A Decomposition of Ricardian Trade Gains under External Economies of Scale

Toru Kikuchi*

Graduate School of Economics, Kobe University, Kobe 657-8501, Japan

and

Ngo Van Long

Department of Economics, McGill University, Montreal H3A 2T7, Canada

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Abstract

Although the one-factor Ricardian trade model with external economies of scale plays a significant role for the understanding of important trade issues under increasing returns, it lacks a compelling graphical representation. We propose a convincing graphical exposition that uses both the PPF and a labor market graph.

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1 Introduction

Increasing returns to scale (IRS) generated by external economies have become a main feature of many economic models that deal with trade gains, endogenous growth, multiplicity of equilibria, and indeterminacy in dynamic models of growth[1]. The simplest route to understand a basic mechanism in these models is the one-sector Ricardian trade model with IRS. Yet, this model lacks a compelling graphical representation.[2] The purpose of this note is to offer such a representation. Our experience indicates that this graphical approach helps the reader to gain, almost effortlessly, a clear understanding of the effect of IRS on trade gains.

* *E-mail addresses:* kikuchi@econ.kobe-u.ac.jp (Kikuchi), ngo.long@mcgill.ca (Long).

[1] Jones (1968), Herberg and Kemp (1969), Kemp (1969, ch. 8), Melvin (1969), Panagariya (1981) and Ethier (1982) are key contributions to the IRS literature in the static trade theory. Helpman and Krugman (1985, ch. 3) provides a comprehensive survey. For indeterminacy in models of trade and growth with IRS, see Nishimura and Shimomura (2002).

[2] Francois and Nelson (2002) is a notable exception. They develop a general graphical framework for division of labor models.

In most textbooks on international economics, the analysis of trade gains under external economies of scale relies either on the utilization of Production Possibility Frontier (PPF) diagrams or partial equilibrium diagrams.[3] One of the great traditions in the analysis of international trade is the exposition of canonical models (e.g., Ricardo, Heckscher-Ohlin-Samuelson) using simple graphical representations. It is useful both providing an intuitive grasp of the theory and for pedagogical purposes. We believe that the focus on the extensive use of trade models with external economies of scale should be accompanied by a canonical geographical representation.

In this note, we propose the use of a combination of the PPF graph and a labor market graph to facilitate the understanding of Ricardian trade gains under external economies of scale.[4] By using these graphs, one can easily decompose Ricardian trade gains under external economies of scale. In addition, for educational purposes, the inclusion of labor market graphs in a lecture on the Ricardian Model lays a useful groundwork for later lectures which deal with the Specific-Factors Model and International Factor Movements.

2 Analysis

Consider a Ricardian economy which produces two goods (good 1 and good 2) with one factor of production, labor.[5] Let Y_i and L_i denote respectively the output and employment levels of industry i.

Good 1 is produced in a competitive industry under external economies of scale. The output of the representative firm (employing ℓ_1 units of labor) is given by

$$y_1 = (L_1)^{\delta}\ell_1, \quad \delta \geq 0, \tag{1}$$

where the firm takes $(L_1)^{\delta}$ as exogenous. The firm perceives that its marginal product of labor is $(L_1)^{\delta}$ which it takes as a constant. Aggregating over all the firms in industry 1, we get

$$Y_1 = (L_1)^{\delta} L_1 = L_1^{1+\delta} \equiv L_1^{\varepsilon}, \qquad \varepsilon \equiv 1 + \delta$$

Therefore at the industry level, the marginal product of labor is $\varepsilon L_1^{\delta} > L_1^{\delta}$ if $\delta > 0$. We call εL_1^{δ} the marginal social product of labor in sector 1, and L_1^{δ} the marginal private product of labor, as perceived by the firms. Here ϵ is the degree of external economies of scale: $\epsilon = 1$ corresponds to the case without external economies of scale. Let a_{Li} denote the amount of labor needed to produce one unit of good i. In sector 1, a_{L1} is defined as follows:

$$a_{L1}(L_1) \equiv \frac{L_1}{Y_1} = (L_1)^{-\delta} = \left(Y_1^{1/\varepsilon}\right)^{-\delta} \tag{2}$$

[3] See Bowen, Hollander, and Viaene (1998), Caves, Frankel, and Jones (2002, ch. 3) for the former examples, while Krugman and Obstfeld (2008, ch. 6), for the latter example.

[4] Kikuchi and Long (2010) apply these methods to the standard Ricardian model without external economies of scale.

[5] The model is taken from Ethier (1982). See also Panagariya (1981).

Thus a greater employment level L_1 results in a lower value of a_{L1}.

Good 2 is the numeraire good. It is produced by a competitive industry under constant-returns-to-scale technology, so that a_{L2} is a constant.

Then the PPF is represented by

$$a_{L1}Y_1 + a_{L2}Y_2 = (Y_1)^{1/\epsilon} + a_{L2}Y_2 = L \tag{3}$$

where L is the total amount of labor. With $\epsilon > 1$, the marginal social cost of good 1 in terms of good 2 falls as Y_1 increases; this gives rise to a convex PPF, VP, depicted at Figure 1 (a).[6] Also, the marginal social cost of good 1 in terms of good 2 is lower than its private counterpart.[7]

Let μ be the expenditure share on good 1. In autarky, the relative price of good 1 is obtained as follows. First, from the definition of μ, and using the facts that in the Ricardian model national income equals the wage bill, wL, and that the price equals labour cost per unit of output, so that $w = p_2/a_{L2}$, we can write

$$p_1Y_1 = \mu(p_1Y_1 + p_2Y_2) = \mu wL = \mu L\frac{p_2}{a_{L2}}$$

Thus in at the autarkic equilibrium

$$Y_1 = \frac{p_2}{p_1a_{2L}}(\mu L)$$

The equalization of the price ratio to the labour cost ratio gives

$$\left(\frac{p_1}{p_2}\right)^A = \frac{a_{L1}(L_1^A)}{a_{L2}} = \frac{1}{a_{2L}}\left(Y_1^{1/\varepsilon}\right)^{-\delta} = \frac{1}{a_{L2}}Y_1^{\frac{1-\varepsilon}{\varepsilon}}$$

or

$$\frac{p_1a_{L2}}{p_2} = \left(\frac{p_2}{p_1a_{L2}}\right)^{\frac{1-\varepsilon}{\varepsilon}}(\mu L)^{\frac{1-\varepsilon}{\varepsilon}}$$

$$\left(\frac{p_1a_{L2}}{p_2}\right)^{\frac{1}{\varepsilon}} = (\mu L)^{\frac{1-\varepsilon}{\varepsilon}}$$

Finally, we obtain

$$\left(\frac{p_1}{p_2}\right)^A = \frac{a_{L1}(L_1^A)}{a_{L2}} = \frac{(\mu L)^{1-\epsilon}}{a_{L2}}, \tag{4}$$

where the superscript A represents the autarkic equilibrium. The autarkic price ratio is obtained as line VA.

[6]When there are more factors, the shape of the PPF will in general be determined by the tension between (1) differences in factor intensities which tend to make the PPF concave, and (2) external economies of scale which tends to make the PPF convex. See, Herberg and Kemp (1969), Kemp (1969, ch. 8), Melvin (1969), and Markusen and Melvin (1984) for discussion.

[7]The marginal social cost of good 1 in terms of good 2 is $(1/\varepsilon a_{L2})Y_1^{-\delta/e}$ and the marginal private cost of good 1 in terms of good 2 is $(1/a_{L2})Y_1^{-\delta/e}$. Both fall as more of good 1 is produced along the PPF.

Now let us add one more panel for a better understanding: Figure 1 (b) depicts labor allocation between sectors. The horizontal axis represents the total labor force, L. The quantity of worker employed in sector 1 (resp. sector 2) is measured from the left (resp. right). The left (resp. right) vertical axis shows the wage rate in the sector 1 (resp. sector 2). Initially, in the autarkic equilibrium, wage rates are equalized between sectors and O_1L_1 workers are hired in sector 1, while L_1O_2 workers are hired in sector 2. The total income (in terms of the numeraire, good 2) is shown by the shaded rectangle.

For later use, we add a curve that represents the relationship between the wage rate (value of average product of labour) in sector 1 and the hypothetical employment level in that sector, L_1, for a given price p_1. This hypothetical curve is denoted by $\tilde{w}_1(L_1;\, p_1)$:

$$\tilde{w}_1(L_1;\, p_1) \equiv \frac{p_1}{a_{L1}(L_1)} = p_1 L_1^{\epsilon-1}. \tag{5}$$

The dotted increasing curve in Figure 1 (b) depicts $\tilde{w}_1(L_1,\, p_1^A)$. Since economies of scale are external to firms (and workers), this curve is not perceived by them.[8] Note that as ϵ approaches unity this curve approaches a horizontal line.

Now let us move to the trading situation with a fixed free-trade price ratio $(p_1/p_2)^T$ (Figure 2). If the terms of trade are given by the slope of the line PD (i.e., the free-trade relative price of good 1 is higher than autarkic price ratio), the economy specializes in good 1 at point P. Assume that consumption occurs at point C located on the consumption possibility frontier PD, so that CBP is the trade triangle (the country exports BP units of good 1 and imports BC units of good 2).

Now we can decompose the movement toward the trading equilibrium into two steps, which is in line with the traditional separation of trade gains into consumption and production gains.[9] Firstly, suppose that in the short run, labor allocation is fixed, and thus the economy is staying at the autarky production point, A, while it is able to trade at the terms of trade $p^T \equiv (p_1/p_2)^T$. Superscript T denotes the trading equilibrium. Then, the consumption possibility frontier expands from VAP to $V'AP$ as in Figure 2 (a).[10] In Figure 2 (b), this change can be illustrated as an increase in the wage rate for the workers employed in sector 1 [i.e., from $p_1^A/a_{L1}(L_1^A)$ to $p_1^T/a_{L1}(L_1^A)$]. Their total increase in wage income is represented by the dotted rectangle in Panel (b), which is shown as VV' in Panel (a). It is also important to note that, as the result of moving from autarkic price to the free-trade price, the curve of hypothetical wage in sector 1, $\tilde{w}_1(L_1,\, p_1)$, shifts up from $\tilde{w}_1(L_1,\, p_1^A)$ to $\tilde{w}_1(L_1,\, p_1^T)$.

Next, let us consider the (long-run) labor movement between sectors. Since sector 1 offers a higher wage rate, workers will gradually move from sector 2 to sector 1 (as shown by the arrows in Figure 2). Due to external economies

[8] Alternatively, we may assume that all agents know this relationship but they do not act on it because individually they are atomistic.

[9] See, for example, Dixit and Norman (1980, pp. 71–72).

[10] At the production point A, under free trade consumers will consume more of good 2 than before, because good 2 becomes relatively cheaper than under autarky, $p^T > p^A$.

of scale, each unit of labor moved from sector 2 to sector 1 generates a larger output increase than the previous unit, which implies a higher wage rate, i.e. a movement along the $\tilde{w}_1(L_1,\ p_1^T)$ curve. In Panel (b), the eventual distribution of labor force will be one of complete sepecialization (with O_1O_2 workers in sector 1), which corresponds to the production point P in Panel (a). The effect of this labor movement among sectors is shown as the expansion of the consumption possibility frontier from $V'AP$ to DP in Panel (a), reflected by the sum of the horizontally shaded area and the vertically shaded area in Panel (b). It is important to note that, by utilizing a labor market graph, the gains from labor movement can be decomposed into (1) direct gains from labor movement (the horizontally shaded area) and (2) indirect gains via external economies of scale (the vertically shaded area). Note that the direct gains from labor movement also exists under constant-returns-to-scale technology. Note also that, the higher is the numerical value of ϵ, the bigger are the indirect gains.

In terms of the numeraire, trade gains are measured by VD in Panel (a), and by the sum of the dotted rectangle, the horizontally shaded rectangle, and the vertically shaded rectangle in Panel (b).

We must now find out what happens when p_1^T is lower than p_1^A (Figure 3). In this case the curve of hypothetical wage in sector 1, $\tilde{w}_1(L_1,\ p_1)$, shifts down from $\tilde{w}_1(L_1,\ p_1^A)$ to $\tilde{w}_1(L_1,\ p_1^T)$. The wage rate in sector 1 becomes lower and workers will gradually moves from sector 1 to sector 2 (shown by the arrows in Figure 3). The eventual distribution of labor force will be one with O_1O_2 workers in sector 2, which corresponds to the production point V in Panel (a). The total income (in terms of the numeraire, good 2) is shown by the sum of the shaded area and the horizontally shaded rectangle, which is same as the total income under the autarky equilibrium. Since the price of good 1 becomes lower in in terms of the numeraire, this implies an increase in total real income.

Now we can restate Ethier's (1982) interesting result on trade gains under external economies of scale.

Proposition 1 (Ethier): *A small country entering into international trade will be driven to specialize in that commodity with the lower autarkic relative price. Regardless of which commodity that is, the small country will gain from free trade relative to autarky.*

Figure 3 also illustrates the possibility of multiple equilibria that can be Pareto-ranked. Although when p_1^T is lower than p_1^A the economy gains relative to its autarky welfare level (at A) by specializing in good 2, it can obtain a larger gain by economy specializing in good 1 if the following condition holds:

$$\tilde{w}_1(L;\ p_1^T) > \frac{p_2^T}{a_{L2}}. \tag{6}$$

(recall that by normalization, $p_2^T = p_2^A = 1$). By shifting labor force toward sector 1 instead of sector 2, the economy can obtain the indirect gain via external economies of scale. When the above inequality holds, the national income in terms of good 2 when the country produces at point P is $\tilde{w}_1(L;\ p_1^T)L$, which is greater than when it produces at point V. In Figure 3 (b) the extra gains from specializing in good 1 is depicted by the vertically shaded rectangle.

When there are multiple equilibria, in general it is not clear which one is likely to prevail, though some sort of stability argument may helps the equilibrium selection.[11] Myerson (2009, p. 1111) points out that the existence of multiple equilibria is "a pervasive fact of life that needs be appreciated and understood, not ignored by economists."[12]

3 Discussion

Using the above illustration confers several advantages. First, since the labor-market diagram emphasizes differences in wage rates between sectors in the short run, it is easier to understand the process of factor movements from the workers' perspectives. Second, it is intuitive to see at first the trade gains in terms of labor income. This is later reinforced by the traditional separation of trade gains into consumption and production gains. Third, the idea of multiple equilibria and the stability considerations can be illustrated in a simple way.

Another major advantage relates to the sequencing of lectures of a typical course of International Economics (or International Trade). Since both the specific-factor model and the analysis of international factor movements make extensive use of the graphs of labor-market (dis)equilibrium, it might be helpful to provide a graph of the labor market in advance. The inclusion of labor market graphs in the Ricardian Model under external economies of scale lecture lays a useful groundwork for the later lectures.

We recognize that there are many alternative ways to show Ricardian trade gains under external economies of scale. Still, we believe that the way presented here provides some helpful tool for understanding (and teaching) Ricardian trade models under external economies of scale.

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[11] Note that Proposition 1 implies that when when p_1^T is lower than p_1^A the country will not be "driven" to the (unstable) equilibrium point P, by the usual stability argument, as illustrated by the arrows in Figure 3. Only an active industrial policy can lead to this equilibrium (e.g. by coordinating expectations)

[12] Multiplicity of equilibria occurs in nature as well. As Stephen Jay Gould (1993, p. 28) noted, "places with apparently identical vegetation, moisture, and temperature might harbor shells of maximally different form."

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Figure 1

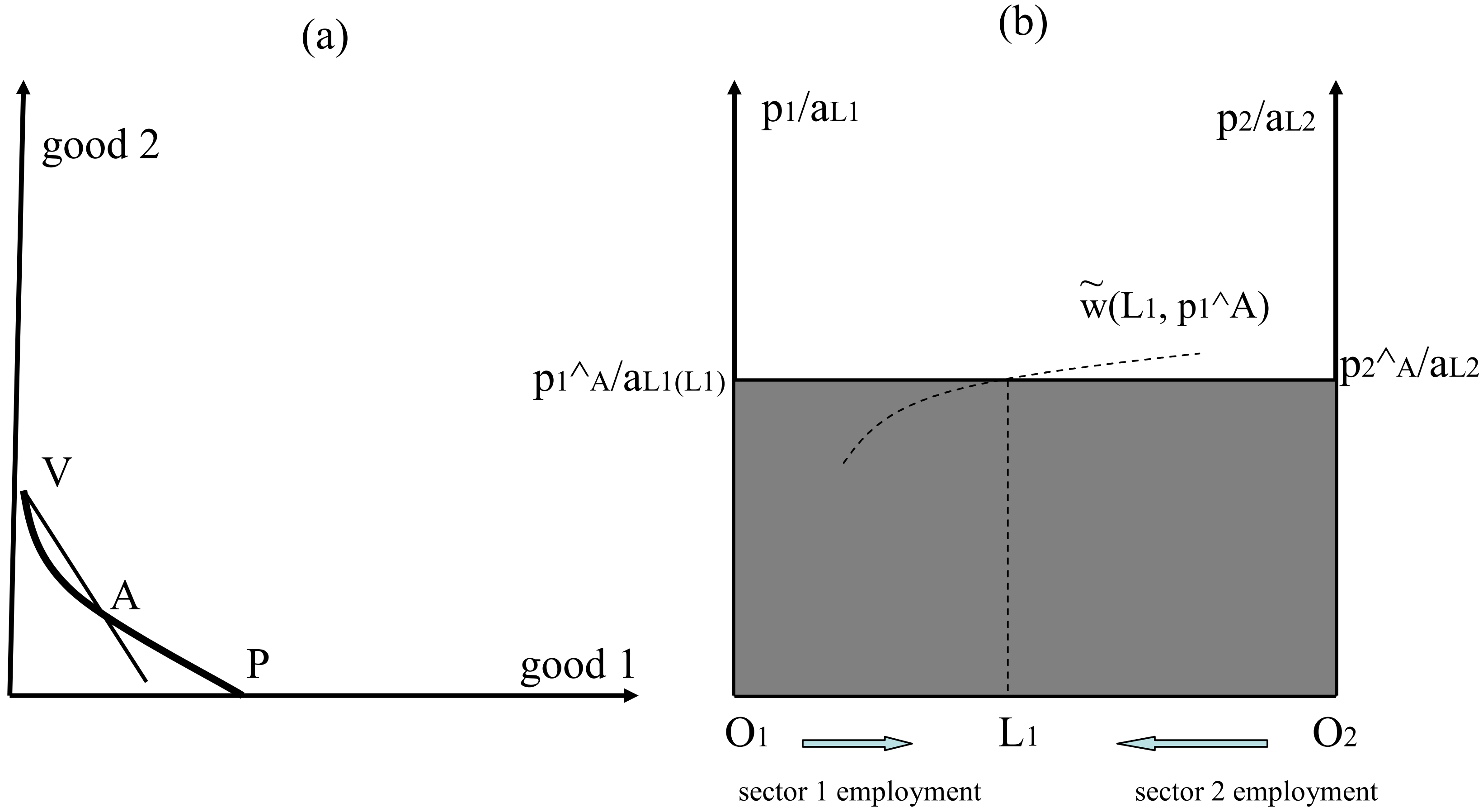

(a)
good 2
V
A
P
good 1
(b)
p1/aL1
p2/aL2
w(L1, p1^A)
p1^A/aL1(L1)
p2^A/aL2
O1
L1
O2
sector 1 employment
sector 2 employment

Figure 2

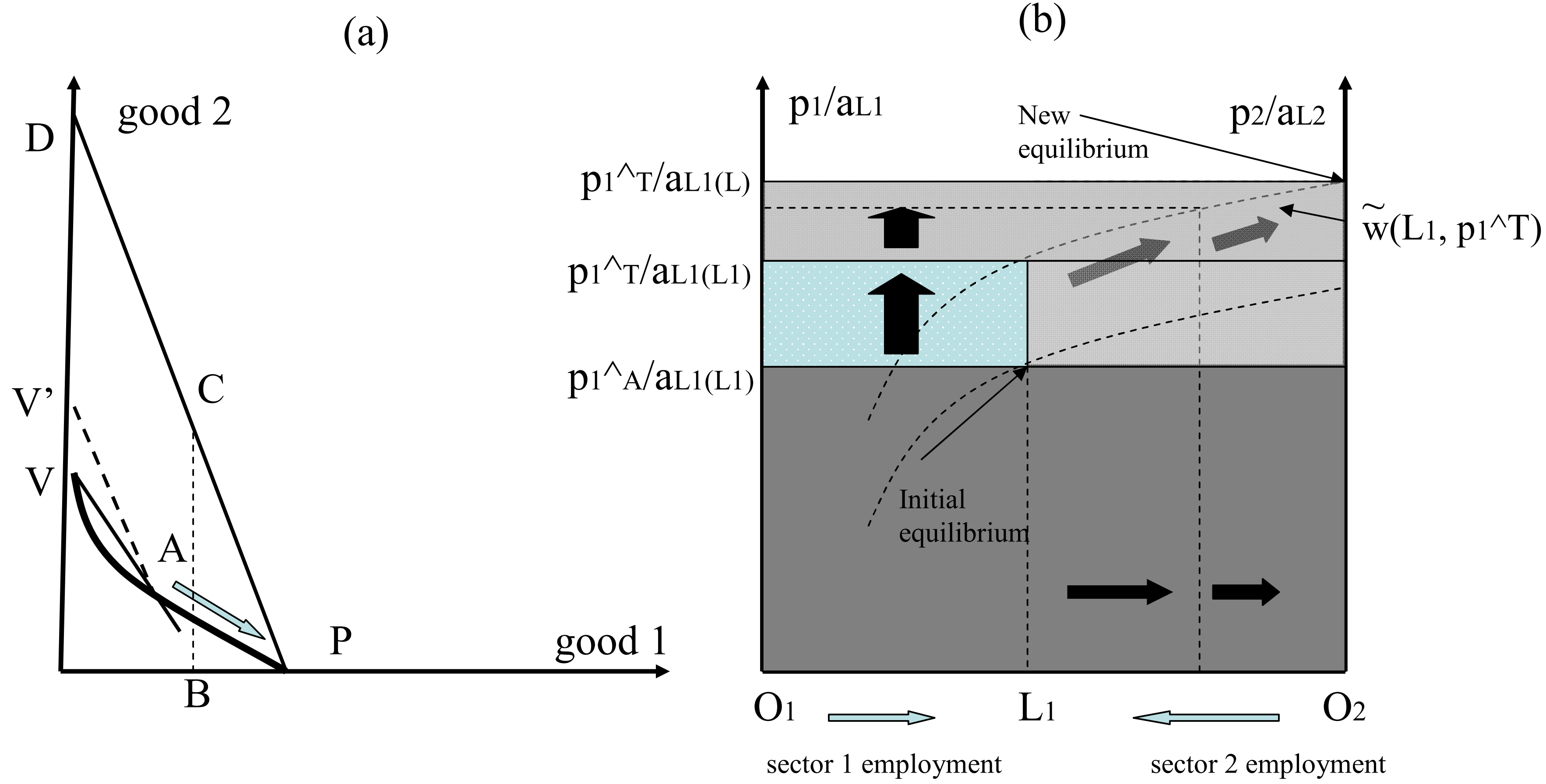
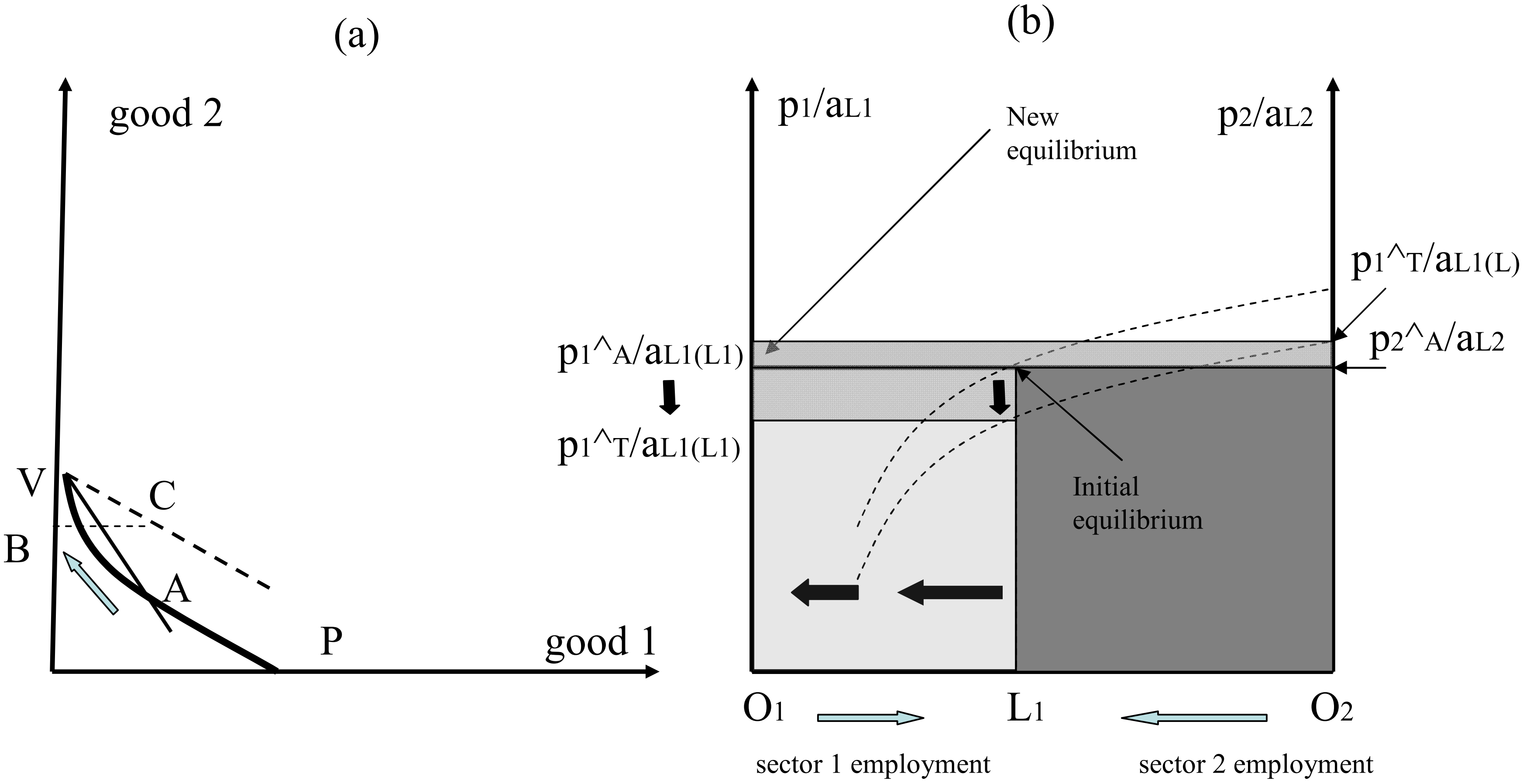

(a)
good 2
D
C
V'
V
A
P
good 1
B
(b)
p1/aL1
New
equilibrium
p2/aL2
p1^T/aL1(L)
p1^T/aL1(L1)
p1^A/aL1(L1)
w̃(L1, p1^T)
Initial
equilibrium
O1
L1
O2
sector 1 employment
sector 2 employment

Figure 3


(a)
good 2
V
C
B
A
P
good 1
(b)
p1/aL1
New equilibrium
p2/aL2
p1^T/aL1(L)
p1^A/aL1(L1)
p2^A/aL2
p1^T/aL1(L1)
Initial equilibrium
O1
L1
O2
sector 1 employment
sector 2 employment