3263	2.5758

Chi-squared distribution: $\chi^2(m)$



$$\alpha = P(\chi^2 > \chi^2_\alpha) = \int_{\chi^2_\alpha}^{\infty} f(\chi^2)\, d\chi^2$$

α / m (Degrees of Freedom)	.995	.99	.975	.95	.90	.10	.05	.025	.010	.005
1	.0000393	.000157	.000982	.00393	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.06	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.65
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43	104.21
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81	140.17