Capacity choice with upstream investment

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February 2022

Discussion Paper No. 2202

GRADUATE SCHOOL OF ECONOMICS

KOBE UNIVERSITY

ROKKO, KOBE, JAPAN

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February 15, 2022

Abstract

We consider a vertically related market with an upstream firm engaging in cost-reducing investment and n downstream firms competing on quantity. We analyze the capacity choice by downstream firms and find that over-capacity occurs in equilibrium if the number of downstream firms is large or the upstream investment is efficient.

JEL codes: D43, L13, M21.

Keywords: capacity, investment, vertical relationship.

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1 Introduction

In a "capacity-then-quantity" game in a pure oligopoly market, it is anticipated that increasing production capacity will also increase the market share; thus, firms choose over-capacity as a strategic device (Dixit, 1980; Brander and Spencer, 1983; Horiba and Tsutsui, 2000). Additionally, in a vertical market, an upstream firm has an incentive to extract rent from a capacity activity; hence, downstream firms choose under-capacity (Choi and Lee, 2020). This study challenges these results by introducing an upstream investment and downstream competition.

Particularly, we consider a three-stage game with an upstream firm and n downstream firms. In the first stage, each downstream firm decides its production capacity. In the second stage, the upstream firm chooses the input price and investment level for marginal cost reduction. In the third stage, the downstream firms compete in terms of quantity.

We find that input price decreases with production capacity if many downstream firms exist or the investment is efficient. Meanwhile, the investment level always increases with the aggregate production capacity. Additionally, downstream firms choose over-capacity if the number of downstream firms is large or the upstream investment is efficient; otherwise, they choose under-capacity. In our analysis, capacity choice is dependent on investment efficiency, and this assumption differs from that of previous research.

The following is the explanation of the intuition for our results. On expanding its production capacity, a downstream firm obtains a larger market share. This effect becomes strong if downstream competition is tough; that is, many downstream firms exist. Hence, downstream firms choose over-capacity. Additionally, the total output increases with the aggregate production capacity. For the upstream firm, an increase of the downstream output strengthens its incentive to invest; thus, the investment level always increases with the production capacity. However, the investment level reduces the upstream firm's marginal cost. Hence, the upstream firm has an incentive to reduce the input price. The input price reduction causes the firms to choose over-capacity. However, this reduction effect plays a limited role when the investment cost is large. Fontana and Guerzoni (2008), using data for European Union countries, show that process R&D becomes less important when market size decreases. Therefore, our result can be inferred indirectly by empirical studies.

Our study is related to previous research that reexamined whether the well-known result of over-capacity always stands in various contexts (Barcena-Ruiz and Garzon, 2010; Chen et al., 2021; Fanti and Meccheri, 2017). Choi and Lee (2020) allow capacity choice to be considered in a vertical market. All these studies consider models without upstream investment.[1]

We first analyze the capacity choice with upstream cost-reducing investment. The assumption that downstream firms decide the production capacities before the upstream firm is crucial for our main result.

The rest of the paper is organized as follows. Section 2 describes the model, Section 3 presents the analysis, and finally, Section 4 concludes the paper.

2 Model

We assume a market with an upstream firm and n downstream firms. The upstream firm produces input and sells it to downstream firms at the input price w. To produce one unit of a homogenous final product, downstream firms use one unit of the input. We denote the output of the downstream firm i $(= 1, \ldots, n)$ as q_i. The inverse demand in the final good is given by $p = 1 - Q$, where p is the price and $Q = \sum_{j=1}^{n} q_j$ is the aggregate output.

We assume that all downstream firms use the same technology. Following the established literature (Nishimori and Ogawa, 2004; Lu and Poddar, 2005; Fanti and Meccheri,

[1]For cost-reducing upstream investment, Hu et al. (2020) analyze the relationship of cross-holdings and upstream R&D.

2017), the cost function of the downstream firm i is denoted by $C_i = wq_i + (x_i - q_i)^2$, where x_i is the production capacity of firm i. This cost function shows that the average cost is minimized when quantity equals production capacity. Thus, both over-capacity ($x_i - q_i > 0$) and under-capacity ($x_i - q_i < 0$) are inefficient. Then, the profit of the downstream firm i is $\pi_i = (p - w)q_i - (x_i - q_i)^2$.

The upstream firm can make investments to reduce its own marginal cost. To reduce the marginal cost by k, it incurs investment cost $\gamma k^2/2$, where γ relates to investment efficiency and c (< 1) is the marginal cost without investment. Hence, the marginal cost of the upstream firm is $c - k$. Then, the profit of the upstream firm is $\pi_U = [w - (c - k)]Q - \gamma k^2/2$. We define

$$\underline{\gamma} = \begin{cases} \dfrac{1+2n}{15} & \text{if} \quad n = 1, 2, \\ \dfrac{1}{2} & \text{otherwise.} \end{cases}$$

With small γ or small c, the marginal cost of the upstream firm may take a non-positive value. Therefore, to guarantee positive equilibrium outcomes, we assume that $\gamma > \underline{\gamma}$ and

$$c > \frac{3n^2[(2\gamma - 1)n + 6\gamma]}{2\gamma[(6\gamma - 3)n^3 + (28\gamma - 5)n^2 + (38\gamma - 4)n + 12\gamma]}.$$

The following is the timing of the game. In the first stage, each downstream firm decides its production capacity. In the second stage, the upstream firm chooses the investment level k and the input price w. In the third stage, each downstream firm competes in terms of quantity. We solve this game using backward induction.

3 Results

3.1 Calculating equilibrium

In the third stage, summing up the first-order conditions, $\partial\pi_i/\partial q_i = 1 - 3q_i - Q - w + 2x_i = 0$, and solving it for Q, we obtain an aggregate output.

$$Q\,(w, X) = \frac{n(1-w) + 2X}{3+n},$$

where $X = \sum_{j=1}^{n} x_j$the aggregate capacity. Substituting the aggregate output into the first-order condition for the downstream firm i, the output of the downstream firm i is

$$q_i\left(w, x_i, X\right) = \frac{3(1-w) + 2x_i(3+n) - 2X}{3(3+n)}.$$

In the second stage, the upstream firm chooses the input price and investment level. Substituting the aggregate output into the profit of the upstream firm and solving the first-order conditions for w and k, we obtain the outcomes in this stage.

$$w(X) = \frac{(3+n)(n+cn+2X)\gamma - n(n+2X)}{n[2(3+n)\gamma - n]}, \quad k(X) = \frac{n(1-c)+2X}{2(3+n)\gamma - n}.$$

Substituting $w(X)$ into $Q(w, X)$, the aggregate output is expressed as follows:

$$Q(X) = \frac{[n(1-c)+2X]\gamma}{2(3+n)\gamma - n}.$$

From $\gamma > \underline{\gamma}$, the denominators of $w(X)$, $k(X)$, and $Q(X)$ are positive. Hence, $k(X)$ and $Q(X)$ increase with X. Differentiating $w(X)$ with respect to X, we obtain

$$w'(X) = \frac{2[(3+n)\gamma - n]}{n[2(3+n)\gamma - n]}.$$

Hence, we obtain the following result:

Lemma 1 *The input price decreases with the aggregate capacity if* $\underline{\gamma} < \gamma < n/(3+n)$*, while the aggregate output and investment level always increase with the aggregate capacity.*

From $\gamma < n/(3+n)$, the input price decreases with the aggregate capacity if the upstream investment technology is efficient or the number of downstream firms is large. The following is an intuition behind this result. Given that, by choosing a larger capacity, downstream firms can make a commitment to expand their production, an upstream firm faces large input demand. Then, the upstream firm decides to increase its investment. Hence, large capacity by downstream firms can have different effects on the input price. Meanwhile, the demand increasing effect raises the input price, but the investment increasing effect reduces it. If the upstream investment technology is efficient, the upstream firm will increase its investment with the aggregate capacity. Additionally, if the number of downstream firms is large, the marginal profitability of investment becomes large. In these cases, the investment increasing effect dominates the demand increasing effect. Therefore, with small γ or large n, the input price decreases with the aggregate capacity.

Finally, we obtain the outcomes in the first stage. Substituting the outcomes in the second and third stages into the profit of the downstream firm i and solving the first-order condition for x_i, we obtain equilibrium capacity and output of the downstream firms as follows:

$$x_i^* = \frac{4(1-c)\gamma\,[(2\gamma-1)n^2 + 4\gamma n + n - 3\gamma]}{3(1-2\gamma)^2 n^3 + 28\gamma(2\gamma-1)n^2 + 4\gamma(19\gamma-2)n + 24\gamma^2},$$

$$q_i^* = \frac{3(1-c)n\gamma[(2\gamma-1)n + 6\gamma]}{3(1-2\gamma)^2 n^3 + 28\gamma(2\gamma-1)n^2 + 4\gamma(19\gamma-2)n + 24\gamma^2}.$$

Note that the denominators of x_i^* and q_i^* are positive because we assume that $\underline{\gamma} < \gamma$.

3.2 Condition for over-capacity investment

Comparing x_i^* with q_i^*, we obtain the main result as follows.

Proposition 1 *The condition in which downstream firms choose over-capacity investment, $x_i^* > q_i^*$, is (i) $n \geq 3$ or (ii) $n \in \{1, 2\}$ and $\underline{\gamma} < \gamma < (n-4)n/[2(n-3)(n+2)]$.*

Proof Comparing x_i^* with q_i^* yields

$$x_i^* - q_i^* = \frac{(1-c)\gamma\,[2(n-3)(n+2)\gamma - (n-4)n]}{3(1-2\gamma)^2 n^3 + 28\gamma(2\gamma-1)n^2 + 4\gamma(19\gamma-2)n + 24\gamma^2}.$$

First, we consider the case with $n \geq 3$. As the denominator is positive, the sign of $x_i^* - q_i^*$ only depends on the terms in square brackets. Rearranging the terms, we obtain

$2(n-3)(n+2)\gamma-(n-4)n=(2\gamma-1)n^2+2n(2-\gamma)-12\gamma$. Solving $(2\gamma-1)n^2+2n(2-\gamma)-12\gamma>0$, we obtain $n<n_0$ or $n>n_1$, where

$$n_0=\frac{\gamma-2-\sqrt{4-16\gamma+25\gamma^2}}{2\gamma-1},\quad n_1=\frac{\gamma-2+\sqrt{4-16\gamma+25\gamma^2}}{2\gamma-1}.$$

Given that n_1 converges to 3 as $\gamma\to\infty$, by showing that n_1 is an increasing function, we can conclude that the least upper bound of n_1 is 3. Differentiating n_1 with respect to γ leads to

$$\frac{\partial n_1}{\partial\gamma}=\frac{3\left(\sqrt{4-16\gamma+25\gamma^2}-3\gamma\right)}{(2\gamma-1)^2\sqrt{4-16\gamma+25\gamma^2}}.$$

As $4-16\gamma+25\gamma^2-(3\gamma)^2=4(2\gamma-1)^2>0$, n_1 increases with γ. Hence, for any γ, $n_1<3$, indicating that, for any $n\geq 3$, we have $x_i^*-q_i^*>0$.

The remaining cases are for $n=1,2$. Solving $x_i^*-q_i^*>0$ for γ, we obtain $\gamma<(n-4)n/[2(n-3)(n+2)]$. Additionally, the threshold value $(n-4)n/[2(n-3)(n+2)$ is larger than $\underline{\gamma}$ at $n=1,2$. Therefore, we obtain this proposition.□

From this proposition, downstream firms choose over-capacity if the upstream investment is efficient or the number of downstream firms is large. The following is an intuition behind this result. Capacity expansion is a commitment to increase production. With the presence of many downstream firms, each downstream firm's share is small. Hence, the effect of one unit of capacity expansion becomes significant. Further, over-capacity investment occurs if the number of downstream firms is large. When downstream firms are few, this effect becomes weak. However, each downstream firm has an incentive to choose over-capacity investment if the upstream investment technology is efficient. From Lemma 1, the input price decreases with the aggregate capacity if the upstream firm's investment is efficient. Given that the lower input price is profitable for the downstream firms, they have incentives to increase capacity, resulting in over-capacity investment.

3.3 Comparative statics of over-capacity investment

By performing comparative statics of over-capacity investment, we can identify a situation wherein overcapacity becomes serious. Differentiating $x_i^* - q_i^*$ with respect to γ, we obtain the following:

$$\frac{\partial(x_i^* - q_i^*)}{\partial\gamma} = \frac{3(1-c)n^2\left[-4\gamma^2(n+2)(n^2-3) + 4\gamma(n-3)(n+2)n - (n-4)n^2\right]}{\left[24\gamma^2 + 3(1-2\gamma)^2n^3 + 28\gamma(2\gamma-1)n^2 + 4\gamma(19\gamma-2)n\right]^2}.$$

The sign of $\partial(x_i^* - q_i^*)/\partial\gamma$ is the same as that of the terms in square brackets in the numerator. Then, we obtain the following result.

Proposition 2 *For $n \geq 2$, the degree of over-capacity investment, $x_i^* - q_i^*$, decreases with γ. With $n = 1$, the degree of over-capacity investment decreases with γ if $\gamma < (2+\sqrt{2})/4$ (≈ 0.854).*

Proof The sign of $\partial(x_i^* - q_i^*)/\partial\gamma$ only depends on $\Psi = -4\gamma^2(n+2)(n^2-3) + 4\gamma(n-3)(n+2)n - (n-4)n^2$.

First, we consider the case with $n \geq 4$. As the coefficient of γ^2 in Ψ is negative, solving $\partial(x_i^* - q_i^*)/\partial\gamma < 0$ for γ, we obtain $\gamma < \gamma_0$ or $\gamma > \gamma_1$, where $\gamma_0 = n(n^2 - n - 6 - \sqrt{6(2+n)})/[2(2+n)(n^2-3)]$ and $\gamma_1 = n(n^2 - n - 6 + \sqrt{6(2+n)})/[2(2+n)(n^2-3)]$. Here, we show $\gamma_1 < 1/2$. Differentiating γ_1 with respect to n, we obtain

$$\frac{\partial\gamma_1}{\partial n} = \frac{6\sqrt{n+2}n^3 + 36\sqrt{n+2} - 3\sqrt{6}n^3 - 4\sqrt{6}n^2 - 6\sqrt{n+2}n - 3\sqrt{6}n - 12\sqrt{6}}{4(n+2)^{3/2}\left(n^2-3\right)^2}.$$

As $n \geq 4$, we have $6\sqrt{n+2}n^3 > 3n^3\sqrt{n+2} + 4n^2\sqrt{n+2} + 32n\sqrt{n+2}$. Then, the numerator of $\partial\gamma_1/\partial n$ is positive and γ_1 increases with n. Additionally, γ_1 converges 1/2 as $n \to \infty$. Hence, for any $n \geq 4$, we obtain $\gamma_1 < \underline{\gamma} = 1/2$, which becomes $\partial(x_i^* - q_i^*)/\partial\gamma < 0$.

Subsequently, we consider the cases with $n = 1$, 2, and 3. Substituting $n = 1$, 2, and

3 into $\partial(x_i^* - q_i^*)/\partial\gamma$ yields

$$\left.\frac{\partial(x_i^* - q_i^*)}{\partial\gamma}\right|_{n=1} = \frac{(1-c)(8\gamma^2 - 8\gamma + 1)}{(1 - 16\gamma + 56\gamma^2)^2},$$
$$\left.\frac{\partial(x_i^* - q_i^*)}{\partial\gamma}\right|_{n=2} = -\frac{3(1-c)(2\gamma^2 + 4\gamma - 1)}{2(3 - 28\gamma + 62\gamma^2)^2} < 0,$$
$$\left.\frac{\partial(x_i^* - q_i^*)}{\partial\gamma}\right|_{n=3} = -\frac{9(1-c)(40\gamma - 3)}{(27 - 200\gamma + 360\gamma^2)^2} < 0.$$

At $n = 1$, solving $\partial(x_i^* - q_i^*)/\partial\gamma < 0$, we obtain $(2 - \sqrt{2})/4$ (≈ 0.146) $< \gamma <$ $(2 + \sqrt{2})/4$ (≈ 0.854). From the assumption $\underline{\gamma} = 1/5$ at $n = 1$, $x_i^* - q_i^*$ decreases with γ if $\gamma < (2 + \sqrt{2})/4$.□

The following is an intuition behind this result. As explained in the intuition for Proposition 1, capacity expansion is a commitment to increase production. Hence, capacity expansion promotes investment by an upstream firm and allows downstream firms to capture their rivals' market share. If the upstream investment is inefficient, the investment enhancing effect becomes weak, leading to small over-capacity investment. However, when $n = 1$, the market share capturing effect disappears. Therefore, downstream firms determine their capacity by comparing the input price reduction effect for the enhanced upstream investment and the input price increasing effect due to the increasing demand. If the upstream firm's investment is efficient, the input price reduction effect due to investment dominates. However, if it is inefficient, the input price increasing effect dominates. In our model, the input price reduction effect is the strongest when the investment efficiency is intermediate. Therefore, $x_i^* - q_i^*$ takes the minimum value.

3.4 Alternative timing

In the previous analysis, we assume that each downstream firm decides its production capacity before the upstream firm. In this subsection, we consider the reversed timing of the game, which is changed to the following. In the first stage, the upstream firm chooses its investment level and input price; in the second stage, each downstream firm decides its production capacity; and in the third stage, downstream firms choose their outputs.

The outcomes in the third stage are the same as in the previous section: $Q(w, X)$ and $q_i(w, x_i, X)$. In the second stage, a downstream firm i maximizes its profit with respect to production capacity x_i yields

$$x_i(w) = \frac{4(2+n)(1-w)}{11+10n+3n^2}.$$

Substituting $x_i = x_i(w)$ and $X = nx_i(w)$ into $q_i(w, x_i, X)$, we obtain the downstream output as

$$q_i(w) = \frac{3(3+n)(1-w)}{11+10n+3n^2}.$$

Comparing $x_i(w)$ with $q_i(w)$ yields

$$x_i(w) - q_i(w) = \frac{(n-1)(1-w)}{11+10n+3n^2} > 0.$$

Hence, we find that, in the reversed timing, a downstream firm always chooses over-capacity. The following is a simple intuition behind this result. When downstream firms choose their capacity, the input price and upstream investment level are fixed. Hence, input price reduction and investment promotion effects disappear. Given that the capacity expansion of each downstream firm will only have the effect of capturing a market share, all downstream firms will choose over-capacity investments.

4 Conclusions

We considered a vertically related market with an upstream firm engaging in cost-reducing investment and n downstream firms. We found that the input price decreases with the production capacity if the number of downstream firm is large or the investment is efficient, while the investment level always increases with the aggregate capacity. Additionally, downstream firms choose over-capacity if the number of downstream firm is large or the upstream investment is efficient; otherwise, they choose under-capacity.

We also analyzed the model wherein the upstream firm makes the input price contract with the downstream firms. It is worthwhile to consider the case where the upstream firm proposes a two-part tariff contract to the downstream firms. Moreover, an extension is possible by analyzing product differentiation. We leave these for future research.

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