Solving a hold-up problem may harm all firms: Downstream R&D and transport-price contracts

Kazuhiro Takauchi
Tomomichi Mizuno

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Kazuhiro Takauchi*
Faculty of Business and Commerce, Kansai University

Tomomichi Mizuno†
Graduate School of Economics, Kobe University

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Abstract

In vertical relations, by raising input price after downstream research and development (R&D) investment, upstream firms can extract the R&D benefit and have an incentive to set higher input price. As downstream firms underinvest for fear of this hold-up by upstream firms, outputs and input-demand shrink, and all firms become worse off. Previous literature emphasizes that a fixed-price contract in which upstream firms first commit themselves to input prices and downstream firms subsequently invest can resolve the hold-up problem and make all firms better off. By contrast, we show that in a vertical relation between firm-specific carriers and exporters, the fixed-price contract of transport price can make all firms worse off because an efficiency improvement in exporters intensifies inter-regional competition. We also discuss the robustness of the result.

Key words: Transport-price contracts; Downstream R&D; Firm-specific carrier; Hold-up problem

JEL classification: L13; F12; O31; R40

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*Faculty of Business and Commerce, Kansai University, 3-3-35 Yamate-cho, Suita, Osaka 564-8680, Japan. E-mail: kazh.takauch@gmail.com; Tel.: +81-6-6368-1817; Fax: +81-6-6339-7704.
†Graduate School of Economics, Kobe University, 2-1 Rokkodai-cho, Nada-ku, Kobe-City, Hyogo 657-8501, Japan. E-mail: mizuno@econ.kobe-u.ac.jp; Tel.: +81-78-803-7245; Fax: +81-78-803-7289.
1 Introduction

In vertical relations, research and development (R&D) investments by downstream firms give upstream agents an incentive for opportunistic behavior. Suppose that, for example, a downstream firm invests to reduce its production cost. After observing the investment activity, by setting higher input-price, the upstream trading partner can extract the R&D benefit from the downstream firm. Some fear that the downstream investment is held-up by the upstream firm. Additionally, the upstream firm’s hold-up reduces downstream investment, and hence, it tends to decrease both upstream and downstream firms’ profit. Other studies find that an effective way to overcome this problem is to fix the input-price through a long-term price agreement, i.e., a fixed-price contract (Banerjee and Lin, 2003; Zikos and Kesavayuth, 2010) because the downstream firm decides its investment level as input-price given, so it does not mind a profit reduction from a higher input price.

This hold-up problem also can appear in a vertical relation between exporting and transporting firms. International transportation is an essential service to ship products overseas, and an exporting firm pays freight rates to cargo carriers to export. Hence, by setting a higher price after the exporting firm’s investment, it is possible for the carrier to extract the R&D benefit. Actually, transport cost is a major trade barrier, as much

\[ \text{1The importance of upstream firms’ opportunistic behavior, which causes downstream firms to underinvest (i.e., the hold-up problem) is widely recognized among researchers. For example, Gilbert and Cvsa (2003) give an example of a key-component supplier such as Intel and the investment by a PC maker such as Dell in the computer industry, and emphasize that this sort of hold-up problem is very likely to occur in supply chains such that downstream firms depend on their trading partners for both knowledge and capacity. Banerjee and Lin (2003) also give a similar example from the computer industry.} \]

\[ \text{2Hummels et al. (2009) empirically show that transport prices, such as the ocean freight rate, is a mark-up price and carriers have monopoly power. This empirical evidence implies that carriers possibly} \]
as or larger than other representative policy barriers\textsuperscript{3} and affects firms’ innovation activities. For example, because a higher transport cost limits access to foreign markets and inhibits export production, it affects incentives to innovate, such as cost-reducing R&D\textsuperscript{4}.

We consider a hold-up problem in international transportation and show that a fixed-price contract that resolves the problem in this transport market has entirely different effects than those found in the existing literature.

Our model is based on a Brander and Krugman (1983)-type two-country duopoly competition. There are two firm-specific carriers upstream and two exporters downstream. Each country has both a carrier and an exporting firm. Each carrier takes a per-unit transport charge from its domestic exporting firm and ships products to the foreign market. Each exporting firm pays a transport charge to its country’s carrier in order to export, while it freely supplies to the domestic market. Suppose that in this market structure, exporters can commit to zero exports. Then, each exporting firm is a monopoly firm in its local market and can thus gain maximum profit. We expect that a commitment to fewer exports benefits exporters.

If there is the hold-up problem, exporter will have a high marginal cost because their investment becomes small. Hence, the hold-up problem is equal to a commitment to fewer exports. At the same time, the marginal cost of domestic production also

\textsuperscript{3}According to Anderson and Van Wincoop (2004), the ad-valorem tax equivalent of transport costs is 10.7\%, while that of tariff and non-tariff barriers is 7 \% in developed nations.

\textsuperscript{4}Innovation incentives are influenced by such factors as market access and the intensity of competition. Since transport cost affects all of these factors, it ultimately affects producer’s innovation incentives. See, for example, Aghion et al. (2004, 2005).
becomes large, which lowers exporters’ profit. Exporters face this trade-off and benefit from the hold-up problem if the commitment effect of fewer exports resulting from the hold-up problem is dominant.

In our analysis, when the fixed-price contract is not employed and exporters can commit to fewer exports, they can maximize their profit because they can create a situation close to domestic monopoly by adjusting the transport price through their investment decision. By contrast, when the fixed-price contract is employed, the exporter’s profit is minimized because carriers set a lower price to promote exporters’ investments and exports become the most active. Furthermore, when the cost-reducing R&D is highly efficient, carriers profit less compared to other contract schemes because carriers set considerably lower prices to promote investments. Thus, the market structure is one in which the fixed-price contract harms all firms. We further discuss the robustness of our results in two cases: one with a positive spillover in R&D, and the other with horizontally differentiated domestic and foreign products. We find that our result holds in these extended cases.

This study is closely related to Takauchi’s (2015a, b) consideration of R&D rivalry with international transportation. Takauchi (2015a) examines the effects of the tech-

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5 Ghosh and Lim (2013), Haaland and Kind (2008), and Long et al. (2011) also consider the relationship between trade costs and innovation. They examine the effects of reduced exogenous trade-cost on R&D investment.

6 Other studies also consider an imperfectly competitive transport sector in international trade (e.g., Abe et al., 2014; Behrens et al., 2009; Francois and Wooton, 2001; Ishikawa and Tarui, 2015; Matsushima and Takauchi, 2014). Francois and Wooton (2001) and Behrens et al. (2009) examine the roles of transport prices and transporting firms’ market power in general equilibrium settings. Using a two-country oligopoly model, Abe et al. (2014) examine the effects of emissions tax in the transport sector, while Ishikawa and Tarui (2015) examine the effects of several trade policies on the transport market. Matsushima and Takauchi (2014) consider how seaport privatization influences their usage fees (trade cost) and welfare in an international oligopoly.
nical efficiency of R&D on exporters’ profit and shows that higher R&D efficiency can reduce their profit. Takauchi (2015b) considers the effects of transport efficiency, R&D spillovers, and transport market competition on investment and welfare. The author shows that competition in the transport market may harm consumers. Although these works share a basic market structure—exporters freely supply to their domestic market while they must pay freight rates for cargo carrier to export—with this study, we incorporate transport price contracts and examine the effects on profit and welfare.

This study is also related to works on input-price contracts with downstream investment (Banerjee and Lin, 2003; Gilbert and Cvsa, 2003; Kesavayuth and Zikos, 2009; Zikos and Kesavayuth, 2010). Banerjee and Lin (2003) show that fixed-price contracts make all firms better off in a market with an upstream monopoly and a downstream oligopoly. Zikos and Kesavayuth (2010) confirm that Banerjee and Lin’s result always holds, even if R&D spillovers exist. Gilbert and Cvsa (2003) consider the role of final demand uncertainty in a supply-chain with one supplier and one buyer. They show that the supplier prefers a commitment to wholesale prices if the demand fluctuations are not too large, and the buyer always profits more when the supplier makes a price commitment. Kesavayuth and Zikos (2009) examine the role of R&D spillovers and the importance of wages for labor unions on an endogenous choice of contract form for wages in a union–firm pair. However, these analyses are all limited to the domestic market and do not consider international trade and transportation. We believe that our analysis compliments the existing literature.

The rest of the paper proceeds as follows. Section 2 presents the model. Section
3 examines transport-price contracts. We discuss the robustness of the main result in Section 4. Section 5 concludes. We provide the proofs in the appendix.

2 Model

We consider a two-way trade model with firm-specific carriers, as in Takauchi (2015a, b). Two symmetric countries, $H$ and $F$, have a homogeneous product market. Each country has a single exporting firm (called firm $i$, $i = H, F$) and a firm-specific cargo carrier (called carrier $i$). The inverse market demand in country $i$ is $p_i = a - q_{ii} - q_{ji}$ ($i, j = H, F; j \neq i$), where $p_i$ is the product price, $q_{ii}$ is firm $i$’s domestic supply, $q_{ji}$ is firm $j$’s exports, and $a > 0$. While firm $i$ freely supplies to the domestic market, it must use carrier $i$ and pay a per-unit transport-price, $t_i$, to ship its product to an overseas market. Before production, firm $i$ invests in R&D to reduce marginal production cost $c$ ($> 0$); after the investment, the marginal cost is $c - x_i$, where $x_i$ is the investment level. We assume that the R&D cost function is $\gamma x_i^2$; $\gamma$ ($> 0$) denotes the technical efficiency in R&D.\footnote{This is a popular setting. See, for example, d’Aprémont and Jacquemin (1988), Ghosh and Lim (2013), Haaland and Kind (2008), and Takauchi (2015a).} Firm $i$’s profit is given by

$$\Pi_i \equiv (a - q_{ii} - q_{ji} - (c - x_i))q_{ii} + (a - q_{jj} - q_{ij} - (c - x_i) - t_i)q_{ij} - \gamma x_i^2,$$

where $i, j = H, F; j \neq i$. Carrier $i$ makes a take-it-or-leave-it offer to firm $i$ and decides its transport price. Each carrier’s profit is $\pi_i \equiv t_i q_{ij}$.\footnote{Our main result does not change, though the other trade cost $\tau$ exists (i.e., $\pi_i \equiv (t_i - \tau)q_{ij}$).}

We consider three transport-price contract schemes. The first is a \textit{fixed-price contract} where each carrier first sets its transport price and firms subsequently invest. The
second is a floating-price contract where firms first invest and each carrier subsequently sets its transport price. The third is a simultaneous move scenario where carriers and firms simultaneously decide transport prices and investment levels. In all schemes, each firm decides its output (i.e., domestic supply and exports) in the final stage of the game and competes à la Cournot in both markets in countries $H$ and $F$. The game is solved by backward induction.

3 Results

In the final stage, each firm decides its outputs to maximize its profit. The first-order conditions (FOCs) for profit maximization are $\partial P_i/\partial q_{ii} = a - c - 2q_{ii} - q_{ji} + x_i = 0$ and $\partial P_i/\partial q_{ij} = a - c - 2q_{ij} - q_{jj} + x_i - t_i = 0$. These yield $q_{ii}(t_j, x) = (a - c + t_j + 2x_i - x_j)/3$ and $q_{ij}(t_i, x) = (a - c - 2t_i + 2x_i - x_j)/3$. Let $x = (x_i, x_j)$.

Fixed-price contract. In the second stage, firm $i$ chooses an investment level, $x_i$, taking $t_i$ as given. From the firm’s FOC, the second-stage investment level is

$$x_i(t) = \frac{4(3\gamma - 4)(a - c) - 4(3\gamma - 2)t_i + 6\gamma t_j}{(3\gamma - 4)(9\gamma - 4)}, \quad j \neq i. \quad (1)$$

Let $t = (t_i, t_j)$. From the third-stage exports $q_{ij}(t_i, x)$ and (1), carrier $i$’s maximization problem is

$$\max_{t_i} t_i \left[ \frac{9\gamma(3\gamma - 4)(a - c) - 2(3\gamma - 1)(9\gamma - 8)t_i + 8(3\gamma - 1)t_j}{3(3\gamma - 4)(9\gamma - 4)} \right].$$

This yields the following equilibrium transport price:

$$t_i^{fx} = \frac{9\gamma(3\gamma - 4)(a - c)}{4(3\gamma - 1)(9\gamma - 10)}. \quad (2)$$
The outcome in the fixed-price contract is labeled “fx.” From (2), we have the equilibrium investment and outputs:

\[ x_i^{fx} = \frac{(189\gamma^2 - 276\gamma + 80)(a - c)}{2(3\gamma - 1)(9\gamma - 10)(9\gamma - 4)}, \]

\[ q_{ii}^{fx} = \frac{3\gamma(135\gamma^2 - 210\gamma + 64)(a - c)}{4(3\gamma - 1)(9\gamma - 10)(9\gamma - 4)} ; q_{ij}^{fx} = \frac{3\gamma(9\gamma - 8)(a - c)}{2(9\gamma - 10)(9\gamma - 4)}. \]

The carrier’s profit and the firm’s profits are

\[ \pi_i^{fx} = \frac{27\gamma^2(3\gamma - 4)(9\gamma - 8)(a - c)^2}{8(3\gamma - 1)(9\gamma - 10)^2(9\gamma - 4)^2}, \]

\[ \Pi_i^{fx} = \frac{\gamma(190269\gamma^5 - 717336\gamma^4 + 1024488\gamma^3 - 686592\gamma^2 + 215808\gamma - 25600)(a - c)^2}{16(3\gamma - 1)^2(9\gamma - 10)^2(9\gamma - 4)^2}. \]

To ensure a positive quantity, we assume the following throughout the analysis.\(^9\)

**Assumption 1.** \(\gamma > 4/3.\)

**Floating-price contract.** In this contract scheme, carrier \(i\) decides its transport price, \(t_i,\) in the second stage of the game. The carrier’s maximization problem yields the following second-stage transport price:

\[ t_i(x) = \frac{1}{4}(a - c - x_j + 2x_i), \quad j \neq i. \]  

From (6) and the firm’s profit, the equilibrium investment level is

\[ \frac{\partial \Pi_i(x)}{\partial x_i} = \frac{1}{72}(43(a-c) - 22x_j - (144\gamma - 65)x_i) = 0, \quad j \neq i \Rightarrow x_i^* = \frac{43(a-c)}{144\gamma - 43}. \]

The outcome in the floating-price contract is labeled “l.” From (7), we get the following

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\(^9\)As long as Assumption 1 holds, the second-order conditions for the carriers’ and firms’ profit maximization are satisfied.
Simultaneous move scenario. In this case, both transport prices and investments are decided in the first-stage of the game simultaneously. Eqs. (1) and (6) yield the following:

\[ t_i^s = \frac{9\gamma(a - c)}{2(18\gamma - 7)}, \]  
\[ x_i^s = \frac{7(a - c)}{18\gamma - 7}, \]  
\[ q_{ii}^s = \frac{15\gamma(a - c)}{2(18\gamma - 7)}; \quad q_{ij}^s = \frac{3\gamma(a - c)}{18\gamma - 7}, \]  
\[ \pi_i^s = \frac{27\gamma^2(a - c)^2}{2(18\gamma - 7)^2}; \quad \Pi_i^s = \frac{\gamma(261\gamma - 196)(a - c)^2}{4(18\gamma - 7)^2}. \]
and firms subsequently invest in the case of a fixed-price contract. Lower transport prices enhance transport demand and, can thus raise investments. As found in the second-stage investment, $x_i(t)$, a lower $t_i$ increases $x_i$, so carrier $i$ sets a lower transport price. The strategic motive of carrier $i$ makes the transport price the lowest among all of the schemes. Corresponding to this low transport price, the investment level becomes the largest. On the one hand, in the simultaneous move scenario, firms cannot directly reduce transport prices by setting a smaller investment; hence, the investment is larger than that in the floating-price contract case ($x_i^a > x_i^l$). Carriers also cannot directly raise investment by setting a lower transport price; thus, the investment is smaller than that in the fixed-price contract case ($x_i^a < x_i^{fx}$). Part (ii) is intuitive. A smaller $\gamma$ improves efficiency in R&D and strengthens innovation incentives. The investment rises as $\gamma$ decreases (see Fig. 1).

\[
\frac{x_i^k}{(a-c)}
\]

![Figure 1: R&D investment](image)

Note: Black line is $k = fx$; Gray line is $k = l$; Dashed line is $k = s$.

Eqs. (2), (8), and (11) yield Lemma 2.
Lemma 2. (i) \( t_s^i > t_l^i > t^{fx}_s \). (ii) If \( \gamma > 2(\sqrt{15} + 5)/3 \approx 5.91532 \), \( \partial t^{fx}_i / \partial \gamma < 0 \), and if \( \gamma < 2(\sqrt{15} + 5)/3 \), \( \partial t^{fx}_i / \partial \gamma > 0 \). \( \partial t_l^i / \partial \gamma < 0 \) and \( \partial t_s^i / \partial \gamma < 0 \).

Part (i) of Lemma 2 has the following intuitive explanation. In the fixed-price contract, carriers commit to lower transport prices in order to increase investment, and therefore, the price in that scheme becomes the lowest. In the floating-price contract and simultaneous move scenario, the firms’ investment level is a given for carriers. In these schemes, if a realized investment becomes larger, firms’ production expands, incentivizing carriers to set a higher price. As shown in Lemma 1, investment in the simultaneous move scenario is larger than that in the floating-price contract, so the transport price in the simultaneous move scenario is higher than that in the floating-price contract.

We next consider part (ii) of Lemma 2. As in part (ii) of Lemma 1, a smaller \( \gamma \) enhances investment incentives. In the fixed-price contract, carriers can raise investments by committing to lower transport prices. If carriers reduce transport prices when \( \gamma \) is small enough, and hence firms’ investment incentives are large enough, it is possible to further increase investments. Hence, when \( \gamma \) falls below a certain level, the transport price also decreases. By contrast, in a floating-price contract and simultaneous move scenario, carriers have no transport price commitment. An increase in investment due to a lower \( \gamma \) promotes production activities, so the transport price increases as \( \gamma \) decreases. Fig. 2 illustrates Lemma 2.

Eqs. (4), (9), and (13) yield Lemma 3.

Lemma 3. (i) For exports: \( q^{fx}_{ij} > q^s_{ij} > q^{d}_{ij} \); for domestic supply: if \( \gamma > (\sqrt{23521} + \)
Note: Black line is $k = f; x; Gray line is $k = l; Dashed line is $k = s.$

\[
\frac{239}{225} \approx 1.74385, \quad q_{ii}^l > q_{ii}^{fx} > q_{ii}^s \quad \text{and if } \gamma < \left(\frac{\sqrt{23521} + 239}{225}\right), \quad q_{ii}^s > q_{ii}^l > q_{ii}^{fx}.
\]

(ii) Comparative statics of exports: $\partial q_{ij}^k / \partial \gamma < 0$ for all $k \in \{f, l, s\}$. Comparative statics of domestic supply: if $\gamma < \hat{\gamma} \approx 1.48449, \partial q_{ii}^{fx} / \partial \gamma > 0$ and if $\gamma > \hat{\gamma}, \partial q_{ii}^{fx} / \partial \gamma < 0. \partial q_{ii}^l / \partial \gamma < 0$ and $\partial q_{ii}^s / \partial \gamma < 0.

We first consider the ranking in output. Although a lower transport price promotes exports and impedes domestic supply, $t_i^s > t_i^l$ and $q_{ij}^s > q_{ij}^l$ hold. These results depend on an investment ranking of $x_i^s > x_i^l.$ In the floating-price contract, firms commit to a smaller investment. This commitment lowers the degree of the production cost reduction, so it does not sufficiently promote the whole production, and hence leads to $q_{ij}^s > q_{ij}^l$ and $q_{ii}^s > q_{ii}^l.$ Comparing the simultaneous move scenario with the fixed-price contract, $q_{ij}^{fx} > q_{ij}^s$ and $q_{ii}^{fx} > q_{ii}^s.$ This corresponds to the fact that the transport price is the highest in the simultaneous move scenario and is the lowest in the fixed-price contract among all schemes, $t_i^s > t_i^l > t_i^{fx}$ (part (i) of Lemma 1). Carriers commit to
a lower transport price in the fixed-price contract. The carrier’s commitment lowers the trade barrier and promotes exports. Because competition in the domestic market becomes more intense, domestic supply decreases. On the one hand, the simultaneous move scenario will have a higher transport price, which impedes exports and promotes domestic supply. From these arguments, the export ranking is $q_{ij}^{fx} > q_{ij}^{s} > q_{ij}^{l}$, and the domestic supply ranking is $q_{ii}^{s} > q_{ii}^{l}$ and $q_{ii}^{s} > q_{ii}^{fx}$.

The domestic supply ranking between the fixed-price and floating-price contracts and part (ii) of Lemma 3 are explained as follows. A smaller $\gamma$ enhances firm’s innovation incentives and raises investment (part (ii) of Lemma 1). Because a decrease in $\gamma$ leads to a reduction in production cost, a decrease in $\gamma$ increases both domestic supply and exports. However, if $\gamma$ is small, the domestic supply in the fixed-price contract decreases as $\gamma$ decreases. This result depends on a change in the transport price for $\gamma$. As in part (ii) of Lemma 2, when $\gamma$ is small, the transport price in the fixed-price contract falls as $\gamma$ decreases. A lower transport price promotes exports, increases competition in the domestic market, and decreases domestic supply. Hence, $\partial q_{ii}^{fx} / \partial \gamma > 0$ if $\gamma$ is small. Furthermore, while a lower $\gamma$ reduces the domestic supply in the fixed-price contract, it increases the domestic supply in the floating-price contract. Therefore, if $\gamma$ is small enough, the domestic supply in the fixed-price contract can be smaller than that in the floating-price contract. Fig. 3 illustrates Lemma 3.

Comparing (5), (10), and (14), we establish Proposition 1.

**Proposition 1.** (i) $\Pi_{i}^{l} > \Pi_{i}^{s} > \Pi_{i}^{fx}$. (ii) If $\gamma < \gamma^{*} \simeq 1.74661$, $\pi_{i}^{s} > \pi_{i}^{l} > \pi_{i}^{fx}$, and if $\gamma > \gamma^{*}$, $\pi_{i}^{s} > \pi_{i}^{fx} > \pi_{i}^{l}$.
This result implies that a better outcome for both carriers and firms can appear when carriers do not commit to a transport price level. The ranking in firm’s profit (part (i) of Proposition 1) inversely corresponds to the ranking in exports (see part (i) of Lemma 3). This is because, in our two-way duopoly model, the prohibitive transport price level gives the highest profit for firms. That is, the profit in a domestic monopoly (there is no export) is higher than the situation with aggressive exports.\textsuperscript{10} When firms can commit to an investment level, it is possible for them to create a situation close to a domestic monopoly because they can adjust the transport price through their investment decision. Hence, the floating-price contract is the best scheme for firms. In contrast, the carrier’s price commitment adjusts the firm’s investments and exports. The carrier’s lower price commitment increases the aggressiveness of the firm’s export activities, and it puts firms in a situation furthest from a domestic monopoly. The fixed-price contract is the worst for firms. In the simultaneous move scenario, carriers

\textsuperscript{10}Using the third-stage outcomes and conditions $x_j = x_i$ and $t_j = t_i$, this fact is immediately found.
and firms have no commitment, so the profit in that scheme is intermediate for firms.

The carrier’s profit ranking (part (ii) of Proposition 1) depends on the transport price and export volume. In the simultaneous move scenario, the transport price is highest among all contract schemes and the exports are of intermediate size, so the profit becomes the largest among all schemes. On the one hand, profits in fixed-price and floating-price contracts can be reversed according to the degree of $\gamma$. This is because, as in part (ii) of Lemma 3, a smaller $\gamma$ increases exports in both schemes and raises the transport price in the floating-price contract, while it can reduce the transport price in the fixed-price contract (see also Fig. 2). Then, a smaller $\gamma$ increases profit in the floating-price contract. In contrast, a smaller $\gamma$ decreases profit in the fixed-price contract when $\gamma$ is small. If $\gamma$ is small enough, (i.e., $\gamma < \gamma^*$), profit in the fixed-price contract is the worst among all contract schemes.

![Figure 4: Graph of profits (firm’s profit on the left; carrier’s profit on the right)](image)

Note: Black line is $k = f x$; Gray line is $k = l$; Dashed line is $k = s$.

**Welfare analysis.** Here, we consider welfare in all contract schemes. Country $i$’s consumer surplus in each contract scheme is given by $CS_i^k = (q_{it}^k + q_{jt}^k)^2 / 2$, where
$k \in \{fx, l, s\}$. This yields the following:

\[
CS_k^x = \frac{9\gamma^2(189\gamma^2 - 276\gamma + 80)^2(a-c)^2}{32(9\gamma - 4)^2(9\gamma - 10)^2(3\gamma - 1)^2}; \quad CS_l^i = \frac{3528\gamma^2(a-c)^2}{(144\gamma - 43)^2},
\]

\[
CS_s^i = \frac{441\gamma^2(a-c)^2}{8(18\gamma - 7)^2}.
\]

Each country’s social surplus consists of consumer surplus $CS_k^i$, firm’s profit $\Pi_k^i$, and carrier’s profit $\pi_k^i$. The social surplus in each contract scheme is

\[
W_k^i = \frac{\gamma(86751\gamma^4 - 251424\gamma^3 + 249120\gamma^2 - 96960\gamma + 12800)(a-c)^2}{32(9\gamma - 10)^2(3\gamma - 1)^2(9\gamma - 4)},
\]

\[
W_l^i = \frac{\gamma(8568\gamma - 1849)(a-c)^2}{(144\gamma - 43)^2}; \quad W_s^i = \frac{7\gamma(153\gamma - 56)(a-c)^2}{8(18\gamma - 7)^2}.
\]

Eqs. (15) and (16) yield the following result.

**Proposition 2.** (i) $CS_k^x > CS_s^i > CS_l^i$ and $W_k^x > W_s^i > W_l^i$. (ii) $\partial CS^k_i / \partial \gamma < 0$ and $\partial W^k_i / \partial \gamma < 0$.

The welfare ranking corresponds to the investment ranking (see part (i) of Lemma 1). When firms increase their investment level, the marginal production cost falls and production efficiencies improve, which undoubtedly increases total sales and consumer surplus. Because the improvement in consumer benefit is dominant, the social surplus also increases. From the welfare point of view, the best scheme is the fixed-price contract in which investment is maximized, and the worst scheme is the floating-price contract in which investment is minimized. Furthermore, a reduction in $\gamma$ enhances efficiencies in R&D and raises investment, so it also increases welfares.
4 Discussion

We have two robustness checks on the main result (Proposition 1). We first relax the assumption of no R&D spillover and examine a case with a positive spillover. Subsequently, we argue a case containing domestic and foreign product differentiation.

4.1 R&D spillovers

Here, we introduce an exogenous spillover rate of R&D, $\delta$, in the previous setting. Since the developed-knowledge of the rival firm is transmitted, firm $i$’s marginal production cost is rewritten as $c - x_i - \delta x_j$ ($i \neq j$).

We assume that the spillover rate is not very large: $0 \leq \delta < (73 - 3\sqrt{377})/44$ ($\approx 0.33529$). We exclude the case with a high spillover rate for the following reason. In the previous section, R&D investments are strategic substitutes. However, under a rather high spillover rate, the investment decisions become strategic complements. Hence, under a high spillover rate, we do not have a mechanism in the previous section and we obtain a significantly different profit ranking. Hence, we omit the case with a high spillover rate.

Firm $i$’s profit is $\Pi_i(\delta) = (a - q_{ii} - q_{ji} - (c - x_i - \delta x_j))q_{ii} + (a - q_{jj} - q_{ij} - (c - x_i - \delta x_j) - t_i)q_{ij} - \gamma x_i^2$ and carrier $i$’s profit is $\pi_i(\delta) = t_i q_{ij}$. We consider the same contract schemes and the same timing of the game. The equilibrium outcomes under
the fixed-price contract are

\[
    x_i^{fx}(\delta) = \frac{(a - c)(\delta - 2)[-189\gamma^2 + 6\gamma(10\delta^2 - 43\delta + 46) + 20(\delta - 2)^2(\delta^2 - 1)]}{(9\gamma + 2\delta^2 - 2\delta - 4)[10\gamma^2 - 39\gamma(\delta - 2)^2 - 10(\delta - 2)^2(\delta^2 - 1)]},
\]

\[
    q_{ii}^{fx}(\delta) = \frac{3\gamma(a - c)[135\gamma^2 - 3\gamma(16\delta^2 - 67\delta + 70) - 16(\delta - 2)^2(\delta^2 - 1)]}{(9\gamma + 2\delta^2 - 2\delta - 4)[10\gamma^2 - 39\gamma(\delta - 2)^2 - 10(\delta - 2)^2(\delta^2 - 1)]},
\]

\[
    q_{ij}^{fx}(\delta) = \frac{3\gamma(a - c)[54\gamma^2 - 3\gamma(4\delta^2 - 19\delta + 22) - 4(\delta - 2)^2(\delta^2 - 1)]}{(9\gamma + 2\delta^2 - 2\delta - 4)[10\gamma^2 - 39\gamma(\delta - 2)^2 - 10(\delta - 2)^2(\delta^2 - 1)]},
\]

\[
    t_i^{fx}(\delta) = \frac{9\gamma(a - c)(3\gamma - 2\delta^2 + 6\delta - 4)}{10\gamma^2 - 39\gamma(\delta - 2)^2 - 10(\delta - 2)^2(\delta^2 - 1)},
\]

\[
    \Pi_i^{fx}(\delta) = [q_{ii}^{fx}(\delta)]^2 + [q_{ij}^{fx}(\delta)]^2 - \gamma[x_i^{fx}(\delta)]^2; \quad \pi_i^{fx}(\delta) = t_i^{fx}(\delta)q_{ij}^{fx}(\delta).
\]

The equilibrium outcomes under the floating-price contract are

\[
    x_i^{f}(\delta) = \frac{(a - c)(43 - 14\delta)}{144\gamma + 14\delta^2 - 29\delta - 43}; \quad q_{ii}^{f}(\delta) = \frac{60\gamma(a - c)}{144\gamma + 14\delta^2 - 29\delta - 43},
\]

\[
    q_{ij}^{f}(\delta) = \frac{24\gamma(a - c)}{144\gamma + 14\delta^2 - 29\delta - 43}; \quad t_i^{f}(\delta) = \frac{36\gamma(a - c)}{144\gamma + 14\delta^2 - 29\delta - 43},
\]

\[
    \Pi_i^{f}(\delta) = [q_{ii}^{f}(\delta)]^2 + [q_{ij}^{f}(\delta)]^2 - \gamma[x_i^{f}(\delta)]^2; \quad \pi_i^{f}(\delta) = t_i^{f}(\delta)q_{ij}^{f}(\delta).
\]

The equilibrium outcomes under the simultaneous move scenario are

\[
    x_i^{s}(\delta) = \frac{7(a - c)(2 - \delta)}{36\gamma + 7(\delta^2 - \delta - 2)}; \quad q_{ii}^{s}(\delta) = \frac{15\gamma(a - c)}{36\gamma + 7(\delta^2 - \delta - 2)},
\]

\[
    q_{ij}^{s}(\delta) = \frac{6\gamma(a - c)}{36\gamma + 7(\delta^2 - \delta - 2)}; \quad t_i^{s}(\delta) = \frac{9\gamma(a - c)}{36\gamma + 7(\delta^2 - \delta - 2)},
\]

\[
    \Pi_i^{s}(\delta) = [q_{ii}^{s}(\delta)]^2 + [q_{ij}^{s}(\delta)]^2 - \gamma[x_i^{s}(\delta)]^2; \quad \pi_i^{s}(\delta) = t_i^{s}(\delta)q_{ij}^{s}(\delta).
\]

Now, we can check the robustness of Proposition 1. Under our assumptions, \(0 \leq \delta < (73 - 3\sqrt{377})/44\) and \(\gamma > 4/3\), we compare equilibrium profit. By numerical calculation, we have \(\Pi_1^f(\delta) > \Pi_1^s(\delta) > \Pi_1^{fx}(\delta)\). Thus, part (i) of Proposition 1 does not change.

We next consider the profit ranking of carrier \(i\). Numerical calculation yields Fig.
5. In the lower left area, we have $\pi_i^s(\delta) > \pi_i^l(\delta) > \pi_i^{fx}(\delta)$; in the other area, we obtain $\pi_i^s(\delta) > \pi_i^{lx}(\delta) > \pi_i^l(\delta)$. Hence, part (ii) of Proposition 1 does not change if the spillover rate is small. On the other hand, a high spillover rate yields only the profit ranking $\pi_i^s(\delta) > \pi_i^{l}(\delta) > \pi_i^{l}(\delta)$.

![Figure 5: Profit ranking for carrier $i$ with spillover](image)

Our results change for the following reason. Under a positive spillover rate, an increase in R&D investment reduces the rival firm’s marginal cost. Then, firms have a small incentive to invest. This case is similar to that with inefficient investment technology (i.e., large $\gamma$). In other words, the effect of an increase in $\delta$ is similar that of an increase in $\gamma$. In our model, the investment level plays an important role so we therefore obtain Fig. 5.
4.2 Differentiated products

We consider the effects of product differentiation here. To exclude the effect of market expansion by product differentiation (Singh and Vives, 1984), we employ the Shubik (1980)-type utility function:11

\[ u_i = a (q_{ii} + q_{ji}) - \left( (1 - \beta) \left( q_{ii}^2 + q_{ji}^2 \right) + \frac{\beta}{2} (q_{ii} + q_{ji})^2 \right), \quad j \neq i. \]

\( \beta \in [0,1] \) is interpreted as the degree of product differentiation. That is, products are homogeneous at \( \beta = 1 \) and are independent at \( \beta = 0 \). Moreover, under this utility function, the aggregate demand in country \( H \) or \( F \), \( q_{ii} + q_{ji} \), does not depend on the degree of product differentiation. In particular, we have \( q_{ii} + q_{ji} = a - (p_{ii} + p_{ji})/2 \).

Hence, we can exclude the market expansion effect.

Solving the utility maximization problem, we have the inverse demand \( p_{ii} = a - (2 - \beta)q_{ii} - \beta q_{ji} \) and \( p_{ij} = a - (2 - \beta)q_{ij} - \beta q_{jj} \). Then, firm \( i \)'s profit is \( \Pi_i(\beta) = (a - (2 - \beta)q_{ii} - (c - x_i))q_{ii} + (a - (2 - \beta)q_{ij} - \beta q_{jj} - (c - x_i) - t_i)q_{ij} - \gamma x_i^2 \) and carrier \( i \)'s profit is \( \pi_i(\beta) = t_i q_{ij} \). We consider the same contract schemes and the same timing of the game. The equilibrium outcomes under the fixed-price contract are

\[ x_{ix}^f(\beta) = \frac{(a - c)(2 - \beta)\Phi_3}{2\Phi_1 \Phi_2}; \quad q_{ii}^f(\beta) = \frac{(a - c)(16 - 16\beta + 3\beta^2)\Phi_1}{4(2 - \beta)\Phi_1 \Phi_2}, \]

\[ q_{ij}^f(\beta) = \frac{(a - c)(16 - 16\beta + 3\beta^2)\gamma \Phi_5}{2\Phi_1 [9\beta^4 \gamma + \beta^3(6 - 96\gamma) + 8\beta^2(-5 + 44\gamma)\beta(88 - 512\gamma) + 64(-1 + 4\gamma)]}, \]

\[ t_i^f(\beta) = \frac{(a - c)(16 - 16\beta + 3\beta^2)\gamma \Phi_6}{4(2 - \beta)\Phi_2}, \]

\[ \Pi_i^f(\beta) = (2 - \beta) \left( [q_{ii}^f(\beta)]^2 + [q_{ij}^f(\beta)]^2 \right) - \gamma [x_{ix}^f(\beta)]^2; \quad \pi_i^f(\beta) = t_i^f(\beta)q_{ij}^f(\beta), \]

\(^{11}\)Our qualitative results do not change when using the utility function proposed by Singh and Vives (1984).
where we define $\Phi_m \ (m = 1, \ldots, 6)$ as in the appendix.

The equilibrium outcomes under the floating-price contract are

\[
x^f_i(\beta) = \frac{(a-c)(320-448\beta+200\beta^2-29\beta^3)}{\Psi_1}; \quad q^f_{ii}(\beta) = \frac{4(a-c)(8-3\beta)(16-16\beta+3\beta^2)\gamma}{\Psi_1},
\]
\[
q^f_{ij}(\beta) = \frac{8(a-c)(2-\beta)(16-16\beta+3\beta^2)\gamma}{\Psi_1}; \quad t^f_i(\beta) = \frac{4(a-c)(16-16\beta+3\beta^2)^2\gamma}{\Psi_1},
\]
\[
\Pi^f_i(\beta) = (2-\beta)\left([q^f_{ii}(\beta)]^2 + [q^f_{ij}(\beta)]^2\right) - \gamma[x^f_i(\beta)]^2; \quad \pi^f_i(\beta) = t^f_i(\beta)q^f_{ij}(\beta),
\]

where $\Psi_1 \equiv 48\beta^4\gamma + \beta^3(29 - 544\gamma) + 8\beta^2(272\gamma - 25) - 448\beta(8\gamma - 1) + 64(32\gamma - 5)$. Here, we consider the robustness of Proposition 1. Under our assumptions, $0 < 4/3$, we compare equilibrium profit. By numerical calculation, we depict the profit ranking of firm $i$ in Fig. 6. On the right, we find $\Pi^f_i(\beta) > \Pi^f_i(\beta) > \Pi^f_i(\beta)$; in the middle, we have $\Pi^f_i(\beta) > \Pi^f_i(\beta) > \Pi^f_i(\beta)$; and on the left, we obtain $\Pi^f_i(\beta) > \Pi^f_i(\beta) > \Pi^f_i(\beta)$. That is, in the case with higher product differentiation, the firms with fixed-price contracts earn larger profits.

The reason is as follows. When products are highly differentiated, competitions between firms become less important. Then, the firms’ profits depend on vertical relationships between the firms and carriers. Hence, a commitment toward aggressive
investment will increase profit in a channel. Therefore, product differentiation brings higher profits to the firms.

Next, we consider the profit ranking of carrier \( i \). Fig. 7 depicts the numerical calculation. In the lower right area, we have \( \pi_i^s(\beta) > \pi_i^l(\beta) > \pi_i^{fx}(\beta) \); in the middle, we find \( \pi_i^s(\beta) > \pi_i^{fx}(\beta) > \pi_i^l(\beta) \); and on the left, we obtain \( \pi_i^{fx}(\beta) > \pi_i^s(\beta) > \pi_i^l(\beta) \). Hence, under higher product differentiation, carriers prefer fixed-price contracts. The intuition behind this result is similar as in the case of firms.

Figure 6: Profit ranking for firm \( i \) with product differentiation
5 Conclusion

In a vertical production relationship, upstream trading firms likely hold up downstream R&D investment. If these upstream firms set a higher input price after observing downstream investment, then they can extract the downstream R&D benefit, and such opportunistic behavior reduces downstream innovation incentives. The previous literature emphasizes that a fixed-price contract for the input price is required to overcome this hold-up problem. In the fixed-price contract, upstream firms commit to an input price level and downstream firms subsequently invest in cost-reducing R&D. By employing the fixed-price contract, upstream firms set a lower input price to promote downstream
investment. Since this lower-price commitment increases outputs and demand for inputs through investment expansion, all firms become better off.

In contrast to this standard theory, we show that the fixed-price contract can harm all firms. We consider a two-country, two-way trade model with two firm-specific carriers upstream, and two exporters downstream. Each country has a carrier and an exporting firm. While each country’s exporting firm freely supplies to the domestic market, it uses a local carrier to export its product. Each carrier charges a transport price and conveys its domestic exporting firm’s product. In this setting, exporters can create a situation close to a domestic monopoly when the transport price is high enough. Although the domestic monopoly is most profitable for exporters, the fixed-price contract lowers the transport price and encourages firms to invest and export aggressively, creating a market furthest from a domestic monopoly. This makes exporters worse off. Furthermore, if R&D efficiency is high enough, carriers set considerably low transport prices in the fixed-price contract. This also makes carriers worse off. Moreover, we examine the robustness of the main result in two different situations: a case with positive R&D spillover and a case with product differentiation. We find that in these extended cases, our main result holds.

This study shows that a fixed-price transportation contract can harm all firms in both upstream and downstream markets when downstream (exporting) firms engage in cost-reducing R&D. Therefore, it would be interesting to investigate other forms of R&D, such as product innovation and product quality improvement to determine whether our result holds. However, this issue is beyond the scope of this study, and we
thus leave it to future research.

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Appendix

Proof of Lemma 1. (i) Comparing investments, we get

\[ x_i^f - x_i^s = \frac{9\gamma(15\gamma - 8)(a - c)}{2(3\gamma - 1)(9\gamma - 10)(9\gamma - 4)(18\gamma - 7)} > 0; \quad x_i^s - x_i^l = \frac{234\gamma(a - c)}{(18\gamma - 7)(144\gamma - 43)} > 0. \]

(ii) Differentiating (3), (7), and (12) w.r.t. \( \gamma \), we obtain

\[ \frac{\partial x_i^f}{\partial \gamma} = -\frac{27(1701\gamma^4 - 4968\gamma^3 + 5130\gamma^2 - 2160\gamma + 320)(a - c)}{2(9\gamma - 4)^2(9\gamma - 10)^2(3\gamma - 1)^2} < 0, \]

\[ \frac{\partial x_i^l}{\partial \gamma} = -\frac{6192(a - c)}{(144\gamma - 43)^2} < 0; \quad \frac{\partial x_i^s}{\partial \gamma} = \frac{126(a - c)}{(18\gamma - 7)^2} < 0. \]

Q.E.D.

Proof of Lemma 2. (i) Comparing transport prices, we get

\[ t_i^s - t_i^l = \frac{117\gamma(a - c)}{2(18\gamma - 7)(144\gamma - 43)} > 0; \quad t_i^l - t_i^f = \frac{27\gamma(27\gamma - 4)(a - c)}{4(3\gamma - 1)(9\gamma - 10)(144\gamma - 43)} > 0. \]

(ii) Differentiating (2), (8), and (11) w.r.t. \( \gamma \), we have

\[ \frac{\partial t_i^s}{\partial \gamma} = \frac{-1548(a-c)}{(144\gamma-43)^2} < 0; \quad \frac{\partial t_i^l}{\partial \gamma} = \frac{-63(a-c)}{(18\gamma-7)^2} < 0; \quad \frac{\partial t_i^f}{\partial \gamma} = \frac{-9(9\gamma^2-60\gamma+40)(a-c)}{4(9\gamma-10)^2(3\gamma-1)^2}. \]

From the last equation, \( \partial t_i^f/\partial \gamma < (\geq 0) \) if \( \gamma > (\leq 2(\sqrt{15} + 5)/3 \approx 5.91532. \)

Q.E.D.

Proof of Lemma 3. (i) Comparing exports, we get

\[ q_{ij}^f - q_{ij}^s = \frac{9\gamma(15\gamma - 8)(a - c)}{2(9\gamma - 10)(9\gamma - 4)(18\gamma - 7)} > 0; \quad q_{ij}^s - q_{ij}^l = \frac{39\gamma(a - c)}{(18\gamma - 7)(144\gamma - 43)} > 0. \]
Comparing domestic supplies, we have
\[ q^i_{ii} - q^l_{ii} = \frac{195\gamma(a-c)}{2(18\gamma-7)(144\gamma-43)} > 0; \quad q^s_{ii} - q^f_{ii} = \frac{9\gamma(3\gamma-2)(15\gamma-8)(a-c)}{4(3\gamma-1)(9\gamma-10)(9\gamma-4)(18\gamma-7)} > 0, \]
\[ q^f_{ii} - q^l_{ii} = \frac{3\gamma(675\gamma^2-1434\gamma+448)(a-c)}{4(3\gamma-1)(9\gamma-10)(9\gamma-4)(144\gamma-43)}. \]
From \( q^f_{ii} - q^l_{ii} \), solving \( 675\gamma^2 - 1434\gamma + 448 \geq 0 \) w.r.t. \( \gamma \), we have \( q^f_{ii} < (\geq) q^l_{ii} \) if \( \gamma < \)
\( (\geq) \sqrt{23521 + 239}/225 \approx 1.74385. \) (ii) From (4), (9), and (13), we get
\[ \frac{\partial q^l_{ii}}{\partial \gamma} = \frac{-1032(a-c)}{(144\gamma-43)^2} < 0; \quad \frac{\partial q^s_{ii}}{\partial \gamma} = \frac{-21(a-c)}{(18\gamma-7)^2} < 0; \quad \frac{\partial q^f_{ii}}{\partial \gamma} = \frac{-3(243\gamma^2-360\gamma+160)(a-c)}{(9\gamma-4)^2(9\gamma-10)^2} < 0, \]
\[ \frac{\partial q^l_{ii}}{\partial \gamma} = \frac{-2580(a-c)}{(144\gamma-43)^2} < 0; \quad \frac{\partial q^s_{ii}}{\partial \gamma} = \frac{-105(a-c)}{2(18\gamma-7)^2} < 0, \]
\[ \frac{\partial q^f_{ii}}{\partial \gamma} = \frac{-3(10935\gamma^4-35316\gamma^3+38484\gamma^2-16800\gamma+2560)(a-c)}{4(3\gamma-1)^2(9\gamma-10)^2(9\gamma-4)^2}. \]
From the last equation, \( \frac{\partial q^f_{ii}}{\partial \gamma} > (\leq) 0 \) if \( \gamma < (\geq) \hat{\gamma} \approx 1.48449. \) Q.E.D.

**Proof of Proposition 1.** (i) Comparing the firm’s profits, we get
\[ \Pi^s_i - \Pi^l_i = \frac{117\gamma^2(5904\gamma - 1945)(a-c)^2}{4(18\gamma-7)^2(144\gamma-43)^2} > 0, \]
\[ \Pi^s_i - \Pi^f_i = \frac{9\gamma^2(15\gamma-8)(87485\gamma^4-12879\gamma^3-864\gamma^2+4440\gamma-1024)(a-c)^2}{16(3\gamma-1)^2(9\gamma-10)^2(9\gamma-4)^2(18\gamma-7)^2} > 0. \]
(ii) Comparing the carrier’s profits, we have
\[ \pi^s_i - \pi^l_i = \frac{3159\gamma^2(32\gamma-11)(a-c)^2}{2(18\gamma-7)^2(144\gamma-43)^2} > 0, \quad \pi^s_i - \pi^f_i = \frac{27\gamma^2(15\gamma-8)(27\gamma-4)(a-c)^2}{8(3\gamma-1)(9\gamma-10)^2(9\gamma-4)(18\gamma-7)^2} > 0, \]
and \( \pi^f_i - \pi^l_i = \frac{27\gamma^2(101088\gamma^3-285309\gamma^2-214602\gamma-43232)(a-c)^2}{8(3\gamma-1)(9\gamma-10)^2(9\gamma-4)(144\gamma-43)^2}. \) From the last equation, we find that \( \pi^f_i - \pi^l_i \leq (>) 0 \) if \( \gamma \leq (>) \gamma^* \approx 1.74661. \) These imply Proposition 1. Q.E.D.

**Proof of Proposition 2.** (i) From (15), we have
\[ CS^f_i - CS^s_i = \frac{81\gamma^3(15\gamma-8)(6804\gamma^3-12717\gamma^2+6816\gamma-1120)(a-c)^2}{32(3\gamma-1)^2(9\gamma-10)^2(9\gamma-4)^2(18\gamma-7)^2} > 0. \]
\[ \text{Definition of } \Phi_m. \text{ We define } \Phi_m (m = 1, \ldots, 6) \text{ as follows: } \Phi_1 \equiv 3\beta^3 \gamma + \beta^2 (4 - 28 \gamma) + 16\beta (5 \gamma - 1) - 64 \gamma + 16, \Phi_2 \equiv 27 \beta^6 \gamma + 27 \beta^5 (1 - 16 \gamma) \gamma + 6 \beta^4 (1 - 55 \gamma + 456 \gamma^2) - 4 \beta^3 (13 - 386 \gamma + 2176 \gamma^2) + 24 \beta^2 (7 - 144 \gamma + 608 \gamma^2) - 16 \beta (15 - 232 \gamma + 768 \gamma^2) + 128 (1 - 12 \gamma + 32 \gamma^2), \\
\Phi_3 \equiv 135 \beta^7 \gamma - 36 \beta^6 \gamma (69 \gamma - 5) + 48 \beta^5 (1 - 53 \gamma + 393 \gamma^2) - 16 \beta^4 (32 - 909 \gamma + 4772 \gamma^2) + 128 \beta^3 (17 - 337 \gamma + 1386 \gamma^2) - 512 \beta^2 (9 - 137 \gamma + 462 \gamma^2) + 256 \beta (19 - 232 \gamma + 656 \gamma^2) - 2048 (1 - 10 \gamma + 24 \gamma^2), \Phi_4 \equiv 81 \beta^7 \gamma - 6 \beta^6 \gamma (252 \gamma - 19) + 8 \beta^5 (4 - 205 \gamma + 1458 \gamma^2) - 8 \beta^4 (44 - 1193 \gamma + 6000 \gamma^2) + 128 \beta^3 (12 - 225 \gamma + 886 \gamma^2) - 256 \beta^2 (13 - 186 \gamma + 600 \gamma^2) + 512 \beta (7 - 80 \gamma + 216 \gamma^2) - 512 (3 - 28 \gamma + 64 \gamma^2), \Phi_5 \equiv -9 \beta^4 \gamma - \beta^3 (8 - 96 \gamma) - 16 \beta^2 (22 \gamma - 3) - \beta (96 - 512 \gamma) - 64 (4 \gamma - 1), \\
\text{and } \Phi_6 \equiv -9 \beta^3 \gamma - \beta^2 (4 - 60 \gamma) - 16 \beta (7 \gamma - 1) + 64 \gamma - 16. \]
References


