Capital income taxation and public debt in an endogenous fertility model

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Abstract

This study constructs an overlapping generations model with fertility choice and unemployment, caused by a constant minimum wage and incorporating public debt. It shows that a higher capital income tax reduces the public debt burden, and hence promotes capital accumulation, which leads to an improvement in unemployment and fertility rates.

JEL classification: H20, J13, J60.

Keywords: Capital income tax, Unemployment, Fertility, Public debt

1. Introduction

Many developed countries face the combination of high unemployment rate and low fertility rate (Fanti and Gori, 2010; Wang, 2015). High labor and childcare costs are considered to cause such economic issues. Fanti and Gori (2010) demonstrated that child tax can improve unemployment and fertility rates in an overlapping generations model. Wang (2015) extended Fanti and Gori (2010)'s study by incorporating pensions and child allowances.

To solve the above mentioned economic issues, this study focuses on capital income tax. The previous studies that have shown that capital income taxation can accelerate economic growth are Uhlig and Yanagawa (1996) and Caballé (1998). Uhlig and Yanagawa (1996) introduced tax reform, which shows that higher capital income tax cuts labor income tax, and demonstrated that such tax reform promotes savings and capital accumulation. Based on Fanti and Gori (2010) and Wang (2015), we consider tax reform of cutting public debt with a higher capital income tax. It shows that increasing capital income tax promotes employment and fertility.

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The rest of the paper is constructed as follows. Section 2 gives a description of the model. Section 3 concludes our study.

2. Model

2.1. Households

We consider an overlapping generation economy where households are identical and experience two periods: young and old. They derive utility from the number of children in the young period, and from consumption in the old period. Following Momota (2000) and De la Croix and Doepke (2004), the utility function is given as:

$$v_t = \log c_{t+1} + \varphi \log n_t, \tag{1}$$

where c_{t+1} is the consumption in the older period, n_t is the number of children, and $\varphi > 0$ is the taste for children. Suppose N_t is the population size of younger agents at period t, and the evolution of the population is described as $N_{t+1} = n_t N_t$.

According to Fanti and Gori (2010) and Wang (2015), a constant minimum wage raises unemployment, and unemployment rates are defined based on a fraction of time. Each household is endowed with one unit of time and provides it to the labor market inelastically in the young period. Households earn wage income, accumulate savings, and invest in childcare in the young period. In the old period, they consume savings. The lifetime budget constraints for households are given as follows:

$$s_t + \varepsilon n_t = (1 - u_t)\overline{w}, \tag{2}$$

$$c_{t+1} = (1 - \tau)(1 + r_{t+1})s_t, \tag{3}$$

where s_t is the savings, $\varepsilon > 0$ is the cost of childcare, u_t is the unemployment rate, $\overline{w} > 0$ is the constant minimum wage, $\tau \in (0,1)$ is the capital income tax, and r_{t+1} is the interest rate. Equation (3) shows that capital income tax is imposed on the gross interest rate, consistent with Yakita (2003) and Uchida and Ono (2021). The optimal allocations are given as follows:

$$n_t = \frac{\varphi(1 - u_t)\overline{w}}{\varepsilon(1 + \varphi)}, \tag{4}$$

$$\frac{s_t}{n_t} = \frac{\varepsilon}{\varphi}. \tag{5}$$

Equation (4) shows that the fertility rate improves if employment increases.

2.2. Firms

Firms produce final goods using capital and labor inputs in a competitive economy. The production technology is given as:

$$Y_t = AK_t^{\alpha} {L_t}^{1-\alpha},\ A > 0;\ 0 < \alpha < 1, \tag{6}$$

where Y_t is the total output, and K_t and L_t are the total capital and labor inputs, respectively. L_t is given as:

$$L_t = (1 - u_t)N_t. \tag{7}$$

Assuming full depreciation, the factor demands are given as follows:

$$\overline{w} = (1-\alpha)Ak_t^{\alpha}(1-u_t)^{-\alpha}, \tag{8}$$

$$1 + r_t = \alpha A k_t^{\alpha-1}(1-u_t)^{1-\alpha}, \tag{9}$$

where $k_t \equiv K_t/N_t$ is the capital per capita. From Equation (8), we obtain

$$1 - u_t = \left[\frac{(1-\alpha)A}{\overline{w}}\right]^{\frac{1}{\alpha}} k_t. \tag{10}$$

It indicates that unemployment reduces with capital accumulation, also explained by Fanti and Gori (2010) and Wang (2015). Substituting Equation (10) into Equation (9), we obtain the following constant interest rate:

$$1 + r_t = \alpha A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\overline{w}}\right)^{\frac{1-\alpha}{\alpha}}. \tag{11}$$

2.3. Government

The government imposes a tax on capital income, issues new bonds to finance public expenditure, and redeems public debt. The government's budgetary constraints are given by:

$$B_{t+1} = (1 + r_t)B_t + G_t - \tau(1 + r_t)s_{t-1}N_{t-1}, \tag{12}$$

where B_{t+1} is the amount of public debt issued at period t, $(1 + r_t)B_t$ is the expenditure for redemption of public debt, G_t is non-productive expenditure that does not contribute to either welfare or productivity, and $\tau(1 + r_t)s_{t-1}N_{t-1}$ is the capital income tax revenue. Following Uhlig and Yanagawa (1996), we assume

$$G_t = \gamma y_t N_t,\ 0 < \gamma < 1, \tag{13}$$

where $y_t \equiv Y_t/N_t = Ak_t^{\alpha}(1-u_t)^{1-\alpha}$ is the per capita output (Equation (6)).

2.4. Equilibrium

We denote $b_t \equiv B_t/N_t$ as the per capita stock of public debt, and it is equal to $b_{t+1} = b_t = b$ in a steady state. In equilibrium, $k_{t+1} + b_{t+1} = s_t/n_t$ holds. Using this equation and Equation (5), we have:

$$k_{t+1} = \frac{\varepsilon}{\varphi} - b_{t+1}. \tag{14}$$

This equation indicates that ε/φ is the upper limit of public debt. Further, we obtain the following equation[3]:

$$b_{t+1} = \frac{\varepsilon(1+\varphi)}{\varphi(1-\alpha)}\left[\frac{(1-\tau)\alpha b_t}{\frac{\varepsilon}{\varphi} - b_t} + \gamma - \alpha\tau\right] \tag{15}$$

In a steady state, we have the following quadratic equation from Equation (15):

$$\frac{\varphi(1-\alpha)}{\varepsilon(1+\varphi)} b^2 + \left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] b + (\gamma - \alpha\tau)\frac{\varepsilon}{\varphi} = 0. \tag{16}$$

Solving this equation, we obtain

$$b = \frac{-\left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] \pm \sqrt{D}}{\frac{2\varphi(1-\alpha)}{\varepsilon(1+\varphi)}} \tag{17}$$

$$D \equiv \left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right]^2 - \frac{4(1-\alpha)(\gamma - \alpha\tau)}{1+\varphi} \tag{18}$$

Suppose b_s and b_u are the two different per capita public debts in a steady state; then, from Equation (17), they are described as follows:

$$b_s = \frac{-\left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] - \sqrt{D}}{\frac{2\varphi(1-\alpha)}{\varepsilon(1+\varphi)}} \tag{19}$$

$$b_u = \frac{-\left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] + \sqrt{D}}{\frac{2\varphi(1-\alpha)}{\varepsilon(1+\varphi)}} \tag{20}$$

If $D > 0$ and $b_s > 0$ hold, there exist two positive equilibria where one is locally stable, and the other is unstable. Figure. 1 captures the dynamics of per capita public debt with two positive equilibria.

[Figure 1 around here]

Figure. 1 describes that b_s is locally stable and b_u is unstable. If $\gamma - \alpha\tau \leq 0$ holds, the dynamics of public debt have two equilibria as captured by Figure. 2. In this case, a locally stable steady state exists though it converges to zero, i.e., $b_s = 0$.

[Figure 2 around here]

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[3] See Appendix for the derivation of Equation (15).

Recall that the present article introduces tax reform that represents cutting public debt with a higher capital income tax. If public debt does not exist in a steady state, tax reform will become impossible. Therefore, we analyze the neighborhoods of the positive and locally stable steady states, i.e., $b_s > 0$, as captured by Figure. 1. Differentiating b_s with respect to τ from Equation (19), we have

$$\frac{db_s}{d\tau} = -\frac{\alpha\varepsilon}{\varphi} D^{\frac{-1}{2}} < 0. \tag{21}$$

Note that D is defined by Equation (18) and $D > 0$ holds if $b_s > 0$ holds. Equation (21) indicates that a higher capital income tax reduces the public debt burden. Substituting Equation (19) into Equation (14), we derive

$$k_s = \frac{\varepsilon}{\varphi} - b_s = \frac{\varepsilon}{\varphi} + \frac{\left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] + \sqrt{D}}{\frac{2\varphi(1-\alpha)}{\varepsilon(1+\varphi)}}, \tag{22}$$

where k_s is the capital per capita in a stable steady state. Differentiating Equation (22) with respect to τ, we derive

$$\frac{dk_s}{d\tau} = -\frac{db_s}{d\tau} = \frac{\alpha\varepsilon}{\varphi} D^{\frac{-1}{2}} > 0. \tag{23}$$

A higher capital income tax decreases public debt stock and hence, promotes capital accumulation. According to Caballé (1998), public debt hinders acquiring capital, which leads to reduction of economic growth. In other words, capital accumulation can increase due to a decline in public debt as shown Equation (23).

We now investigate how capital income taxes affect unemployment. Suppose u_s is the long-run unemployment rate in a stable steady state. By using Equations (10), (14), (19), and (22), we obtain the following unemployment rate as a generic function of capital income tax (τ):

$$u_s = u\{k[b(\tau)]\}. \tag{24}$$

From Equations (10), (21), (23) and (24), we have

$$\frac{du_s}{d\tau} = \overbrace{\underbrace{\frac{du_s}{dk_s}}_{<0}\underbrace{\frac{dk_s}{db_s}}_{<0}\underbrace{\frac{db_s}{d\tau}}_{<0}}^{-}. \tag{25}$$

In this equation, increasing capital income tax reduces public debt, which in turn promotes capital accumulation, and this reduces unemployment. Substituting Equation (22) into Equation (10), we derive

$$u_s = 1 - \left[\frac{(1-\alpha)A}{\overline{w}}\right]^{\frac{1}{\alpha}} k_s,$$

$$= 1 - \left[\frac{(1-\alpha)A}{\overline{w}}\right]^{\frac{1}{\alpha}} \left\{ \frac{\varepsilon}{\varphi} + \frac{\left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] + \sqrt{D}}{\frac{2\varphi(1-\alpha)}{\varepsilon(1+\varphi)}} \right\}. \quad (26)$$

From this equation, we derive

$$\frac{du_s}{d\tau} = -\left[\frac{(1-\alpha)A}{\overline{w}}\right]^{\frac{1}{\alpha}} \frac{\alpha\varepsilon}{\varphi} D^{\frac{-1}{2}} < 0. \quad (27)$$

Using Equation (27), we obtain the following proposition:

Proposition 1

A higher capital income tax promotes employment.

Next, we investigate how capital income tax affects fertility rates. n_s denotes the long run fertility rate in a locally stable steady state. By using Equations (4) and (24), the generic form of n_s is written as:

$$n_s = n\{u[k(b(\tau))]\}. \quad (28)$$

From Equations (4), (10), (21), (23), and (28), we obtain

$$\frac{dn_s}{d\tau} = \overbrace{\underbrace{\frac{dn_s}{du_s}}_{<0} \underbrace{\frac{du_s}{dk_s}}_{<0} \underbrace{\frac{dk_s}{db_s}}_{<0} \underbrace{\frac{db_s}{d\tau}}_{<0}}^{+}. \quad (29)$$

As denoted above, increasing capital income tax promotes employment, and hence, it also improves the long run fertility rate from Equation (4). Substituting Equation (26) into Equation (4), the long-run fertility rate can be written as:

$$n_s = \frac{\varphi(1-u_s)\overline{w}}{\varepsilon(1+\varphi)},$$

$$= \frac{\overline{w}^{1-\frac{1}{\alpha}}[(1-\alpha)A]^{\frac{1}{\alpha}}}{(1+\varphi)} \left\{ 1 + \frac{\left[\alpha - \gamma - \frac{(1-\alpha)}{1+\varphi}\right] + \sqrt{D}}{\frac{2(1-\alpha)}{(1+\varphi)}} \right\}. \quad (30)$$

Differentiating Equation (30) with respect to τ, we have

$$\frac{dn_s}{d\tau} = \frac{\overline{w}^{1-\frac{1}{\alpha}}\alpha[(1-\alpha)A]^{\frac{1}{\alpha}}}{(1+\varphi)} D^{\frac{-1}{2}} > 0. \quad (31)$$

Thus, we obtain the following proposition:

Proposition 2

A higher capital income tax improves fertility.

A numerical example is presented in Table. 1. The parameters are set as follows: $\alpha = 0.25, \varphi = 0.1, \varepsilon = 0.1, \gamma = 0.105, \overline{w} = 3.2,$ and $A = 4.2$.

[Table 1 around here]

3. Conclusion

Many developed countries face high unemployment rates and a decline in fertility rates. The current study developed an overlapping generations model with unemployment and fertility choice. The results show that capital income taxation is effective in improving employment and fertility rates.

Appendix

Dividing Equation (12) with N_t and considering Equation (13), we have:

$$b_{t+1}n_t = (1+r_t)b_t + \gamma y_t - \tau(1+r_t)\frac{s_{t-1}}{n_{t-1}}. \quad \text{(A.1)}$$

Recall that the per capita output is $y_t \equiv Y_t/N_t = Ak_t^{\alpha}(1-u_t)^{1-\alpha}$. From Equations (8) and (9), we derive:

$$(1-u_t)\overline{w} = (1-\alpha)y_t, \quad \text{(A.2)}$$

$$(1+r_t)k_t = \alpha y_t, \quad \text{(A.3)}$$

In equilibrium, $k_{t+1} + b_{t+1} = s_t/n_t$ holds. Substituting Equations (4), (A.2), (A.3), and $k_t + b_t = s_{t-1}/n_{t-1}$ into Equation (A.1), we derive:

$$b_{t+1}\frac{\varphi(1-\alpha)y_t}{\varepsilon(1+\varphi)} = \frac{\alpha y_t b_t}{k_t} + \gamma y_t - \tau\alpha y_t - \frac{\tau\alpha y_t b_t}{k_t} \quad \text{(A.4)}$$

It can be rewritten as follows:

$$b_{t+1} = \frac{\varepsilon(1+\varphi)}{\varphi(1-\alpha)}\left[\frac{(1-\tau)\alpha b_t}{k_t} + \gamma - \alpha\tau\right] \quad \text{(A.5)}$$

Adding one period to Equation (A.5), we have:

$$b_{t+2} = \frac{\varepsilon(1+\varphi)}{\varphi(1-\alpha)}\left[\frac{(1-\tau)\alpha b_{t+1}}{k_{t+1}} + \gamma - \alpha\tau\right] \quad \text{(A.6)}$$

By substituting Equation (14) into Equation (A.6) and going back by one period, we obtain Equation (15).

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Figure 1

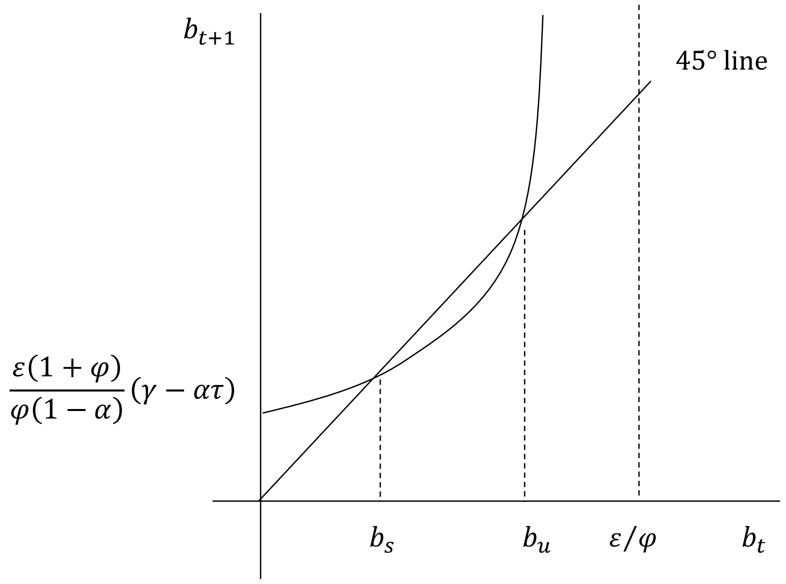

Figure 2

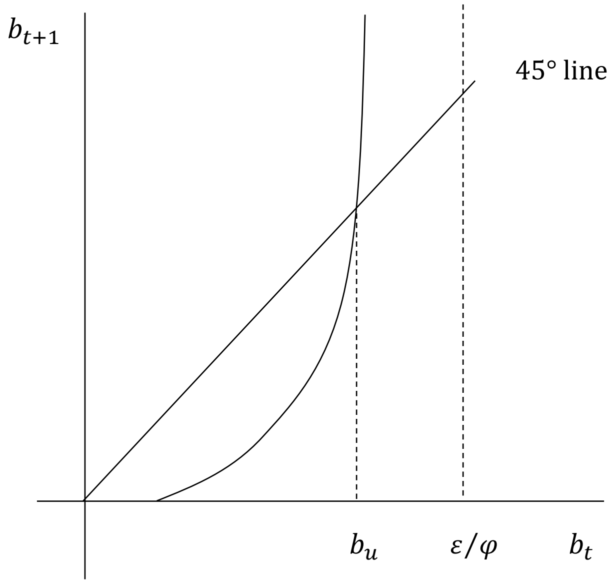

Table 1

τ	0	0.1	0.2	0.3	0.4
u_s	0.40	0.25	0.17	0.12	0.07
n_s	1.74	2.19	2.40	2.57	2.71

$\alpha = 0.25, \varphi = 0.1, \varepsilon = 0.1, \gamma = 0.105, \overline{w} = 3.2, A = 4.2$