The Conditions for Indeterminacy in Two Types of Balanced Budget Rules − Reconsidered

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Abstract

When governments levy taxes on labor income on the basis of a balanced budget rule, and if a countercyclical income tax is adopted, two steady states in an economy exist, of which one can cause indeterminacy paths and a saddle path. Conversely, without a countercyclical income tax, indeterminacy disappears, as does one steady state, even if a balanced budget rule is adopted. This holds regardless of labor supply adjustment.

*I thank Professor Jeffrey Herrick for checking my English in this paper. Naturally, I accept full responsibility for any errors.

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1 Introduction

The literature on indeterminacy is substantial. Many scholars, like Benhabib and Farmer, deal with this hot topic from the viewpoint of naturally produced economic outcomes, such as externality and monopolistic competition. But we are interested in artificial determinants such as fiscal policy. Along this line, Guo and Harrison (2004) argues that fiscal policies alone can cause indeterminacy to disappear. The foundation for this is based on a balanced budget rule with fixed tax rates. In general, that rule can be classified into two categories: one in which government spending is fixed and tax rates are correspondingly determined in order to be equal to that fixed spending, the other in which tax rates are fixed and government spending is equal to the revenue determined under these fixed tax rates. Below, we denote the former as balanced budget rule I, the latter as balanced budget rule II.

Following Schmitt-Grohé and Uribe (1997), Guo and Harrison (2004) argues that under the second category this indeterminacy never occurs, while under the first category it can occur. However, when they make this assertion, they base it on two economies, one with indivisible labor, one with divisible labor. A decisive difference between these two economies exists: the former has full participation in the labor market, while the latter always has partial participation in the labor market. Since Schmitt-Grohé and Uribe (1997) deals with an economy with indivisible labor, which derives from Hansen (1985) and Rogerson (1988), uncertainty remains in the labor market. This phenomenon apparently affects their logic. If one seeks an explanation for the disappearance of indeterminacy, one should keep other factors unchanged, except for the fact postulated as a cause. In this context, Guo and Harrison should have assumed and analyzed only an economy with divisible labor. We analyze their logic from this viewpoint.

In what follows, our model is introduced in the next section. In section 3, the model is converted to dynamics and a linearization system is established. Subsequently in section 4, two diagrams in relation to economic movements are depicted. Finally, section 5 is devoted to conclusions.

2 The model

Guo and Harrison (2004) offers a model with a different assumption from that of Schmitt-Grohé and Uribe (1997), as follows:its maximization problem is expressed as

$$\max_{C_t, H_t} \int_0^\infty e^{-\rho t} \left(\log \mathrm{C}_t - \frac{A H_t^{\gamma+1}}{\gamma+1} \right) dt \tag{1}$$

and assumes the budget constraint as

$$\dot{K}_t = (1-\tau_h) H_t w_t + (1-\tau_k)(r_t - \delta) K_t + T_t - C_t.$$

Here, C_t means individual household consumption, H_t hours worked, γ the inverse of the intertemporal elasticity of substitution in the labor supply, and ρ the subjective discount rate.

Furthermore, K_t means the household's capital stock, w_t the real wage rate, r_t the real rental rate of capital, τ_h the labor income tax rate and τ_k the capital income tax rate. Both tax rates are constant. δ means the depreciation rate of capital and T_t the lump-sum transfers.

At this stage, it is crucial that Guo and Harrison (2004) assumes an economy with divisible labor, contrary to that in Schmitt-Grohé and Uribe (1997). The former implies an economy with intensive arrangements in the labor supply, while the latter implies one with extensive arrangements in the labor supply. In this situation, Guo and Harrison insist that in the former economy indeterminacy disappears because of balanced budget rule II alone, while in the latter economy indeterminacy can appear due to balanced budget rule I alone.

In general, in order to discover a cause, one should form a hypothesis associated with the cause, in this case, the difference between the balanced budget rules, with other factors unchanged. Otherwise, other factors may cause the difference in the phenomenon which the hypothesis assumes. Guo and Harrison (2004) deals with indeterminacy problems based on different economies. In order to avoid this defect, it is better to discuss the economies with the same properties except for the balanced budget rule alone. Though Guo and Harrison (2004) deals with an economy with divisible labor and balanced budget rule II, it should demonstrate the appearance of indeterminacy in an economy with divisible labor and balanced budget rule I, like Schmitt-Grohé and Uribe (1997). Guo and Harrison mention a model with indivisible labor only for analytical tractability, but the two economies definitely are different.[1] To highlight this, Takata (2017) offers a demonstration with detailed analyses, stating what conditions can cause the indeterminacy in a divisible labor economy.

[1]See footnote 6 in Guo and Harrison (2004).

So, first, we will convert Guo and Harrison (2004) to a form comparable with that demonstration.

(1) is modified as

$$\max_{C_t, H_t} \int_0^\infty e^{-\rho t} \left(\log \mathrm{C}_t - \frac{H_t^{1-\chi}}{1-\chi} \right) dt. \tag{2}$$

Here, A is assumed to be 1, γ is denoted as -χ.

$$\dot{K}_t = (1-\tau) H_t w_t + u_t K_t - C_t. \tag{3}$$

Here, $\tau_h = \tau$, $\tau_k = 0$, $r_t = u_t$, $\delta = 0$ and $T_t = 0$. The modification described above implies that capital tax is omitted, capital is not depreciated and that there are no lump-sum transfers. These assumptions are for simplicity and to focus attention on labor and labor income taxation.

Second, there are many identical firms, which are competitive. The representative firm produces Y_t utilizing a Cobb-Douglas production function, which has a constant-to-scale property, as follows: $Y_t = F(K_t, H_t) = K_t^\alpha H_t^{1-\alpha}$, where $0 < \alpha < 1$ implies the share of capital in output. The firm aims to maximize its profit and consequently $w_t = (1-\alpha)k_t^\alpha$ and $u_t = \alpha k_t^{\alpha-1}$ hold. Additionally, we assume the output price is 1.

Third, the government spends all labor income taxes, so government spending fully depends on labor income taxes. This features balanced budget rule I. We can describe the Hamiltonian $\mathcal{L}$ as

$$\mathcal{L} = e^{-\rho t} \left(\log C_t - \frac{H_t^{1-\chi}}{1-\chi} \right) + \mu_t \left(K_t u_t + (1-\tau) H_t w_t - C_t \right). \tag{4}$$

Based on (4), we obtain the first-order conditions as

$$H_t = \left[\frac{(1-\tau) w_t}{C_t} \right]^{-\frac{1}{\chi}}, \tag{5}$$

$$\frac{\dot{C}_t}{C_t} = u_t - \rho. \tag{6}$$

(5) shows the labor supply, (6) the consumption Euler equation, respectively. The transversality condition can be expressed as

$$\lim_{t \to \infty} \frac{k_t e^{-\rho t}}{C_t} = 0.$$

3 Dynamics

Our dynamic system can be expressed as two simultaneous differential equations.

$$\dot{\lambda}_t = \frac{(\chi((1-\alpha)\tau - 1) + \alpha)e^{(\alpha-1)\lambda_t} + \chi((\alpha-1)(\tau-1))^{1/\chi} e^{\frac{(\chi-1)\delta_t}{\chi}} e^{\frac{(\alpha-\chi)\lambda_t}{\chi}} - \rho}{\alpha - \chi}, \tag{7}$$

and

$$\dot{\delta}_t = \alpha \mathrm{e}^{(\alpha-1)\lambda_t} - \rho. \tag{8}$$

Here, $\lambda_t = \log(k_t)$, $\delta_t = \log(C_t)$.[2]

This differential system has a unique fixed point $(\lambda^*, \delta^*)^3$. The result is

$$\begin{aligned} \lambda^* &= \frac{\log\left(\frac{\rho}{\alpha}\right)}{\alpha - 1}, \\ \delta^* &= \frac{\alpha \log\left(\frac{\rho}{\alpha}\right)}{\alpha - 1} + \frac{\log\left(\frac{(1-(1-\alpha)\tau)^\chi}{(1-\alpha)(1-\tau)}\right)}{\chi - 1}. \end{aligned} \tag{9}$$

At this stage, we intend to establish a linear approximation system around (9). The Jacobian matrix J is showned as

$$J = \begin{pmatrix} 0 & a_{12} \\ a_{21} & a_{22}. \end{pmatrix}$$

Additionally, we postulate that $a_{12} = \dfrac{\partial f_1}{\partial \lambda}$, $a_{21} = \dfrac{\partial f_2}{\partial \delta}$, and $a_{22} = \dfrac{\partial f_2}{\partial \lambda}$.

Moreover, a_{12}, a_{21} and a_{22} are respectively values evaluated at (9).

We calculate a_{12}, a_{21} and a_{22} in order:

$$a_{12} = (-1 + \alpha)\rho,$$

$$a_{21} = \left(\frac{\rho}{\alpha}\right) \frac{(\chi - 1)((\alpha - 1)\tau + 1)}{(\alpha - \chi)},$$

$$a_{22} = \left(\frac{\rho}{\alpha}\right) \frac{(1-\alpha)\tau(\alpha - (2-\alpha)\chi) + (\alpha - 2)(\alpha - \chi)}{\chi}.$$

[2] For deduced processes in the above, see Appendix I.

[3] For a proof, see Appendix II.

We would like to know the signs of the products of eigenvalues of J, which can be identified by $|J|$.

$$|J| = -a_{12} \cdot a_{21} = -(-1+\alpha)\rho \cdot \left(\frac{\rho}{\alpha}\right) \frac{(\chi-1)((\alpha-1)\tau+1)}{(\alpha-\chi)} < 0 \qquad (10)$$

Since the determinant of J implies the product of two eigenvalues of J, we can know that one eigenvalue is positive, the other negative and that our system has a saddle path. Indeterminacy obviously disappears in the case of balanced budget rule II.

4 Diagrams

Our aim is to identify the reason for the appearance and disappearance of indeterminacy in budget rules I and II. So, we utilize diagrams, in which the determinant and the trace of the Jacobian matrix in both economies are drawn, when τ is chosen.

First, in the case of balanced budget rule II, we have established the determinant as

$$D_J(\tau) = -\left(\frac{\rho}{\alpha}\right) \frac{(\alpha-1)\rho(\chi-1)((\alpha-1)\tau+1)}{(\alpha-\chi)},$$

which has properities such as

$$\frac{d\ D_j(\tau)}{d\tau} = \left(\frac{\rho}{\alpha}\right) \frac{(1-\alpha)(\alpha-1)\rho(\chi-1)}{(\alpha-\chi)} > 0,$$

$$D_J(0) = \left(\frac{\rho}{\alpha}\right) \frac{(1-\alpha)\rho(\chi-1)}{\alpha(\alpha-\chi)} < 0, \quad \text{and}$$

$$D_J(1) = \frac{(1-\alpha)\rho^2(\chi-1)}{\alpha-\chi} < 0.$$

On the other hand, the trace $T_j(\tau)$ has been deduced as

$$T_j(\tau) = \left(\frac{\rho}{\alpha}\right) \frac{1}{\chi}[(\alpha-1)((2-\alpha)\chi-\alpha)\tau + (\alpha-2)(\alpha-\chi)]$$

which has properties such as

$$\frac{dT_j(\tau)}{d\tau} = \left(\frac{\rho}{\alpha}\right) \frac{1}{\chi}(\alpha-1)((2-\alpha)\chi-\alpha) < 0,$$

$$T_j(0) = \frac{1}{\chi}\left(\frac{\rho}{\alpha}\right)(\alpha - 2)(\alpha - \chi) > 0 \text{ and}$$

$$T_j(1) = \rho\left(-\alpha - \frac{1}{\chi} + 2\right) > 0.$$

Based on the above analyses, we can show the diagrams below.

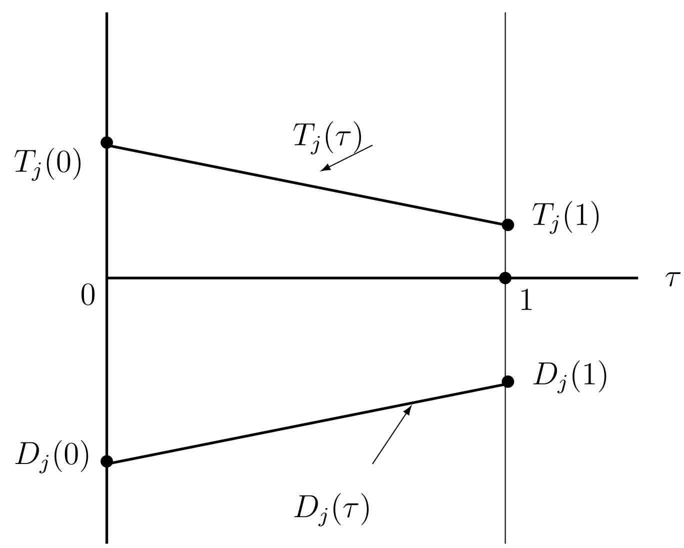

Figure 1: In a range of $0 < \tau < 1$, a unique saddle path occurs.

Second, in the case of balanced budget rule I, we obtained figure 2.

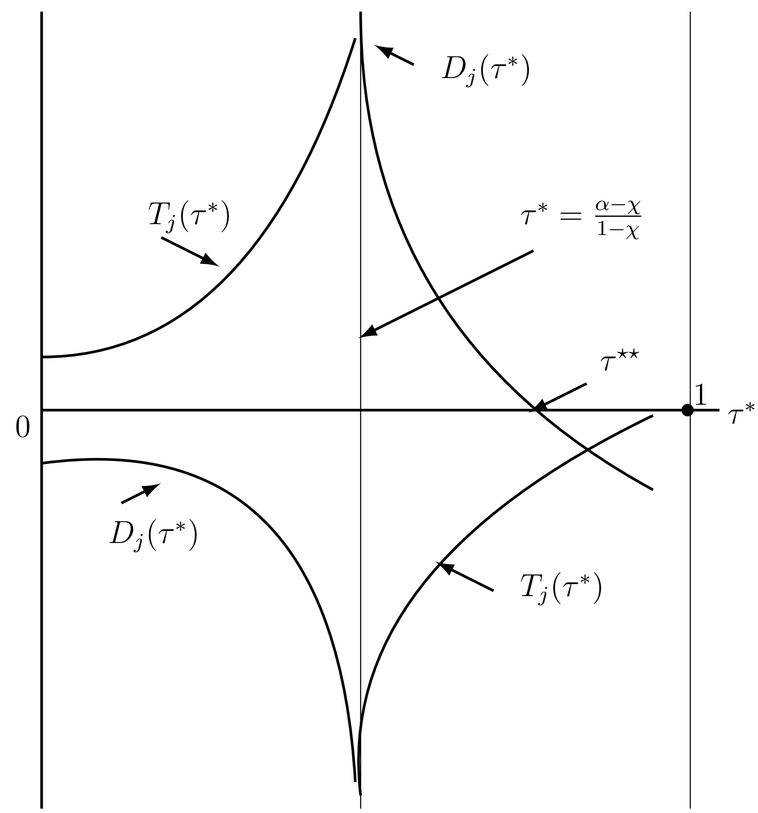

Figure 2: Given $\dfrac{\alpha-\chi}{\mathbf{1}-\chi} < \tau^* < \tau^{\star\star}$, indeterminacy paths exist; when $\mathbf{0} < \tau^* < \dfrac{\alpha-\chi}{\mathbf{1}-\chi}$, or $\tau^{\star\star} < \tau^* < \mathbf{1}$, a saddle path occurs.

Figure 2 shows a possibility of indeterminacy. In addition, in Figure 1, τ is an arbitrary tax rate imposed by the government, while in Figure 2, τ^* shows the corresponding values at steady states when the government determines its spending.[4]

5 Conclusions

We demonstrated that in a economy with balanced budget rule I, indeterminacy can appear, while in a economy with rule II, it does not. Our demonstration is based on analyses focusing on whether tax rates are given or not. Guo and Harrison (2004) briefly says, "Suppose that the future return on capital is expected to increase. Without taxes, indeterminacy cannot occur since a higher capital stock is associated with a lower rate of return under constant returns-to-scale. However, a balanced-budget rule with countercyclical income taxes can cause the after-tax return on capital to rise, thus validating agents' initial optimistic expectations." This statement, as far as it goes, is true.

However, Guo and Harrison discuss different economies, that is, one with divisible labor and the other with indivisible labor. They deploy divisible economy, then compare the result with the indivisible economy analyzed by Schmitt–Grohé and Uribe (1997). To make their conclusion accurate, they need to analyze an economy with divisible labor alone, as we demonstrated in this paper.[5] But if they conjecture that both economies have the same features in relation to movement, they should demonstrate it.[6]

6 Appendices

6.1 Appendix I

We establish our final system, which consists of two variables, λ and δ. Based on (3) and the definition of k_t, we can deduce the following:

$$\dot{k}_t = -\frac{C_t}{H_t} - \frac{\dot{H}_t k_t}{H_t} + k_t u_t + (1-\tau) w_t$$

[4]See Takata (2017) for details.
[5]See Takata (2017).
[6]See Mulligan (1999).

which is transformed to

$$\dot{k}_t = -\frac{k_t\left(\frac{\dot{C}_t}{C_t} - \frac{\dot{w}_t}{w_t}\right)}{\chi} - C_t\left(\frac{C_t}{(1-\tau)w_t}\right)^{-1/\chi} + k_t u_t + (1-\tau)w_t.$$

Here, (5) was utilized. Moreover, substituting the following relationships

$$u_t = \alpha k_t^{\alpha-1},\ w_t = (1-\alpha)k_t^{\alpha},\ \frac{\dot{C}_t}{C_t} = \alpha k_t^{\alpha-1} - \rho,$$

we finally obtain

$$\frac{\dot{k}_t}{k_t} = \frac{\chi((\alpha-1)(\tau-1))^{1/\chi} C_t^{\frac{\chi-1}{\chi}} k_t^{\frac{\alpha}{\chi}-1} + (\alpha(-\tau)\chi + \alpha + (\tau-1)\chi)k_t^{\alpha-1} - \rho}{\alpha - \chi}.$$

Moreover, when we replace $\log(k_t)$ with λ_t and $\log(C_t)$ with δ_t, we reach (7). (8) obviously holds.

6.2 Appendix II

Next, we proceed to (9). In order to obtain the steady state, we respectively set $\dot{\lambda} = 0$ in (7) and $\dot{\delta} = 0$ in (8). This manipulation means that we make the right side in (7) and that in (8) equal to null. Then, we solve these two equations for λ and δ. In these processes, it is convenient to transform variables such as $e^{\frac{\delta(\chi-1)}{\chi}} = q$ and $e^{\frac{\lambda(\chi-1)}{\chi}} = y$. Finally, we can obtain (9).

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