

**Bias Composition and Information Transmission in
Organizations**

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Bias Composition and Information Transmission in Organizations *

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Abstract

We study information transmission within organizations, focusing on how biased Subordinates should be combined to elicit truthful information. We assume that the directions of the Subordinates' biases are common knowledge, whereas their magnitudes are privately known to each Subordinate. We show that a homogeneous composition of Subordinates is preferable to a heterogeneous one. The intuition is that, under a homogeneous composition, the effect of an individual false report is amplified by false reports from other Subordinates. Anticipating this amplification effect reduces each Subordinate's incentive to misreport and thereby promotes truthful information transmission.

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1 Introduction

In their seminal book, Cyert and March [7] view an organization as a communication system and emphasize that information transmitted within organizations is often biased because organizational members may strategically manipulate information in an attempt to influence organizational decisions. Motivated by this observation, this paper studies information transmission within organizations using a cheap-talk framework à la Crawford and Sobel [6]. In particular, we ask how an organization should combine members with different directional biases in order to elicit truthful information from them. A key feature of our model is uncertainty about the magnitude of each member's bias.

Krishna and Morgan [16] provide a seminal analysis of strategic information transmission between a receiver and multiple senders. They consider a sequential cheap-talk model with two senders and compare heterogeneous senders, whose biases are in opposite directions, with homogeneous senders, whose biases are in the same direction.¹ Their main result is that, under relatively mild conditions, heterogeneous senders are preferable to homogeneous senders from the receiver's perspective.

A crucial assumption in Krishna and Morgan [16], however, is that the senders' biases are common knowledge. In many organizational settings, it may be natural to know the direction in which a colleague would like to influence the organization's decision while remaining uncertain about the strength of that preference. We therefore depart from Krishna and Morgan by assuming that the *directions* of the senders' biases are common knowledge, whereas their *magnitudes* are privately known. We show that this seemingly modest change in the informational environment reverses the main conclusion of Krishna and Morgan.

More specifically, we consider a game between one Boss and multiple Subordinates. The Subordinates act as senders of cheap-talk messages, while the Boss is the receiver. The underlying state is binary. There are two types of Subordinates: upward biased and downward biased Subordinates. The biases of upward biased Subordinates are drawn from a positive range, whereas those of downward biased Subordinates are drawn from a negative range. Our main result is that the Boss prefers a completely homogeneous

¹Krishna and Morgan consider a one-dimensional state space. For a multidimensional state space, see Battaglini [2].

composition of Subordinates to any other composition.

The intuition behind this result is as follows. Consider first a composition that is not completely homogeneous. When a Subordinate contemplates sending a false report, he anticipates that Subordinates with the opposing directed biases send reports in the opposite direction. Because conflicting reports lead the Boss to discount the informational content of each message, the effect of an individual false report on the Boss’s decision is limited. This weakens the cost of misreporting and thereby makes deception more attractive. By contrast, when all Subordinates are biased in the same direction, a false report by one Subordinate is reinforced by false reports from others, potentially inducing a larger distortion in the Boss’s decision. Anticipating this amplification effect makes each Subordinate more cautious about misreporting and strengthens the incentive to report truthfully. This mechanism generates a novel rationale for homogeneous information sources within organizations.

1.1 Related Literature

Cheap talk models with one sender whose bias is uncertain have been analyzed by Morgan and Stocken [23], Dimitrakas and Sarafidis [11], Li and Madarász [20] for static situations, and Sobel [33], Benabou and Laroque [3], Morris [24] for dynamic situations. Cheap talk models with multiple senders of which biases are uncertain also have been analyzed by Li [18, 19], Le Queument [28], Karakoç [15]. However, they have not addressed the combination of directions of senders’ biases.

The combination of directions of senders’ biases has been addressed by Rantakari [29, 30] and Shimizu [31].² Rantakari [29, 30] consider a model in which sender’s private information pertains to the probability of his own project or idea which is independent from that of another sender. On the other hand, in our model senders’ private information is one common state of the world and thus they are completely correlated. Shimizu [32] is the most closely related paper to the present one. I investigate the combination of directions of senders’ biases when they are imperfectly informed of the realized state by adopting Austen-Smith [1] as baseline model. The present model is simpler in the sense that a state space is binary. Instead, it allows

²Mechtenberg and Münster [22] discuss the combination of directions of sender’s and strategic mediator’s biases.

much more general environments, especially concerning bias distributions and the number of Subordinates.

The homogeneity/heterogeneity in principal-agent relationships has been discussed by many papers.³ Most of them, however, consider the homogeneity/heterogeneity between principal and agent. The exceptions are Prasad and Tomaino [27], Prasad and Tanase [26], and Rantakari [29, 30] referred above. The first two papers examine the divergence between two agents. Their models are, however, much different from ours in the sense that there is no asymmetric information in Prasad and Tomaino and there is no strategic information transmission in Prasad and Tanase.

1.2 Organization of the Paper

In Section 2, we present our model and refers to strategy and equilibrium concept in our setting. In Section 3, we present a basic result in the case of 2 Subordinates and uniformly distributed biases. In Section 4, we show that the previous results are extended to more general environments, more precisely, those with any arbitrary number of Subordinates and more generally distributed biases. In Section 5, we show the robustness of our results by discussing the possibility of their extensions to more general environments. In Section 6, we conclude the paper. In Appendix, we present a few proofs omitted in the main text.

2 Model

2.1 Environment

There are $n + 1$ players where $n \geq 2$. Player 0 is called Boss (female), who is the receiver of cheap talk messages. Players $1, 2, \dots, n$ called Subordinates (males), who are the senders of cheap talk. The state of the world is denoted by t , which takes binary values 0 or 1 with equal probabilities. $a \in \mathbb{R}$ is an action chosen by Boss. For ease of exposition, we denote the state space and the action space by $T := \{0, 1\}$ and $A := \mathbb{R}$, respectively.

³There are mainly two lines of research: those which deal with separation of decision and implementation (Blanes i Vidal and Möller [35], Bester and Kräbmer [4], Landier et al. [17], Marino et al. [21], Van den Steen [10], Ishihara and Miura [13], and Itoh and Morita [14]) and those which deal with information acquisition (Szalay [34], Hori [12], Che and Kartik [5], Van den Steen [9], Omiya et al. [25], and de Bettignies and Zábojník [8]).

We assume that player i 's payoff function is $U_i(t, b_i, a) := -(t + b_i - a)^2$ where b_i is called player i 's bias. We assume that $b_0 = 0$ for normalization. This implies that Boss's best response is to choose the action equal to Boss's belief on $t = 1$. Subordinate i 's bias is distributed by a distribution function $F_i(b_i)$, specified in detail later. We assume that $t, b_1, b_2, \dots, b_n$ are mutually independent.

We assume b_0 is common knowledge, while b_i for $i \neq 0$ is Subordinate i 's private information. A distribution function F_i for $i \neq 0$ is, however, assumed to be common knowledge.

According to Krishna and Morgan [16], we consider a sequential information transmission protocol. The timing of the game is as follows:

Stage 0: A state t is realized, which is only observed by Subordinates. Also, for each $i \neq 0$, b_i is realized, which is only observed by Subordinate i .

Stage i ($i = 1, \dots, n$): Subordinate i publicly announces a cheap talk message $m_i \in M_i$, which is heard by Boss and other Subordinates.

Stage $(n + 1)$: Based on the received message profile, Boss chooses an action $a \in A$.

We assume that each Subordinate's message space M_i contains 2 or more messages.

Let F be a baseline distribution function. We focus on F satisfying the following assumptions.

Assumption 1

- F is continuous.
- $F\left(\frac{1}{4}\right) > 0$ and $F\left(\frac{1}{2}\right) < 1$.
- $\text{Supp } F \subseteq [0, 1]$.

The first assumption is made for the existence of pure strategy equilibrium. The second assumption is made for excluding degenerate cases. Roughly speaking, a random variable is widely distributed, not clustered at a particular point. The third assumption is the most critical. The most important thing is that it never takes a negative value.

By using a baseline distribution function, we define two types of Subordinates as follows. The first type Subordinates are called *upward biased*,

where their bias distributions F_i is equal to F . The second type Subordinates are called *downward biased* where their bias distributions satisfy $F_i(b) = 1 - F(-b)$ for any b . Combined with the assumption that bias distributions are common knowledge, these assumptions implies that the other players know the direction of a Subordinate while they do not know the realized value. To put it differently, it is common knowledge that a bias of upward or downward biased Subordinate is non-negative or non-positive, respectively. Also, it is assumed that Subordinates are symmetric except their bias directions. We assume $F_1(b) = F(b)$ without loss of generality.

2.2 Equilibrium

In order to avoid technical difficulties, we focus on Subordinates' pure strategies. Generally, Subordinate i pure strategy is a function μ_i such that $\mu_i : T \times B_i \times M_1 \times \cdots \times M_{i-1} \rightarrow M_i$. We, however, focus on the class of Subordinates' pure strategies that are characterized by some thresholds. b_+ or b_- is a threshold for upward or downward biased Subordinates, respectively. For ease of exposition, we denote them such that $b_+ \geq 0$ and $b_- \geq 0$. In this class, there are two messages each of which corresponds to each state. We denote a message corresponding to $t = 0$ or 1 by $m_i = 0$ or 1 , respectively.

Using this notation, we define a strategy for an upward biased Subordinate i as follow. When $t = 1$ is realized, he always sends a truthful message $m_i = 1$ in any equilibrium since he has no incentive to lie. On the other hand, when $t = 0$ is realized, the size of his bias matters. When his bias is sufficiently small such that $b_i \leq b_+$, he sends a truthful message $m_i = 0$. When his bias is sufficiently large such that $b_i > b_+$, he sends a false message $m_i = 1$. Note that this strategy is history-independent and order-independent.

Similarly, we define a strategy for a downward biased Subordinate i as follows. When $t = 0$ is realized, he always sends a truthful message $m_i = 0$ in any equilibrium. On the other hand, when $t = 1$ and $|b_i| \leq b_-$, he sends a truthful message $m_i = 1$. When $t = 1$ and $|b_i| > b_-$, he sends a false message $m_i = 0$. This strategy is also history-independent and order-independent.

These thresholds b_+ and b_- can be said to measure the degree of organizational governance concerning the Subordinates' incentives to manipulate information relevant to the organization decision. In other words, the larger the equilibrium thresholds are, the more efficient information Boss can elicit from Subordinates.

Generally, Boss's pure strategy is a function α such that $\alpha : M_1 \times \cdots \times$

$M_n \rightarrow A$. It is, however, much more simply written when Subordinates follow history-independent and order-independent threshold strategies defined above. Boss considers $m_i = 1$ (resp. $m_i = 0$) dubious when it is sent by upward (resp. downward) biased Subordinate i . It follows that Boss's best response is depending upon the numbers of dubious messages. To be more precise, Boss's best response is defined as $a(\tilde{k}, \tilde{\ell})$ where $\tilde{k}$ is the number of dubious messages sent by upward biased Subordinates and $\tilde{\ell}$ is the number of dubious messages sent by downward biased Subordinates.

We adopt Perfect Bayesian equilibrium as solution concept. A PBE associated with the threshold strategy, which we simply call it a *threshold strategy equilibrium* hereafter, is defined a triple, $(b_+, b_-, a(\cdot, \cdot))$.

I would like to make a few remarks. First, we can prove that there exists no fully revealing PBE in this environment. This is done in Appendix A. Second, our focus on the threshold strategy equilibrium is not restrictive. To be exact, in Appendix B, we show that any PBE is essentially outcome-equivalent to some threshold strategy equilibrium under some mild condition.

2.3 Organizational mode

In this paper, we call a combination of Subordinates as *organizational mode*. To be more precise, organizational mode (n, k) means a combination of Subordinates in which there are n Subordinates in total, and among them, k Subordinates are upward biased and $(n - k)$ Subordinates are downward biased. Particularly, in the case of $n = 2$, organizational mode $(2, 2)$ is called as *homogeneous (Subordinates) mode*, while organizational mode $(2, 1)$ is called as *heterogeneous (Subordinates) mode*. Due to the symmetry referred above, it suffices to only consider the case of $k \geq \frac{n}{2}$ without loss of generality.

Hereafter, by n in organizational mode (n, k) , we mean that there are n Subordinates active in information transmission. For example, when there are 2 Subordinates, but only Subordinate 1 plays an influential strategy and the other Subordinate always babbles, this organizational mode is $(1, 1)$.

In this paper, we compare threshold strategy equilibria among various organizational modes. We denote an equilibrium variable x under organizational mode (n, k) by $x^{(n, k)}$.

3 2 Subordinates with Uniformly Distributed Biases

In this section, we consider the case in which there are 2 Subordinates with uniformly distributed biases, i.e., $n = 1, 2$ and $F(b) = b$ for $b \in [0, 1]$.

3.1 Single Subordinate Mode

First, we consider organizational mode $(1, 1)$. As referred previously, this is the situation in which only one Subordinate plays an influential threshold strategy and the other Subordinate always babbles. This mode can be also interpreted as the situation in which Boss intentionally ignores a particular Subordinate's message.

We obtain

$$\begin{aligned} a^{(1,1)}(0, 0) &= 0, \\ a^{(1,1)}(1, 0) &= \frac{1}{1 + (1 - b_+^{(1,1)})}. \end{aligned}$$

The Subordinate's incentive condition requires

$$-\left(0 + b_+^{(1,1)} - a^{(1,1)}(0, 0)\right)^2 = -\left(0 + b_+^{(1,1)} - a^{(1,1)}(1, 0)\right)^2,$$

which boils down to

$$b_+^{(1,1)} = \frac{a^{(1,1)}(1, 0)}{2}.$$

Therefore, $b_+^{(1,1)}$ is the solution of the following equation:

$$G^{(1,1)}(b) := b(1 - b) - \left(\frac{1}{2} - b\right) = 0.$$

Solving this, we obtain

$$b_+^{(1,1)} = 1 - \frac{1}{\sqrt{2}} \doteq 0.29289.$$

The receiver's equilibrium expected payoff is

$$\begin{aligned} EU_0^{(1,1)} &= -\frac{1}{2} \left(1 - b_+^{(1,1)}\right) \left(a^{(1,1)}(1, 0)\right)^2 - \frac{1}{2} \left(1 - a^{(1,1)}(1, 0)\right)^2 \\ &\doteq -0.20711. \end{aligned}$$

3.2 Homogeneous Subordinates Mode

Next, we consider organizational mode $(2, 2)$. We obtain

$$\begin{aligned} a^{(2,2)}(0, 0) &= a^{(2,2)}(1, 0) = 0, \\ a^{(2,2)}(2, 0) &= \frac{1}{1 + \left(1 - b_+^{(2,2)}\right)^2}. \end{aligned}$$

Subordinate's message is influential only if he is pivotal, in other words, the other Subordinate sends a false message. Therefore, the Subordinates' incentive condition requires that

$$-\left(0 + b_+^{(2,2)} - a^{(2,2)}(1, 0)\right)^2 = -\left(0 + b_+^{(2,2)} - a^{(2,2)}(2, 0)\right)^2,$$

which boils down to

$$b_+^{(2,2)} = \frac{a^{(2,2)}(2, 0)}{2}.$$

Therefore, $b_+^{(2,2)}$ is the solution of the following equation:

$$G^{(2,2)} := b(1 - b)^2 - \left(\frac{1}{2} - b\right) = 0.$$

Numerically, it is verified that $b_+^{(2,2)} \doteq 0.3522$. The receiver's equilibrium expected payoff is

$$\begin{aligned} EU_0^{(2,2)} &= -\frac{1}{2} \left(1 - b_+^{(2,2)}\right)^2 \left(a^{(2,2)}(2, 0)\right)^2 - \frac{1}{2} \left(1 - a^{(2,2)}(2, 0)\right)^2 \\ &\doteq -0.14780. \end{aligned}$$

3.3 Heterogeneous Subordinates Mode

Lastly, we consider organizational mode $(2, 1)$. We obtain

$$\begin{aligned} a^{(2,1)}(0, 1) &= 0, \\ a^{(2,1)}(1, 1) &= \frac{\left(1 - b_-^{(2,1)}\right)}{\left(1 - b_-^{(2,1)}\right) + \left(1 - b_+^{(2,1)}\right)} \\ a^{(2,1)}(1, 0) &= 1. \end{aligned}$$

Then, by the Subordinates' incentive conditions, we obtain

$$\begin{aligned} b_+^{(2,1)} &= \frac{a^{(2,1)}(1,1)}{2}, \\ b_-^{(2,1)} &= \frac{1 - a^{(2,1)}(1,1)}{2}. \end{aligned}$$

Therefore, $b_+^{(2,1)}$ is the solution of the following equation:

$$G^{(2,1)}(b) := b(1-b) - \left(\frac{1}{2} - b\right) \left\{1 - \left(\frac{1}{2} - b\right)\right\} = 0.$$

Solving this, we obtain

$$b_+^{(2,1)} = b_-^{(2,1)} = \frac{1}{4}.$$

The receiver's equilibrium expected payoff is

$$\begin{aligned} EU_0^{(2,1)} &= -\frac{1}{2} \left(1 - b_+^{(2,1)}\right) \left(a^{(2,1)}(1,1)\right)^2 - \frac{1}{2} \left(1 - b_-^{(2,1)}\right) \left(1 - a^{(2,1)}(1,1)\right)^2 \\ &= -0.1875. \end{aligned}$$

3.4 Comparison Results

Comparing the above three organizational modes, we obtain the following comparison results:

- $b_+^{(2,1)} < b_+^{(1,1)} < b_+^{(2,2)}$
- $EU_0^{(1,1)} < EU_0^{(2,1)} < EU_0^{(2,2)}$

As for the comparison between homogeneous mode and heterogeneous mode, the advantage in the equilibrium threshold leads to the advantage in the Boss's payoff. This is basically similar as in the comparison between homogeneous mode and single mode. However, in the comparison between heterogeneous mode and single mode, the ordering is reversed. This is because, in heterogeneous mode, the effect of having more information sources dominates the effect of smaller threshold.

4 Multiple Subordinates with Generally Distributed Biases

In this section, we consider the case of $n \geq 2$ with generally distributed biases.

4.1 Equilibrium Conditions and Payoff

Given senders' threshold strategy $(b_+^{(n,k)}, b_-^{(n,k)})$, the receiver's best response is as follows.

$$a^{(n,k)}(\tilde{k}, \tilde{\ell}) = \begin{cases} 0 & \text{if } \tilde{k} < k, \tilde{\ell} = n - k, \\ \frac{(1 - F(b_-^{(n,k)}))^{n-k}}{(1 - F(b_-^{(n,k)}))^{n-k} + (1 - F(b_+^{(n,k)}))^k} & \text{if } \tilde{k} = k, \tilde{\ell} = n - k, \\ 1 & \text{if } \tilde{k} = k, \tilde{\ell} < n - k. \end{cases}$$

Similarly in Section 3, Subordinate's incentive matters only when he is pivotal. Therefore, the Subordinates' incentive condition requires that

$$\begin{aligned} b_+^{(n,k)} &= \frac{a^{(n,k)}(k, n - k)}{2}, \\ b_-^{(n,k)} &= \frac{1 - a^{(n,k)}(k, n - k)}{2}. \end{aligned}$$

Combining these conditions, we obtain $b_+^{(n,k)}$ as a solution of the following equation.

$$G^{(n,k)}(b) := b(1 - F(b))^k - \left(\frac{1}{2} - b\right) \left(1 - F\left(\frac{1}{2} - b\right)\right)^{n-k} = 0.$$

Remark 1 In the case of general distributions, the uniqueness of the equilibrium threshold is no longer guaranteed because the above equation may have multiple solutions. For example, consider the F such that

$$F(b) = \begin{cases} 0.04b & \text{if } b < \frac{1}{4}, \\ 0.01 & \text{if } \frac{1}{4} \leq b < 0.365, \\ 49b - 17.875 & \text{if } 0.365 < b < 0.385, \\ 0.99 & \text{if } 0.385 < b \leq \frac{1}{2}, \\ 0.02b + 0.98 & \text{if } b > \frac{1}{2}. \end{cases}$$

In this case, $G^{(2,1)}(b) = 0$ has three solutions: $b \doteq 0.25126, 0.37866, 0.49505$.

The receiver's equilibrium expected payoff is

$$EU_0^{(n,k)}(b_+^{(n,k)}) = -\frac{1}{2} \left(1 - F(b_+^{(n,k)})\right)^k (a^{(n,k)}(k, n-k))^2 \\ - \frac{1}{2} \left(1 - F(b_-^{(n,k)})\right)^{n-k} (1 - a^{(n,k)}(k, n-k))^2.$$

By using the equilibrium condition, this is also written as

$$EU_0^{(n,k)}(b_+^{(n,k)}) = -b_+^{(n,k)} \left(1 - F(b_+^{(n,k)})\right)^k, \quad (1)$$

$$EU_0^{(n,k)}(b_+^{(n,k)}) = -\left(\frac{1}{2} - b_+^{(n,k)}\right) \left(1 - F\left(\frac{1}{2} - b_+^{(n,k)}\right)\right)^{n-k}. \quad (2)$$

Particularly, the latter expression leads to the following simple expression:

$$EU_0^{(n,n)}(b_+^{(n,n)}) = b_+^{(n,n)} - \frac{1}{2}. \quad (3)$$

This makes later analysis much easier.

4.2 Comparison

First, we confirm the existence of equilibrium and the region where equilibrium thresholds are lying.

Proposition 1 For any $n \geq 2$ and any k such that $n \geq k \geq \frac{n}{2}$, there exists $b_+^{(n,k)}$, and moreover,

- $0 < b_+^{(n,k)} < \frac{1}{2}$, especially
- $\frac{1}{4} < b_+^{(n,n)} < \frac{1}{2}$.

Proof:

For any $n \geq 2$ and any k such that $n \geq k \geq \frac{n}{2}$,

- $G^{(n,k)}(0) < 0$ and
- $G^{(n,k)}(b) > 0$ for $b \geq \frac{1}{2}$

hold, which implies that $b_+^{(n,k)}$ exists and $b_+^{(n,k)} \in (0, \frac{1}{2})$. Especially,

- $G^{(n,n)}(b) < 0$ for $b \leq \frac{1}{4}$

holds, which implies $b_+^{(n,n)} \in (\frac{1}{4}, \frac{1}{2})$. ■

Next, consider the comparison among completely homogeneous modes. The next result shows that completely homogeneous mode with full participation of Subordinates has an equilibrium in which Subordinates' thresholds and Boss's payoff are larger than any one in homogeneous mode with partial participation of Subordinates. In other words, an equilibrium in which some Subordinates babble is inferior to the equilibrium in which Boss listens to all Subordinates.

Proposition 2 For any $n \geq 2$ and any $b_+^{(n-1,n-1)}$, there exists $b_+^{(n,n)}$ such that $b_+^{(n,n)} > b_+^{(n-1,n-1)}$ and $EU_0^{(n,n)}(b_+^{(n,n)}) > EU_0^{(n-1,n-1)}(b_+^{(n-1,n-1)})$.

Proof:

$$G^{(n,n)}(b_+^{(n-1,n-1)}) = -\left(\frac{1}{2} - b_+^{(n-1,n-1)}\right) F(b_+^{(n-1,n-1)}) < 0,$$

holds. Combined with the fact that $G^{(n,n)}(\frac{1}{2}) > 0$ and Proposition 1, this implies the existence of $b_+^{(n,n)}$ such that $b_+^{(n,n)} > b_+^{(n-1,n-1)}$. Combined with (3), this directly implies $EU_0^{(n,n)}(b_+^{(n,n)}) > EU_0^{(n-1,n-1)}(b_+^{(n-1,n-1)})$. ■

As a corollary to this proposition, we obtain the following limit results. In other words, as the number of Subordinates with the same directed biases increases, the losses incurred by Boss approach zero.

Corollary 1 There exists a strictly increasing sequence $\{b_+^{(n,n)}\}_{n \geq 2}$ such that

- $\lim_{n \rightarrow \infty} b_+^{(n,n)} = \frac{1}{2}$ and
- $\lim_{n \rightarrow \infty} EU_0^{(n,n)}(b_+^{(n,n)}) = 0$.

Proof:

This corollary follows from the fact that, for any small $\varepsilon > 0$, there exists $\bar{n}$ such that $G^{(n,n)}\left(\frac{1}{2} - \varepsilon\right) < 0$ for any $n > \bar{n}$. $\blacksquare$

Next, we show that Subordinates become more disciplined as the organizational composition shifts toward a larger number of Subordinates with biases in the same direction.

Proposition 3 For any $n \geq 2$, any k such that $n - 1 \geq k \geq \frac{n}{2}$, and any $b_+^{(n,k)}$, there exists $b_+^{(n,k+1)}$ such that $b_+^{(n,k+1)} > b_+^{(n,k)}$.

Proof:

$$\begin{aligned}
& G^{(n,k+1)}\left(b_+^{(n,k)}\right) \\
&= -\left(\frac{1}{2} - b_+^{(n,k)}\right) \left(1 - F\left(\frac{1}{2} - b_+^{(n,k)}\right)\right)^{n-k-1} \\
&\quad \times \left[1 - \left(1 - F\left(b_+^{(n,k)}\right)\right) \left(1 - F\left(\frac{1}{2} - b_+^{(n,k)}\right)\right)\right] \\
&< 0,
\end{aligned}$$

holds. Combined with the fact that $G^{(n,k+1)}\left(\frac{1}{2}\right) > 0$ and Proposition 1, this implies the existence of $b_+^{(n,k+1)}$ such that $b_+^{(n,k+1)} > b_+^{(n,k)}$. $\blacksquare$

The intuition behind this result is as follows. The key is the sensitivity of the Boss's response to false messages. Consider, for example, an upward biased Subordinate. If his bias is small, he would like to induce the Boss to choose a somewhat higher action, but not an excessively high one. Therefore, the more sensitive the Boss's response is to a false message, the weaker the Subordinate's incentive to manipulate her decision. This observation provides a key mechanism for eliciting truthful information through cheap talk communication.

Within the class of threshold strategy equilibria, however, a Subordinate's incentive matters only when he is pivotal. Hence, an increase in the number of Subordinates whose biases are in the same direction as his amplifies the effect of his false report on the Boss's response. As a result, the Subordinate has a weaker incentive to misreport.

The comparison of the Boss's equilibrium payoff, however, is less clear-cut. This is because $b_+^{(n,k+1)} > b_+^{(n,k)}$ implies $b_-^{(n,k+1)} < b_-^{(n,k)}$. In other words, increasing the number of upward biased Subordinates makes all upward biased Subordinates more disciplined, while it makes all downward biased Subordinates less disciplined. The Boss therefore faces a trade-off between these two effects.

This trade-off disappears under a completely homogeneous organizational mode, because there are no Subordinates with biases in the opposite direction. Consequently, the completely homogeneous mode admits an equilibrium in which the Subordinates' threshold levels, as well as the Boss's expected payoff, are higher than in any equilibrium under any other organizational mode.

Proposition 4 For any $n \geq 2$, any k such that $n - 1 \geq k \geq \frac{n}{2}$, and any $b_+^{(n,k)}$, there exists $b_+^{(n,n)}$ such that $b_+^{(n,n)} > b_+^{(n,k)}$ and $EU_0^{(n,n)}(b_+^{(n,n)}) > EU_0^{(n,k)}(b_+^{(n,k)})$.

Proof:

The proof of Proposition 4 is based on the following lemma.

Lemma 1 For any $n \geq 2$, any k such that $n - 1 \geq k \geq 1$, and any $b_+^{(n,k)} \in [\frac{1}{4}, \frac{1}{2})$, there exists $b_+^{(n,n)}$ such that $EU_0^{(n,n)}(b_+^{(n,n)}) > EU_0^{(n,k)}(b_+^{(n,k)})$.

Proof:

Throughout the proof, we denote

- $x := b_+^{(n,k)}$,
- $y := 1 - F(x)$, and
- $z := 1 - F(\frac{1}{2} - x)$.

Then, the equilibrium condition $G^{(n,k)}(b_+^{(n,k)}) = 0$ is written as $xy^k = (\frac{1}{2} - x)z^{n-k}$. Then, $z < 1$, which follows from Assumption 1, implies that $\frac{1}{2} - xy^k > x$, and $y \leq z$, which follows from the assumption $b_+^{(n,k)} \in [\frac{1}{4}, \frac{1}{2})$,

implies that $x \geq \frac{y^{n-k}}{2(y^{n-k} + y^k)}$. The Boss's equilibrium payoff is also written as $EU_0^{(n,k)}(b_+^{(n,k)}) = -xy^k$. By using these facts, we obtain

$$\begin{aligned}
G^{(n,n)}\left(EU_0^{(n,k)}(b_+^{(n,k)}) + \frac{1}{2}\right) &= G^{(n,n)}\left(\frac{1}{2} - xy^k\right) \\
&= \left(\frac{1}{2} - xy^k\right) \left(1 - F\left(\frac{1}{2} - xy^k\right)\right)^n - xy^k \\
&\leq \left(\frac{1}{2} - xy^k\right) y^n - xy^k \\
&= \frac{1}{2}y^n - xy^k(1 + y^n) \\
&\leq \frac{1}{2}y^n - y^k(1 + y^n) \frac{y^{n-k}}{2(y^{n-k} + y^k)} \\
&= -\frac{y^n(1 - y^{n-k})(1 - y^k)}{2(y^{n-k} + y^k)} \\
&< 0.
\end{aligned}$$

Combined with the fact that $G^{(n,n)}(\frac{1}{2}) > 0$, this implies the existence of $b_+^{(n,n)}$ such that $b_+^{(n,n)} > EU_0^{(n,k)}(b_+^{(n,k)}) + \frac{1}{2}$. Combined with (3), this implies $EU_0^{(n,n)}(b_+^{(n,n)}) > EU_0^{(n,k)}(b_+^{(n,k)})$. ■

Proof of Proposition 4:

Proposition 3 implies the existence of $\bar{b}_+^{(n,n)}$ such that $\bar{b}_+^{(n,n)} > b_+^{(n,k)}$.

On the other hand, Proposition 1 means that $b_+^{(n,k)} \in (0, \frac{1}{2})$. If $b_+^{(n,k)} \in [\frac{1}{4}, \frac{1}{2})$, Lemma 1 directly implies the existence of $\hat{b}_+^{(n,n)}$ such that $EU_0^{(n,n)}(\hat{b}_+^{(n,n)}) > EU_0^{(n,k)}(b_+^{(n,k)})$. If $b_+^{(n,k)} \in (0, \frac{1}{4})$, then there exists $b_+^{(n,n-k)} = \frac{1}{2} - b_+^{(n,k)} \in (\frac{1}{4}, \frac{1}{2})$. Moreover, by (1) and (2), we obtain $EU_0^{(n,n-k)}(b_+^{(n,n-k)}) = EU_0^{(n,k)}(b_+^{(n,k)})$. Therefore, again, Lemma 1 implies the existence of $\hat{b}_+^{(n,n)}$ such that $EU_0^{(n,n)}(\hat{b}_+^{(n,n)}) > EU_0^{(n,k)}(b_+^{(n,k)})$.

By these facts and (3), it is clear that $\max\{\bar{b}_+^{(n,n)}, \hat{b}_+^{(n,n)}\}$ satisfies the statement of Proposition. ■

4.3 Other Comparison Results

The comparison among organizational modes other than the completely homogeneous one is less clear-cut.

For the later analysis in this subsection, we restrict our attention to the equilibrium thresholds larger than or equal to $\frac{1}{4}$. The existence of such a threshold is guaranteed by Proposition 3 and $b = \frac{1}{4}$ necessarily satisfies $G^{(n, \frac{n}{2})}(b) = 0$.

Since $-b(1 - F(b))^{k+1} > -b(1 - F(b))^k$ holds for any $b \in (0, \frac{1}{2})$, (1) and Proposition 3 imply that

$$\frac{d}{db} [-b(1 - F(b))^k] \geq 0 \quad \forall b \in \left[\frac{1}{4}, \frac{1}{2}\right)$$

is a sufficient condition for the existence of $b_+^{(n, k+1)}$ such that $EU_0^{(n, k+1)}(b_+^{(n, k+1)}) > EU_0^{(n, k)}(b_+^{(n, k)})$. If F has a density function f , then this is equivalent to

$$kbf(b) - (1 - F(b)) \geq 0 \quad \forall b \in \left[\frac{1}{4}, \frac{1}{2}\right).$$

This implies that, in the case of the uniform distribution, there exists such a $b_+^{(n, k+1)}$ for any $n \geq 6$ and for any k such that $n - 1 \geq k \geq \frac{n}{2}$.

Remark 2 The above sufficient condition is satisfied only if $f(b)$ is sufficiently large. As a counter-example, consider the following baseline distribution function:

$$F(b) = \begin{cases} \frac{b}{100000000} & \text{if } 0 \leq b \leq \frac{1}{2} \\ b + \frac{99999999}{100000000}(b - 1) & \text{if } \frac{1}{2} < b \leq 1. \end{cases}$$

Given this distribution function, we can numerically verify that $EU_0^{(40, 20)}(b_+^{(40, 20)}) > EU_0^{(40, 21)}(b_+^{(40, 21)})$, while $b_+^{(40, 20)} < b_+^{(40, 21)}$.

On the other hand, for the case of uniform distribution and lower n , we can numerically derive the equilibrium thresholds and Boss's equilibrium payoffs (Tables 1 and 2). It then follows that the existence of such $b_+^{(n, k+1)}$ is guaranteed for any $n \geq 2$ in the case of the uniform distribution.

k / n	1	2	3	4	5	6
1	0.292893211	0.25				
2		0.352201038	0.319448451	0.25		
3			0.417477852	0.40612275	0.386369991	0.25
4				0.461113738	0.459000081	0.456444304
5					0.482027692	0.481637276
6						0.491503242

Table 1: Equilibrium Thresholds in the Case of Uniform Distribution

k / n	1	2	3	4	5	6
1	-0.207106778	-0.187500049				
2		-0.147798875	-0.147952681	-0.140625018		
3			-0.082522194	-0.08506431	-0.089273623	-0.105459195
4				-0.03888629	-0.039318918	-0.039844098
5					-0.017972343	-0.018025566
6						-0.008496802

Table 2: Boss's Equilibrium Payoffs in the Case of Uniform Distribution

Proposition 5 Suppose the baseline distribution is the uniform distribution over $[0, 1]$. Then, for any $n \geq 2$, any k such that $n - 1 \geq k \geq \frac{n}{2}$, and any $b_+^{(n,k)} \geq \frac{1}{4}$, there exists $b_+^{(n,k+1)}$ such that $b_+^{(n,k+1)} > b_+^{(n,k)}$ and $EU_0^{(n,k+1)}(b_+^{(n,k+1)}) > EU_0^{(n,k)}(b_+^{(n,k)})$.

5 Extensions

We can extend our previous results to other environments.

5.1 Heterogeneous Bias Distributions

We can drop the assumption that baseline bias distributions are common between Subordinates in different indexes, as long as we keep the assumption of symmetry between upward and downward biased Subordinates in the same index. We relegate the formal results to Appendix C. The next example illustrates how crucial to our results the latter assumption.

Example 1 Consider the case of $n = 2$. We denote upward and downward

biased Subordinates 2 by $2+$ and $2-$, respectively. The bias distribution of each Subordinate is

- $b_i \sim U[0, w_i]$ for $i = 1, 2+$, and
- $b_{2-} \sim U[-w_{2-}, 0]$.

We assume $w_i \in (\frac{1}{2}, 1)$ for $i = 1, 2+, 2-$.

It is verified that, if $w_{2-} = w_{2+}$, then Boss prefers the homogeneous mode to the heterogeneous mode. Moreover, we can show that there exists $\hat{w}_{2-} \in (\frac{1}{2}, w_{2+})$ such that

$$\begin{aligned} EU_0^{(+,+)} &> EU_0^{(+,-)} && \text{if } w_{2-} > \hat{w}_{2-}, \\ EU_0^{(+,+)} &< EU_0^{(+,-)} && \text{if } w_{2-} < \hat{w}_{2-}. \end{aligned}$$

As w_2 decreases, downward biased Subordinate 2 has more tendency to send a truthful message, while upward biased Subordinate 1 becomes less disciplined. This is the trade-off for Boss.

5.2 Slightly Overlapping Biases

We assumed that the bias support for upward biased Subordinate does not overlap with one for downward biased Subordinate. The previous results would not change, however, even if a small overlapping would be introduced.

Example 2 Consider the case of $(2, 1)$. We assume that the baseline distribution function is the uniform distribution over $[-z, 1 - z]$ where $z \in [0, \frac{1}{2})$. If $z < \frac{1}{4}$, the strategy profile described in Subsection 3.3 also constitutes an equilibrium.

5.3 Simultaneous Information Transmission Protocol

The previous results would not change even if we assume the cheap talk stage would proceed according to a simultaneous protocol. This is because each Subordinate follows history-independent and order-independent strategy.

5.4 Common Knowledge among Subordinates

The equilibria we focus on would also remain even if their biases were common knowledge among Subordinates as long as we keep the assumption that Boss does not know. Of course, the latter assumption is crucial.

6 Conclusion

We consider the information transmission problem within organization, and especially we focus on how to combine biased subordinates to elicit truthful information from them. We assume that the directions of Subordinates' biases are common knowledge, but not their magnitudes. This leads to the results that completely homogeneous composition of Subordinates are better than any other composition. The intuition is that, under a homogeneous composition, the effect of an individual false report is amplified by false reports from other Subordinates. Anticipating this amplification effect reduces each Subordinate's incentive to misreport and thereby promotes truthful information transmission. This is a new mechanism for information transmission within organization.

Compared with Krishna and Morgan [16], our analysis allows for an arbitrary number of Subordinates but is restricted to a binary state space. Extending the analysis to a more general state space is left for future research.

References

- [1] David Austen-Smith. Interested experts and policy advice: Multiple referrals under open rule. *Games and Economic Behavior*, 5(1):3–43, 1993.
- [2] Marco Battaglini. Multiple referrals and multidimensional cheap talk. *Econometrica*, 70(4):1379–1401, 2002.
- [3] Roland Bénabou and Guy Laroque. Using privileged information to manipulate markets: Insiders, gurus, and credibility. *Quarterly Journal of Economics*, 107(3):921–958, 1992.
- [4] Helmut Bester and Daniel Kräbmer. Delegation and incentives. *The RAND Journal of Economics*, 39(3):664–682, 2008.
- [5] Yeon-Koo Che and Navin Kartik. Opinions as incentives. *Journal of Political Economy*, 117(5):815–860, 2009.
- [6] Vincent P. Crawford and Joel Sobel. Strategic information transmission. *Econometrica*, 50(6):1431–1451, 1982.

- [7] Richard M. Cyert and James G. March. *A Behavioral Theory of the Firm*. Englewood Cliffs, NJ: Princeton-Hall, 1963.
- [8] Jean-Etienne de Bettignies and Jan Zábajník. Information sharing and incentives in organizations. *The Journal of Law, Economics, and Organization*, 35(3):619–650, 2019.
- [9] Eric Van den Steen. Culture clash: The costs and benefits of homogeneity. *Management Science*, 56(10):1718–1738, 2010.
- [10] Eric Van den Steen. Interpersonal authority in a theory of the firm. *American Economic Review*, 100(1):466–490, 2010.
- [11] Vassilos Dimitrakas and Yianis Sarafidis. Advice from an expert with unknown motives. mimeo., 2005.
- [12] Kazumi Hori. The role of private benefits in information acquisition. *Journal of Economic Behavior & Organization*, 68:626–631, 2008.
- [13] Akifumi Ishihara and Shintaro Miura. Delegation and strategic silence. mimeo., 2021.
- [14] Hideshi Itoh and Kimiyuki Morita. Information acquisition, decision making, and implementation in organizations. *Management Science*, 69(1):446–463, 2023.
- [15] Gülen Karakoç. Cheap talk with multiple experts and uncertain biases. *The B.E. Journal of Theoretical Economics*, 2021.
- [16] Vijay Krishna and John Morgan. A model of expertise. *Quarterly Journal of Economics*, 116(2):747–775, 2001.
- [17] Augustin Landier, David Sraer, and David Thesmar. Optimal dissent in organizations. *The Review of Economic Studies*, 76(2):761–794, 2009.
- [18] Ming Li. Two (talking) heads are not better than one. *Economics Bulletin*, 3(63):1–8, 2008.
- [19] Ming Li. Advice from multiple experts: A comparison of simultaneous, sequential, and hierarchical communication. *The B.E. Journal of Theoretical Economics: Topics*, 10(1), 2010. Article 18.

- [20] Ming Li and Kristóf Madarász. When mandatory disclosure hurts: Expert advice and conflicting interests. *Journal of Economic Theory*, 139(1):47–74, 2008.
- [21] Anthony M. Marino, John G. Matsusaka, and Jan Zábajník. Disobedience and authority. *The Journal of Law, Economics, & Organization*, 26(3):427–459, 2010.
- [22] Lydia Mechtenberg and Johannes Münster. A strategic mediator who is biased in the same direction as the expert can improve information transmission. *Economics Letters*, 117(2):490–492, 2012.
- [23] John Morgan and Phillip C. Stocken. An analysis of stock recommendations. *The RAND Journal of Economics*, 34(1):183–203, 2003.
- [24] Stephen Morris. Political correctness. *Journal of Political Economy*, 109(2):231–265, 2001.
- [25] Shungo Omiya, Yasunari Tamada, and Tsung-Sheng Tsai. Optimal delegation with self-interested agents and information acquisition. *Journal of Economic Behavior & Organization*, 137:54–71, 2017.
- [26] Suraj Prasad and Sebastian Tanase. Competition, collaboration and organization design. *Journal of Economic Behavior & Organization*, 183:1–18, 2021.
- [27] Suraj Prasad and Marcus Tomaino. Resources and culture in organizations. *Journal of Economics & Management Strategy*, 29(4):854–872, 2020.
- [28] Mark Thordal-Le Quement. The (human) sampler’s curses. *American Economic Journal: Microeconomics*, 8(4):115–148, 2016.
- [29] Heikki Rantakari. A simple model of project selection with strategic communication and uncertain motives. *Journal of Economic Behavior & Organization*, 102(1):14–42, 2014.
- [30] Heikki Rantakari. Managerial influence and organizational performance. *Journal of the European Economic Association*, 19(2):1116–1161, 2021.

- [31] Takashi Shimizu. Cheap talk with an exit option: A model of exit and voice. Kobe University, Graduate School of Economics, Discussion Paper No.1607, 2016.
- [32] Takashi Shimizu. Which is better for the receiver between senders with like biases and senders with opposing biases? mimeo., 2016.
- [33] Joel Sobel. A theory of credibility. *Review of Economic Studies*, 52(4):557–573, 1985.
- [34] Dezső Szalay. The economics of clear advice and extreme options. *The Review of Economic Studies*, 72(4):1173–1198, 2005.
- [35] Jordi Blanes I. Vidal and Marc Möller. When should leaders share information with their subordinates. *Journal of Economics & Management Strategy*, 16(2):251–283, 2007.

A Non-Existence of Fully Revealing PBE

Given μ , we define $m_i^*(t, b_1, \dots, b_i)$ recursively as follows:

$$\begin{aligned} m_1^*(t, b_1) &:= \mu_1(t, b_1) \\ m_i^*(t, b_1, \dots, b_i) &:= \mu_i(t, b_i, m_1^*(t, b_1), \dots, m_{i-1}^*(t, b_1, \dots, b_{i-1})) \quad \text{for } i > 1. \end{aligned}$$

Using this notation, We define a fully revealing strategy profile.

Definition 1 (μ^*, α^*) is *fully revealing* if $\alpha^*(m_1^*(t, b_1), \dots, m_n^*(t, b_1, \dots, b_n)) = t$ for any $t \in T$ and any $b_1 \in B_1, \dots, b_n \in B_n$.

Similarly, given μ , we define $\gamma_i(t, m_1, \dots, m_i, b_{i+1}, \dots, b_n)$ as follows:

$$\gamma_i(t, m_1, \dots, m_i, b_{i+1}, \dots, b_n) := (\hat{m}_{i+1}, \dots, \hat{m}_n)$$

where

$$\begin{aligned} \hat{m}_{i+1} &= \mu_{i+1}(t, b_{i+1}, m_1, \dots, m_i) \\ \hat{m}_{i+2} &= \mu_{i+2}(t, b_{i+2}, m_1, \dots, m_i, \hat{m}_{i+1}) \\ &\vdots \\ \hat{m}_n &= \mu_n(t, b_n, m_1, \dots, m_i, \hat{m}_{i+1}, \dots, \hat{m}_{n-1}). \end{aligned}$$

Proposition 6 For any $n \geq 2$, there is no fully revealing PBE.

Proof:

Suppose to the contrary that there exists a fully revealing PBE (μ^*, α^*) . For any i , we pick up $\hat{b}_i \in B_i$ such that $|\hat{b}_i| > \frac{1}{2}$. With a slight abuse of notation, we simply denote $m_i^*(t) := m_i^*(t, \hat{b}_1, \dots, \hat{b}_i)$ and $\gamma_i(t, m_1, \dots, m_i) := \gamma_i(t, m_1, \dots, m_i, \hat{b}_{i+1}, \dots, \hat{b}_n)$.

Lemma 2 For any $i < n$ and any $m_1 \in M_1, \dots, m_{i-1} \in M_{i-1}$, there exists no $\hat{m}_i \in M_i$ such that

$$\begin{aligned} \alpha^*(m_1, \dots, m_{i-1}, \hat{m}_i, \gamma_i(1, m_1, \dots, m_{i-1}, \hat{m}_i)) &= 1 \text{ and} \\ \alpha^*(m_1, \dots, m_{i-1}, \hat{m}_i, \gamma_i(0, m_1, \dots, m_{i-1}, \hat{m}_i)) &= 0. \end{aligned}$$

Proof:

Suppose to the contrary that there exists $(\hat{m}_1, \dots, \hat{m}_i)$ such that

$$\begin{aligned}\alpha^*(\hat{m}_1, \dots, \hat{m}_i, \gamma_i(1, \hat{m}_1, \dots, \hat{m}_i)) &= 1 \text{ and} \\ \alpha^*(\hat{m}_1, \dots, \hat{m}_i, \gamma_i(0, \hat{m}_1, \dots, \hat{m}_i)) &= 0.\end{aligned}$$

We prove that this makes contradiction for any $i < n$ by backward induction.

Step 1: When $i = n - 1$ and Subordinate n is upward biased, since $U_n(0, \hat{b}_n, a) > U_n(0, \hat{b}_n, 0)$ for any $a \in (0, 1]$, he has no incentive to follow $\gamma_{n-1}(0, \hat{m}_1, \dots, \hat{m}_{n-1}) = \mu_n^*(0, \hat{b}_n, \hat{m}_1, \dots, \hat{m}_{n-1})$ at $t = 0$.

Step 2: When $i = n - 1$ and Subordinate n is downward biased, since $U_n(0, \hat{b}_n, a) > U_n(0, \hat{b}_n, 1)$ for any $a \in [0, 1)$, he has no incentive to follow $\gamma_{n-1}(1, \hat{m}_1, \dots, \hat{m}_{n-1}) = \mu_n^*(1, \hat{b}_n, \hat{m}_1, \dots, \hat{m}_{n-1})$ at $t = 1$.

Step 3: When $i < n - 1$, the claim of Lemma holds for $i + 1$, and Subordinate $i + 1$ is upward biased, since $U_{i+1}(0, \hat{b}_{i+1}, a) > U_{i+1}(0, \hat{b}_{i+1}, 0)$ for any $a \in (0, 1]$, it must follow

$$\alpha^*(\hat{m}_1, \dots, \hat{m}_i, m_{i+1}, \gamma_{i+1}(0, \hat{m}_1, \dots, \hat{m}_i, m_{i+1})) = 0 \quad \forall m_{i+1} \in M_{i+1}.$$

On the other hand, there must exists $\hat{m}_{i+1} \in M_{i+1}$ such that

$$\alpha^*(\hat{m}_1, \dots, \hat{m}_i, \hat{m}_{i+1}, \gamma_{i+1}(1, \hat{m}_1, \dots, \hat{m}_i, \hat{m}_{i+1})) = 1.$$

This is contradiction to the supposition that the claim of Lemma holds for $i + 1$.

Step 4: When $i < n$, the claim of Lemma holds for $i + 1$, and Subordinate $i + 1$ is downward biased, since $U_i(1, \hat{b}_{i+1}, a) > U_{i+1}(1, \hat{b}_{i+1}, 1)$ for any $a \in [0, 1)$, it must follow

$$\alpha^*(\hat{m}_1, \dots, \hat{m}_i, m_{i+1}, \gamma_{i+1}(1, \hat{m}_1, \dots, \hat{m}_i, m_{i+1})) = 1 \quad \forall m_{i+1} \in M_{i+1}.$$

On the other hand, there must exists $\hat{m}_{i+1} \in M_{i+1}$ such that

$$\alpha^*(\hat{m}_1, \dots, \hat{m}_i, \hat{m}_{i+1}, \gamma_{i+1}(0, \hat{m}_1, \dots, \hat{m}_i, \hat{m}_{i+1})) = 0.$$

This is contradiction to the supposition that the claim of Lemma holds for $i + 1$.

■

Lemma 3 For any $i > 1$ and any $t \in T$,

$$\alpha^*(m_1^*(1), \dots, m_{i-1}^*(1), \gamma_{i-1}(t, m_1^*(1), \dots, m_{i-1}^*(1))) > 0.$$

Proof:

We prove this by backward induction.

Step 1: When $i = n$ and Subordinate n is upward biased, since $U_n(0, \hat{b}_n, a) > U_n(0, \hat{b}_n, 0)$ for any $a \in (0, 1]$, then, even at $t = 0$, Subordinate n has no incentive to choose $\hat{m}_n$ such that

$$\alpha^*(m_1^*(1), \dots, m_{n-1}^*(1), \hat{m}_n) = 0.$$

Step 2: When $i = n$ and Subordinate n is downward biased, since $U_n(1, \hat{b}_n, a) > U_n(1, \hat{b}_n, 1)$ for any $a \in [0, 1)$, it must follow

$$\alpha^*(m_1^*(1), \dots, m_{n-1}^*(1), m_n) = 1 \quad \forall m_n \in M_n.$$

Step 3: When $i < n$, the claim of Lemma holds for $i + 1$, and Subordinate i is upward biased, since $U_i(0, \hat{b}_i, a) > U_i(0, \hat{b}_i, 0)$ for any $a \in (0, 1]$ and $\alpha^*(m_1^*(1), \dots, m_{i-1}^*(1), m_i^*(1), \gamma_i(t, m_1^*(1), \dots, m_{i-1}^*(1), m_i^*(1))) > 0 \quad \forall t \in T$, then, even at $t = 0$, Subordinate has no incentive to choose $\hat{m}_i$ such that

$$\alpha^*(m_1^*(1), \dots, m_{i-1}^*(1), \hat{m}_i, \gamma_i(0, m_1^*(1), \dots, m_{i-1}^*(1), \hat{m}_i)) = 0.$$

Step 4: When $i < n$, the claim of Lemma holds for $i + 1, \dots, n$, and Subordinate i is downward biased, since $U_i(1, \hat{b}_i, a) > U_i(1, \hat{b}_i, 1)$ for any $a \in [0, 1)$, it must follow

$$\alpha^*(m_1^*(1), \dots, m_{i-1}^*(1), m_i, \gamma_i(1, m_1^*(1), \dots, m_{i-1}^*(1), m_i)) = 1 \quad \forall m_i \in M_i.$$

Then, by Lemma 2, this implies

$$\alpha^*(m_1^*(1), \dots, m_{i-1}^*(1), m_i, \gamma_i(0, m_1^*(1), \dots, m_{i-1}^*(1), m_i)) > 0 \quad \forall m_i \in M_i.$$

■

Proof of Proposition 6: By Lemma 3, Subordinate 1 has no incentive to choose $\mu_1^*(0)$ at $t = 0$. ■

B Essentially Outcome-Equivalence to Threshold Strategy Equilibrium

In this Appendix, we use the following notations:

- $M^i := M_1 \times \cdots \times M_i$ where $M^0 = \emptyset$
- $\mathcal{M}_i(t, m^{i-1}) := \{m_i \in M_i \mid \exists \hat{B}_i \subseteq B_i \text{ s.t. } \rho_i(\hat{B}_i) > 0 \text{ and } m_i = \mu_i(t, b_i, m^{i-1}) \forall b_i \in \hat{B}_i\}$ where ρ_i is the probability measure of random variable b_i ⁴
- $\mathcal{M}_{i0}(m^{i-1}) := \mathcal{M}_i(0, m^{i-1}) / \mathcal{M}_i(1, m^{i-1})$
- $\mathcal{M}_{i1}(m^{i-1}) := \mathcal{M}_i(1, m^{i-1}) / \mathcal{M}_i(0, m^{i-1})$
- $\mathcal{M}_{im}(m^{i-1}) := \mathcal{M}_i(0, m^{i-1}) \cap \mathcal{M}_i(1, m^{i-1})$
- $\mathcal{H}_i(t) := \{m^{i-1} \in M^{i-1} \mid m_1 \in \mathcal{M}_1(t), m_2 \in \mathcal{M}_2(t, m_1), \dots, m_{i-1} \in \mathcal{M}_{i-1}(t, m^{i-2}) \text{ where } \mathcal{H}_1(t) = \emptyset\}$
- $\mathcal{C}_i(t, m^i) := \{(m_{i+1}, \dots, m_n) \in M_{i+1} \times \cdots \times M_n \mid m_{i+1} \in \mathcal{M}_{i+1}(t, m^i), m_{i+2} \in \mathcal{M}_{i+2}(t, m^i, m_{i+1}), \dots, m_n \in \mathcal{M}_n(t, m^i, m_{i+1}, \dots, m_{n-1})\}$ where $\mathcal{C}_n(t, m^n) = \emptyset$

To avoid technical difficulties, we assume that for any state and any Subordinate, there exists at least one message sent by the Subordinate with some positive measure set of biases.⁵

Assumption 2 $\mathcal{M}_i(t, m^{i-1}) \neq \emptyset$ for $t = 0, 1$, any $i = 1, \dots, n$, and any $m^{i-1} \in M^{i-1}$

In order to express the situation where all Subordinates are active in information transmission, we define the concept of fully active PBE as follows.

Definition 2 A PBE (μ^*, α^*) is *fully active* if, for any i , there exist $t \in T$, $h_i \in \mathcal{H}_i(t)$, $m_i, m'_i \in \mathcal{M}_i(t, h_i)$, $c_i \in \mathcal{C}_i(t, h_i, m_i)$, and $c'_i \in \mathcal{C}_i(t, h_i, m'_i)$ such that $\alpha^*(h_i, m_i, c_i) \neq \alpha^*(h_i, m'_i, c'_i)$.

⁴In other words, $F_i(b) = \rho_i([0, b])$ if Subordinate i is upward biased, and $F_i(b) = \rho_i((-\infty, b])$ if Subordinate i is downward biased.

⁵If all Subordinates' messages are finite, this assumption is automatically satisfied.

An *outcome* is a function that maps the state and the profile of biases into the action realized along the equilibrium path. We say that an equilibrium is *essentially outcome-equivalent* to another equilibrium if the outcomes induced by the two equilibria coincide for all bias profiles except on a set of measure zero. Under Assumption 2, we can show that any fully active PBE is essentially outcome-equivalent to some threshold-strategy equilibrium.

Proposition 7 Under Assumption 2, any fully active PBE (μ^*, α^*) is essentially outcome-equivalent to some threshold strategy equilibrium.

Proof:

The proof proceeds by the following series of Lemmas.

Lemma 4 Given any i . Suppose that $\hat{m}^{i-1} \in M^{i-1}$ such that $\hat{m}_1 \in \mathcal{M}_{1m}, \dots, \hat{m}_{i-1} \in \mathcal{M}_{i-1,m}(\hat{m}^{i-2})$.

- (i) If Subordinate i is upward biased, then $\mathcal{M}_{i1}(\hat{m}^{i-1}) = \emptyset$.
- (ii) If Subordinate i is downward biased, then $\mathcal{M}_{i0}(\hat{m}^{i-1}) = \emptyset$.
- (iii) $\mathcal{M}_{im}(\hat{m}^{i-1}) \neq \emptyset$

Proof:

We prove this by backward induction.

Step 1: Suppose that Subordinate n is upward biased and $\mathcal{M}_{n1}(\hat{m}^{n-1}) \neq \emptyset$. This implies $\alpha^*(\hat{m}^{n-1}, m_n) = 1$ for any $m_n \in \mathcal{M}_{n1}(\hat{m}^{n-1})$. If $\mathcal{M}_{nm}(\hat{m}^{n-1}) \neq \emptyset$, since $\alpha^*(\hat{m}^{n-1}, m_n) \neq 1$ for any $m_n \in \mathcal{M}_{nm}(\hat{m}^{n-1})$, Subordinate n has no incentive to choose any $m_n \in \mathcal{M}_{nm}(\hat{m}^{n-1})$ at $t = 1$. Therefore, $\mathcal{M}_{nm}(\hat{m}^{n-1}) = \emptyset$. This implies $\mathcal{M}_{n0}(\hat{m}^{n-1}) \neq \emptyset$. Nevertheless, since $\alpha^*(\hat{m}^{n-1}, m_n) = 0$ for any $m_n \in \mathcal{M}_{n0}(\hat{m}^{n-1})$, Subordinate n with $b_n > \frac{1}{2}$ has no incentive to choose any $m_n \in \mathcal{M}_{n0}(\hat{m}^{n-1})$ at $t = 0$. This is contradiction, which means we have proven that $\mathcal{M}_{n1}(\hat{m}^{n-1}) = \emptyset$. By Assumption 2, this implies $\mathcal{M}_{nm}(\hat{m}^{n-1}) \neq \emptyset$.

Step 2: Similarly, we can prove that, if Subordinate n is downward biased, then $\mathcal{M}_{n0}(\hat{m}^{n-1}) = \emptyset$ and $\mathcal{M}_{nm}(\hat{m}^{n-1}) \neq \emptyset$.

Step 3: Suppose that Subordinate $i < n$ is upward biased, the claim of Lemma holds for $i + 1, \dots, n$, and $\mathcal{M}_{i1}(\hat{m}^{i-1}) \neq \emptyset$. This implies $\alpha^*(\hat{m}^{i-1}, m_i, c_i) = 1$ for any $m_i \in \mathcal{M}_{i1}(\hat{m}^{i-1})$ and any $c_i \in \mathcal{C}_i(1, \hat{m}^{i-1}, m_i)$.

Suppose $\mathcal{M}_{im}(\hat{m}^{i-1}) \neq \emptyset$. Then, for any $m_i \in \mathcal{M}_{im}(\hat{m}^{i-1})$, there exists $(\hat{m}_{i+1}, \dots, \hat{m}_n) \in \mathcal{C}_i(1, \hat{m}^{i-1}, m_i)$ such that $\hat{m}_{i+1} \in \mathcal{M}_{i+1.m}(\hat{m}^i, m_i)$ and $\hat{m}_{i'} \in \mathcal{M}_{i'm}(\hat{m}^{i-1}, m_i, \hat{m}_{i+1}, \dots, \hat{m}_{i'-1})$ for any $i' > i + 1$. Since $\alpha^*(\hat{m}^{i-1}, m_i, \hat{m}_{i+1}, \dots, \hat{m}_n) \neq 1$, Subordinate i has no incentive to choose such m_i at $t = 1$. This is contradiction, which means we have proven $\mathcal{M}_{im}(\hat{m}^{i-1}) = \emptyset$

This implies $\mathcal{M}_{i0}(\hat{m}^{i-1}) \neq \emptyset$. Nevertheless, since $\alpha^*(\hat{m}^{i-1}, m_i, c_i) = 0$ for any $m_i \in \mathcal{M}_{i0}(\hat{m}^{i-1})$ and any $c_i \in \mathcal{C}_i(0, \hat{m}^{i-1}, m_i)$, Subordinate i with $b_i > \frac{1}{2}$ has no incentive to choose any $m_i \in \mathcal{M}_{i0}(\hat{m}^{i-1})$ at $t = 0$. This is contradiction, which means we have proven that $\mathcal{M}_{i1}(\hat{m}^{i-1}) = \emptyset$. By Assumption 2, this implies $\mathcal{M}_{im}(\hat{m}^{i-1}) \neq \emptyset$.

Step 4: Similarly, we can prove that, if Subordinate $i < n$ is downward biased and the claim of Lemma holds for $i+1, \dots, n$, then $\mathcal{M}_{i0}(\hat{m}^{i-1}) = \emptyset$ and $\mathcal{M}_{im}(\hat{m}^{i-1}) \neq \emptyset$.

■

Lemma 5 There exists $a^* \in (0, 1)$ such that $a^* = \alpha^*(m)$ for any m such that $m_1 \in \mathcal{M}_{1m}, \dots, m_n \in \mathcal{M}_{nm}(m^{n-1})$.

Proof:

We can find such a^* backward.

Step 1: Given any m^{n-1} such that $m_1 \in \mathcal{M}_{1m}, \dots, m_{n-1} \in \mathcal{M}_{n-1.m}(m^{n-2})$. Suppose Subordinate n is upward biased. When $t = 1$, he must choose m_n^* such that $m_n^* \in \arg \max \alpha^*(m^{n-1}, m_n)$, and Lemma 4 implies $m_n^* \in \mathcal{M}_{nm}(m^{n-1})$. We denote the maximum value by $a_n^*(m^{n-1})$.

In contrast, suppose Subordinate n is downward biased. When $t = 0$, he must choose m_n^* such that $m_n^* \in \arg \min \alpha^*(m^{n-1}, m_n)$, and Lemma 4 implies $m_n^* \in \mathcal{M}_{nm}(m^{n-1})$. We denote the minimum value by $a_n^*(m^{n-1})$.

Step 2: Given any $i < n$ and any m^i such that $m_1 \in \mathcal{M}_{1m}, \dots, m_{i-1} \in \mathcal{M}_{i-1.m}(m^{i-1})$. Suppose Subordinate i is upward biased. When $t = 1$, he must choose m_i^* such that $m_i^* \in \arg \max a_{i+1}^*(m^{i-1}, m_i)$, and Lemma 4 implies $m_i^* \in \mathcal{M}_{im}(m^{i-1})$. We denote the maximum value by $a_i^*(m^{i-1})$.

In contrast, suppose Subordinate i is downward biased. When $t = 0$, he must choose m_i^* such that $m_i^* \in \arg \min a^*(m^{i-1}, m_i)$, and Lemma 4 implies $m_i^* \in \mathcal{M}_{im}(m^{i-1})$. We denote the minimum value by $a_i^*(m^{i-1})$.

Step 3: We define $a^* := a^1$. It is verified that this a^* satisfies the conditions of Lemma. ■

Lemma 6 Given any i . Suppose that $\hat{m}^{i-1} \in M^{i-1}$ such that $\hat{m}_1 \in \mathcal{M}_{1m}, \dots, \hat{m}_{i-1} \in \mathcal{M}_{i-1,m}(\hat{m}^{i-2})$.

- (i) If Subordinate i is upward biased, then $\mathcal{M}_{i0}(\hat{m}^{i-1}) \neq \emptyset$.
- (ii) If Subordinate i is downward biased, then $\mathcal{M}_{i1}(\hat{m}^{i-1}) \neq \emptyset$.

Proof:

By Lemmas 4 and 5, this must hold because a PBE is fully active. ■

Lemma 7 Given any i . Suppose that $\hat{m}^{i-1} \in M^{i-1}$ such that $\hat{m}_1 \in \mathcal{M}_{1m}, \dots, \hat{m}_{i-1} \in \mathcal{M}_{i-1,m}(\hat{m}^{i-2})$.

- (i) $\alpha^*(m^{i-1}, m_i, c_i) = 0$ for any $m_i \in \mathcal{M}_{i0}(\hat{m}^{i-1})$ and any $c_i \in \mathcal{C}_i(0, \hat{m}^{i-1}, m_i)$.
- (ii) $\alpha^*(m^{i-1}, m_i, c_i) = 1$ for any $m_i \in \mathcal{M}_{i1}(\hat{m}^{i-1})$ and any $c_i \in \mathcal{C}_i(1, \hat{m}^{i-1}, m_i)$.

Proof:

This follows from the previous Lemmas. ■

Proof of Proposition 7: By the previous Lemmas, any fully active PBE is essentially outcome-equivalent to some threshold strategy equilibrium. ■

C Heterogeneous Baseline Distributions

In this appendix, we extend the previous results to the case in which baseline distributions for Subordinates in the different indexes are not common.

Let F_i be a baseline distribution function for Subordinate i . We focus on F_i satisfying the following assumptions.

Assumption 3 For any i , the following conditions hold:

- F_i is continuous.
- $F_i\left(\frac{1}{4}\right) > 1$ and $F_i\left(\frac{1}{2}\right) < 1$.
- $\text{Supp } F_i \subseteq [0, 1]$.

We denote upward and downward biased Subordinates by $+$ and $-$, respectively. For any $n \geq 2$, let $\mathcal{O}^n := \{+, -\}^n$. An organizational mode is characterized by (n, o) where $o \in \mathcal{O}^n$. We denote the numbers of upward and downward Subordinates among o by $+(o)$ and $-(o)$, respectively. We also denote the index of i th upward and downward biased Subordinate by $+_i(o)$ and $-_i(o)$, respectively.⁶ We define $\bar{\mathcal{O}}^n := \mathcal{O}^n \setminus (\{(+, \dots, +)\} \cup \{(-, \dots, -)\})$, which is the set of combinations of Subordinates other than completely homogeneous combinations. Due to the symmetry, $(n, (-, \dots, -))$ have symmetric thresholds and gives the same equilibrium payoff to Boss as $(n, (+, \dots, +))$. Based on this reason, in presenting the results concerning completely homogeneous mode, we only consider $(+, \dots, +)$, which is denoted by $\hat{o}^n$.

C.1 Equilibrium Conditions and Payoff

We denote a message profile by m . We also denote the message profile in which all Subordinates send their dubious messages by $\hat{m}$. Then, the receiver's strategy is as follows: given m ,

- if all downward biased Subordinates send 0 and at least one upward biased Subordinate sends message 0, then $a^{(n,o)}(m) = 0$,

⁶For example, given $n = 3$ and $o = (+, -, +)$, then $+(o) = 2$, $+_1(o) = 1$, $+_2(o) = 3$, $-(o) = 1$, and $-_1(o) = 2$.

- if all Subordinates send their dubious messages, i.e., $m = \hat{m}$, then

$$a^{(n,o)}(\hat{m}) = \frac{\Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)} \left(b_{-}^{(n,o)} \right) \right)}{\Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)} \left(b_{-}^{(n,o)} \right) \right) + \Pi_{i=1}^{+(o)} \left(1 - F_{+i(o)} \left(b_{+}^{(n,o)} \right) \right)},$$

and,

- if all upward biased Subordinates send 1 and at least one downward biased Subordinate sends message 1, then $a^{(n,o)}(m) = 1$.

Then, the senders' thresholds are

$$b_{+}^{(n,o)} = \frac{a^{(n,o)}(\hat{m})}{2},$$

$$b_{-}^{(n,o)} = \frac{1 - a^{(n,o)}(\hat{m})}{2}.$$

Therefore, $b_{+}^{(n,o)}$ is the solution of the following equation:⁷

$$G^{(n,o)}(b) := b \Pi_{i=1}^{+(o)} \left(1 - F_{+i(o)}(b) \right) - \left(\frac{1}{2} - b \right) \Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)} \left(\frac{1}{2} - b \right) \right).$$

The receiver's equilibrium expected payoff is

$$EU_0^{(n,o)} \left(b_{+}^{(n,o)} \right) = -\frac{1}{2} \Pi_{i=1}^{+(o)} \left(1 - F_{+i(o)} \left(b_{+}^{(n,o)} \right) \right) \left(a^{(n,o)}(\hat{m}) \right)^2$$

$$- \frac{1}{2} \Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)} \left(\frac{1}{2} - b_{+}^{(n,o)} \right) \right) \left(1 - a^{(n,o)}(\hat{m}) \right)^2.$$

By using the equilibrium condition, this is also written as

$$EU_0^{(n,o)} \left(b_{+}^{(n,o)} \right) = -b_{+}^{(n,o)} \Pi_{i=1}^{+(o)} \left(1 - F_{+i(o)} \left(b_{+}^{(n,o)} \right) \right), \quad (4)$$

$$EU_0^{(n,o)} \left(b_{+}^{(n,o)} \right) = - \left(\frac{1}{2} - b_{+}^{(n,o)} \right) \Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)} \left(\frac{1}{2} - b_{+}^{(n,o)} \right) \right). \quad (5)$$

Particularly, the latter expression leads to the following simple expression:

$$EU_0^{(n,\hat{\sigma}^n)} \left(b_{+}^{(n,\hat{\sigma}^n)} \right) = b_{+}^{(n,\hat{\sigma}^n)} - \frac{1}{2}. \quad (6)$$

⁷We assume $\Pi_{i=1}^0 A_i = 1$.

C.2 Comparison Results

First, we confirm the existence of equilibrium and the region where equilibrium thresholds are lying.

Proposition 8 For any $n \geq 2$ and any $o \in \mathcal{O}^n$, there exists $b_+^{(n,o)}$, and moreover,

- $0 < b_+^{(n,o)} < \frac{1}{2}$, especially
- $\frac{1}{4} < b_+^{(n,\hat{o}_n)} < \frac{1}{2}$.

Proof:

For any $n \geq 2$ and $o \in \mathcal{O}^n$,

- $G^{(n,o)}(0) < 0$, and
- $G^{(n,o)}(b) > 0$ for $b \geq \frac{1}{2}$,

hold, which implies that $b_+^{(n,o)} \in (0, \frac{1}{2})$. Especially,

- $G^{(n,\hat{o}_n)}(b) < 0$ for $b \leq \frac{1}{4}$,

holds, which implies $b_+^{(n,\hat{o}_n)} \in (\frac{1}{4}, \frac{1}{2})$. ■

The next result shows that completely homogeneous mode with full participation of Subordinates has an equilibrium in which Subordinates' thresholds and Boss's payoff are larger than any one in homogeneous mode with partial participation of Subordinates.

Proposition 9 Suppose $n \geq 3$. For any $\tilde{i}$ such that $1 \leq \tilde{i} \leq n$, we consider the organizational mode $(n-1, \tilde{o})$ such that

$$\tilde{o}_i = \begin{cases} \hat{o}_i^n & \text{if } i < \tilde{i}, \\ \hat{o}_{i+1}^n & \text{if } i \geq \tilde{i}. \end{cases}$$

Then, for any $b_+^{(n-1,\tilde{o})}$, there exists $b_+^{(n,\hat{o}^n)}$ such that $b_+^{(n,\hat{o}^n)} > b_+^{(n-1,\tilde{o})}$ and $EU_0^{(n,\hat{o}^n)}(b_+^{(n,\hat{o}^n)}) > EU_0^{(n-1,\tilde{o})}(b_+^{(n-1,\tilde{o})})$.

Proof:

$$G^{(n,\hat{o}^n)}(b_+^{(n-1,\hat{o})}) = - \left(\frac{1}{2} - b_+^{(n-1,\hat{o})} \right) F_i(b_+^{(n-1,\hat{o})}) < 0,$$

holds. Combined with the fact that $G^{(n,\hat{o}^n)}(\frac{1}{2}) > 0$ and Proposition 8, this implies the existence of $b_+^{(n,\hat{o}^n)}$ such that $b_+^{(n,\hat{o}^n)} > b_+^{(n-1,\hat{o})}$. Combined with (6), this directly implies $EU_0^{(n,\hat{o}^n)}(b_+^{(n,\hat{o}^n)}) > EU_0^{(n-1,\hat{o})}(b_+^{(n-1,\hat{o})})$. ■

Lastly, consider the comparison between homogeneous mode and any other mode. In doing so, the next lemma is very helpful.

Lemma 8 For any $n \geq 2$, any $o \in \bar{\mathcal{O}}^n$, and any $b_+^{(n,o)} \in [\frac{1}{4}, \frac{1}{2})$, there exists $b_+^{(n,\hat{o}^n)}$ such that $EU_0^{(n,\hat{o}^n)}(b_+^{(n,\hat{o}^n)}) > EU_0^{(n,o)}(b_+^{(n,o)})$.

Proof:

Let $k = +(o) < n$. Without loss of generality, we can relabel the order of Subordinates such that

$$o_i = \begin{cases} + & \text{if } i \leq k, \\ - & \text{if } i \geq k+1. \end{cases}$$

Throughout the proof, we denote

- $x := b_+^{(n,o)}$,
- $y_i := 1 - F_i(x)$ for $i = 1, \dots, n$, and
- $z_i := 1 - F_i(\frac{1}{2} - x)$ for $i = k+1, \dots, n$.

Then, the equilibrium condition $G^{(n,o)}(b_+^{(n,o)}) = 0$ is written as $xy_1 \cdots y_k = (\frac{1}{2} - x)z_{k+1} \cdots z_n$. Then, $z_i < 1$ for $i = k+1, \dots, n$, which follows from Assumption 3, implies that $\frac{1}{2} - xy_1 \cdots y_k > x$, and $y_i \leq z_i$ for $i = k+1, \dots, n$, which follows from the assumption $b_+^{(n,o)} \in [\frac{1}{4}, \frac{1}{2})$, implies that $x \geq \frac{y_{k+1} \cdots y_n}{2(y_1 \cdots y_k + y_{k+1} \cdots y_n)}$. The Boss's equilibrium payoff is also written as

$EU_0^{(n,o)}(b_+^{(n,o)}) = -xy_1 \cdots y_k$. By using these facts, we obtain

$$\begin{aligned}
G^{(n,\delta^n)} \left(EU_0^{(n,o)}(b_+^{(n,o)}) + \frac{1}{2} \right) &= G^{(n,\delta^n)} \left(\frac{1}{2} - xy_1 \cdots y_k \right) \\
&= \left(\frac{1}{2} - xy_1 \cdots y_k \right) \Pi_{i=1}^n \left(1 - F_i \left(\frac{1}{2} - xy_1 \cdots y_k \right) \right) - xy_1 \cdots y_k \\
&\leq \left(\frac{1}{2} - xy_1 \cdots y_k \right) y_1 \cdots y_n - xy_1 \cdots y_k \\
&= y_1 \cdots y_k \left[\frac{y_{k+1} \cdots y_n}{2} - x(1 + y_1 \cdots y_n) \right] \\
&\leq y_1 \cdots y_k \left[\frac{y_{k+1} \cdots y_n}{2} - \frac{y_{k+1} \cdots y_n (1 + y_1 \cdots y_n)}{2(y_1 \cdots y_k + y_{k+1} \cdots y_n)} \right] \\
&= -\frac{y_1 \cdots y_n (1 - y_1 \cdots y_k) (1 - y_{k+1} \cdots y_n)}{2(y_1 \cdots y_k + y_{k+1} \cdots y_n)} \\
&< 0.
\end{aligned}$$

Combined with the fact that $G^{(n,\delta^n)}(\frac{1}{2}) > 0$, this implies the existence of $b_+^{(n,\delta^n)}$ such that $b_+^{(n,\delta^n)} > EU_0^{(n,o)}(b_+^{(n,o)}) + \frac{1}{2}$. Combined with (6), this implies $EU_0^{(n,\delta^n)}(b_+^{(n,\delta^n)}) > EU_0^{(n,o)}(b_+^{(n,o)})$. $\blacksquare$

Based on this lemma, we can prove the next proposition, which means that homogeneous mode has an equilibrium in which Subordinates thresholds and Boss's expected payoffs are larger than any equilibrium in any other organizational mode.

Proposition 10 For any $n \geq 2$, any $o \in \bar{\mathcal{O}}^n$, and any $b_+^{(n,o)}$, there exists $b_+^{(n,\delta^n)}$ such that $b_+^{(n,\delta^n)} > b_+^{(n,o)}$ and $EU_0^{(n,\delta^n)}(b_+^{(n,\delta^n)}) > EU_0^{(n,o)}(b_+^{(n,o)})$.

Proof:

$$\begin{aligned}
G^{(n,\delta^n)}(b_+^{(n,o)}) &= -\left(\frac{1}{2} - b_+^{(n,o)} \right) \left[1 - \left(\Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)}(b_+^{(n,o)}) \right) \right) \left(\Pi_{i=1}^{-(o)} \left(1 - F_{-i(o)} \left(\frac{1}{2} - b_+^{(n,o)} \right) \right) \right) \right] \\
&< 0,
\end{aligned}$$

holds. Combined with the fact that $G^{(n,\hat{o}^n)}(\frac{1}{2}) > 0$ and Proposition 8, this implies the existence of $\bar{b}_+^{(n,\hat{o}^n)}$ such that $\bar{b}_+^{(n,\hat{o}^n)} > b_+^{(n,o)}$.

On the other hand, Proposition 8 means that $b_+^{(n,o)} \in (0, \frac{1}{2})$. If $b_+^{(n,o)} \in [\frac{1}{4}, \frac{1}{2})$, Lemma 8 directly implies the existence of $\hat{b}_+^{(n,\hat{o}^n)}$ such that $EU_0^{(n,\hat{o}^n)}(\hat{b}_+^{(n,\hat{o}^n)}) > EU_0^{(n,o)}(b_+^{(n,o)})$. Suppose $b_+^{(n,o)} \in (0, \frac{1}{4})$. We define $\tilde{o} \in \bar{\mathcal{O}}^n$ as

$$\tilde{o}_i = \begin{cases} - & \text{if } o_i = +, \\ + & \text{if } o_i = -. \end{cases}$$

Then, there exists $b_+^{(n,\tilde{o})}$ such that $b_+^{(n,\tilde{o})} = \frac{1}{2} - b_+^{(n,o)} \in (\frac{1}{4}, \frac{1}{2})$. Moreover, by (4) and (5), we obtain $EU_0^{(n,\tilde{o})}(b_+^{(n,\tilde{o})}) = EU_0^{(n,o)}(b_+^{(n,o)})$. Therefore, again, Lemma 8 implies the existence of $\hat{b}_+^{(n,\hat{o}^n)}$ such that $EU_0^{(n,\hat{o}^n)}(\hat{b}_+^{(n,\hat{o}^n)}) > EU_0^{(n,o)}(b_+^{(n,o)})$.

By these facts and (6), it is clear that $\max\{\bar{b}_+^{(n,\hat{o}^n)}, \hat{b}_+^{(n,\hat{o}^n)}\}$ satisfies the statement of Proposition. ■