When is the city sustainable?: A unified model of von Thünen and Melitz

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Abstract

We develop a continuous location-space model with heterogeneous firms and investigate the conditions under which the spatial concentration of economic activities is sustainable. Introducing firm heterogeneity, we can assess the difference of the number of consumable goods among locations, and show that it makes cities more attractive and strengthens their *lock-in effect*. Furthermore, the present model can analyze how the spatial structure depends on the transport technology and population size in the economy. Particularly, if transport costs of agricultural goods are too high, the supply of agricultural goods to the single city will never be sufficiently large to maintain the city for any population size.

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1. Introduction

The purpose of this paper is to develop a monopolistic competition model of a spatial economy with heterogeneous firms and to ascertain the conditions under which the spatial concentration of economic activities is sustainable.

Today, perhaps, many economists can agree that the most important feature of economic geography is the spatial concentration of economic activities in cities. Of course, even now that many metropolitan areas exist throughout the world, the concentration of economic activities is not only a present issue but also is a historical phenomenon.[1] Therefore, it is natural that the spatial structure of economic activities has persisted as a major concern of regional scientists and urban economists for a long time. Particularly, urban economists have pondered two fundamental questions, i.e., *Why are cities formed?* and *Why are these cities maintained for such a long time?*

Urban economists, such as Henderson (1974) have usually emphasized the presence of externalities related to the agglomeration of economic activities, called the *agglomeration economies*, which make cities more attractive and productive places because of denser markets, increased consumption diversity, nearly costless transportation, knowledge spillovers and other characteristics. However even though the agglomeration economies can explain why and how cities form in response to economic forces, the fundamental questions have remained unanswered because agglomeration economies remain as black boxes.

Abdel-Rahman (1998) and Fujita (1988) presented pioneering studies undertaken to explain the agglomeration of economic activities without treating the agglomeration economies as black boxes. Both introduced Dixit and Stiglitz (1977) type monopolistic competition into Alonso (1964) and von Thünen (1826) type models featuring a continuous location-space.[2] Krugman (1991) developed two symmetric region model in which no region has spatial extent. Since then, various extensions have been developed to analyze the spatial structure of economic activities. Collectively, they are called the new economic geography (NEG).[3]

To address the questions of why cities are formed and maintained, the NEG model employs two key concepts: *forward linkage* and *backward linkage*. The availability of a large variety of consumption goods in a location increases the real income of workers there (forward linkage). Numerous consumers in a location supports numerous specialized firms there (back ward linkage).[4] Based on this circular causality, a small initial disparity between

[1]For example, Rome and Pompeii of the Roman Empire, Moenjo-Daro of the Indus civilization, and Ur of the Mesopotamian civilization. These examples are evidence that cities existed even in ancient times.

[2]A continuous location-space model often used in the urban economics (see Fujita (1989) for details). In NEG, the continuous location-space type model is classified into two types. One type resembles that of von Thünen (1826). Each region is represented by a point on the real number line, which we call point location. This type of model, in NEG, was first developed by Fujita (1988). Since then, some extensions have been made: Behrens (2007), Fujita and Hamaguchi (2001), Fujita and Krugman (1995), Fujita et al. (1999a), Fujita and Mori (1996), and Fujita and Mori (1997). The other is like Alonso (1964). In this type model, regions are expressed discretely, but each region has continuous space and workers commute to a central business district in each region. See Krugman and Elizondo (1996), Murata and Thisse (2005), Ottaviano et al (2002) and Tabuchi (1998) for examples.

[3]See Baldwin et al. (2003), Combes et al. (2008), Fujita et al. (1999b), and Fujita and Thisse (2002) for details related to the NEG model.

[4]These linkages are related to the Marshallian externalities. See Marshall (1920). The explanation of

regions tends to expand over time. Furthermore, for whatever reason, once it is formed, the city tends to be sustainable. This mechanism, which causes the concentration of economic activities, is plausible and NEG models provide amazing insight into determination of the spatial structure. Even so, however, it must be said that NEG models present some weak points.

In usual NEG models, firms in the manufacturing industry are assumed as homogeneous for simplicity. However, some empirical studies using firm and plant-level data suggest that there are the persistent productivity differences among firms and that firm's export status is related closely with its productivity (e.g., see Bernard and Jensen (1999) and Roberts and Tybout (1997)). Therefore, we must treat results obtained using NEG models carefully because these results might depend strongly on firm homogeneity.

Another, perhaps more important, weak point is that the number of consumable goods is the same at every region in usual NEG models. In the real world, an important difference between cities and peripheral areas is the number of consumable goods. For example, a person will have more opportunities to see art exhibitions, movies and plays when living in a big city because there might be no art museum, movie theater, and playhouse in a small town or rural area. After all, The Metropolitan Museum is only in New York City! Consequently, if consumers have a taste for variety, then they are attracted to big cities.[5] The usual model in NEG cannot explain this phenomenon. Surely, it also refers to the importance of accessibility to consumption goods and the love of variety in consumer tastes. However, actually, the usual type model in NEG does not consider the difference in the number of consumable goods between regions, and the accessibility to consumption goods is expressed only by high-low goods prices (consumers are attracted to the location in which goods prices are low). Therefore, it seems necessary to develop a model in which the love of variety in consumer tastes has more important role in city formation.[6]

The present investigation examines why cities are maintained for a long time, developing a NEG model permitting firm heterogeneity and difference in the number of consumable goods between regions. To do so, we produce a continuous location-space model with the monocentric spatial structure based on Fujita and Krugman (1995) (FK model), introducing the Melitz (2003) type firm heterogeneity. The basic structure of the present model is the following: Fixed and variable transport costs are imposed on each firm to transport its

forward and backward linkages presented here is based on Fujita and Krugman (1995).

[5]Since the work of Dixit and Stiglitz (1977), the concept of *love of variety* in consumer's tastes has served a central role in the theory of monopolistic competition. Usually, the theory of monopolistic competition assumes that each consumer has a taste for variety and therefore has an incentive to consume goods of many kinds in as well-balanced a way as possible: This is the love of variety. This in turn gives each firm an incentive to differentiate their products from other. The demand for products of each firm becomes small when the market is crowded with competitors. Therefore, consumer's tastes for variety has an influence on the market outcome. Benassy (1996) produced a simple model to analyze the role of consumer taste for variety in the theory of monopolistic competition.

[6]Of course, the love of variety in consumer tastes plays a key role in the NEG model of the usual type, too. A reason that each firm can specialize in production of particular product is caused by the love of variety. Therefore, the love of variety closely related to the backward linkage in the usual type model of NEG. As explained later, the present model expands the usual type model to a model in which the love of variety also bears a relation with the forward linkage.

products, and the variable transport costs increase with respect to the transport distance. Furthermore, the productivities of firms mutually differ. Consequently, less-productive firms cannot supply products to locations further away from the production location. In addition, because the selection of heterogeneous firms is weak in the large market, firms have an incentive to locate in cities. Distant locations are supplied with goods of few kinds. Because of the love of variety, cities are more attractive for consumers and lead an inflow of them to cities. However, because the inflow of consumers increases the market size of cities, cities will have goods of more kinds. This circular causality makes cities sustainable.

Under this structure, we investigate the conditions under which the monocetric spatial structure is sustainable. We will present *lock-in effects* of two types, which play important roles in making the monocentric spatial configuration sustainable. Furthermore, we will demonstrate that the spatial structure depends strongly on the transport technology and population size in the economy. The monocentric spatial configuration will never be sustainable when the population size is sufficiently small because the supply of agricultural goods is too small to maintain the monocentric spatial configuration. This phenomenon does not appear in the original FK model. Particularly, if transport costs imposed on agricultural goods are too high, then the supply of the agricultural goods to the single city is always insufficient to maintain it.

The remainder of this paper is organized as follows. In Section 2, we present the continuous location-space model with the monocentric spatial structure and describe the production and supply side of the model. In Section 3, we determine the monocentric equilibrium. Then we define the (*market*) *potential function* and investigate the sustainability of the monocentric spatial configuration in Section 4. The last section presents both conclusions and discussions of the salient results and implications.

2. The Model

We consider an economy represented as a boundless, one-dimensional location space, $X \equiv \mathbb{R}$. The land density at each $x \in X$ is set to one, and the quality of land is homogeneous everywhere.[7]

The economy has two sectors, the manufacturing sector and the agricultural sector, which are designated respectively as the M-sector and A-sector. The M-sector is characterized by Chamberlinian monopolistic competition. Firms in the M-sector produce horizontally differentiated goods (M-goods) using increasing returns technology, with labor only. All production of M-goods takes place in the city located at $c \in X$, and M-goods are exported from the city to each location of the agricultural hinterland. Because no M-goods are produced at $x \neq c$, without loss of generality, we can label locations such that the city is located at the origin of X. The A-sector is characterized by perfect competition. It produces a homogeneous agricultural good (A-good) at each $x \in X$ of the agricultural hinterland, using constant returns technology with labor and land. Production of the A-good extends among the agricultural hinterland until a farmer can pay land rent, and A-good is exported from each $x \in X$ to the city.

[7]Description of the present model is based on Fujita and Krugman (1995) except the production structure of the manufacturing goods.

There is a continuum of homogeneous workers with a given size L in the economy. Each worker is endowed with one unit of labor, and each can freely choose any location for residence. However, workers can supply labor only to their location of residence. The consumers of the economy consist of workers and landlords. All landlords are attached to their land. They consume the revenue received from land at their location.

2.1. Preference and Demand

All consumers have the common preference represented by the following utility function

$$u = M^{\alpha_M} A^{\alpha_A}, \quad \alpha_M, \alpha_A > 0, \quad \alpha_M + \alpha_A = 1, \tag{1}$$

where A denotes the amount of A-good. M is the quantity index of M-goods defined as

$$M \equiv \left[\int_{\gamma \in \mathit{\Gamma}} q(\gamma)^\rho d\gamma \right]^{1/\rho}, \quad 0 < \rho < 1, \quad \sigma \equiv \frac{1}{1-\rho} > 1,$$

where $q(\gamma)$ is the consumption of each variety γ of M-goods. $\mathit{\Gamma}$ is the set of available varieties and the measure of the set $\mathit{\Gamma}$ represents the number of available M-goods. σ is the elasticity of substitution between varieties. Let $p_A(x)$ and $p_M(\gamma, x)$ be the prices for A-good and each variety γ of M-goods at $x \in X$, respectively. Moreover, let $Y(x)$ be the income at $x \in X$. Then, from the usual utility maximization problem with (1), we can obtain demand functions for each variety γ and the A-good at $x \in X$ as follows:

$$q(x, \gamma) = \alpha_M Y(x) p_M(x, \gamma)^{-\sigma} P(x)^{\sigma - 1}, \quad \gamma \in \mathit{\Gamma}, \tag{2}$$

$$A(x) = \frac{\alpha_A Y(x)}{p_A(x)}. \tag{3}$$

Therein, $P(x)$ is the price index of M-goods at $x \in X$ defined as

$$P(x) \equiv \left[\int_{\gamma \in \mathit{\Gamma}^*(x)} p_M(x, \gamma)^{1-\sigma} d\gamma \right]^{\frac{1}{1-\sigma}}, \tag{4}$$

where $\mathit{\Gamma}^*(x) \subset \mathit{\Gamma}$ is the subset of varieties that are supplied to location x.

2.2. Production

2.2.1. Production of A-good

We assume that the transport costs of A-good take Samuelson's *iceberg* form: if one unit of A-good is shipped over a distance d, only $e^{-\tau_A d}$ units actually arrive. Let the price of A-good at the city be normalized such that $p_A(c) = 1$. Then, for all excess A-goods produced at $x \in X$ transported to the city, the A-good price curve must hold that $p_A(x) = e^{-\tau_A |x|}$.

One unit of A-good is produced using a_A units of labor and one unit of land. Therefore, by the zero profit condition in the A-sector at each $x \in X$, $\pi_A(x) = e^{-\tau_A |x|} - w(x) a_A - R(x) = 0$, the land rent $R(x)$ at $x \in X$ is obtainable as below:

$$R(x) = e^{-\tau_A |x|} - a_A w(x), \quad x \in X. \tag{5}$$

Therein, $w(x)$ represents the wage rate at $x \in X$. R is called the *bid rent* function which gives the maximum rent a farmer can offer and is represented as a downward-sloping curve with respect to the distance from the city. If $R(x) < 0$, then no farmer can produce the A-goods at $x \in X$. On the other hand, if $R(x) > 0$, then some farmers can produce A-goods at $x \in X$. Consequently, land rents will be zero at fringe location h. From (5), therefore, it must be true that

$$w(h) = \frac{e^{-\tau_A |h|}}{a_A}. \tag{6}$$

2.2.2. Production of M-goods

Turning to the production of M-goods, variety γ is produced by a firm indexed by γ under increasing-returns technology, using labor only, and each firm behaves as monopolist. To enter the market, each firm must use f_E units of labor as a fixed input.[8] The levels of marginal labor input, a_M, differ among firms. G is a distribution function of a_M supported on $(0, \bar{a}_M] \subset \mathbb{R}$. All firms know the shape of the distribution function G whether or not they enter the market, but they cannot know the level of their own marginal labor input until they enter the market. After firms enter the market, they draw their level of marginal labor input from G. Then they decide the prices for their own products. A firm entering the market, must use $f(x)$ units of labor as fixed input to supply any products of the M-goods for each location $x \in X$. If a firm that enters the market is attached to a large marginal labor input (i.e., low productivity), then it might be not able to cover the fixed cost. Therefore, a threshold $a_M^*(x)$ exists and if a firm's marginal labor input is higher than this threshold, a firm would not supply product for location x. Hereafter, for simplicity, we assume that $f(x) = f$ for any $x \in X$.[9]

Like A-good, the *iceberg form of* transport costs to transport products of M-goods is assumed: i.e., if one unit of any products of M-goods is transported from a location $s \in X$ to a location $x \in X$, only $e^{-\tau_M |x-s|}$ units of them actually arrive. Therefor, each firm located at s must produce $e^{\tau_M |x-s|}$ units of its product to supply one unit of it for a location x.

Since we assume that all production of the M-sector takes place in the city which is located at the origin, we can express the operating profit of firm γ attached to marginal labor input a_M as

$$\pi(\gamma, a_M) = \pi(c, \gamma, a_M) + \int_{-h}^{h} \pi(x, \gamma, a_M) dx, \tag{7}$$

where $\pi(c, \gamma, a_M)$ and $\pi(x, \gamma, a_M)$ respectively represent operating profits earned by supplying the product to the city and to the location $x \in X$.[10] They are defined as follows

$$\begin{aligned}
\pi(c, \gamma, a_M) &\equiv \max \left[p_M(c, \gamma) q(c, \gamma) - w(c) \left(f + a_M q(c, \gamma) \right), 0 \right], \\
\pi(x, \gamma, a_M) &\equiv \max \left[p_M(x, \gamma) q(x, \gamma) - w(c) \left(f + a_M e^{\tau_M |x|} q(x, \gamma) \right), 0 \right], x \neq c,
\end{aligned}$$

[8] f_E is often interpreted as the initial investment, product development, and production start-up costs.

[9] $f(x)$ is interpreted, for example, as a research cost for the market of location $x \in X$. In this context, the assumption of $f(x) = f$ for any $x \in X$, implies that costs of research for the market of each location do not depend on the distance from the city.

[10] The city is distinguished from location $0 \in X$ of the agricultural hinterland.

where $w(c)$ is the wage rate at the city. If firm γ produces an M-product to $x \in X$, it chooses a price $p_M(x,\gamma)$ to maximize its profit at the Chamberlinian equilibrium. Consequently, by the equality of the marginal revenue and marginal cost, the pricing rules for the firm can be obtained as

$$p_M(x,\gamma) = p_M(x,a_M) \equiv \frac{w(c)a_M e^{\tau_M|x|}}{\rho}, \quad x \in X, \tag{8}$$

which represents the familiar result that each monopolistic firm will charge its price at a markup over the marginal cost. The presence of manufacturing trade costs means that the prices of M-goods increase as we move to locations further away from the city.

From (8), any firm having the same level of marginal labor input would offer equal prices for their products. Therefore, we can use a_M, instead of the index γ, to characterize the varieties in the M-sector. Note that any entering firm that draws a level of marginal labor input $a_M(> a_M^*(x))$ will never supply its product to location $x \in X$ because they cannot cover fixed costs. Thus, let us define the conditional distribution of a_M as shown bellow.[11]

$$\mu(x,a_M) = \begin{cases} \frac{G'(a_M(x))}{G(a_M^*(x))} & \text{if } 0 < a_M \leq a_M^*(x), \\ 0 & \text{if } a_M^*(x) < a_M \leq \bar{a}_M. \end{cases} \tag{9}$$

We then rewrite the price index (4) as

$$P(x) = \left[\int_{a_M \in (0,\bar{a}_M]} p_M\left(x,a_M\right)^{1-\sigma} n(x)\mu(x,a_M)da_M \right]^{\frac{1}{1-\sigma}}, \quad x \in X,$$

where $n(x)$, the measure of $\mathit{\Gamma}^*(x)$, is interpreted as the number of firms which supply their products to location x. Substitution of (8) into the equation above yields that

$$P(x) = n(x)^{\frac{1}{1-\sigma}} p_M(x,\tilde{a}_M(x)), \quad x \in X, \tag{10}$$

where $\tilde{a}_M(x)$ is the average level of marginal labor input among firms supplying products to location $x \in X$ and $\tilde{a}_M(x)$ is defined as

$$\tilde{a}_M(x) \equiv \left[\int_0^{a_M^*(x)} {a_M}^{1-\sigma} \mu(x,a_M)da_M \right]^{\frac{1}{1-\sigma}}, \quad x \in X. \tag{11}$$

A simple calculation yields $\partial\tilde{a}_M(x)/\partial a_M^*(x) > 0$ (see Appendix A), which implies that the average productivity of firms which are supplying products to location $x \in X$ will be higher (i.e., lower $\tilde{a}_M(x)$) when the cutoff level $a_M^*(x)$ is lower.

2.2.3. Zero Profit Cutoff Condition

Using (2) and (8), the operating profits can be written as

$$\pi(x,\gamma,a_M) = \pi(x,a_M) \equiv \frac{p_M(x,a_M)q(x,a_M)}{\sigma} - w(c)f, \quad x \in X. \tag{12}$$

[11] We assume that the distribution function G is of class C^1.

For any $x \in X$, the cutoff level $a_M^*(x)$ is determined by the zero profit cutoff condition:

$$\pi\left(x, a_M^*(x)\right) = 0 \quad \Longleftrightarrow \quad p(x, a_M^*(x))q(x, a_M^*(x)) = \sigma w(c) f. \tag{13}$$

Meanwhile, from (8) and (10), we rewrite the demand function for M-goods (2) as follows:

$$q(x, a_M) = \left(\frac{\tilde{a}_M(x)}{a_M}\right)^{\sigma-1} \frac{\alpha_M Y(x)}{n(x)p(x, a_M)}, \quad x \in X.$$

The number of entrants, n_E is given as $n_E = n(x)/G(a_M^*(x))$. Then, substituting the equation above into (13), $a_M^*(x)$ is determined implicitly by the following equation:

$$\left(\frac{\tilde{a}_M(x)}{a_M^*(x)}\right)^{\sigma-1} = \frac{\sigma f}{\alpha_M}\frac{n(x)w(c)}{Y(x)} = \frac{\sigma f}{\alpha_M}\frac{n_E G(a_M^*(x))w(c)}{Y(x)}. \tag{14}$$

Consequently, the distance from the city will affect the cutoff level $a_M^*(x)$ (hence $\tilde{a}_M(x)$) only through the market size $Y(x)$.

2.3. Free Entry Equilibrium

Before entry, the expected firm profit Π_E is

$$\Pi_E = \int_0^{a_M^*(c)} \pi(c, a_M) dG(a_M) + \int_{-h}^{h} \left[\int_0^{a_M^*(x)} \pi(x, a_M) dG(a_M)\right] dx - w(c) f_E.$$

If Π_E were negative, no firm would enter the market. Otherwise, there would be some new entrants. In the equilibrium, as long as some firms produce, the expected profit is driven to zero by the unrestricted entry of new firms. Using (12), the free entry condition obtained: $\Pi_E = 0 \Longleftrightarrow$

$$\int_0^{a_M^*(c)} p_M(c, a_M)q(c, a_M) dG(a_M) + \int_{-h}^{h} \left[\int_0^{a_M^*(x)} p_M(x, a_M)q(x, a_M) dG(a_M)\right] dx$$
$$= \sigma w(c) \left[f_E + f G(a_M^*(c)) + \int_{-h}^{h} f G(a_M^*(x)) dx\right]. \tag{15}$$

In equilibrium, the total revenue of the M-sector must be equal to the total labor income of the M-sector. The following equation is then satisfied:

$$n_E = \frac{L(c)}{\sigma\left[f_E + f G(a_M^*(c)) + \int_{-h}^{h} f G(a_M^*(x)) dx\right]}, \tag{16}$$

where $L(c)$ represents the population size in the city.

Given the production technology referenced by G and the fringe distance of the agricultural area h, we obtain following relation.[12]

$$\frac{\tilde{a}_M(x)}{\tilde{a}_M(s)} \gtreqless 1 \Longleftrightarrow \frac{a_M^*(x)}{a_M^*(s)} \gtreqless 1 \Longleftrightarrow \frac{Y(x)}{Y(s)} \gtreqless 1, \quad x, s \in X. \tag{17}$$

[12]The proof is given in Appendix B.

Consequently, both the cutoff and average productivities of firms supplying to location $x \in X$ will be lower (i.e. higher $a_M^*(x)$ and $\tilde{a}_M(x)$) when location x has a larger market size $Y(x)$.

Although the average productivity of supplying firms is lower, consumers will gain higher welfare when they reside in a location that has a large market. Because a larger market induces weaker selection (i.e. higher $a_M^*(\cdot)$),[13] even low-productivity firms can supply their products to a large market. This results implies that the number of firms supplying the products to a large market is large (i.e. large $n(\cdot)$), and that consumers can consume a large number of M-goods in a large market. Thus, holding other conditions constant, the price index is lower in the large market (see (10)). This reflects the *love of variety* in consumer tastes. Of course, higher $a_M^*(\cdot)$ engenders lower average productivity, which engenders a higher price index. Effects of the expansion of the number of supplying firms, however, are sufficiently large to decrease the price index (see (B.3)). Particularly because the market size decreases with respect to the origin (note $Y(x) = e^{-\tau_A|x|}$), the number of the consumable M-goods decreases and the price index increases as we move away from the center.

There is a mechanism that induces agglomeration of economic activities in the presented model. Because more varieties are supplied to the location that has a larger market, consumers gain higher welfare in such locations, reflecting the love of variety. This induces an inflow of consumers to the city. Then the market size of the location becomes larger, and even more varieties are supplied to the location. This circulatory effect induces agglomeration. This mechanism differs from that of usual models in NEG. Surely, in the usual type model of NEG, a larger market size engenders a lower price index, as with the presented model. However it is merely reflective of the effect of saving the transport costs because the number of the consumable varieties is the same everywhere. The presented model has the effect of the expanding the number of the consumable M-goods, including the effect of saving transport costs. The mechanism of the model presented here is more appropriate for analysis of the role of the love of variety in relation to agglomeration.

3. The Monocentric Equilibrium

We described the demand and production side of the economy in the previous section. In this section, we characterize the location decisions of workers, and determine the spatial equilibrium of the monocentric economy for a given population size of labor force L.

3.1. Location Decision of Workers

The indirect utility function for a worker residing in each location is given as follows:

$$V(x) = \alpha_M^{\alpha_M} \alpha_A^{\alpha_A} w(x) P(x)^{-\alpha_M} p_A(x)^{-\alpha_A}, \quad x \in X.$$

We have assumed that workers can freely choose where they live. Therefore, it is natural to assume that workers move to the location with higher welfare. Then, in equilibrium, workers

[13] It differs with Melitz and Ottaviano (2008). In Melitz and Ottaviano (2008), larger markets induced tougher selection. This is a result from linear demand system employed in Melitz and Ottaviano (2008).

must achieve the same utility levels everywhere. Using (8), (10), and (11) we have

$$
\begin{aligned}
w(x) =& \frac{1}{a_A}\left(\frac{G(a_M^*(h))}{G(a_M^*(x))}\right)^{\frac{\alpha_M}{\sigma-1}}\left(\frac{\tilde{a}_M(x)}{\tilde{a}_M(h)}\right)^{\alpha_M} \\
&\times \exp\left((\alpha_M\tau_M - \alpha_A\tau_A)|x| - \alpha_M(\tau_M+\tau_A)|h|\right), \quad x \in X.
\end{aligned}
$$

Using (14), it is satisfied that

$$
\frac{\tilde{a}_M(x)}{\tilde{a}_M(h)} = \frac{a_M^*(x)}{a_M^*(h)}\left(\frac{G(a_M^*(x))}{G(a_M^*(h))}\frac{Y(h)}{Y(x)}\right)^{\frac{1}{\sigma-1}}, \quad x \in X.
$$

Substitution of this into equation above yields

$$
\begin{aligned}
w(x) =& \frac{1}{a_A}\left(\frac{a_M^*(x)}{a_M^*(h)}\right)^{\alpha_M}\left(\frac{Y(h)}{Y(x)}\right)^{\frac{\alpha_M}{\sigma-1}} \\
&\times \exp\left((\alpha_M\tau_M - \alpha_A\tau_A)|x| - \alpha_M(\tau_M+\tau_A)|h|\right), \quad x \in X.
\end{aligned} \tag{18}
$$

3.2. The Monocentric Equilibrium

We are now ready to characterize the monocentric equilibrium. First, one unit of A-good is produced at each location $x \in (-h, h) \subset X$. Then, from (3) and $Y(x) = e^{-\tau_A|x|}$, the excess supply of the A-goods at each x becomes α_M. Consequently, the total supply of the A-good to the city is given as presented below:

$$
Q_A(c) = \alpha_M \int_{-h}^{h} e^{-\tau_A|x|}dx. \tag{19}
$$

From the market clearing condition, $Q_A(c) = A(c)$, we can obtain the following condition:

$$
L(c) = L_c^A(h) \equiv \frac{\alpha_M}{\alpha_A}\frac{\int_{-h}^{h} e^{-\tau_A|x|}dx}{w(c)}.
$$

$L_c^A(h)$ represents the number of urban consumers that satisfies the A-good market clearing condition. Because all firms locate at the city, the income is $Y(c) = w(c)L(c)$ and $Y(x) = e^{-\tau_A|x|}$ at locations c and x, respectively. In equilibrium, from the Cobb-Douglas utility function (1), the following equation must hold

$$
\frac{Y(c)}{Y(c) + \int_{-h}^{h} Y(x)dx} = \frac{w(c)L(c)}{w(c)L(c) + \int_{-h}^{h} e^{-\tau_A|x|}dx} = \alpha_M.
$$

Then, using (18), we can write the wage rate at the city as shown bellow:

$$
w(c) = \frac{1}{a_A}\left(\frac{a_M^*(h)}{a_M^*(c)}\right)^{-\alpha_M}\left[\frac{\alpha_M}{\alpha_A}\frac{2(1-e^{-\tau_A|h|})}{\tau_A}\right]^{\frac{-\alpha_M}{\sigma-1}} e^{-\alpha_M(\tau_M+\tau_A+\frac{\tau_A}{\sigma-1})|h|}.
$$

The number of urban consumers that satisfies the market clearing condition for A-good is therefore

$$
L_c^A(h) = a_A\left(\frac{a_M^*(h)}{a_M^*(c)}\right)^{\alpha_M}\left[\frac{\alpha_M}{\alpha_A}\frac{2(1-e^{-\tau_A h})}{\tau_A}\right]^{\frac{\alpha_M+\sigma-1}{\sigma-1}} e^{\alpha_M(\tau_M+\tau_A+\frac{\tau_A}{\sigma-1})h}. \tag{20}
$$

However, because a_A units of labor are used in the A-sector at each $x \in X$, the urban population size can also be written as

$$L(c) = L_c^L(h) \equiv L - 2a_A h. \tag{21}$$

$L_c^L(h)$ is the size of the urban workers that satisfies the market clearing condition for labor.

Using (20) and (21), the monocentric equilibrium is then defined as

$$L_c^A(h) = L_c^L(h). \tag{22}$$

3.3. Parametrization

To date, any distribution function G has been permitted in the discussion. Hereafter, to simplify and clarify the some of the subsequent analyses, we specify the distribution function. According to earlier reports in the relevant literature, we adopt the Pareto distribution of

$$G(a_M) = \left(\frac{a_M}{\bar{a}_M}\right)^k, \quad a_M \in (0, \bar{a}_M], \quad k > 1, \quad k > \sigma - 1, \tag{23}$$

where k is the "shape" parameter. When k converges to 1, the distribution of marginal labor input converges to a uniform distribution on $(0, \bar{a}_M]$. As k increases, the distribution is more concentrated at higher levels. Especially, the distribution will degenerate into a point $\bar{a}_M$, when k goes to infinity.

Given this parametrization, the cutoff and average levels of marginal labor input and the equilibrium number of entrants are given by the following equations:

$$\begin{aligned} G(a_M^*(c)) &= \frac{\kappa}{\sigma f}\frac{\alpha_M L(c)}{n_E} && (24)\\ G(a_M^*(x)) &= \frac{\kappa}{\sigma f}\frac{\alpha_M e^{-\tau_A|x|}}{n_E w(c)}, \quad x \in X,\, x \neq c, && (25)\\ \tilde{a}_M(x) &= \kappa^{\frac{1}{\sigma-1}} a_M^*(x), \quad x \in X, && (26)\\ n_E &= \frac{\rho}{k}\frac{L(c)}{f_E}. && (27) \end{aligned}$$

where $\kappa \equiv (k + 1 - \sigma)/k > 0$. This parameterization also yields simple derivations for the equilibrium wage rate in (18) as

$$\begin{aligned} w(c) &= \frac{1}{a_A}\left[\frac{\alpha_M}{\alpha_A}\frac{2\left(1 - e^{-\tau_A|h|}\right)}{\tau_A e^{-\tau_A|h|}}\right]^{\frac{\alpha_M \kappa}{1-\sigma}} e^{-\alpha_M(\tau_M+\tau_A)|h|}, \\ w(x) &= \frac{1}{a_A} e^{-\varphi|x| - \alpha_M\left(\tau_M + \tau_A + \frac{\tau_A \kappa}{\sigma - 1}\right)|h|}, \quad x \in X,\, x \neq o, \end{aligned} \tag{28}$$

where φ is defined as presented below:

$$\varphi \equiv \alpha_A \tau_A - \alpha_M \tau_M - \frac{\kappa}{\sigma - 1}\alpha_M \tau_A. \tag{29}$$

The equilibrium wage rate is a decreasing (increasing) function with respect to the distance from the city if $\varphi > 0\,(< 0)$. In fact, φ consists of three terms. The first term of the RHS of (29) represents the effect of change in the A-good price on the wage rate. Because the transport costs are imposed on shipping the A-good, the A-good price decreases with the distance from the origin. Thus, holding other conditions constant, consumers will gain high welfare when the distance from the center is large. For that reason, the equilibrium wage rate must decrease with respect to the distance from the center because all workers must have the same utility level everywhere in the equilibrium. The second term represents the effect of change in M-goods prices. Because each product of M-goods increases when we move away from the center, holding other conditions constant, welfare decreases with respect to the distance from the origin. To compensate all the workers with the same utility level, the equilibrium wage rate must increase when the distance from the center becomes large. The final term of the RHS of (29) represents the effect of change in the number of the consumable M-goods. The market size of each location decreases with respect to the distance from the center. Consequently, as we have described, consumers can consume less variety when they move to locations further away from the origin. The salient implication is that, holding other conditions constant, consumers can gain higher welfare when they are closer to the city. Therefore, to achieve identical utility everywhere, the equilibrium wage rate must increase as the distance from the city becomes greater.

Summarizing the discussion presented above, the first term represents the advantage to moving away from the city although the second and final term represents the disadvantage. If the advantage is larger than the disadvantage, then the equilibrium wage rate will decrease when the distance from the city becomes large (i.e. $\varphi > 0$). Otherwise, the equilibrium wage rate will increase (i.e. $\varphi < 0$). The reason for this is that the equilibrium wage rate is determined as a reservation wage rate that ensures all consumers can obtain the same utility everywhere. The sign of φ is determined as follows.

$$\varphi \lesseqgtr 0 \iff \frac{\bar{\kappa} - \alpha_M}{\alpha_M \bar{\kappa}} \lesseqgtr \frac{\tau_M}{\tau_A}, \quad \bar{\kappa} \equiv \frac{k(\sigma-1)}{k(\sigma-1)+k+1-\sigma} \in (\rho, 1). \tag{30}$$

In that equation, $\bar{\kappa}$ converges to 1 when k converges to $\sigma - 1$. In this case, φ converges to $\alpha_A\tau_A - \alpha_M\tau_M$. Meanwhile, $\bar{\kappa}$ converges to ρ when k goes to infinity. In this case, if α_M is larger than ρ, then φ is always negative for any positive value of τ_A and τ_M. We explain in Section 4 that condition $\alpha_M < \rho$ is the necessary and sufficient condition under which the monocentric spatial structure becomes unstable when the population size in the economy is sufficiently large. This condition is called the *no-black-hole* condition in the original FK model.

Using the parameterization, $L_c^A(h)$ is written as follows.

$$L_c^A(h) = a_A \frac{\alpha_M}{\alpha_A} \frac{2\left(1 - e^{-\tau_A h}\right)}{\tau_A} \left[\frac{\alpha_M}{\alpha_A} \frac{2\left(1 - e^{-\tau_A h}\right)}{\tau_A e^{-\tau_A h}}\right]^{\frac{\alpha_M \kappa}{\sigma - 1}} e^{\alpha_M(\tau_M + \tau_A)h}. \tag{31}$$

The monocentric equilibrium is then given as (22), which determines the equilibrium urban population size and fringe distance uniquely. This in turn determines all other unknowns by (24)-(28).

4. Sustainability of the Monocentric Spatial structure

To date, we have simply assumed that production of M-goods takes place exclusively in the city. However, this spatial configuration might be not sustainable because some firms might have an incentive to enter the market at a location that is not the city. Consequently, in this section, we explore the spatial structure sustainability.

4.1. The Potential Function

To check the sustainability condition, it is helpful to use the properties of the (*market*) *potential function* of the M-sector. In this subsection, we define the market potential function and then describe the necessary and sufficient condition for sustainability of the monocentric spatial configuration.

First, consider a firm entering the market at $s \in (0, h]$.[14] If the firm draws a_M from G, the operating profit of the firm obtained by supplying product to location $r \in X$ is given as

$$\pi(s, r, a_M) = \max\left[0, \frac{w(s)}{\sigma - 1}\left[a_M e^{\tau_M |r-s|} q(s, r, a_M) - (\sigma - 1) f\right]\right], \tag{32}$$

where $q(s, r, a_M)$ is the demand for the product of this firm in location r:

$$q(s, r, a_M) = \frac{\alpha_M Y(r)}{n(r) p(r, \tilde{a}_M(r))} \left(\frac{\tilde{a}_M(r) w(c)}{a_M w(s)}\right)^{\sigma} e^{-\sigma \tau_M (|r-s| - |r|)}, \quad r \in X.$$

If $\pi(s, r, a_M) < 0$, the firm will not supply its product to r. Because $\pi(s, r, a_M)$ is a decreasing function with respect to a_M, the threshold $a_M^*(s, r)$, which satisfies $\pi(s, r, a_M^*(s, r)) = 0$, uniquely exists and the firm will supply its product for r iff $a_M \leq a_M^*(s, r)$. The expected operating profit, $\pi(s, h)$, is then given as

$$\pi(s, h) = \int_0^{a_M^*(s,c)} \pi(s, c, a_M) dG(a_M) + \int_{-h}^{h} \left[\int_0^{a_M^*(s,r)} \pi(s, r, a_M) dG(a_M)\right] dr.$$

To derive the necessary and sufficient condition for sustainability of the monocentric spatial configuration, we next define the (*market*) *potential function*, Ω. Using the equation presented above, we define Ω at each location $s \in (0, h]$ as

$$\Omega(s, h) \equiv \frac{\pi(s, h)}{w(s) f_E}.$$

Therein, $\Omega(s, h)$ measures the relative profitability of each location $s \in (0, h]$ (for M-sector) in comparison with that in the city location. Before entry into the market at s, the expected profit is $\Pi_E(s, h) = \pi(s, h) - w(s) f_E$. By definition, it is readily apparent that

$$\Pi_E(s) \lesseqqgtr 0 \Longleftrightarrow \Omega(s, h) \lesseqqgtr 1.$$

[14] Because the spatial configuration is symmetric with respect to the origin, it is sufficient that we specifically examine the right-hand side of the origin.

If $s = c$, then $\Omega(c,h) = 1$ and thus $\Pi_E(c,h) = 0$. Therefore, we conclude that, given population size L (and thereby knowing the fringe location h), the monocentric equilibrium is stable if and only if the following condition is satisfied.

$$\Omega(s;h) < 1 \quad \text{for any } s \in (0,h]. \tag{33}$$

A simple but messy calculation rewrites the potential function as

$$\Omega(s,h) \equiv \alpha_M \frac{G(a_M^*(s,c))}{G(a_M^*(c))} + \alpha_A \int_{-h}^{h} \psi(r) \frac{G(a_M^*(s,r))}{G(a_M^*(r))} dr,\ \psi(r) \equiv \frac{e^{-\tau_A|r|}}{\int_{-h}^{h} e^{-\tau_A|r|dr}}, \tag{34}$$

where $G(a_M^*(s,r))/G(a_M^*(r))$ is

$$\frac{G(a_M^*(s,r))}{G(a_M^*(r))} = \underbrace{\left(\frac{w(c)}{w(s)}\right)^{\frac{\sigma}{\sigma-1}k}}_{\mathbf{a}} \underbrace{e^{-k\tau_M(|r-s|-|r|)}}_{\mathbf{b}}, \quad r \in X. \tag{35}$$

Therefore, the potential function is a weighted average of $G(a_M^*(s,r))/G(a_M^*(r))$.

$G(a_M^*(s,r))/G(a_M^*(r)) > 1 (< 1)$ implies that the possibility the firm can supply its product to location r is high (low) when the firm enters the market at location s compared to the city. The term **a** of (35) represents the effect of saving production costs. By (28), if φ are positive, then the wage rate will be lower than that of the city when the firm production takes place in a remote area. Consequently, the incentive to enter the market at the remote area is strong because the production cost is low. If φ are negative, then the incentive is weak because production costs become high in remote area. Term **b** of (35) represents the effect of change in the transport costs by changing production location from the city to location s. If $|r-s| > |r|$, then the production costs increase by changing the production location. Therefore, the incentive to change the production location is weak. In contrast, the incentive to change the production location is strong when $|r-s| < |r|$.

Although (33) is a necessary and sufficient condition for sustainability of the monocentric spatial configuration, it is not explicit. Thus, in the following subsections, we investigate parametrical conditions under which the monocentric configuration is sustainable. For this purpose, we present the explicit form of the potential function. Substitution of (28) and (35) into (34) yields

$$\Omega(s,h) = D(h) e^{\frac{\sigma}{\sigma-1}\varphi ks} \left[\left(\frac{1+\alpha_M}{2}\right) e^{-k\tau_M s} + \frac{\alpha_A}{2} \Psi(s,h) e^{k\tau_M s} \right], \tag{36}$$

where

$$D(h) \equiv \left[\frac{\alpha_A}{\alpha_M} \frac{\tau_A}{2(1-e^{-\tau^A h})} \right]^{\frac{\alpha_M}{\rho}\frac{k+1-\sigma}{\sigma-1}},$$

$$\Psi(s,h) \equiv 1 - \frac{\int_0^s e^{-\tau_A x}\left[1 - e^{-2k\tau_M(s-x)}\right] dx}{\int_0^h e^{-\tau_A x} dx} \in [0,1].$$

4.2. Case of Fujita and Krugman

To derive the parametrical conditions for the sustainability of the monocentric spatial configuration, first, let us consider a specific case. The shape parameter k converges to $\sigma-1$. In this case, the potential function (36) converges to

$$\Omega^{FK}(s,h) \equiv e^{\sigma(\alpha_A\tau_A-\alpha_M\tau_M)s}\left[\left(\frac{1+\alpha_M}{2}\right)e^{-(\sigma-1)\tau_M s} + \frac{\alpha_A}{2}\Psi^{FK}(s,h)e^{(\sigma-1)\tau_M s}\right], \tag{37}$$

where

$$\Psi^{FK}(s,h) \equiv 1 - \frac{\int_0^s e^{-\tau_A x}\left[1-e^{-2(\sigma-1)\tau_M(s-x)}\right]dx}{\int_0^h e^{-\tau_A x}dx}.$$

Ω^{FK} is the potential function in the FK model. This implies that our model is a generalized version of the FK model. It asymptotically includes the FK model.[15] In this case, we can obtain some stability conditions as shown in Table 1.[16]

Insert Table 1

From Table 1, when $k \to \sigma - 1$, the monocentric spatial configuration is sustainable only if

$$\frac{\tau_M}{\tau_A} \geq \frac{\alpha_A}{(1+\rho)\alpha_M} \tag{38}$$

is satisfied. This condition ensures that Ω^{FK} has a non-positive gradient in the neighborhood of the origin as we move away from the center. It is a necessary condition for the monocentric spatial configuration that it be sustainable when $k \to \sigma - 1$ because $\Omega^{FK}(c,h) = \lim_{s\to 0^+}\Omega^{FK}(s,h) = \Omega^{FK}(0,h) = 1$. Here, (38) implies that the relative transportation costs, τ_M/τ_A, is important for ensuring the agglomeration of the M-sector.[17]

4.3. Lock-in Effect of the City

To reveal the properties of the potential function, it is helpful to examine the behavior of the potential function in the limiting case. First, we define the value of the potential function in the neighborhood of the origin as $\Omega(0;h) \equiv \lim_{s\to 0^+}\Omega(s,h)$.[18] It is then satisfied that

$$\Omega(0;h) = D(h) \equiv \left[\frac{\alpha_A}{\alpha_M}\frac{\tau_A}{2(1-e^{-\tau_A h})}\right]^{\frac{\alpha_M}{\rho}\frac{k+1-\sigma}{\sigma-1}}.$$

[15]In fact, the equilibrium wage rate, city size, and fringe distance also converge to the case of FK model, when $k \to \sigma - 1$.

[16]See Fujita and Krugman (1995) for the proof.

[17]Davis (1998) shows that the home market effect might disappear when transportation costs to the homogeneous goods sector is introduced in Krugman's model (Krugman (1991)). It suggests that agglomeration of certain sector depends on the relation of transportation costs between sectors.

[18]City location, c is distinguished from the location $0 \in X$ of the agricultural area and, in general, $\Omega(c,h) \neq \Omega(0;h)$.

Therefore, we have a necessary condition for the sustainability of the monocentric spatial configuration as follows.

$$\frac{\alpha_M}{\alpha_A}\frac{2(1-e^{-\tau_A h})}{\tau_A} > 1. \tag{39}$$

This condition ensures that the market size of city $Y(c)$ is the largest in X, and it implies a larger number of varieties supplied to the city than to any other location. Form (21), (22), and (31), the fringe distance h is understood to increase with respect to the population size L and

$$D \to \infty \quad \textbf{as} \quad h \to 0 \quad \textbf{as} \quad L \to 0,$$

holds. Consequently, the condition (39) is violated when L is sufficiently small. Particularly, if $\alpha_A\tau_A \geq 2\alpha_M$, then the monocentric spatial configuration is always unstable for any L. Therefore, for the monocentric configuration to be sustainable, it must hold that

$$\frac{\alpha_A}{\alpha_M} < \frac{2}{\tau_A}. \tag{40}$$

Condition (40) requires that the transport costs incurred in the A-sector be sufficiently small or that the demand of M-goods be large compared to that of A-good (i.e., α_M/α_A is sufficiently large). This condition is necessary to maintain the monocentric spatial structure for the following reason: the total supply of the A-good to the city given by (19) is a function of the fringe distance h, and $2\alpha_M/\tau_A$ is the least upper bound of it. Meanwhile, the total demand for A-good in the city is $\alpha_A Y(c)$. Consequently, at least, $\alpha_A Y(c) < 2\alpha_M/\tau_A$ must hold to maintain the city. When condition (40) is violated, $Y(c)$ is smaller than one. However, $Y(0) = 1$ implies that some locations exist in the neighborhood of the origin and that these locations have a larger market than that of the city has. Because the larger number of the M-goods is supplied to these locations, the wage rate in these locations became lower than that in the city. Therefore, firms can find it more profitable to enter the market there. $\alpha_A\tau_A \geq 2\alpha_M$ means that the supply of the A-goods to the city is insufficient to maintain the city for any population size L. For that reason, condition (40) is necessary. This fact suggests that not only the relative value of the transport costs between A-good and M-goods but also that the absolute value of the transport costs of A-good is important to maintain the monocentric spatial configuration.

Next, we explore the gradient of the potential function in the neighborhood of the origin. It is readily found by differentiation as follows:

$$\lim_{s\to 0^+}\frac{\partial \Omega}{\partial s}(s,h) = D(h)k\left(\frac{\sigma}{\sigma-1}\varphi - \alpha_M\tau_M\right).$$

Thus, by the definition of $\bar{\kappa}$ (see (30)), we have

$$\lim_{s\to 0^+}\frac{\partial \Omega}{\partial s}(s,h) \leq 0 \iff \varphi \leq \rho\alpha_M\tau_M \iff \frac{\bar{\kappa}-\alpha_M}{(1+\rho)\alpha_M\bar{\kappa}} \leq \frac{\tau_M}{\tau_A}.$$

Consequently, the following condition must hold so that the potential function has a non-positive gradient in the neighborhood of the origin:

$$\frac{\tau_M}{\tau_A} \geq \frac{\bar{\kappa}-\alpha_M}{(1+\rho)\alpha_M\bar{\kappa}}. \tag{41}$$

The meaning of this condition is the same as that in the case of Fujita and Krugman (see (38)). However, it is noteworthy that condition (38) is more strict compared with the condition (41). In fact, because $\alpha_A > (\bar{\kappa} - \alpha_M)/\bar{\kappa}$ holds, (38) implies (41). When k converges to $\sigma - 1$, (38) and (41) are equivalent because RHS of (41) converges to RHS of (38). Therefore, we conclude that condition (38) implies that $\lim_{s \to 0^+} \partial \Omega / \partial s < 0$.

Under conditions (39) and (41) (or (38)), two types of *lock-in effect* of the city in the present model contribute to sustainability of the monocentric spatial configuration. To show the first type of effect, consider that the condition (41) does not hold. It implies that $\varphi > \rho \alpha_M \tau_M > 0$. If φ is positive, then the wage rate is decreasing with distance from the city. Therefore, That (41) is not true means that the wage rate is decreasing sufficiently fast when we move a short distance away from the origin, and a firm finds it more profitable to move away from the center.[19] If (41) is strictly satisfied or if (38) holds, then the potential function has negative gradient in the neighborhood of the origin. Therefore, in the neighborhood of the origin, no firm has an incentive to move away from the origin. This is the first type of lock-in effect of the city.

To elucidate the second type of the lock-in effect of the city, we assume that the condition (39) does not hold. If (39) is not true, then there will be some locations in the neighborhood of the origin where the market size (and thus the number of supplied varieties) is greater than the area of the city. Consequently, because of the love of variety, some locations will exist in the neighborhood of the origin, and at these locations consumers can have a higher utility level than that of the city. As explained earlier, the equilibrium wage rate is determined as a reservation wage that ensures an identical utility level for workers in any location. Therefore, if (39) does not hold, there will be some locations in the neighborhood of the origin where the market size is larger and the wage rate is lower than those of the city. It is readily apparent that these locations are more suitable for the production of M-goods than the city. For that reason, (39) must hold to maintain the monocentric spatial configuration. If (39) holds, then the value of the potential function is less than unity in the neighborhood of the origin. Therefore, in the neighborhood of the origin, no firm has an incentive to enter the market at any other location because production of M-goods is unprofitable at those locations. This is the second type of the lock-in effect of the city, and it is not considered in Fujita and Krugman (1995) or any other literature.[20]

Because the fringe distance h increases with respect to the population size L, the second type of the lock-in effect becomes strong when L is larger. The gap of the market size between the city and any other location increases with respect to the population size. Therefore, the gap of the number of consumable varieties between the city and any other location also increases when L increases. This is the reason that the second type of the lock-in effect becomes large when L becomes strong. In the existence of both types of the lock-in effect of the city, new cities never emerge in the neighborhood of the origin as long as both conditions (39) and (41) hold.

Finally, let us consider the case in which population size L diverges to infinity. In this

[19] For this reason, (38) must hold to ensure the sustainability of the monocentric spatial configuration when $k \to \sigma - 1$.

[20] e.g., see Behrens (2007), Fujita and Hamaguchi (2001), Fujita and Mori (1996), and Fujita et al. (1999a).

case, the fringe distance also diverges to infinity. Therefore, we can rewrite the potential function as

$$\begin{aligned}\Omega(s;\infty) &\equiv \lim_{h\to\infty}\Omega(s,h)\\ &= D(\infty)e^{\left(\frac{\sigma}{\sigma-1}\varphi-\tau_M\right)ks}\left[\frac{\alpha_M+1}{2}+\frac{\alpha_A}{2}\frac{2k\tau_M e^{(2k\tau_M-\tau_A)s}-\tau_A}{2k\tau_M-\tau_A}\right],\end{aligned} \tag{42}$$

where

$$D(\infty)\equiv\lim_{h\to\infty}D(h)=\left(\frac{\alpha_A}{\alpha_M}\frac{\tau_A}{2}\right)^{\frac{\alpha_M}{\rho}\frac{k+1-\sigma}{\sigma-1}}.$$

The term outside the brackets in (42) is smaller than unity and decreases with respect to s when (40) and (41) (or (38)) are satisfied. Consequently, this term works as a damping force for the potential function when we move away from the center. The term inside the brackets in (42) works as a booster of the potential function because it increases with s.

4.4. The conditions for the stability of the monocentric spatial configuration

We now are ready to derive the condition for the monocentric spatial configuration to be sustainable. In the circumstances under which (40) and (41) (or (38)) are satisfied, $\Omega(s;\infty)$ has a non-positive gradient in the neighborhood of the origin and the number of its local minimum point is only one at most on $(0,h]$. Therefore, when the population size is sufficiently large, the monocentric spatial configuration is sustainable if and only if

$$\lim_{s\to\infty}\Omega(s;\infty)<1 \iff \lim_{s\to\infty}e^{\left[2k\tau_M-\tau_A+\left(\frac{\sigma}{\sigma-1}\varphi-\tau_M\right)k\right]s}\leq 1,$$

is satisfied. It is readily apparent that

$$2k\tau_M-\tau_A+\left(\frac{\sigma}{\sigma-1}\varphi-\tau_M\right)k=(\rho-\alpha_M)\frac{k\tau_A}{\rho}\left[\frac{\tau_M}{\tau_A}+\frac{k-\rho}{\rho k}\right],$$

and therefore we can eventually conclude that the monocentric spatial configuration is sustainable for sufficiently large L if and only if $\alpha_M\geq\rho$ is satisfied.[21] Given this condition, the associated curves of Ω^{FK} are always below 1 everywhere under large L (see Table 1). To rule out this case, the *no-black-hole* condition $\alpha_M<\rho$ must hold.[22] We can summarize the stability conditions under which (41) or (38) is satisfied as in Table 2.

Insert Table 2

When $\alpha_A\tau_A<2\alpha_M$ and $\alpha_M<\rho$ hold, the monocentric spatial configuration is unstable for a sufficiently small or large population size. However, the reasons differ greatly according to the cases. When the population size is too small, the monocentric spatial structure become unstable because the supply of the A-good to the city is too small to maintain it. Particularly, if the transport costs of A-good are too high, then the supply of the A-good to the city is always insufficient and the monocentric spatial configuration is never sustainable

[21] This condition is equivalent with the condition so that $\Omega(s;\infty)$ has a non-positive gradient on $(0,h]$.

[22] For details about the no-black-hole condition, see Fujita et al. (1999b).

for any population size.[23] When the population size is sufficiently large, the monocentric spatial structure become unstable because the periphery market is sufficiently large to ensure that a firm finds it more profitable to enter the market at the remote area.

An interesting case arises, when $\alpha_A \tau_A < 2\alpha_M$ and $\alpha_M < \rho$ hold. The monocentric spatial configuration is unsustainable with sufficiently small population size which in tern, becomes sustainable with medium population size, and un sustainable when the population size sufficiently large.

Insert Figure 1

Fig.1 illustrates this point. In this numerical example, we assign $\alpha_A = \alpha_M = 0.5$, $\tau_A = 0.9$, $\tau_M = 1.0$, $\sigma = 4.0$, $k = 4.5$ and $a_A = 0.5$. We can see that when the population size L increases from $0.2e$, the potential curve first exceeds 1 in the neighborhood of the origin; then the potential curve becomes below 1 everywhere; and finally it exceeds 1 in the periphery.

5. Conclusion and Discussion

In this paper, we presented a continuous location-space NEG model based on Fujita and Krugman (1995) with heterogeneous firms. We examined conditions under which all manufacturing productions will concentrate in a single city.

Introducing Melitz type firm heterogeneity, we can consider differences of two types: one is the difference of productivity among firms; another is the difference in the number of consumable goods among locations. Furthermore, we have described the existence of the difference of the number of consumable goods among locations makes the city more attractive and strengthens the *lock-in effect* which leads to the monocentric spatial configuration to a sustainable state. In addition, we have shown how the sustainability of the monocentric spatial structure depends on the transport technology and population size in the economy. When the population size in the economy is too small, the monocetric spatial structure will be unstable because the supply of the A- good to the city is insufficient to maintain the city. Particularly when the transport costs of A-good are too high, the supply of the A-good to the city will always be insufficient and the monocentric spatial configuration will never be sustainable for any population size.

These results lend historical insight into economic geography. When the population size is small, economic activities would disperse in the economy and ratio of urban population to total population in the economy would be low. Even if population grew gradually, as long as transport technology level for agricultural goods was low, there would be no large city and the ratio of urban population to total population would still be low. When the

[23] In the original FK model, the monocentric spatial configuration also becomes unstable when the transport costs of A-good are sufficiently high relatively to that of M-goods. However the reason differs from that of the present model. High transport costs lead the high A-good price relative to M-goods in the city. Therefore, workers have an incentive to leave the city. Because the equilibrium wage is determined as a reservation wage which ensures identical utility level for workers in any locations, the wage rate decreases when we move a short distance away from the center. Therefore, the monocentric spatial structure becomes unstable because the potential function has the positive gradient in the neighborhood of the center.

population size became sufficiently large and the transportation cost of agricultural goods became sufficiently low, then a large city would be expected to form.

Our analysis of the model presented in this paper, however, is preliminary. Some important works remain to be conducted in the future. We have examined the monocentric spatial configuration and have shown that it might become unsustainable according to the population size. In the analysis of the present model, however, we have not described what happened when the monocentric spatial structure became unstable. Because, perhaps, the economy will have more than one city when the monocentric spatial structure becomes unstable, one avenue of the future studies is to investigate spatial configurations including multiple cities, as did Fujita and Mori (1997) and Fujita et al. (1999a).

Venables (1996) shows the vertical linkage between the manufacturing and intermediate-good sectors becomes one important force to concentrate economic activities. We have ignored this force in the present model. It seems that our model can be modified to include the vertical linkage between the manufacturing and intermediate-good sectors, introducing the final goods sector, which uses a continuum of intermediate goods as input (e.g., Fujita and Hamaguchi (2001)). Therefore, a second direction of future studies is to develop a model that has a vertical linkage between the manufacturing and intermediate-goods sectors.

One other directions of the future studies is the development of a model that can yield an answer to the question of *why the economy tends to move toward agglomeration*. In the analysis of the present model, we have assumed that all production of manufacturing goods in the economy initially concentrates in a single city. However, we have not questioned why this structure emerges initially. It is merely an assumption. Initially, some cities might emerge simultaneously or there might be no city. Consequently, it is necessary to develop a model that can analyze conditions under which a symmetric spatial structure is unstable and the economy moves to agglomeration, under the same framework with the present model. That point is left as a subject for future research.

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Appendix A.

Differentiating $\tilde{a}_M(x)$ with respect to $a^*_M(x)$, we can obtain the following equation

$$\frac{a^*_M(x)}{\tilde{a}_M(x)}\frac{\partial \tilde{a}_M(x)}{\partial a^*_M(x)} = \frac{a^*_M(x)\mu(x, a^*_M(x))}{\sigma - 1}\left[1 - \left(\frac{\tilde{a}_M(x)}{a^*_M(x)}\right)^{\sigma-1}\right], \quad x \in X. \tag{A.1}$$

Since, by definition, $\tilde{a}_M(x)$ is smaller than $a^*_M(x)$, $\partial \tilde{a}_M(x)/\partial a^*_M(x)$ is positive.

Appendix B.

Substitution of (16) into (14) yields that

$$\left(\frac{a_M^*(x)}{\tilde{a}_M(x)}\right)^{\sigma-1} = \frac{\alpha_M \left[f_E + fG(a_M^*(c)) + \int_{-h}^{h} fG(a_M^*(z))dz\right]}{fG\left(a_M^*(x)\right)} \frac{Y(x)}{Y(c)}, \quad x \in X. \tag{B.1}$$

It implicitly determines the cutoff level $a_M^*(x)$.[24] The above equation also implies that $a_M^*(x)$ and $\tilde{a}_M(x)$ are functions of relative market size $Y(x)/Y(c)$, and $a_M^*(c)$ and $\tilde{a}_M(c)$ do not depend on $Y(c)$ and $Y(x)$.

Using the implicit function theorem and (A.1), the following equation holds:

$$\begin{aligned}\frac{Y(x)/Y(c)}{a_M^*(x)} \frac{\partial a_M^*(x)}{\partial(Y(x)/Y(c))} &= \left[a_M^*(x)\mu(x, a_M^*(x))\left(\frac{\tilde{a}_M(x)}{a_M^*(x)}\right)^{\sigma-1} + \sigma - 1\right]^{-1} \\ &> 0, \quad x \in X, x \neq c. \end{aligned} \tag{B.2}$$

Thus, we obtain (17).

Using from (8) to (11) and $n(x) = n_E G(a_M^*(x))$, the price index is given by

$$P(x) = \frac{n_E^{\frac{1}{1-\sigma}} w(c) e^{\tau_M |x|}}{\rho} \left[\int_0^{a_M^*(x)} a_M^{1-\sigma} G'(a_M) da_M\right]^{\frac{1}{1-\sigma}}, \quad x \in X.$$

Thus,

$$\frac{dP(x)/da_M^*(x)}{P(x)} = \frac{-1}{\sigma-1} \frac{a_M^*(x) G(a_M^*(x))}{\int_0^{a_M^*(x)} a_M^{1-\sigma} G'(a_M) da_M} < 0. \tag{B.3}$$

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[24]Note that $\tilde{a}_M(x)$ is also a function of $a_M^*(x)$. See (11).

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Table 1. Stability conditions for the monocentric equilibrium ($k \to \sigma - 1$)

$(1+\rho)\alpha_M \tau_M \geq \alpha_A \tau_A$		$(1+\rho)\alpha_M \tau_M < \alpha_A \tau_A$
$\alpha_M \geq \rho$	$\alpha_M < \rho$	
Always stable	Stable for small L	Always unstable

Source: Table 1 of Fujita and Krugman

Table 2. Stability conditions for the monocentric equilibrium in general case

$\alpha_A \tau_A < 2\alpha_M$			$\alpha_A \tau_A \geq 2\alpha_M$
For small L	For large L		
	$\alpha_M \geq \rho$	$\alpha_M < \rho$	
Unstable	Stable	Unstable	Always Unstable

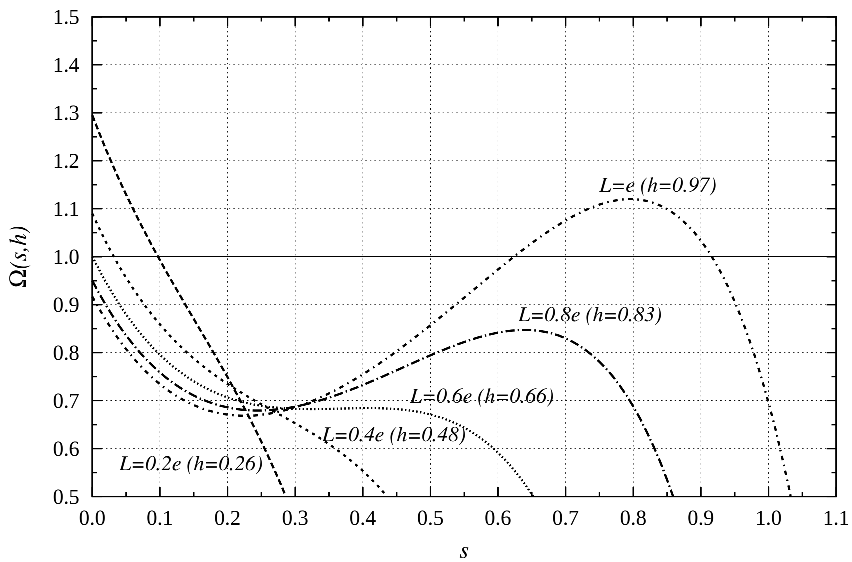

Figure 1: Potential Curve

Note: $\alpha_A = \alpha_M = 0.5$, $\tau_A = 0.9$, $\tau_M = 1.0$, $\sigma = 4.0$, $k = 4.5$, and $a_A = 0.5$.