Real-Time Estimate of the Equilibrium Real Interest Rate: Evidence from Japan

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1 Introduction

Most central banks in developed countries adopt a short-term nominal interest rate as a policy rate to stabilize economic activity and financial systems: the Uncollateralized Overnight Call Rate in Japan and Federal Funds Target Rate in the United States are examples of such rates. Under such a consensus of monetary policy rates, when the central bank sets a target rate in real-time monetary policy, the real equilibrium interest rate (ERR), which is the real interest rate consistent with output equal to the potential level, can be regarded a benchmark rate among monetary policy rates [2]. When the actual real rate is equal to the ERR, the actual output is coincident with the potential output and prices become stable. For central banks, if they raise their policy rates, then the actual real rate exceeds the ERR and the economy contracts. However, if they decrease their policy rates, then the actual real rate is below the ERR and the economy is stimulated upward. Therefore, for central banks that can control the actual real rate, perceiving the real-time ERR is significant for stabilization of economic activity and prices. Therefore, one can readily imagine that the real-time ERR reflects the economy of the moment and that it varies over time. Nevertheless, many studies which have conducted monetary policy analysis have shown that the ERR is assumed to be as constant as the long-term equilibrium rate. In stark contrast, some evidence presented herein shows that the ERR has a character of sufficient

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2 Some authors refer to the ERR as the natural rate of interest (NRI) in the same meaning. Wicksell (1896) presented a pioneer study of both the ERR and NRI. Blinder (1998) differently names the ERR as the neutral rate.

time-variation to the degree that it is difficult to estimate it accurately[3].

As previous works have discussed, it is difficult to measure the true ERR. Nevertheless, many researchers continue to study estimation of the ERR because estimating the ERR is believed to enable the sensing of economic conditions and support the appropriate conduct of monetary policy. For instance, the central bank might consider the reliable real-time ERR as a benchmark rate to stabilize the economy when conducting monetary policy in real time. However, as Orphanide and van Norden (2002) proposed when they estimated the real-time output gap in the United States, the monetary authority might confront two uncertainties from available data, such that the ERR is discredited in real time. First, the data that are used in estimating the ERR are often revised. Therefore, the estimated ERR using real-time data differs from the second ERR using revised data for the same time. Secondly, even if the data are not revised, additional data become available in a subsequent quarter. Therefore, the previously estimated ERR might vary according to the acquisition of new data[4]. These possibilities imply that re-estimation of the ERR perturbs the central bank. Consequently, it cannot conduct monetary policy appropriately[5]. Orphanide and van Norden (2002) reported that uncertainty in measuring the output gap is caused by data revision and accumulation, especially real GDP data. Kamada (2009) also examined the uncertainty of the Japan's ERR caused by the revision of real GDP[6].

However, the uncertainty that occurs because of variation of the ERR from acquisition of additional data over time is unavoidable. Nevertheless, it is necessary to control the uncertainty by which the variation of the ERR is caused by the revision of data. As is well known, from Faust et al. (2005), the feature of the Japan's data revision, that is, the reason why the preliminary GDP data differ from the true value is 'noise.' And, when one estimates the true value of GDP as the latent variable from the featured GDP data, the studies by Mankiw et al. (1984) and Mankiw and Shapiro (1986) revealed that the filtering approach is useful. Hence, the estimation model for the ERR as a latent variable is applied to the Kalman filter. Our main goal in this paper is to obtain credible estimates of the ERR

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[3] For the Japanese case, see Oda and Muranaga (2003) and Kamada (2009). For the United States, see Laubach and Williams (2003), Clark and Kozicki (2004) and Trehan and Wu (2007).

[4] The latter is generally regarded as the *end-of-sample problem.*

[5] Of course, Orphanide and van Norden (2002) also present a third uncertainty: model uncertainty.

[6] Real GDP data entail several estimates in Japan: the first preliminary (quick) estimate, the second preliminary (quick) estimate, the semi-final estimate and the final estimate.

and other latent variables using a conventional method. Indeed, few methods exist to measure the ERR yet. Consequently, from some candidate models, it is not productive to select the best estimation model that holds the smallest uncertainty. Instead, it is necessary to incorporate into a conventional model those ideas and artifices that reduce uncertainty of the ERR estimates and which improve the credibility of the ERR estimates. Therefore, this paper specifically applies the method proposed by Laubach and Williams (2003) (LW) model[7].

The main objective is divided into two more parts. First, to ascertain the degree of uncertainty by revising GDP data, the uncertainties generated by an original LW model are evaluated when the ERR is measured in real time. Using a different method from that presented herein, Kamada (2009), based on the classification of four uncertainties by Orphanides and van Norden (2002), reported that the uncertainty of the data revision is a fraction of 1% in the estimated ERR. This might mean that this uncertainty causes a policy misunderstanding under the nominal zero lower bound in Japan. However, in Kamada (2009), the greatest uncertainty was the *end-of-sample* problem. These results were identical to those presented by Orphanides and van Noden (2002), which evaluated the uncertainties in the US real-time output gap using various methods. Moreover, the result reported by Kamada (2005) after examination of Japan’s real-time output gap resembled that obtained by Orphanides and van Norden (2002).

Second, data used herein are reviewed based upon hints offered by Kamada (2005, 2009) to reduce their associated uncertainty. The baseline LW model is modified if necessary. Furthermore, the uncertainties of the modified LW model are evaluated using Kamada’s conceptualization. In real-time estimation of the ERR, Kamada (2009) proposed the use of the revision-free data instead of real GDP data. There has not been such a proposition to reduce uncertainty into the ERR in previous studies. The usual GDP data must estimate the latent variables as the potential output or the ERR. However, when the ordinary GDP data is not used, the revision-free data are expected to hold the characteristic that the data reflect the level of aggregate demand or the output gap. Fortunately, full revision-free data for Japan are readily accessible, for example the *Short-Term Economic Survey of Enterprises* in Japan (*TANKAN*).

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[7] As methods to measure the ERR, a simple Hodrick–Prescott (HP) filter to estimate the trend of actual real rate is also available. Meanwhile, the ERR can be shown from a dynamic stochastic general equilibrium (DSGE) model. See Neiss and Nelson (2003), Smets and Wouter (2004) and Edge et al. (2007).

Results of this study can be summarized as follows. As a report by Kamada (2009) and other reports of previous studies have described, the real-time estimate obtained from the baseline LW model includes considerable uncertainty because of the inconsistency between vintage data and updated data for the real GDP. Moreover, the baseline model shows that the components used to construct the ERR amplify the uncertainty. Therefore, to reduce this uncertainty attributable to the estimates, the baseline LW model was modified. Then the usage of data that can be revised was reviewed. Consequently, the real-time ERR, as estimated by the modified LW and the usage of a new data, can be shown to have lower uncertainty than that of the baseline model. In general, only a few ERR estimation methods exist. Nevertheless, without eliminating the idea of the baseline model, a modified approach that reduces the uncertainties, which are an obstacle to policymakers, can be presented.

The remainder of the paper is organized as follows. Section 2 presents a description of the baseline Laubach–Williams model. Section 3 presents evaluation of the uncertainties from the baseline LW model in real time and presents some issues for consideration. Section 4 presents the revision-free data and the modified models, with comparison of the uncertainties from the modified model with those of the baseline model. Section 5 concludes this presentation.

2 ERR estimation model

2.1 Baseline LW model

The equilibrium real interest rate (ERR) is estimated using a model resembling that of Laubach and Williams (2003). The dynamics of the output gap that are expressed as the IS equation are a backward-looking formulation.

$$y_t - y_t^* = a_{y,1}(y_{t-1} - y_{t-1}^*) + a_{y,2}(y_{t-2} - y_{t-2}^*)$$

$$- \frac{a_{r,3}}{2}\left[(r_{t-1} - r_{t-1}^*) + (r_{t-2} - r_{t-2}^*)\right] + \varepsilon_{y,t}, \qquad (1)$$

where y_t is the real logarithm of GDP and y_t^* is the potential output. Consequently, the output gap $y_t - y_t^*$ is defined as the difference between y_t and y_t^*. r_t is the real uncollateralized call rate and r_t^* denotes the equilibrium real interest rate. Similarly, the real interest rate gap is expressed as the difference between r_t and r_t^*. Therefore, an IS equation is constructed from the lags of the output gap, the lagged real interest rate and a serially uncorrelated shock.

Second, the AS equation based on the Phillips curve is presented as follows.

$$\pi_t = b_{\pi,1}\pi_{t-1} + \frac{b_{\pi,2}}{3}\sum_{i=2}^{4}\pi_{t-i} + \frac{1-b_{\pi,1}-b_{\pi,2}}{4}\sum_{i=5}^{8}\pi_{t-i}$$
$$+ b_{\pi,3}(y_{t-1} - y_{t-1}^*) + b_{\pi,4}\pi_t^l + b_{\pi,5}\pi_{t-1}^o + \varepsilon_{\pi,t}. \quad (2)$$

The current inflation π_t is explained by the first through eighth lagged inflation. The sum of all coefficients of the terms of inflation is assumed to be one. Moreover, two restrictions are imposed on the inflation terms in the AS equation. First, the coefficients of the second through fourth lagged variable are assumed to be equal. Similarly, the coefficients of the fifth through eighth lagged variable are assumed to be equal. Gordon (1998) and Laubach and Williams (2003) are followed to set such restrictions. Two variables are set: core import price inflation π^l (excluding petroleum, computers and semiconductors) and lagged crude imported oil price inflation π^o. Relative oil price shocks are measured using them. Finally, in addition, AS equation consists of the lagged output gap and a serially uncorrelated error.

As described by optimal growth theory and New Keynesian theory, ERR is usually correlated with the growth rate trend. It also depends upon the household's rate of time preference. In this study, therefore, the form of the ERR is defined as follows.

$$r_t^* \equiv c \cdot g_t + z_t. \quad (3)$$

In that equation, g_t signifies the annualized trend growth rate of real output, and z_t is definable as the factor that influence on household's consumption demand. Therefore, z_t is regarded as the demand component, for which, if it holds high persistency, the process shown below follows.

$$z_t = z_{t-1} + \varepsilon_{z,t} \quad (4)$$

Consequently, z_t is set as a random walk process.

Finally, the data generating processes of the potential output and the trend growth rate are given as

$$y_t^* = y_{t-1}^* + g_t + \varepsilon_{y^*,t}, \tag{5}$$

$$g_t = g_{t-1} + \varepsilon_{g,t}. \tag{6}$$

Therein, g_t is defined as the annualized trend growth rate of the potential output. Furthermore, $\varepsilon_{y^*,t}$ and $\varepsilon_{g,t}$ are assumed as serially uncorrelated and contemporaneously uncorrelated error. Our formulation of the trend growth rate is given by Kuttner (1994) and is adjusted by many previous studies. In this form, the trend growth rate is assumed to vary at small variance every quarter because it is difficult to assume that the trend growth is constant.

The baseline LW model includes some unobservable variables. This model is designated as an unobserved-component (UC) model. A Kalman filter is useful to estimate the UC model and to generate unobserved data. This paper presents the stat-space representation of baseline LW model and applies Kalman filtering to this representation.

2.2 Estimation results

The estimate of ERR for Japan and the data used are shown as quarterly from 1980Q3 to 2010Q2 in Japan. The logarithmic real GDP (seasonal adjusted) is used as the measure of output and consumer price index (excluding fresh food, seasonal adjusted) as the measure of prices. Expected inflation is calculated from the forecast of the univariate AR(3) model. The nominal interest rate is the quarterly average of the overnight uncollateralized call rate, expressed in units of percent per year. The price index of non-oil imports is not available in Japan. The imported price index is applied to π^I and π^o.

As Laubach and Williams (2003) described, the maximum likelihood estimates of the standard deviations of the innovations to the trend growth rate g_t and the demand components z_t, σ_g and σ_z are likely to be biased toward zero. This phenomenon is the '*pile-up*' problem that Stock (1994) and Stock and Watson (1998) pointed out. To address this problem, this study uses the medium unbiased estimator suggested by Stock and Watson (1998). Consistent estimates of the ratios, $\lambda_g \equiv \sigma_g / \sigma_{y*}$ and $\lambda_z \equiv a_{r,3}(\sigma_z / \sigma_y)$, are obtained in the first estimation. In the final estimation, these restrictions are imposed to obtain the maximum likelihood estimation.

Table 1 presents estimation results of the baseline model parameters for different model specifications. In all cases, the coefficients relating the output gap to the real rate gap, $a_{r,3}$, are negative and statistically significant. The second and third columns report results in the models in which the baseline LW model is modified. Especially in the third column, the coefficient $a_{r,3}$ is quite small compared to those reported by Kamada (2009) in Japan and Clark and Kozicki (2005) in the US, which were obtained assuming the same model. However, when the baseline and a different specification model are estimated in a sub-sample from 1999Q1, even then Japan's nominal interest rate remains at around zero, which confirms that each estimation result of model parameters is robust.

3 Evaluation of uncertainty

3.1 Definitions of uncertainty

The uncertainty caused by data revision and accumulation can affect the action of a central bank. In this context of the output gap, Orphanides and van Norden (2005) define output gap estimates of four kinds: *real time*, *quasi-real*, *quasi-final* and *final* output gaps. In this case of the ERR, the ERR is estimated using only the vintage of data available for each point in time. Consequently, the *real-time* ERR is obtained. Next, when the counterfactual assumption is made that the ERR at period t is calculated using the full sample data series 1 through $T(>t)$, the *quasi-real* ERR is estimated. For the *quasi-final* ERR, another counterfactual assumption is made that the ERR at period t is calculated using the estimated model parameters at period T and the final data series. Then, the *quasi-final* ERR is calculated. The *final* ERR is a series that is estimated from all data in period t. It is used in the usual analysis. Moreover, for UC models, the quasi-final ERR is a filtered estimate, whereas the final ERR is a smoothed estimate.

As discussed in some previous reports of studies, three uncertainties are defined through each difference in the above four ERR estimates. First, the difference between the real-time and quasi-real ERR estimates is attributable to the effect of data revision. This study refers to the effect of the first uncertainty as *Error 1*. Second, the difference between the quasi-real and quasi-final ERR estimates is attributable to the effect of model parameters revision. This study specifically examines the effect of the second uncertainty as *Error 2*. Finally, the difference between the quasi-final and final ERR estimates is generally attributable to the effect of the *sample-end problem*. This study refers to the effect of the third uncertainty as *Error 3*.

3.2 Results

Complete real-time GDP data for Japan were unavailable. A dataset such as the US real-time dataset by the Federal Reserve Bank of Philadelphia is unfortunately not published as historical data. Therefore, the first published real GDP data were used, that is the first quick estimate, from the *Nikkei* (*Nihon Keizai Shinbun*). Table 2 below presents those results:

1. Error 1 in most estimated state variables is larger than Error 2 or 3.
2. Error 1 in the potential output that is apparently influenced directly by the revision of GDP is noticeable.
3. Two components of which the ERR consists, ‘ g ’ and ‘ z ’, are apparently sources of high uncertainty in the ERR.
4. Because of the data-generating processes of g and z, the uncertainty of the ERR can be amplified to a greater degree than necessary.

Results (a) and (b) present that the real-time latent variables are affected by the revision of real GDP. From the first column of Table 1, the uncertainty of the ERR (Error 1) is 0.58%. Considering the situation of long-run low interest rate in the estimate period, this value is large. Similarly, Error 1 in the potential output is larger than other errors. However, Error 1 of the ERR is larger than that of the potential output. Moreover, the two components, ‘ g ’ and ‘ z ’, that must be estimated first to calculate the ERR have respective uncertainties that are greater than 1%. Comparison of the results obtained by Kamada

(2009) must be done in light of an important point: the results obtained in this study reflect Japan's long-run slump after the bubble burst, whereas Kamada (2009) includes the bubble period. Furthermore, it is noteworthy that results reported by Kamada (2009) differ with those of this study in terms of the use of real-time data.

3.3 Implications

According to the results described above, it is necessary to reduce the uncertainty of ERR in real-time estimate as summarized in the following two points: (1) rethinking data that have a possibility of revision and (2) reviewing the data generating processes of 'g' and 'z'.

The best means to reduce uncertainty is to avoid using data that have a possibility of revision, but this way of measuring the ERR is not feasible. Data that have a possibility of revision, such as real GDP, are important to estimate latent economic indicators. Therefore, if real GDP is not used, then data must be prepared that are not revised and which have information about current economic activity such as real GDP. Kamada (2005, 2009) reported such data in the context that it remedies the real-time output gap. Such supplementary data are taken from the *Short-term Economic Survey of Enterprise* in Japan, *TANKAN*. The *TANKAN* aims at contributing to the appropriate implementation of monetary policy by capturing business trends of enterprises accurately. Kamada (2005, 2009) also suggests the use of a *Business Conditions diffusion index* (DI) of 10 items in the *TANKAN*. This DI indicates judgment of general business conditions of the responding enterprise in light of individual profits.

However, this DI is not applied for this study. Instead, the *weighted* DI is proposed. It is frequently seen on the minutes of monetary policy meetings because this DI is superior to the *business condition* DI in that a DI that reflects the output gap is desired here. The *weighted* DI actually consists of two DIs:

the *weighted* DI = $(1-\alpha)\times$ *Production Capacity* DI $+\alpha\times$ *Employment Condition* DI,

In that equation, α is a labour income share[8]. The *Production Capacity* DI is a

[8] It is assumed to be 0.64.

judgment of excessiveness, adequacy, or shortage of production capacity or business equipment of the responding enterprise[9]. The *Employment Conditions* DI is a judgment of excessiveness, adequacy, or shortage of the number of employees at the responding enterprise[10].

When the revision-free data are used and the processes of both ‘ *g* ’ and ‘ *z* ’ are reconsidered, it is necessary to modify the baseline LW model. Herein, the following two versions of the modified LW model are proposed:

Version 1 is to use both the *weighted* DI and ordinary GDP data, and to reconsider both ‘ *g* ’ and ‘ *z* ’ processes.

Version 2 is to use the *weighted* DI only and to reconsider both the ‘ *g* ’ and ‘ *z* ’ processes.

4 Modifications and re-evaluation

First of all, let me consider the relation between the TANKAN and e *noise*f before applying the TANKAN to the ERR estimation model. Publicly, the real-time data set has not yet been published in Japan. So, in this paper, I gathered both the first preliminary and second preliminary GDP published in 2006Q1-2011Q2 (benchmark year 2000). I ensured that the GDP data published first at some point takes around three years to be the stable number. Therefore, if the GDP published three years later is the true one, the difference between each revised GDP and the three-years-later GDP is regarded to be noise. Figure 1 illustrates the relation between noise and TANKAN. The figure displays a positive correlation. Namely, I ensure that TANKAN is helpful to estimate the true output gap.

Based upon the proposition of the two types of baseline LW model in the previous section, for **Version 1**, the two measurement equations of the baseline model are retained with no change and modify the transition ones only. This model uses both data that have a possibility of revision (ordinary real GDP) and non-revised data (the *weighted* DI). As discussed in the preceding section, because the published *weighted* DI already includes a

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[9] This DI excludes a shortage caused by temporary conditions such as a factory closure for regular repairs.

[10] the *Business Condition* DI and the *Employment Condition* DI are available from 1980Q1. However, the *Production Capacity* DI is available from 1990Q1. Consequently, the business condition DI 1980Q3 through 1989Q4 is used. Subsequently, the *weighted* DI is adopted.

gap form, it plays a role as a guide of the dynamics of the output gap, as

$$r_t^* = 4c(y_t^* - y_{t-1}^*), \tag{7}$$

$$(y_t^* - y_{t-1}^*) = fx_t + \varepsilon_{y^*,t}, \tag{8}$$

where variable x_t corresponds to the *weighted* DI in the data series and $\varepsilon_{y^*,t}$ is a serially uncorrelated error. This model is designated as the *Version 1* model, which can estimate the potential output simultaneously.

However, when the *weighted* DI is fully reviewed as an alternative measure of the output gap, the *weighted* DI can be substituted for the output gap $y - y^*$ in Eq. (1). Then the baseline LW model transforms into a very simple model. The Phillips curve, Eq. (2), drops from the baseline LW model and the ERR is simply a random walk process, as in specifications explained by Clark and Kozicki (2005).

$$x_t = a_{x,1}x_{t-1} + a_{x,2}x_{t-2} + \frac{a_{r,3}}{2}\left[(r_{t-1} - r_{t-1}^*) + (r_{t-2} - r_{t-2}^*)\right], \tag{9}$$

$$r_t^* = r_{t-1}^* + \varepsilon_{r^*,t}, \tag{10}$$

Therein, the variable x_t which corresponds to the *weighted* DI in the data series is the output gap and $\varepsilon_{r^*,t}$ is a serially uncorrelated error. This model is designated as the *Version 2* model. Unlike the *Version 1* model, the *Version 2* model can not estimate the potential output simultaneously.

Table 3 shows that most uncertainties in the ERR and output gap become smaller than those in the baseline LW model[11]. For the *Version 1* model, the ERR's uncertainty caused by the data revision (*Error 1*) is decreased by 85% relative to that obtained using the baseline model. In addition, uncertainty in the output gap drops to a much lower value. The *sample-end problem* in both ERR and output gap (*Error 3*) gets low. From results obtained for the *Version 2* model, of course, because real GDP is not used, the uncertainty by the data revision does not rise. Instead, the *Error 3* increases by 2.6%.

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[11] The case of the United States could not be examined because no statistical survey exists similar to the *TANKAN* in US.

5 Conclusion

Although knowledge of the usefulness of the ERR is widely shared, the problem of the real-time estimation that Orphanides and van Norden (2002) pointed out also arises in estimating the ERR when the central bank attempts to capture the dynamics of the ERR in real time using all available data in time. Results of this study showed that the real-time estimate of the ERR by LW model includes several types of uncertainty that arise through processes of data revision and accumulation. Especially, the effect of the data revision is not negligible in the context of a zero bound interest rate, and the model specification amplifies such effects. However, to reduce the uncertainties of the real-time ERR estimate, when the revision-free data (the *weighted* DI for *TANKAN*) were used as an indicator of the output gap and the LW model was modified appropriately, most uncertainties in the modified LW models became lower than those of the baseline LW model.

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Table 1: Parameter Estimates

Parameters	baseline LW model Estimates	t -value	**Version 1** Estimates	t -value	**Version 2** Estimates	t -value
$a_{y,1}$	0.660	2.508	1.025	26.746	0.986	16.484
$a_{y,2}$	0.285	1.083	0.007	6.571	0.005	1.357
$a_{r,3}$	-0.114	-2.253	-0.006	-8.665	-0.015	-4.214
$b_{\pi,1}$	0.688	5.516	1.305	0.571		
$b_{\pi 2}$	0.109	0.595	0.198	0.146		
$b_{y,3}$	1.026	2.078	0.047	4.590		
$b_{\pi^m,4}$	0.008	1.006	0.039	0.857		
$b_{\pi^m,5}$	0.006	0.883	-0.017	-0.484		
c	0.948	0.795	0.681	3.593		
f			0.060	1.786		
σ_y	0.948	2.472	0.362	2.880	0.413	9.063
σ_π	0.619	4.990	0.491	6.526		
σ_{y^*}	0.846	9.940	0.233	4.461		
σ_g	0.177					
σ_z	0.886					
σ_{r^*}					0.552	

MUE	$\lambda_g = 0.210$ $\lambda_z = 0.224$	$\lambda_{r^*} = 0.020$

Note: $\lambda_g = \sigma_g / \sigma_{y*}, \lambda_z = a_{r,3}(\sigma_z / \sigma_y), \lambda_{r^*} = a_{r,3}(\sigma_{r^*} / \sigma_y)$.

All estimates are for 1980Q3–2010Q2.

Table 2: Evaluation of the uncertainty on the baseline LW model.

	Error 1	Error 2	Error 3
r^*	0.584	0.022	0.739
g	1.363	0.041	0.515
z	1.377	0.016	0.603
$y - y^*$	0.299	0.015	0.268

Note: All numbers are root square means of the revision series shown. All statistics are for 1993Q1–2010Q2.

Table 3: Evaluation of uncertainty of the modified models.

		Error 1	Error 2	Error 3
Version 1	r^*	0.083	0.208	0.258
	$y - y^*$	0.0002	0.002	0.258
Version 2	r^*		0.069	2.602

Note: All numbers are root square means of the revision series shown. All statistics are for 1993Q1–2010Q2.

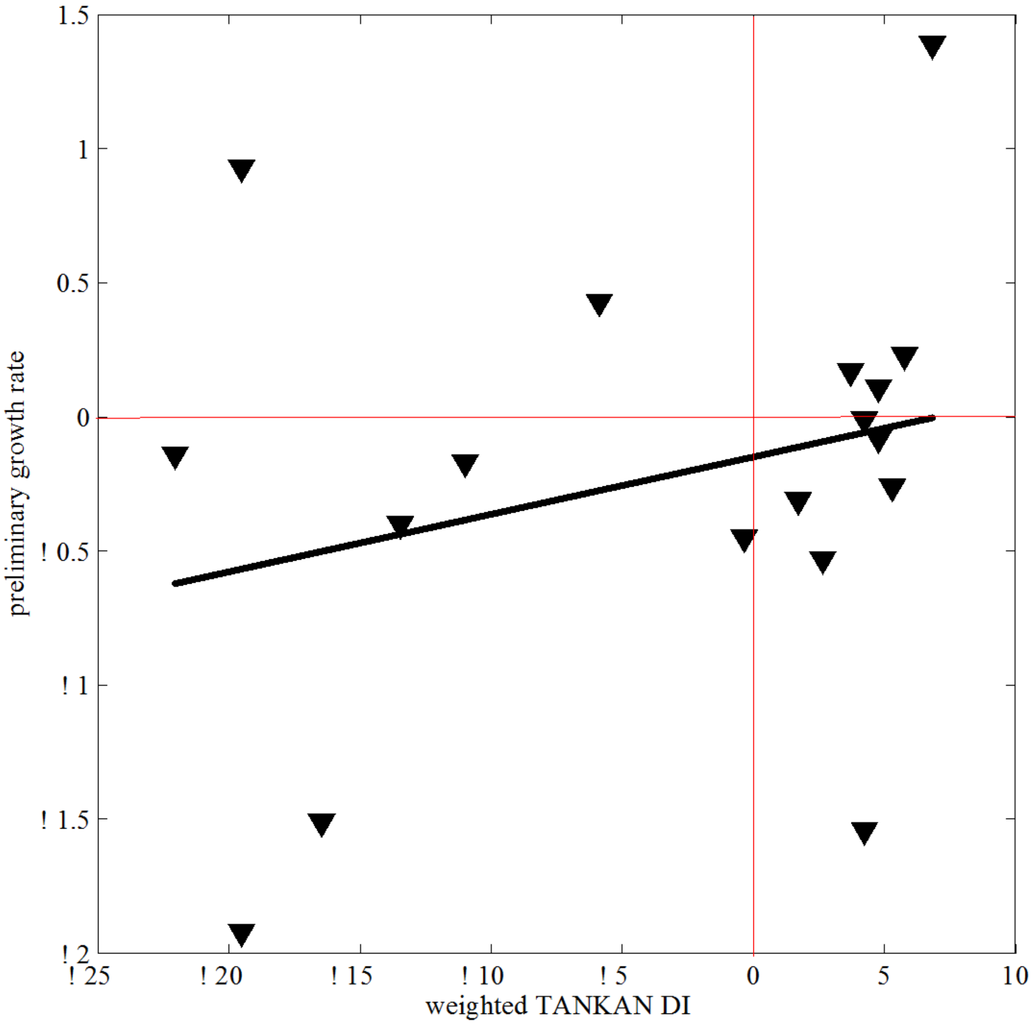

Figure 1: Preliminary data revision and the *weighted* TANKAN DI.