Consumer-benefiting transport cost: The role of product innovation in a vertical structure

Kazuhiro Takauchi
Tomomichi Mizuno

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KOBE UNIVERSITY

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Consumer-benefiting transport cost: The role of product innovation in a vertical structure

Kazuhiro Takauchi*† and Tomomichi Mizuno‡

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Abstract

We study the effects of a reduction of transport cost on the firm’s activity of product innovation and on consumer welfare. Firms engage in product R&D that increases the degree of product differentiation, purchase intermediate inputs from an exclusive supplier, and export their products to the foreign market paying a per-unit transport cost. Trade theory commonly asserts that zero transport cost maximizes consumer surplus. Contrary to this standard belief, we show that a positive transport cost can maximize consumer surplus. We also consider more general effects of R&D and show that our main results hold even in such case.

Key words: Transport cost; Consumer surplus; Product R&D

JEL classification: L13; F12; O31

*Corresponding author: Kazuhiro Takauchi, Faculty of Business and Commerce, Kansai University, 3-3-35 Yamate-cho, Suita, Osaka 564-8680, Japan. E-mail: kazh.takauch@gmail.com; Tel.: +81-6-6368-1817.
†Research Fellow, Graduate School of Economics, Kobe University, 2-1 Rokkodai-cho, Nada-ku, Kobe-City, Hyogo 657-8501, Japan.
‡Graduate School of Economics, Kobe University, 2-1 Rokkodai-cho, Nada-ku, Kobe-City, Hyogo 657-8501, Japan. E-mail: mizuno@econ.kobe-u.ac.jp.
1 Introduction

Trade barriers such as transportation cost and tariffs affect consumer welfare and the incentive of firms to innovate. When market access to foreign firms is facilitated, inexpensive imported goods become available to domestic consumers, which benefits them.\(^1\) Furthermore, if a reduction in shipping costs facilitates the access to the foreign market, then exporting firms can enjoy a larger market size. The growth in the market size implies that a firm’s sales opportunities increase. Then, the possibility of increasing production capability can increase the incentive to invest in research and development (R&D).\(^2\)

Trade theory commonly argues that reducing transport cost, for example, through trade liberalization, makes consumers better off. In the case of intra-industry trade, Helpman and Krugman (1985, p. 108) state: “Trade has a procompetitive effect: each firm’s exports do not displace an equal volume of shipments from the other firm to its home market, so total output and consumption rise and the price falls.” According to this argument, a decline in transport cost facilitates exports and, thereby, increases total output while reducing prices.\(^3\) Hence, the rationale in standard theory would lead one to expect that zero transport cost, that is, free trade, maximizes consumer surplus.

This study examines the effects of transport cost on a firm’s product innovation\(^4\) and on consumer welfare in a two-country, two-way trade model. In each country, the firm exporting the final good engages in product R&D (which increases the degree of differentiation of the product), and purchases input from an exclusive supplier. We show that a lower transport

\(^1\) For example, Nicita (2009) empirically finds that Mexican tariff liberalization during the 1990s reduced consumer prices for agricultural and manufactured goods and raised consumers’ purchasing power in about 1.8%.

\(^2\) Some recent empirical works strongly support this view. For example, Lileeva and Treffer (2010) find that trade liberalization leads firms to engage in more product innovation. Bustos (2011) also finds that regional trade liberalization (Brazil’s tariff cut) fosters firms’ innovation (technology upgrading by Argentinian firms).

\(^3\) This result holds in other extended cases. For example, Naylor (1998) shows that transport cost reduction always increases consumer surplus, even if the reciprocal market model incorporates unionized labor markets. Maiti and Mukherjee (2013) also find this result in a unionized oligopoly model.

\(^4\) This setting is partially consistent with the empirical evidence. According to a survey by Fontana and Gueorguieva (2008) in major manufacturing industries, most firms consider product innovation as their most important form of innovation.
cost leads the final-good exporting firms to invest in product R&D. However, contrary to the standard belief, we show that a positive transport cost can maximize consumer surplus.

The key assumptions to our results are “strategic complements in R&D” and “input-price jumps due to investment.” When the R&D cost is too high, no firm will invest in R&D. However, when the R&D cost is small, the incentive to invest in R&D depends on the transport cost. When the home and foreign exporting firms invest in R&D, they benefit from the product differentiation, which moderates competition. However, if the transport cost is high, the market size in the other country is small, and the benefits of product differentiation are also small. Hence, if the transport cost is high, non-investment is the dominant strategy. Conversely, if the transport cost is low, then the benefit of product differentiation in the other market is large, and investment is the dominant strategy. When the transport cost is at an intermediate level, there is no dominant strategy, and multiple equilibria, the “everyone invests” and “no one invests” regimes, appear.

In our model, each exporting firm uses its domestic supplier’s input. R&D activities take a much longer time, so exporting firms first invest in R&D, then input suppliers, subsequently, charge their prices. In this case, by charging a higher price after observing the exporting firm’s investment behavior, the input suppliers can extract R&D benefits. Hence, the investment by the exporting firm not only increases the input price, but also the production cost. Due to these higher costs, each firm’s production falls and total output decreases, so the final-good price rises. Therefore, R&D investment reduces consumer surplus. Furthermore, since the strategic complements effect influences input prices, the home input supplier raises its price when the other country’s input supplier raises its price. As the number of investing firms increases, input prices rise and the total output decreases, so consumer surplus also drops as the number of investing firms increases. The “everyone invests” regime has the lowest consumer surplus level of all the regimes. If the transport cost decreases below a certain level, then consumer surplus drops
because the equilibrium switches from the “no one invests” regime to the “everyone invests” regime.

We believe that our analysis can contribute to the field of trade and competition. In competition policy, the consumer welfare standard is often adopted (Viscusi et al. 2018), and its purpose is to protect consumers’ wellbeing and enhance consumer surplus. Hence, if a reduction in transport cost increases consumer surplus, the authority in charge of competition policy has an incentive to decrease transport cost. Because the reduction in transport cost tends to be seen as beneficial to consumers, it might be actually implemented. However, our result shows that a reduction in transport cost does not always bring about the desirable result of increased consumer welfare, and points out to a problematic issue in competition policy.

Whilst our result opposes the conventional wisdom in trade theory, it is not necessarily inconsistent with empirical findings. Examples of price rises due to the promotion of competition have been reported by several empirical studies (Caves et al. 1991, Grabowski and Vernon 1992, and Thomadsen 2007). Transport cost reduction lowers the export barrier to foreign firms and enhances competition in the domestic market, so it undoubtedly has a competition-enhancing effect. The result that the promotion of competition raises the final-good price, and hence, harms consumers, is broadly consistent with empirical evidence.

This paper is closely related to Kabiraj and Marjit’s (2003) study. By considering technology transfer through licensing, they show the existence of a tariff rate that maximizes consumer surplus in an importing country. In their model, a high tariff rate hinders the exports of a foreign firm, so the foreign firm has an incentive to license its low-cost production technology to the high-cost local firm and earn license royalties. As a result, a high tariff of the importing country increases total output, thereby increasing the consumer surplus above that of the zero tariff case. Although Kabiraj and Marjit (2003) find the existence of a consumer-benefiting tariff policy, their analysis is limited to the case of one-way trade. By contrast, here we examine the
two-way trade case.\(^5\)

This paper is also related to some works on the nexus between trade liberalization and product innovation (Bastos and Straume, 2012; Braun, 2008; Hwang et al., 2018).\(^6\) In the context of oligopolistic intra-industry trade, Bastos and Straume (2012) and Braun (2008) use a product R&D model based on Lin and Saggi (2002). However, they focus on the role of skilled and unskilled workers and consider the effects of trade liberalization on wage inequalities and labour demands through the impact on firm’s innovation. Hwang et al. (2018) employ a type of product R&D in which investment affects consumers’ willingness to pay, and show that trade liberalization may reduce R&D investments.

The rest of the paper proceeds as follows. Section 2 describes the model. In Section 3, we derive the equilibrium outcomes and the main results. In Section 4, we consider a general case with a smaller degree of product differentiation due to R&D compared to the case in Section 3. Section 5 concludes the paper.

2 Model

We consider a reciprocal market as in Brander and Krugman (1983). There are two symmetric countries, \(H\) and \(F\), with final-good markets. Each country \(i\) \((i = H, F)\) has an input supplier (called supplier \(i\)) and an innovative final-good exporting firm (called firm \(i\)). To produce one unit of the final good, firms employ one unit of input. We assume that firm \(i\) buys its input from an exclusive supplier \(i\). That is, firm \(i\) is technologically unable to use the input produced by (foreign) supplier \(j\) \((j \neq i)\). This is assumed because the intermediate input employed by

\(^5\)Furthermore, in contexts different from our analysis, some studies examine the effects on consumer welfare arising from different organizational forms among firms and market structure. In a free-entry situation, Marjit and Mukherjee (2015) indicate that a transport cost reduction may decrease consumer surplus. Cao and Mukherjee (2017) find that, if a labour union exists and technology transfer is implemented among firms, then consumer surplus may increase. Using a third-market model with export policy, Mukherjee and Sinha (2019) show that cooperation between exporting firms harms consumers in the third market.

\(^6\)In addition, several studies focus on the nexus between process innovation (i.e., cost-reducing R&D) and trade liberalization. See, for example, Haaland and Kind (2008) and Takauchi (2015).
each firm can differ in several ways (e.g., in quality) since final goods are differentiated.\textsuperscript{7} While the firms freely supply their products domestically, they incur a per unit transport cost of $\tau \geq 0$ to export.

The representative consumer’s utility function in country $i$ is

$$U_i = y_i + q_{ii} + q_{ji} - \frac{1}{2}(q_{ii}^2 + q_{jj}^2 + 2bq_{ii}q_{ji}), \quad i \neq j; \quad i, j = H, F,$$

where $y_i$ is the numeraire good. This utility function yields the following inverse demands:

$$p_{ii} = 1 - q_{ii} - bq_{ji} \quad (1)$$

$$p_{ij} = 1 - q_{ij} - bq_{jj}.$$  

In equation (1), $p_{ii}$ ($p_{ij}$) denotes firm $i$’s product price in country $i$ ($j$), $q_{ii}$ ($q_{jj}$) denotes firm $i$’s ($j$’s) domestic supply, and $q_{ij}$ ($q_{ji}$) denotes firm $i$’s ($j$’s) exports to country $j$ ($i$). The parameter $b \in [0, 1]$ measures the degree of product substitutability between the final goods of firms $H$ and $F$. If $b$ is zero, then the firms become monopolists; whereas if $b = 1$, firm $i$’s product becomes perfectly substitutable by firm $j$’s product. We assume that $b$ is determined by firm $i$’s investment in product R&D, $d_i \in \{0, 1/2\}$, as follows: $b = 1 - (d_H + d_F)$\textsuperscript{8}.

We consider a binary-choice case in which firms choose whether to invest in R&D by expending a fixed cost or not. It is well-known that fixed costs are needed in order to conduct R&D (Desmet and Parente, 2010; Dubois et al., 2015; Tang, 2006; Aboal and Garda, 2016). Hence, firms often opt for “do not invest in R&D” due to such fixed costs. For example, in the manufacturing and service industries in Germany, there is a number of firms which R&D expenditures are close to zero (Czarnitzki and Hottenrott, 2011; Peters, 2009).\textsuperscript{9} Despite these empirical facts, the framework in which firms decide their investment level of R&D is prevalent,

\textsuperscript{7}Even if these two inputs were perfectly homogeneous, firms use only their domestic input when there is a sufficiently high tariff on imported inputs.

\textsuperscript{8}This formulation of $b$ is the same type employed by Lin and Saggi (2002). However, they assume that firm’s investment in product R&D $d_i$ is a continuous variable, and the cost function of product R&D is convex with respect to $d_i$. Moreover, although Lin and Saggi (2002) further add to their model a stage-game in which firms conduct cost-reducing R&D investment as in d’Aspremont and Jacquemin (1988), from the purpose of our analysis, we do not consider cost-reducing R&D investment.

\textsuperscript{9}In the United Kingdom and Germany, Bond et al. (2005) similarly found that there are some firms that do not conduct R&D in the high tech sector.
leading existing studies to exclusively focus on positive investment levels (i.e., an interior equilibrium), and the option “do not invest in R&D” is virtually ignored (d’Aspremont and Jacquemin, 1988; Lin and Saggi, 2007; Rosenkranz, 2003; Takauchi, 2015). To explicitly consider the option “do not invest in R&D”, Lambertini and Rossini (1998) build a binary-choice model of product R&D and examine on the firm’s strategic decision whether to invest in R&D or not. Similar to their analysis, we focus on the actual options of firms, and consider the situation in which firms strategically choose whether to invest in R&D or not.\(^{10}\)

Let \(I\) be the strategy “invest in R&D” and \(N\) the strategy “do not invest in R&D”. If firm \(i\) chooses \(I\), then \(d_i = 1/2\) and it pays the fixed investment cost \(k > 0\). If it chooses \(N\), then \(d_i = 0\) and it has no investment cost. Hence, when all firms invest, \(b = 0\); while \(b = 1\) when no firm invests. If only one firm invests, then \(b = 1/2\). In Section 4, we relax the assumption on \(d_i\) and consider a general case where \(d_i = d \in (0, 1/2]\) if firm \(i\) invests. Under this extended setting, our main results basically hold.

Firm \(i\)’s gross profit (excluding \(k\)) is \(\Pi_i \equiv (p_{ii} - w_i)q_{ii} + (p_{ij} - w_i - \tau)q_{ij}\), where \(w_i\) is the price of the input produced by supplier \(i\). Supplier \(i\) makes a take-it-or-leave-it offer, and its profit is \(\pi_i \equiv (w_i - \overline{w})(q_{ii} + q_{ij})\), where \(\overline{w}\) is the unit cost. For simplicity, we set \(\overline{w}\) equal to zero.\(^{11}\)

We consider the following three-stage game: In the first stage, each firm independently and simultaneously chooses whether to invest in product R&D \((I)\) or not \((N)\). In the second stage, each supplier decides its input price. In the third stage, firms compete \(à la\) Cournot in the \(H\) and \(F\) markets. R&D activity takes a significant amount of time; firms generally implement R&D activity as a long-term project and it is impossible to frequently change decisions within a short period of time. Thus, firms make decisions on R&D investment in the first stage of the game. In contrast, the production quantity decision is easier to change, and firms, therefore,

\(^{10}\)Matsushima and Mizuno (2009), Mukherjee and Mukherjee (2013), Poddar and Bibhas (2010), and Zanchettin and Mukherjee (2017) also adopt a binary-choice of product R&D.

\(^{11}\)This setting does not alter our results.
determine their outputs in the last stage.

Since firms have two options, four cases can arise: \( II \), \( IN \), \( NI \) and \( NN \). All firms invest in \( II \). In \( IN \) (\( NI \)), firm \( H \) chooses \( I \) (\( N \)) and firm \( F \) chooses \( N \) (\( I \)). No firm invests in \( NN \). The solution concept is the subgame perfect Nash equilibrium (SPNE).

3 Results

We solve the game by backward induction, so we first consider the third stage of the game.

3rd stage: The first-order conditions to maximize firm \( i \)'s profit are

\[
\frac{\partial \Pi_i}{\partial q_{ii}} = 0 \iff 1 - bq_{ii} - 2q_{ii} - w = 0, \\
\frac{\partial \Pi_i}{\partial q_{ij}} = 0 \iff 1 - 2q_{ij} - bq_{jj} - w_i - \tau = 0 \text{ for } i \neq j.
\]

These yield the third-stage outputs:

\[
q_{ii}(w_i, w_j) = \frac{2 - b + b(\tau + w_j) - 2w_i}{4 - b^2}; \quad q_{ij}(w_i, w_j) = \frac{2 - b + bw_j - 2(\tau + w_i)}{4 - b^2}.
\]

2nd stage: Using the third-stage outputs, the profit maximization problem \( \max_w \pi(w_i, w_j) \), yields the following best response function of supplier \( i \):

\[
w_i = BR_i(w_j, b) \equiv \frac{1}{8}((2 - b)(2 - \tau) + bw_j) \text{ for } i \neq j.
\]

Hence, the input price shows strategic complementarity. We can explain this result as follows. Suppose that supplier \( j \) raises its price; then, a rise in \( w_j \) increases firm \( j \)'s production cost and reduces outputs. A reduction in the rival’s outputs tends to increase firm \( i \)'s outputs, which then increases the input demand for firm \( i \), influencing supplier \( i \) to raise its price. Therefore, there is a strategic complementarity between input prices.

From \( BR_i(w_j, b) \), the second-stage input price becomes

\[
w_i(b) = \frac{(2 - b)(2 - \tau)}{2(4 - b)}.
\]
1st stage: Plugging the second-stage outcomes into the profit of firm \( i \), we have

\[
\Pi_i(b) = \frac{16(2 - b)^2(1 - \tau) + b^4 - 4b^3 - 8b^2 + 16b + 80\tau^2}{2(4 - b)^2(4 - b^2)^2}.
\]

Substituting \( b = 1 - (d_F + d_H) \) in (2) and using the four possible combinations of \( d_H \) and \( d_F \),

\[
(d_H, d_F) = \{(0, 0), (0, 1/2), (1/2, 0), (1/2, 1/2)\},
\]

we obtain firm \( i \)'s equilibrium profit:

\[
\begin{align*}
\Pi_{iN} &= \frac{85\tau^2 - 16\tau + 16}{162}; \\
\Pi_{iI} &= \frac{5\tau^2 - 4\tau + 4}{32}; \\
\Pi_{iNI} &= \frac{2(1369\tau^2 - 576\tau + 576)}{11025},
\end{align*}
\]

where the superscripts on the variables denote the equilibrium regimes.

The equilibrium input prices are

\[
\begin{align*}
w_{iNN}^I &= \frac{2 - \tau}{6}; & w_{iN}^I &= \frac{2 - \tau}{4}; & w_{iNI}^N &= w_{iI}^N &= \frac{3(2 - \tau)}{14}.
\end{align*}
\]

Firm \( i \)'s equilibrium outputs are

\[
\begin{align*}
q_{iNN}^I &= \frac{4 + 7\tau}{18}; & q_{iN}^I &= \frac{4 - 11\tau}{18}; & q_{iN}^I &= \frac{2 + \tau}{8}; & q_{iI}^I &= \frac{2 - 3\tau}{8}, \\
q_{iNI}^I &= q_{iNN}^N = \frac{24 + 23\tau}{105}; & q_{iN}^N &= q_{iI}^N &= \frac{24 - 47\tau}{105}.
\end{align*}
\]

To ensure a positive quantity in all regimes, we assume the following.

**Assumption 1.** \( \tau < \frac{4}{11} \).

Equations (3) and (4) yield the following lemmas.

**Lemma 1.** (i) Suppose that firm \( j \) chooses \( N \). Then, if \( \phi_l(\tau) > k \), firm \( i \) (\( i \neq j \)) chooses \( I \); otherwise, it chooses \( N \). (ii) Suppose that firm \( j \) chooses \( I \). Then, if \( \phi_u(\tau) \leq k \), firm \( i \) (\( i \neq j \)) chooses \( N \); otherwise, it chooses \( I \). Here,

\[
\begin{align*}
\phi_l(\tau) &= \Pi_{iN}^I - \Pi_{iN}^N = \frac{1136 - 1136\tau - 54841\tau^2}{198450}; & \phi_u(\tau) &= \Pi_{iI}^N - \Pi_{iN}^N = \frac{7236 - 7236\tau - 32491\tau^2}{352800}.
\end{align*}
\]

**Proof.** From (3), \( \Pi_{iN}^I - k - \Pi_{iN}^N > (\leq) 0 \Leftrightarrow \Pi_{iI}^N - k - \Pi_{iN}^N > (\leq) 0 \Leftrightarrow \phi_l(\tau) > (\leq) k \) and \( \Pi_{iI}^I - k - \Pi_{iN}^I \leq (>) 0 \Leftrightarrow \Pi_{iI}^I - k - \Pi_{iN}^I \leq (>) 0 \Leftrightarrow \phi_u(\tau) \leq (>) k. \)

**Lemma 2.** (i) \( w_{iI}^I > w_{iN}^I = w_{iI}^N > w_{iNN}^N \). (ii) \( \frac{\partial w_{iI}^I}{\partial \tau} < 0 \), where \( r = II, NI, IN, NN \).
Proof. (i) From (4), \( w_i^{II} - w_i^{NI} = (2 - \tau)/28 > 0 \) and \( w_i^{NI} - w_i^{NN} = (2 - \tau)/21 > 0 \). (ii) \( \partial w_i^{NN}/\partial \tau = -1/6, \partial w_i^{II}/\partial \tau = -1/4 \) and \( \partial w_i^{NI}/\partial \tau = -3/14 \). □

Lemma 1 yields Proposition 1.

**Proposition 1.** 1. Suppose that \( \tau < \bar{\tau} \equiv \frac{4(105\sqrt{355} - 142)}{54841} \). (i) If \( k < \phi_l(\tau) \), then II occurs. (ii) If \( \phi_l(\tau) \leq k \leq \phi_u(\tau) \), then NN and II can occur. (iii) If \( k > \phi_u(\tau) \), then NN occurs. 2. Suppose that \( \tau \geq \bar{\tau} \). (i) If \( k < \phi_u(\tau) \), then NN and II can occur. (ii) If \( k \geq \phi_u(\tau) \), then NN occurs.

Proof. From Lemma 1, \( \phi_u(\tau) > 0 \ \forall \tau \in [0, 4/11], \phi_u(\tau) - \phi_l(\tau) = \frac{46948(1-\tau)+585037\tau^2}{3175200} > 0 \), \( \partial \phi_l(\tau)/\partial \tau = -\frac{568+54851\tau}{99225} < 0 \) and \( \phi_l(0) = \frac{568}{99225} \equiv k \). Solving \( \phi_l(\tau) = 0 \) with respect to \( \tau \), we have \( \bar{\tau} = \frac{4(105\sqrt{355} - 142)}{54841} \approx 0.13394 \). □

![Figure 1: SPNE of the game in (\( \tau, k \))-space](image)

We first consider Lemma 2. Part (i) states that input prices increase with the number of investing firms. This result depends on the opportunistic behavior by the input suppliers. Each firm invests in R&D in the first stage of the game, and each input supplier charges its price in the second stage. Then, if supplier \( i \) charges a higher input price after observing firm \( i \)'s investment, then the supplier can extract the R&D benefit of firm \( i \).\(^{12}\) Therefore, when firm \( i \) invests in

\(^{12}\)Prior studies show that opportunistic behavior by upstream agents may cause a hold-up problem in down-
R&D, the input price \( w_i \) becomes higher. In addition, as we find in the best response function \( BR_i(w_j, b) \), there is a strategic complementarity between home and foreign inputs. Hence, if firm \( F \) invests in the case in which firm \( H \) invests, the input prices in both \( H \) and \( F \) are higher than those when only one of the firms invests.

Part (ii) is intuitive. A rise in the transport cost \( \tau \) continuously reduces input prices because a rise in \( \tau \) hinders exports and discourages production, thereby reducing the input demand. Therefore, the supplier tries to restore input demand by lowering its price.

As we show in Proposition 1, only “NN”, “II” or “NN and II” can become equilibrium outcomes of the game. Hereafter, we call \( NN \) the “\textit{no one invests}” regime and \( II \) the “\textit{everyone invests}” regime.

The intuition behind Proposition 1 is as follows. The R&D motive depends on the transport and investment costs. When investment cost \( k \) decreases, the R&D motive intensifies because the net benefit of product differentiation increases. Furthermore, firms gain the differentiation benefit from both domestic and foreign markets. When \( \tau \) falls, the R&D motive intensifies because a lower \( \tau \) raises exports and increases the differentiation benefit gained from the foreign market. Hence, if \( \tau \) and \( k \) are small enough, \( I \) becomes the dominant strategy and the “everyone invests” regime appears. If both costs are large enough, \( N \) becomes the dominant strategy and the “no one invests” regime appears (see Fig. 1).

When \( \tau \) is high and \( k \) is at an intermediate level, both “no one invests” and “everyone invests” regimes can appear. If firm \( i \) deviates from the “everyone invests” regime, \( w_i \) falls. However, \( w_j \) also falls due to strategic complementarity. The deviation makes competition tougher and the rival’s cost does not increase; hence, firm \( i \) does not deviate from the “everyone invests” regime. Additionally, the deviation from the “no one invests” regime raises the input price, but it brings a differentiation benefit to the deviator. On the other hand, in this case, \( \tau \) is high and the size

\footnote{stream investment. Banerjee and Lin (2003) present a long-term input price agreement (i.e., a fixed-price contract) to resolve this hold-up problem. In contrast, Takauchi and Mizuno (2019) show that solving the hold-up problem with a long-term input price agreement can harm all firms.}
of the foreign market is small, and hence the differentiation benefit from the foreign market is small. Since the R&D benefit is relatively small, there is no incentive to deviate from the “no one invests” regime.

Next, we examine the effects of the transportation cost on consumers. From the utility function, the formula for consumer surplus is

\[ CS_i = U_i - (y_i + p_i q_{ii} + p_j q_{ji}) \]

\[ = \frac{1}{2} (q_{ii}^2 + q_{ji}^2 + 2b_{ij} q_{ji}). \]

The above formula, (1) and (5) yield

\[ CS_{iN}^N = \frac{2(2 - \tau)^2}{81}; \quad CS_{iI}^I = \frac{4 - 4\tau + 5\tau^2}{64}. \] (6)

From (6), we establish the following.

**Proposition 2.** (Consumer-benefiting transport cost)

1. Suppose \( k \in \left[0, \frac{9329}{10672200}\right] \). If NN appears within the area in which the multiple equilibria of “NN and II” arise in Figure 1, then \( \tau = \phi_i^{-1}(k) \) maximizes the consumer surplus.

2. Suppose \( k \in \left(\frac{9329}{10672200}, \bar{k}\right) \). (i) If NN appears within the area in which the multiple equilibria of “NN and II” arise in Figure 1, then \( \tau = \phi_i^{-1}(k) \) maximizes the consumer surplus; (ii) if II appears in that area, then \( \tau = \phi_u^{-1}(k) \) maximizes the consumer surplus.

3. Suppose \( k \in \left(\frac{9329}{10672200}, \bar{k}\right) \). If II appears within the area in which the multiple equilibria of “NN and II” arise in Figure 1, then \( \tau = \phi_u^{-1}(k) \) maximizes the consumer surplus.

Here, \( k \equiv \frac{568}{99225} \approx 0.00572436 \) and \( \bar{k} \equiv \frac{201}{9800} \approx 0.0205102 \).

**Proof.** First, from Proposition 1 and its proof, NN&II appear if \( k < \bar{k} \equiv \frac{201}{9800} = \phi_u(0) \), whereas only NN appears if \( k \geq \bar{k} \). In the interval \( (0, \bar{k}) \), the equilibrium transition is divided into the following three types: for \( k \in \left(0, \frac{9329}{10672200}\right) \), NN&II \( \rightarrow \) II as \( \tau \) decreases; for \( k \in \left(\frac{9329}{10672200}, \bar{k}\right) \), NN \( \rightarrow \) NN&II \( \rightarrow \) II as \( \tau \) decreases; and for \( k \in \left(\frac{9329}{10672200}, \bar{k}\right) \), NN \( \rightarrow \) NN&II \( \rightarrow \) II as \( \tau \) decreases. Here,
\[ \phi_u(\frac{4}{11}) = \frac{9329}{10672200} \approx 0.00087414 \text{ and } \bar{k} \equiv \phi_u(0) = \frac{201}{9800} \approx 0.020510. \]

Second, \(\partial CS_i^{NN}/\partial \tau = -\frac{4(2-\tau)}{81} < 0\) and \(\partial CS_i^{II}/\partial \tau = -\frac{2-5\tau}{3\tau} < 0\), so the consumer surplus in both aforementioned regimes is monotonically decreasing for \(\tau\). Simple algebra yields
\[ \max CS_i^{II} = CS_i^{II}\big|_{\tau=0} = \frac{1}{17}, \min CS_i^{NN} = CS_i^{NN}\big|_{\tau=4/11} = \frac{8}{121} \text{ and } \min CS_i^{NN} - \max CS_i^{II} = \frac{7}{1996} > 0. \]
These imply Proposition 2. □

![Figure 2: The Relationship between CS and k: k = 0.0045.](image)

The input price discontinuously rises as the number of investing firms increases (part (i) of Lemma 2). Hence, the input price in the “everyone invests” regime is highest among all regimes. The firms’ production cost also sharply rises due to the sharp increase in input prices. Because this rapid increase in the cost suppresses the production of final good, the total output plummets. Consequently, for consumer surplus, \(\min CS_i^{NN} > \max CS_i^{II}\) holds. (See proof of Proposition 2.) For example, when \(k < \frac{k}{\bar{k}} = 0.0045\), the equilibrium transition \(NN \& II \rightarrow II\) arises as \(\tau\) decreases (see Fig. 1). In any regime, a decrease in \(\tau\) raises consumer surplus; hence, the threshold at which the equilibrium switches from \(NN \& II\) to \(II\) maximizes consumer surplus if \(NN\) appears in \(NN \& II\) (see Fig. 2).

Proposition 2 has a significant policy implication associated with competition (antitrust) policy. In the United States, for example, the consumer welfare standard is currently used, and this position can be summarized by the statement: “If consumers are made worse off then the
practice is to be prohibited. Otherwise, it is to be allowed.” (Viscusi et al., 2018, p. 97.) A reduction in the transport cost usually lowers the entry barrier for foreign competitors, facilitates imports, promotes competition in the domestic market, and decreases the price, so one could think that transport cost reduction makes consumers better off. Therefore, when the consumer welfare standard is employed, it is naturally assumed that the authority in charge of competition policy has an incentive to support a transport cost reduction and promote competition in order to protect consumers. However, Proposition 2 indicates that there is a case in which it is not desirable to use the consumer welfare standard, and that such case is not necessarily rare. In this regard, our result provides a new insight into the context of competition (antitrust) policy and international trade.

While Proposition 2 shows a result that contradicts with the common wisdom in trade theory, it is broadly consistent with empirical findings. Some empirical studies (Caves et al. 1991, Grabowski and Vernon 1992, and Thomadsen 2007) have already found several examples in which the promotion of competition leading to new entries of firms into the market, and the resulting increase in the number of operating firms, ends up raising prices, potentially harming consumers. Generally, the transport cost is equivalent to an entry barrier that shuts foreign exporting firms out if the level is high enough. Hence, a transport cost reduction lowers the barrier hindering supplying activities by foreign firms and promotes trade, so a transport cost reduction intensifies the domestic market competition. Proposition 2 states that the promotion of competition through a transport cost reduction can raise prices and harm consumers. Therefore, it constitutes a theoretical evidence that supports empirical findings showing that the enhancement of competition can raise prices.

13 Caves et al. (1991) empirically show that market entry can lead to an increase in drug prices. Grabowski and Vernon (1992) also shows that an entry causes a rise in the price of pharmaceuticals. Using a simulation, Thomadsen (2007) demonstrates that duopoly price can be higher than monopoly prices in a first-food market.
Social welfare

Unlike consumer surplus, social surplus tends to increase with transport cost reduction. We verify this fact here.

Excluding the investment cost $k$, the social surplus in country $i$, $SW_i$, is the sum of the consumer surplus, $CS_i$, the profit of firm $i$, $\Pi_i$, and the profit of the input supplier $i$, $\pi_i$. The equilibrium outcome becomes

$$SW_i^{NN} = \frac{56 - 56\tau + 95\tau^2}{162}; \quad SW_i^{II} = \frac{28 - 28\tau + 19\tau^2}{64}.$$ 

As we show in Proposition 2, the consumer surplus is largest in the “no one invests” regime. However, the profit ranking of firm $i$ is ambiguous because in the “everyone invests” regime, the firm’s profit rises as the transport cost falls, while in the “no one invests” regime, its profit can decrease as the transport cost falls. In the “everyone invests” regime, each firm becomes a monopolist in both the home and foreign markets, so the firm’s profit increases as the transport cost falls. In contrast, in the “no one invests” regime, the firms’ products are homogenous, and each firm faces tough competition. Because high transport cost shuts out the rival’s exports, each firm can enjoy a domestic monopoly. A prohibitive transport cost maximizes each firm’s profit.\textsuperscript{14}

Since the input price jumps with the number of investing firms (Lemma 2), the profit of supplier $i$, $\pi_i$, is the largest in the “everyone invests” regime. In our model, the input supplier’s profit strongly affects the welfare ranking. Comparing the welfare levels between $II$ and $NN$, we have

$$(SW_i^{II} - k) - SW_i^{NN} = \left(\frac{476 - 476\tau - 1501\tau^2}{5184}\right) - k.$$ 

As long as $k$ is small enough, $SW_i^{II} > SW_i^{NN}$ holds.

\textsuperscript{14}This property of a firm’s profit is the same as in Brander and Krugman (1983). For a more detailed argument, see Takauchi and Mizuno (2019).
In the “everyone invests” regime, consumer surplus, firm’s profit and the input supplier’s profit are monotonically decreasing with respect to the transport cost, while in the “no one invests” regime, a change in the firm’s profit can be the opposite of the other items.\textsuperscript{15} For this reason, in the “no one invests” regime, the social surplus is U-shaped for the transport cost.\textsuperscript{16}

Figure 3 illustrates the relationship between social surplus and transport cost. If the investment cost is small enough, that is, $k < k_\ast$, and the “everyone invests” regime appears, then the welfare level may jump upwards due to the reduction in transport cost.\textsuperscript{17}

As we show in our arguments so far, the government has an incentive to activate firms’ innovation by reducing trade barriers, while consumers do not necessarily have such an incentive. There is a market structure in which consumers prefer a certain level of trade barriers, so our result points out that the government needs to be careful when promoting trade liberalization.

\textsuperscript{15}In fact, $\partial SW_{i}^{II}/\partial \tau = (19\tau - 14)/32 < 0$. On the other hand, $\partial SW_{i}^{NN}/\partial \tau = (95\tau - 28)/81; \partial SW_{i}^{NN}/\partial \tau > 0$ for $\tau > 28/95 (< 4/11)$.

\textsuperscript{16}$SW_{i}^{NN}$ is U-shaped with respect to the transport cost, and it has the same property as in Brander and Krugman (1983). Helpman and Krugman (1985) also illustrate a U-shaped welfare curve in Figure 5.11, page 110 of their work.

\textsuperscript{17}Even if we consider $k \in (k_\ast, \bar{k})$, a similar result holds.
4 Extension

The degree of product differentiation

In the previous section, we assumed that $d_i = 1$ if firm $i$ invests in product R&D. Here, we relax this assumption and consider a more general case in which $d_i = d \in (0, 1/2]$ if firm $i$ invests, and $d_i = 0$ otherwise. The other settings are the same as in the previous section. We will see that even if we consider that $d_i$ is within the range $(0, 1/2]$, our main previous results basically hold.

We first examine the equilibrium of the game. In the first stage of the game, each firm decides whether to invest ($I$) or not ($N$). Substituting $d_i = d$ into (2), we obtain the gross profit of firm $i$ in the “everyone invests” regime:

$$
\Pi_i^{II}(d) = \frac{(16d^4 - 56d^2 + 16d + 85)\tau^2 - 16(2d + 1)^2\tau + 16(2d + 1)^2}{2(3 + 2d)^2(3 + 4d - 4d^2)^2}.
$$

By taking $d_i = d$ and $d_j = 0$, the gross profits of firm $i$ in $IN$ and $NI$ become

$$
\Pi_i^{IN}(d) = \Pi_i^{IN}(d) = \frac{(d^4 - 14d^2 + 8d + 85)\tau^2 - 16(d + 1)^2\tau + 16(d + 1)^2}{2(9 + 9d - d^2 - d^2)^2}.
$$

We obtain the profit of firm $i$ in the “no one invests” regime by substituting $d_i = 0$ into (2), just as we did with $\Pi_i^{NN}$ in equation (3).

By using the above equations for the firms’ profit, we now investigate the incentives in the investment decision. When the rival firm does not invest, firm $i$ invests if $\Pi_i^{IN}(d) - k \geq \Pi_i^{NN}$. Then, the minimum value of $k$ that yields investment is $k = \Phi_i(\tau, d) \equiv \Pi_i^{IN}(d) - \Pi_i^{NN}$. Similarly, when the rival firm invests, firm $i$ also has an incentive to invest if $\Pi_i^{II}(d) - k \geq \Pi_i^{NI}(d)$. Solving this inequality for $k$, we have the minimum value of $k$ with investment: $k = \Phi_i(\tau, d) \equiv \Pi_i^{II}(d) - \Pi_i^{NI}(d)$. Comparing these thresholds, we obtain $\Phi_i(\tau, d) \geq \Phi_i(\tau, d)$. Then, we establish Proposition 3.

**Proposition 3.** 1. Suppose that $\tau < \tau(d)$. (i) If $k < \Phi_i(\tau)$, then $II$ appears. (ii) If $\Phi_i(\tau) \leq \tau(d)$ and $k \geq \Phi_i(\tau)$, then $IN$ appears. (iii) If $\Phi_i(\tau) \leq \tau(d)$ and $k \leq \Phi_i(\tau)$, then $NI$ appears.
\( k \leq \Phi_u(\tau), \) then NN and II can appear. (iii) If \( k > \Phi_u(\tau), \) then NN appears. 2. Suppose that \( \tau \geq \tau(d). \) (i) If \( k < \Phi_u(\tau), \) then NN and II can appear. (ii) If \( k \geq \max\{\Phi_u(\tau), 0\}, \) then NN appears.

Proof. See Appendix A.

Next, we consider the relationship between consumer surplus and transport cost. Because we added the new parameter \( d \) into the model, our analysis is more complicated. Thus, to provide a simple confirmation on whether a result similar to that of Proposition 2 holds, we here assume that the investment cost \( k \) takes a discrete value, \( k \in \{0.001, 0.002, 0.003, \ldots\}. \) From Proposition 3, we find that only the “no one invests” regime, “everyone invests” regime and the “no one invests” and “everyone invests” regimes can appear in equilibrium. Hence, we compare the consumer surplus of the two regimes. Country \( i \)'s consumer surplus in the “everyone invests” regime is

\[
CS_i^{II}(d) = \frac{16d^5\tau^2 - 8d^3(11\tau^2 - 8\tau + 8) + d(93\tau^2 - 48\tau + 48) + 4(\tau - 2)^2}{2(2d + 3)^2(3 + 4d - 4d^2)^2}.
\]

\( CS_i^{NN} \) is given by equation (6).

Since \( \Phi_u(\tau, d) \) is a decreasing function of \( \tau, \) \( \Phi_u(\tau, d) \) takes the maximum value at \( \tau = 0. \) Moreover, with \( \tau = 0, \) \( \Phi_u(0, d) \) is maximized at \( d = 1/2. \) Since \( \Phi_u(0, 1/2) = 201/9800 \simeq 0.205, \) from Proposition 3, for any \( k > 0.205, \) no firm decides to invest. In addition, since \( CS_i^{NN} \) decreases with \( \tau, \) consumer surplus is maximized at \( \tau = 0. \)

The remaining cases are \( k \in \{0.001, 0.002, \ldots, 0.204\}. \) We define the inverse functions of \( \Phi_u^{-1}(\tau, d) \) and \( \Phi_l^{-1}(\tau, d) \) by \( \Phi_u^{-1}(k, d) \) and \( \Phi_l^{-1}(k, d), \) respectively. Numerically comparing \( CS_i^{NN} \) with \( CS_i^{II}(d), \) we establish the following result.

**Proposition 4.** Let \( k \in \{0.001, 0.002, \cdots\}. \) 1. Suppose that NN appears in the NN and II regimes. (i) For \( k \leq \Phi_l(0, d), \) consumer surplus is maximized at \( \tau = \Phi_l^{-1}(k, d). \) (ii) For \( k > \Phi_l(0, d), \) consumer surplus is maximized at \( \tau = 0. \)
2. Suppose that $II$ appears in the NN and II regimes. (i) For $k \leq \Phi_u(4/11, d)$, consumer surplus is maximized at $\tau = 0$ if $d > 0.479$; otherwise, it is maximized at $\tau = \Phi_u^{-1}(k, d)$ if $d \leq 0.479$. (ii) For $\Phi_u(4/11, d) < k \leq \Phi_u(0, d)$, consumer surplus is maximized at $\tau = \Phi_u^{-1}(k, d)$. (iii) For $k > \Phi_u(0, d)$, consumer surplus is maximized at $\tau = 0$.

Proof. See Appendix B.

5 Conclusion

Reducing transport cost in two-way trade increases each country’s total outputs and lowers prices due to the promotion of competition in the domestic market, so a reduction in transport cost will certainly make consumers better off. Thus, a zero transportation cost, that is, free trade, maximizes consumer surplus. Although we could consider this assertion as theoretically obvious, it is not necessarily true. We show that when an exporting firm that deals exclusively with its domestic input supplier engages in product R&D, a positive transport cost can maximize consumer surplus. We find this result in a setting where R&D race and input price are present. When the investment cost is not too high, the equilibrium pattern also depends on the transport cost. Whilst the “everyone invests” regime appears if both the investment and transport costs are low, the “no one invests” regime appears if both costs are high. When both costs are at an intermediate level, then, multiple equilibria, the “everyone invests” and “no one invests” regimes, appear. The input suppliers can extract R&D benefits by charging a higher price after observing their customers’ investments, so input prices increase when firms invest. In addition, there is a strategic complementarity between the domestic and foreign input prices; hence, the input price increases with the number of investing firms. Therefore, the “everyone invests” regime yields the worst production efficiency of all regimes and the smallest consumer surplus through a decline in total outputs. In the case of low investment cost, if the transport cost declines below a certain level, the consumer surplus decreases because the equilibrium can switch from the “no
one invests” regime to the “everyone invests” regime.

Our model is based on a product R&D model in which investment promotes a degree of horizontal product differentiation (Lin and Saggi, 2002; Lambertini and Rossini, 1998), so we do not examine the vertical product differentiation case. Also, considering quality-improving R&D, for example, may be fruitful, but it is beyond the scope of this study and is left to future research.

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Appendix A. Proof of Proposition 3

To prove this proposition, we show that (i) \( \Phi_l(\tau, d) < \Phi_u(\tau, d) \), (ii) \( \Phi_l(0, d) > 0 \) and \( \Phi_u(0, d) > 0 \), (iii) \( \partial \Phi_l/\partial \tau < 0 \) and \( \partial \Phi_u/\partial \tau < 0 \), (iv) \( \Phi_l(0, d) > \Phi_u(4/11, d) \), (v) \( \tau(d) \in [0, 4/11] \) such that \( \Phi_l(\tau(d), d) = 0 \) and (vi) \( \Phi_u(4/11, d) > 0 \) for \( d > 0.479 \) and \( \Phi_u(4/11, d) \leq 0 \) for \( d \leq 0.479 \). From these conditions, we can create a similar figure as in Figure 1, so it is enough to show them to complete the proof.

First, we show that \( \Phi_l(\tau, d) < \Phi_u(\tau, d) \). The difference between the two is

\[
\Phi_u(\tau, d) - \Phi_l(\tau, d) = \frac{d^2(\psi_2 \tau^2 + \psi_1 \tau + \psi_0)}{81(3 + 2d)^2(3 + 4d - 4d^2)^2(9 + 9d - d^2 - d^3)^2},
\]

where \( \psi_2 \equiv 2720d^{10} + 8160d^9 - 56896d^8 - 183408d^7 + 337514d^6 + 1298646d^5 - 504621d^4 - \)
and \( \psi_0 \equiv 16(2d^2 + 3d + 1)^2(8d^6 - 180d^4 + 81d^2 + 1458) \). Thus, the sign of \( \Phi_u(\tau, d) - \Phi_l(\tau, d) \) depends on the numerator. The numerator is a quadratic function of \( \tau \) and the coefficient of \( \tau^2 \) is positive. Further, its discriminant is \( \psi_1^2 - 4\psi_2\psi_0 = -5184d^4(2d^2 + 3d + 1)^2(2d^2 + 6d + 3)(4d^4 - 45d^2 + 81)^2(8d^6 - 180d^4 + 81d^2 + 1458) < 0 \), and hence \( \Phi_l(\tau, d) < \Phi_u(\tau, d) \).

Second, we consider the signs of \( \Phi_l(0, d) \) and \( \Phi_u(0, d) \). Substituting \( \tau = 0 \) into \( \Phi_l(\tau, d) \) and \( \Phi_u(\tau, d) \), we have
\[
\Phi_l(0, d) = \frac{24d(18 - 5d^2)}{81(9 - d)^2} > 0 \quad \text{and} \quad \Phi_u(0, d) = \frac{8d(18 - d)}{81(9 - d)^2} > 0.
\]

Third, we consider the sign of the first derivatives of \( \Phi_l(\tau, d) \) and \( \Phi_u(\tau, d) \) with respect to \( \tau \).

We have
\[
\frac{\partial \Phi_l(\tau, d)}{\partial \tau} = \frac{d(\psi'_l + \psi'_0)}{(2d + 3)^2(3 + 4d - 4d^2)^2(3d^3 + 4d^2 - 9d - 9)^2},
\]
where \( \psi'_l \equiv -48d^3 - 32d^8 + 840d^7 - 7935d^5 - 3114d^4 + 24618d^3 + 15876d^2 - 19467d - 13122 < 0 \) and \( \psi'_0 \equiv -24d(2d^2 + 3d + 1)^2(18 - 5d^2) < 0 \). Note that we confirm the first inequality by using a numerical calculation. Since \( \psi'_l < 0 \) and \( \psi'_0 < 0 \), the above derivative takes a negative value: \( \partial \Phi_l(\tau, d)/\partial \tau < 0 \).

We consider the sign of \( \partial \Phi_u(\tau, d)/\partial \tau \).
\[
\frac{\partial \Phi_u(\tau, d)}{\partial \tau} = \frac{\psi''_l + \psi''_0}{81(9 + 9d - d^2 - d^3)^2},
\]
where \( \psi''_l \equiv -85d^5 - 170d^4 + 1526d^3 + 3060d^2 - 6489d - 13122 < 0 \) and \( \psi''_0 \equiv -8d(1 + d)^2(18 - d^2) < 0 \). Note that we find the first inequality by using a numerical calculation. Hence, we have \( \partial \Phi_u(\tau, d)/\partial \tau < 0 \).
Fourth, we show that $\Phi_t(0, d) > \Phi_u(4/11, d)$.

\[
\Phi_t(0, d) - \Phi_u(4/11, d) = \frac{8d}{9801(3 - 2d)^2(3 - d)^2(d + 1)^2(2d + 1)^2(2d + 3)^2} > 0,
\]

To show the above inequality, we use a numerical calculation.

Fifth, we consider $\tau(d)$ such that $\Phi_t(\tau(d), d) = 0$. Solving $\Phi_t(\tau, d) = 0$ for $\tau$, we have

\[
\tau = \tau(d) = -\frac{4\left(2d^5 + 4d^4 - 34d^3 - 72d^2 - 36d - 9(d - 3)(d + 1)(d + 3)\sqrt{d(d + 2)(18 - d^2)}\right)}{85d^5 + 170d^4 - 1526d^3 - 3060d^2 + 6489d + 13122}.
\]

From numerical calculation, we obtain $0 \leq \tau(d) < 40(3219\sqrt{14053} - 21620)/134273093 \simeq 0.107$.

Finally, we consider the sign of $\Phi_u(4/11, d)$. We have

\[
\Phi_u(4/11, d) = -\frac{8d}{121(2d + 3)^2(-4d^2 + 4d + 3)^2(-d^2 - d^2 + 9d + 9)^2}
\]

The sign of $\Phi_u(4/11, d)$ depends only on the terms in the square brackets of the numerator. We solve the equation numerically and find that $\Phi_u(4/11, d) > 0$ if and only if $d > 0.479$. Therefore, we complete the proof of Proposition 3. \(\square\)

**Appendix B. Proof of Proposition 4**

We show that (i) $CS_t^{NN} > CS_t^{II}(d)$, (ii) $CS_t^{NN}$ and $CS_t^{II}(d)$ decrease with $\tau$ and (iii) at $\tau = 0$, $CS_t^{NN}$ and $CS_t^{II}(d)$ take positive values.

First, we consider $CS_t^{NN} - CS_t^{II}(d)$.

\[
CS_t^{NN} - CS_t^{II}(d) = \frac{d}{162(3 - 2d)^2(1 + 2d)^2(3 + 2d)^2}
\]

The sign of $CS_t^{NN} - CS_t^{II}(d)$ depends only on the terms in the square brackets. Using numerical
calculation, we find that the coefficient of $\tau^2$ in the square brackets takes a negative value and that two roots for $\tau$ satisfying $CS_i^{NN} - CS_i^{II}(d) = 0$ are not in $[0, 4/11)$. Hence, for any $\tau \in [0, 4/11), CS_i^{NN} - CS_i^{II}(d) > 0$ holds.

Second, we show that $CS_i^{NN}$ and $CS_i^{II}(d)$ decrease with $\tau$. The first derivative of $CS_i^{NN}$ with respect to $\tau$ is 
\[
\frac{\partial CS_i^{NN}}{\partial \tau} = \frac{(16d^5 - 88d^3 + 93d + 4)\tau - 8(1 - d)(2d + 1)^2}{(3 + 2d)^2(3 + 4d - 4d^2)^2}.
\]
The numerator determines the sign of the above equation. Since the coefficient of $\tau$ is positive and $\tau \in [0, 4/11)$, the numerator takes the supremum at $\tau = 4/11$. Substituting $\tau = 4/11$ into the numerator, we have $4(-18 + 27d + 16d^2)/11$, which takes a negative value over $d \in [0, 1/2]$. Hence, $\partial CS_i^{II}(d)/\partial \tau < 0$.

Finally, at $\tau = 0$, we have $CS_i^{NN} = 8/81 > 0$ and $CS_i^{II}(d) = 8(1 - d)/(9 - 4d^2)^2 > 0$.

Now, we are ready to prove Proposition 4. First, we consider a case where $NN$ appears in the $NN$ and $II$ regimes. Because $\partial CS_i^{NN}/\partial \tau < 0$ and $\partial CS_i^{II}(d)/\partial \tau < 0$, the candidates that maximize consumer surplus are $\tau = 0$ and $\tau = \Phi_l^{-1}(k, d)$, where $\Phi_l^{-1}(k, d)$ is an inverse function of $k = \Phi_l(\tau, d)$.

If $k \geq \Phi_l(0, d)$, then for any $\tau$, the $NN$ regime appears. Hence, $\tau = 0$ maximizes consumer surplus. On the other hand, if $k < \Phi_l(0, d)$, we need to compare the consumer surplus at $\tau = 0$ and $\tau = \Phi_l^{-1}(k, d)$.

Since $\Phi_l(\tau, d)$ decreases with $\tau$, $\Phi_l(\tau, d)$ is maximized at $\tau = 0$. In addition, since $\Phi_l(0, d)$ increases with $d$, the maximum value of $\Phi_l(\tau, d)$ is $\Phi_l(0, 1/2) = 568/99225 \simeq 0.0057$. Hence, for any $k \in \{0.006, 0.007, \ldots \}$, $NN$ is the only regime, which means that $\tau = 0$ maximizes consumer surplus.

The remaining cases are in $k \in \{0.001, \ldots , 0.005\}$. Because $\partial \Phi_l(0, d)/\partial d = 32d/(9-d^2)^3 > 0$, there exists a $d'$ such that for any $d \in [0, d')$, we have $k > \Phi_l(0, d)$. This case also leads to the maximum consumer surplus with $\tau = 0$. Since we assume that $k$ takes a discrete value, we can
numerically solve $k = \Phi_d(0, d')$ for $d'$ by fixing $k$ to a certain value.

Over $d \in [d', 1/2]$, there always exists $\tau = \Phi_d^{-1}(k, d)$, which is the candidate that maximizes consumer surplus. Fixing $k$ to a certain value, $k \in \{0.001, \ldots, 0.005\}$, and substituting $\tau = 0$ and $\tau = \Phi_d^{-1}(k, d)$ into $CS_{II}^i(d)$ and $CS_{NN}^i$, respectively, we can compare the consumer surplus in these cases. Since $CS_{II}^i(d)$ and $CS_{NN}^i$ depend only on $d$, we can compare them numerically and find that for any $k \in \{0.001, \ldots, 0.005\}$, $CS_{NN}^i > CS_{II}^i(d)$. Hence, $\tau = \Phi_d^{-1}(k, d)$ maximizes consumer surplus. Summarizing the discussion above, we obtain the first part of Proposition 4.

Next, we show the second part. We assume that $II$ occurs in the $NN$ and $II$ regime. Since for some $d$, $\Phi_u(\tau, d)$ takes a positive value at $\tau = 4/11$, we calculate this threshold value. Numerically solving $\Phi_u(4/11, d) > 0$ for $d$, we obtain $d > 0.4879$.

To show the second part, we consider two cases: (i) $k < \Phi_u(4/11, d)$ and (ii) $k \geq \Phi_u(4/11, d)$. Note that we can consider case (i) only for $d > 0.4879$. In case (i), only the $II$ regime appears. Hence, $\tau = 0$ maximizes consumer surplus because $CS_{II}^i(d)$ decreases with $\tau$.

We now consider case (ii). Since $\Phi_u(\tau, d)$ decreases with $\tau$ and $\Phi_u(0, d)$ increases with $d$, the maximum value is $\Phi_u(0, 1/2) = 201/9800 \simeq 0.0205$. Hence, if $k \in \{0.021, 0.022, \ldots\}$, for any $\tau$, the $II$ regime occurs, which means that $\tau = 0$ maximizes consumer surplus.

Next, we consider the case with $k \in \{0.001, \ldots, 0.020\}$. Here, we have two candidates that maximize consumer surplus: $\tau = 0$ and $\tau = \Phi_u^{-1}(k, d)$. Given some value of $k$ and substituting $\tau = 0$ and $\tau = \Phi_u^{-1}(k, d)$ into $CS_{II}^i(d)$ and $CS_{NN}^i$, respectively, we can compare the consumer surplus in these cases. Since $CS_{II}^i(d)$ and $CS_{NN}^i$ depend only on $d$, we can compare them numerically and find that for any $k \in \{0.001, \ldots, 0.020\}$, $CS_{NN}^i > CS_{II}^i(d)$. Therefore, we obtain the second part of Proposition 4. Combining these results, we complete the proof. □
References


