The non-negative constraint on the nominal interest rate and the effects of monetary policy*

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Abstract

This paper analyzes the effects of monetary policy shock when there is a non-negative constraint on the nominal interest rate. I employ two algorithms: the piecewise linear solution and Holden and Paetz's (2012) algolithm (the HP algorithm). I apply these methods to a dynamic stochastic general equilibrium (DSGE) model which has sticky prices, sticky wages, and adjustment costs of investment. The main findings are as follows. First, the impulse responses obtained with the HP algorithm do not differ much from those obtained with the piecewise linear solution. Second, the non-negative constraint influences the effects of monetary policy shocks under the Taylor rule under some parameters. In contrast, the constraint has little effects on the response to money growth shocks. Third, wage stickiness contributes to the effects of the non-negative constraint through the marginal cost of the product.

The result of money growth shock suggests that it is important to analyze the effects of the zero lower bound (ZLB) in a model which generates a significant liquidity effect.

Keywords: Zero lower bound; Monetary policy shock; Wage stickiness; Liquidity effect

JEL Classification: E47; E49; E52

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1 Introduction

After the colossal financial crisis in 2008, the short term nominal interest rates stayed at zero. This prompts the question of how the zero lower bound (ZLB) influences the effects of monetary policy shocks. Many papers have derived the impulse responses and analyzed the economic behaviors in the dynamic stochastic general equilibrium (DSGE) literature, but most of that research did not include the non-negative constraint on nominal interest rates. This paper analyzes the effects of monetary policy shocks when there is a non-negative constraint on nominal interest rates in a typical DSGE framework.

Several authors have described models including a non-negative constraint on nominal interest rates in the optimal monetary policy literature. Their analyses focused on how to avoid going into the *liquidity trap* and on the effectiveness of monetary policy through expected inflation[1].(Eggertsson and Woodford 2003, Jung, et al. 2005, Kato and Nishiyama 2005, Adam and Billi 2006, 2007, Nakov 2008). More recent analyses used a strand of occasionally binding constraint to tackle non-linear problem.(Christiano, et al. 2010 , Fernández-Villaverde, et al. 2012, Nakata 2012) The studies in both the DSGE literature and the optimal monetary policy literature did not analyze the effects of monetary policy shocks when there is a non-negative constraint on the nominal rate of interest.

Holden and Paetz (2012) created an algorithm dealing with the ZLB. Holden and Paetz's algorithm (henceforth, the HP algorithm) employs news shocks (Holden and Paetz called these the "shadow shock") to deal with the ZLB. They added this algorithm which generates impulse responses to news shocks to their Dynare code to derive the extended versions of impulse responses.

The intuition for algorithm is as follows. If there is a ZLB constraint, nominal interest rates maybe zero for some periods. The boundaries of nominal interest rate affect the economic behavior. For example, if the nominal interest rate binds, output and inflation decrease more. These effects are expressed by anticipated components that are created by news shocks. In other words, if the nominal interest rate binds, the effects which are created by anticipated shocks are allocated to other macro-variables. Since news shock is the shcok that agents know when the shock materialize, agents can behave rationally considering the occurence of shocks. This feature is used for the behavior of an agent who knows when the nominal rate of interest will reach the lower bound.

The HP algorithm allocates the anticipated component to macro variables by solv-

[1]Nakajima (2008), Fujiwara, Sudo and Teranishi (2010), and Ida (2013) analyze the optimal monetary policy with ZLB in an open economy.

ing the complementary condition with slackness. Impulse responses accommodating ZLB consist of an unanticipated component and an anticipated component with weight-parameters. Solving the complementary problem, we obtain optimal weight-parameters.

In the present analysis, I used both the HP algorithm and the piecewise linear solution, which is an algorithm that interpolates impulse responses with other impulse responses. The picewise linear solution replace the periods during which the nominal interest rate might hit the lower bound with another impulse response that accommodates the model structure which nominal interest rate binds. The period which is replaced by another impulse response is determined by guess and verify method.

Here I use these two algorithms and analyze the effects of monetary policy shocks when there is a non-negative constraint on the nominal interest rate. I employ a DSGE model which has sticky prices, sticky wages and adjustment costs of investment. First, I show that strong reductions in the nominal interest rate play a significant role in the dynamics of the economy after policy shock when there is the ZLB constraint in this model. The nominal interest rate decreases significantly when the monetary policy rule is the Taylor type. On the other hand, the nominal interest rate decreases tvery small when the monetary policy rule is the money growth rule.

Second, I remove the wage stickiness to test how it contributes to the effects of the ZLB on monetary policy shocks under the Taylor rule. The response of the inflation in an economy under flexible wages becomes larger than in an economy under sticky wages. Then, inflation can absorb the relatively large effects of the ZLB on the case without the wage stickiness. The impulse responses results indicate that effects of the ZLB in the economy under flexible wages is smaller than in the economy under sticky wages.

Third, I manipulate the persistence of the monetary policy shock under the Taylor rule. An increase in the persisitece of a monetary policy shock significantly reduces the effects of the ZLB by reducing the response of the nominal interest rate. The increase in the persistence of a monetary policy shock gives a long term feature to nominal interest rates. This decreases the reduction in the nominal rate of interest in response to the policy shock, and then the absence of significant easings are mitigated more so than in the case in which the persistence of shocks is low.

In the remainder of the paper, I explain the model in Section 2, and I derive the impulse responses by the HP algorithm and the piecewise linear solution in Section 3. My conclusions and directions for future research are presented in Section 4.

2 The model

I use the medium scale DSGE model, presented by Christiano et al. (2005), Smets and Wouters (2003), and others, to analyze the effects of the ZLB on monetary policy shocks. The model economy has the sectors of households, final goods firms, intermediate firms, and the government . The firms in the intermediate goods sector produce differentiated goods and set the price following the Calvo (1987) pricing rule. Workers supply a differentiated labor force to the intermediate-goods sector. The firms maximize their profit as evaluated by marginal utility following the Calvo pricing rule.

Throughout the optimization in each sector, the aggregate demand equation, the aggregate supply with inflation dynamics, and the labor supply equation with inflation and wage dynamics are derived.

The fiscal policy is Ricardian and the central bank employs the Taylor (1993) rule and money growth rule. The disturbance term is the only monetary policy shock in this economy.

2.1 Households

I assume that the household is a continuum and indexed by h in $(0, 1)$. The households get the utility from the consumption $C_t(h)$ and real money balances $M_t(h)/P_t$ and gets disutility from the labor supply $N_t(h)$.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t(h)^{1-\sigma}}{1-\sigma} + \frac{(M_t(h)/P_t)^{1-\varsigma}}{1-\varsigma} - \eta N_t(h)^2 \right). \tag{1}$$

Where, M_t and P_t are nominal money and aggregate price, respectively. The household's budget constraint is

$$\begin{aligned} &P_t C_t(h) + P_t I_t(h) + B_t(h) + M_t(h) \\ &\qquad \leq W_t(h) N_t(h) + r_t^k P_t K_{t-1}(h) + R_{t-1} B_{t-1}(h) + M_{t-1}(h) + D_t(h), \end{aligned} \tag{2}$$

where, I_t, K_t, B_t, and D_t denote the investment, capital, government bond, and dividend from the profit, respectively. The investment assumed to follow the accumulative process with adjustment cost.

$$K_t(h) = (1-\delta) K_{t-1}(h) + I_t(h) - S\left(\frac{I_t(h)}{I_{t-1}(h)} \right) I_t(h), \tag{3}$$

where, $S(\cdot)$ denotes the adjustment function of the investment and satisfies the property $S(1) = S'(1) = 0$. The household's first order conditions are as follows.

$$
\begin{aligned}
C_t(h)^{-\sigma} &= \lambda_t(h), \\
m_t(h)^{-\varsigma} &= \lambda_t(h) - \beta E_t\left[\frac{\lambda_{t+1}(h)}{\Pi_{t+1}}\right], \\
\lambda_t(h) &= \beta R_t E_t\left[\frac{\lambda_{t+1}(h)}{\Pi_{t+1}}\right], \\
\lambda_t(h) &= \psi_t(h)\left[1 - S\left(\frac{I_t(h)}{I_{t-1}(h)}\right) - S'\left(\frac{I_t(h)}{I_{t-1}(h)}\right)\frac{I_t(h)}{I_{t-1}(h)}\right] \\
&\quad + \beta E_t\left[\psi_{t+1}(h)S'\left(\frac{I_{t+1}(h)}{I_t(h)}\right)\left(\frac{I_{t+1}(h)}{I_t(h)}\right)^2\right], \\
\psi_t(h) &= \beta E_t\left[\lambda_{t+1}(h)r^k_{t+1} + \psi_{t+1}(h)(1-\delta)\right].
\end{aligned}
$$

where, λ_t and ψ_t are lagrange multipliers associated with the household's budget constraint and capital accumulation equation, respectively. These conditions are reduced into

$$1 = \beta E_t\left[\frac{R_t}{\Pi_{t+1}}\left(\frac{C_{t+1}(h)}{C_t(h)}\right)^{-\sigma}\right], \tag{4}$$

$$\frac{R_t - 1}{R_t} = \frac{m_t(h)^{-\varsigma}}{C_t(h)^{-\sigma}}, \tag{5}$$

$$q_t(h) = E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r^k_{t+1} + (1-\delta)q_{t+1}(h)\right]\right\}, \tag{6}$$

$$
\begin{aligned}
&q_t(h)\left[1 - S\left(\frac{I_t(h)}{I_{t-1}(h)}\right) - S'\left(\frac{I_t(h)}{I_{t-1}(h)}\right)\frac{I_t(h)}{I_{t-1}(h)}\right] \\
&= 1 - E_t\left[q_{t+1}(h)\frac{\lambda_{t+1}(h)}{\lambda_t(h)}S'\left(\frac{I_{t+1}(h)}{I_t(h)}\right)\left(\frac{I_{t+1}(h)}{I_t(h)}\right)^2\right],
\end{aligned} \tag{7}
$$

Eq.(4) is a Euler equation which describes the household's intertemporal decision rule of savings. Eq.(5) is money demand equation showing that the opportunity cost of holding money equals the nominal interest rate. Eq.(6) shows the asset price determination, and Eq.(7) is the process of the investment associated adjustment costs. The term q_t denotes Tobin's marginal q, and it is defined as ψ_t/λ_t.

Following Erceg et, al. (2000), I focus on the symmetric equilibrium, i.e. $C_t(h) = C_t$, $M_t(h) = M_t$, $I_t(h) = I_t$, $K_t(h) = K_t$, $q_t(h) = q_t$. Then, I log-linearize the first order conditions around the steady state. The resulting expressions are as follows.

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}), \tag{8}$$

$$r_t = \frac{\sigma(1-\beta)}{\beta} c_t - \frac{\varsigma(1-\beta)}{\beta} m_t., \tag{9}$$

$$q_t = E_t \pi_{t+1} - r_t + \frac{r^k}{1 + r^k - \delta} r_t^k + \frac{1-\delta}{1 + r^k - \delta} E_t q_{t+1} \tag{10}$$

$$i_t = \frac{\beta}{1+\beta} E_t i_{t+1} + \frac{1}{1+\beta} i_{t-1} + \frac{\kappa\beta}{1+\beta} q_t. \tag{11}$$

where, I define $\kappa \equiv 1/S''(1)$, and all variables are log-deviated from the steady state. Eq.(11) is the log-linearized version of Eq.(7). It is reduced into the simple form significantly since I give the property, $S(1) = S'(1) = 0$, to the adjustment function, $S(\cdot)$.

2.2 Wage decision

I give the sticky wage into the labor supply. There are infinite continuum labor $N(h,j), h, j \in (0,1)$ and the aggregate labor supply is defined by

$$\begin{aligned} N_t(j) &= \left(\int_0^1 N_t(h,j)^{\frac{\theta_w - 1}{\theta_w}} dh \right)^{\frac{\theta_w}{\theta_w - 1}}, \\ N_t &= \int_0^1 N_t(j) dj. \end{aligned} \tag{12}$$

where, $N(h,j)$ denotes the h type of labor supply to the firm j. I define the intratemporal profit maximization problem as follows.

$$\max_{N(h,j)} \; W_t N_t(j) - \int_0^1 W_t(h) N_t(h,j) dh \;\; \text{s.t.} \;\; N_t(j) = \left(\int_0^1 N_t(h,j)^{\frac{\theta_w - 1}{\theta_w}} dh \right)^{\frac{\theta_w}{\theta_w - 1}}$$

The first order condition is

$$N(h,j) = \left(\frac{W_t(h)}{W_t} \right)^{-\theta_w} N_t(j). \tag{13}$$

Eq.(13) is the demand function for h type of labor by firm j. Substituting Eq.(13) into the zero-profit condition yields

$$W_t = \left(\int_0^1 W_t(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta_w}}. \tag{14}$$

Next, I define the optimal wage setting for workers. The workers set their wages to maximize the difference between disutilty from the labor supply and their real wages evaluated by marginal utility. Each worker has an opportunity to change his wage with probability ω. Then, the worker's optimization problem is defined by

$$
\begin{aligned}
&\max_{W_t(h)} E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left[\lambda_{t+s}(h) \frac{W_t(h) X_{t,s}}{P_{t+s}} N_{t+s}(h) - \eta N_{t+s}(h)^2 \right], \\
&\text{s.t. } N_{t+s}(h) = \left(\frac{\tilde{W}_t X_{t,s}}{W_{t+s}} \right)^{-\theta_w} N_{t+s}.
\end{aligned}
\tag{15}
$$

where,

$$
X_{t,s} = \begin{cases} \Pi_1 \times \Pi_2 \times \cdots \times \Pi_t & t \geq 1 \\ 1 & t = 0 \end{cases}
$$

I assume the indexation of the unchanged wage. Then, unchanged wages are shifted by past inflation π_{t-1}. The first order condition is

$$
E_t \sum_{s=0}^{\infty} (\omega\beta)^s \lambda_{t+s}(h) \left[\frac{\tilde{W}_t X_{t,s}}{P_{t+s}} - \mathcal{M}_w \frac{N_{t+s}(h)}{\lambda_{t+s}(h)} \right] N_{t+s}(h) = 0, \tag{16}
$$

where $\mathcal{M}_w \equiv \theta_w/(\theta_w - 1)$. From Eq.(14), the aggregate wage is given by the Dixit-Stigltz form.

$$
W_t = \left[(1-\omega) \tilde{W}_t^{1-\theta_w} + \omega (\pi_{t-1} W_{t-1})^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{17}
$$

There is π_{t-1} in the second term of the bracket since I assume the indexation of unchanged wages. Log-linearinzing Eq.(16) and Eq.(17) around the steady state and combining both equations yield the wage Philips curve (hereafter, the WPC).

$$
\begin{aligned}
\kappa_w w_t = \beta E_t w_{t+1} + w_{t-1} + \beta E_t (\pi_{t+1} - \pi_t) - (\pi_t - \pi_{t-1}) \\
+ \frac{(1-\mathcal{M}_w)\sigma}{b_w \omega} c_t - \frac{1-\mathcal{M}_w}{b_w \omega} n_t.
\end{aligned}
\tag{18}
$$

where

$$
\kappa_w \equiv \frac{b_w(1+\beta\gamma^2) - \mathcal{M}_w}{b_w \omega}, \quad b_w \equiv \frac{2\mathcal{M}_w - 1}{(1-\omega)(1-\beta\omega)}
$$

Since I assume the indexation in the wage setting, there is lagged variable in both wage and inflation in the WPC. The WPC denotes the relationship between wages and the labor supply.

2.3 Final goods sector

There is an infinite continuum of intermediate goods $Y_t(j),\ j \in (0,1)$. The final goods sector produces its output by combining the intermediate goods.

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \tag{19}$$

The optimization of the final goods firm is defined as the intratemporal profit maximization.

$$\max_{Y_t(j)} \ P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj \ \text{ s.t. } \ Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$$

The first order condition is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t. \tag{20}$$

Eq.(20) is the demand function for type j intermediate goods for any $j \in (0,1)$. Substituting Eq.(20) into Eq.(19) yields the aggregate price index.

$$P_t = \left(\int_0^1 P(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \tag{21}$$

2.4 Intermediate goods sector

In this subsection, I derive the dynamics of inflation log-linearized around the steady state. The intermediate firm j has a production technology given by

$$Y_t(j) = K_{t-1}(j)^{\alpha} N_t(j)^{1-\alpha}. \tag{22}$$

By the cost minimization problem, the total cost of firm's product is

$$\left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} (r_t^k)^{\alpha} (w_t)^{1-\alpha} Y_t(j),$$

and the marginal cost is

$$s_t = \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} (r_t^k)^{\alpha} (w_t)^{1-\alpha}. \tag{23}$$

The real marginal cost s_t is independent of the index j. The intermediate firm's profits at t are replaced into

$$\left[\frac{P_t(j)}{P_t} - s_t \right] P_t Y_t(j).$$

The intermediate firms dynamically maximize their profits evaluated by the household's marginal utility by setting their optimal price considering that they cannot change their price forever with probability γ. I define $\tilde{P}_t$ as the price which can be set optimal in the period t. Then, the intermediate firm's optimal price setting is

$$\begin{aligned} &\max_{\tilde{P}_t} E_t \sum_{s=0}^{\infty} (\gamma\beta)^s \lambda_{t+s} \left[\frac{\tilde{P}_t X_{t,s}}{P_{t+s}} - s_{t+s} \right] P_{t+s} Y_{t+s}(j), \\ &\text{s.t. } Y_{t+s}(j) = \left(\frac{\tilde{P}_t X_{t,s}}{P_{t+s}} \right)^{-\theta} Y_{t+s}. \end{aligned} \tag{24}$$

The first order condition is

$$E_t \sum_{s=0}^{\infty} (\gamma\beta)^s v_{t+s} \left[\frac{\tilde{P}_t X_{t,s}}{P_{t+s}} - \mathcal{M} s_{t+s}(j) \right] P_{t+s} \left(\frac{\tilde{P}_t X_{t,s}}{P_{t+s}} \right)^{-\theta} Y_{t+s} = 0, \tag{25}$$

where, $\mathcal{M} \equiv \theta/(\theta-1)$. From Eq.(21), the aggregate price is given by the Dixit-Stiglitz type CES aggregator and it is divided into the changed price component and the unchanged price component.

$$P_t = \left[(1-\gamma)\tilde{P}_t^{1-\theta} + \gamma(\pi_{t-1}P_{t-1})^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{26}$$

Since I assume price indexation on the unchanged prices, there is the past inflation in the second term of the bracket. Log-linearizing Eq.(25) and Eq.(26) and combining the two equation yield the dynamic equation of inflation.

$$\pi_t = \frac{\beta}{1+\beta} E_t \pi_{t+1} + \frac{1}{1+\beta} \pi_{t-1} + \frac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\beta)} s_t, \tag{27}$$

Equation (27) is the *New Keynesian Philips curve* (hereafter, the NKPC), which describes the supply side of the economy, the terms of inflation appear because of the sticky price in the intermediate firm sector. There is lagged inflation in Eq. (27) since I assumed indexiation of the unchanged prices. The effects of stickiness are on s_t, which is the marginal cost of intermediate firms. As γ becomes large, the coefficient of s_t becomes small. Moreover, the log-linear version of real maginal cost is given by

$$s_t = \alpha r_t^k + (1-\alpha) w_t. \tag{28}$$

The inflation dynamics may become small since the sticky wage is present in this economy. The sticky wage lowers the dynamics of s_t and then the inflation dynamics becomes smaller.

2.5 Monetary policy

I derive the impulse responses to both the money growth rule and the Taylor (1993) rule. First, when the monetary policy rule is the Taylor type,

$$r_t = \psi_\pi \pi_t + \psi_y y_t + v_t^r,$$
$$v_t^r = \rho_r v_{t-1}^r + \epsilon_t^r, \; \epsilon_t^r \sim i.i.d(0, \sigma_r^2).$$

where, $\psi_\pi \geq 1$ is called 'the Taylor principle'.

Second, the monetary policy is the money growth rule

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_t^\mu, \;\; \epsilon_t^\mu \sim i.i.d\ (0, \sigma_\mu^2), \tag{29}$$

where, $\mu_t \equiv M_t - M_{t-1}$ denotes the money growth rate; Eq.(29) is the log-linearized form. The relationship between money growth and the real money rate can be described as follows by using the definition of real balances, $m_t = M_t - p_t$.

$$\mu_t = m_t - m_{t-1} + \pi_t.$$

Finally, I give the ZLB constraint explicitly[2] .

$$r_t \geq -\ln(1/\beta). \tag{30}$$

2.6 Aggregation

I aggregate the household's budget constraint and the intermediate firms' budget constraint in h and j.

$$P_t C_t + P_t I_t + B_t + M_t = W_t N_t + P_t r_t^k K_{t-1} + R_{t-1} B_{t-1} + M_{t-1} + D_t.$$
$$D_t = P_t Y_t - W_t N_t - P_r r_t^k K_{t-1}.$$

From the above equations and the government's budget constraint, the goods market clearing condition can be obtained.

$$Y_t = C_t + I_t. \tag{31}$$

[2]Since impulse responses indicate percent deviates from the steady state, the lower bound of the nominal interest rate becomes $-\ln(1/\beta)$ in the Figures.

2.7 Steady-state conditions

Next, I derive the steady-state conditions. First, from the equler equation,

$$R = \frac{1}{\beta}$$

The capital rental rate is given by

$$r^k = R + \delta - 1.$$

The optimal price equation at the steady state gives the steady state value of real wage,

$$w = \left(\frac{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{\mathcal{M}(r^k)^{\alpha}} \right)^{\frac{1}{1-\alpha}}.$$

From the intermediate firm's cost minimization problem, the output-capital ratio at the steady state is derived as follows.

$$\frac{K}{Y} = \left(\frac{(1-\alpha)r^k}{w} \right)^{\alpha-1}.$$

Then, the output-investment ratio for the steady state is given by capital accumulation.

$$\frac{I}{Y} = \delta\frac{K}{Y}$$

Finally, the steady state value for C/Y is

$$\frac{C}{Y} = 1 - \frac{I}{Y},$$

3 Simulation

In this section, I derive the impulse responses to the monetary policy shock dealing with the non-negative constraint on the nominal interest rate. First, I show the intuition of the HP algorithm and the piecewise linear solution. Second, I show that strong reductions in the nominal interest rate play a significant role in the dynamics of the economy in response to the policy shock when there is a ZLB constraint in this model. The nominal interest rate decreases significantly when the monetary policy rule is the Taylor type under the benchmark parameters. On the other hand, the nominal interest rate decreases to a very low rate in response to the money growth shock.

Third, I remove the wage stickiness to investigate how it contributes to the effects of the ZLB in response to monetary policy shocks under the Taylor rule. The response of the inflation in the economy under flexible wages becomes larger than that in the economy under sticky wages. Then, inflation can absorb the relatively large effects of the ZLB in the case without wage stickiness. The impulse responses results indicate that the effects of the ZLB in an economy under flexible wages is smaller than that in an economy under sticky wages.

Fourth, I change the persistence of the monetary policy shock under the Taylor rule. An increase in the persisitece of monetary policy shock significantly reduces the effects of the ZLB by reducing the response of the nominal interest rate. The increase in the persistence of the monetary policy shock gives a long term feature to nominal interest rates. This decreases the reduction in the nominal rate of interest in response to the policy shock, and then the absence of significant easings is mitigated more than in the case which the persistence of shocks is low.

3.1 Algorithms dealing with the ZLB

3.1.1 The HP algorithm

I explain the HP algorithm intuitively in this subsection.[3]

First, I need to solve the rational expectation model. In this paper I use the 'Sims (2002) form'. Then, I derive impulse responses to unanticipated policy shocks. I define impulse responses as U_l — where, U_l is the $T \times 1$ matrix, T is the period of simulation and l corresponds to each variables in the model.

Second, I derive the impulse responses to anticipated shocks. I introduce the news shock to the equation in which I want to set an inequality. If the ZLB constraint is present, the nominal rates maybe zero for some periods. The HP algorithm introduces news shocks to accommodate this. Agents know when a neews shock will materialize, and thus the agents can behave rationally given the information about the time that shock will occur. This structure is applied to explorations of how an economy behaves if it knows when the nominal rate of interest binds. In other words, the HP algorithm replaces 'future ZLB' with 'anticipated shock'. Holden and Paetz (2012) call this 'shadow shocks'. Shadow shocks are added into equations which include inequality-constrained variables because we want to know how the economy behaves in response to the dynamics of the nominal interest rate.

In this paper, I add the shadow shock term to the Taylor rule and to the money

[3] See Holden and Paetz (2012) for more details.

growth rule.

$$r_t = \psi_\pi \pi_t + \psi_y y_t + v_t^r + \sum_{s=0}^{T^*-1} v_{t,s}^n \tag{32}$$

or

$$r_t = \sigma c_t + \varsigma m_t + \sum_{s=0}^{T^*-1} v_{t,s}^n \tag{33}$$

where, $v_{t,s}^n$ denotes news shocks. $v_{t,s}^n$ is a shock which is known at $t-s$ and materializes at t. For example, $v_{t,s}^n$ is expressed as follows in an AR(1) system when $s = 3$.

$$\begin{gathered} x_t = \rho_x x_{t-1} + v_t + v_{t,3}^n, \\ v_{t,3}^n = \mathrm{news3}_t \\ \begin{bmatrix} \mathrm{news3}_t \\ \mathrm{news2}_t \\ \mathrm{news1}_t \end{bmatrix} = \begin{bmatrix} \mathrm{news2}_{t-1} \\ \mathrm{news1}_{t-1} \\ \epsilon \end{bmatrix} \end{gathered} \tag{34}$$

Thus $\sum_{s=0}^{T^*-1} v_{t,s}^n$ means there are T^* systems like (34). In other words, we must derive all impulse responses to shocks ϵ since we need all behavior of an agent when nominal rates are binding, where, $T^* \leq T$. Using a news shock algorithm, I derive the extended version of impulse responses. I define impulse responses to news shock as A_l, where l denotes each variable, and A_l are $T \times T^*$ matrices.

Next, I derive the impulse responses dealing with ZLB. The HP algorithm uses the idea of complementary conditions employing slack variables. First, it is necessary to define the impulse responses. The result of the impulse response of the nominal rate must satisfy

$$r_t^z = \max\{-\ln(1/\beta), r_t\}. \tag{35}$$

Holden and Paetz (2012) converted the form of Eq.(35) into the following parameter weighted form.

$$r_t^z = U_{r,t} + \ln(1/\beta) + A_{r,t}\alpha. \tag{36}$$

where, U_r is the impulse response of the nominal interest rate to an unanticipated policy shock and A_r denotes the impulse responses to T^* news shocks. α is a $T^* \times 1$ vector. Thus, the impulse responses accommodating the ZLB constraint consist of unanticipated and anticipated components. The anticipated component is amplified by α to deal with $r_t + \ln(1/\beta) \geq 0$.

Eventually, the problem is replaced to find the optimal value of parameter vector α. Holden and Paetz (2012) uses the idea of a complmentary slackness type condition,

$$\alpha^\top (U_r + \ln(1/\beta) + A_r\alpha) = 0 \tag{37}$$

and then define the problem;

$$\begin{aligned} \alpha^* = &\arg\min\ \alpha^\top (U_r + \ln(1/\beta) + A_r\alpha) \\ &\text{s.t. } \alpha \geq 0,\ U_r + \ln(1/\beta) + A_r\alpha \geq 0. \end{aligned} \tag{38}$$

If the objective function is close to zero, it regards α^* as satisfying the complementary condition. MATLAB has a quadratic optimization function `quadprog.m` in its *optimization toolbox*. Since `quadprog.m` requires the initial value of α and α^* is obtained only when the objective function converges to zero, it is necessary to change the initial value of α or the number of news shocks impulse responses if the objective function does not converge zero.

Finally, the responses dealing with ZLB for each variables are obtained as follows.

$$U_l + A_l\alpha^*, \tag{39}$$

3.1.2 The piecewise linear solution

Here, I explain piecewise linear solution. Guerierri and Iacoviello (2013) created the MATLAB codes for the piecewise linear solution. They provide the codes on the web.[4]

The piecewise linear solution is an algorithm that replaces periods in which the nominal rate might bind with another impulse responses at which the nominal interest rate binds. The algorithm needs two regimes as follows:

$$AE_tX_{t+1} + BX_t + CX_{t-1} + F\mathcal{E}_t = 0, \tag{40}$$

$$A^*E_tX_{t+1} + B^*X_t + C^*X_{t-1} + D^* = 0, \tag{41}$$

where, X_t is a vertical vector of variables. D^* is a vertical vector which includes the deviated threshold value from the steady state at which the nominal interest rate binds. A, B, C, F, A^*, B^* and C^* are structual matrices that include coefficients. I suppose that

[4]See Guerrieri and Iacoviello (2013) for more details. They provide Dynare codes, occbin_20130531.zip on `https://www2.bc.edu/~iacoviel/`

α	β	γ	ω	θ	θ_w	ψ_π	ψ_y	ρ_μ	ρ_r
0.3	0.99	0.7	0.8	6	6	1.5	0.5	0.5	0.5

Table 1: Calibration

Eq.(40) satisfies the Blanchard-Kahn condition and the inequality constraint does not bind. The rational expected solution for this regime is

$$X_t = \Phi X_{t-1} + \Psi \mathcal{E}_t. \tag{42}$$

I also suppose that regime (41) does not always satisfy the Blanchard-Kahn condition and the inequality constraint always binds. Now I suppose that an agent guesses that regime (41) starts from T_s and finishes at T_f. The guessed solution for $[T_s, T_f]$ is obtained as follows: Since the agent guessed that regime (41) finishes at T_f, the solution (42) is applied after T_f. Then, $E_t X_{T_f+1} = \Phi X_{T_f}$. Substituting this into Eq.(41) yields

$$X_{T_f} = \Phi_{T_f} X_{T_f-1} + \Gamma_{T_f}. \tag{43}$$

where,

$$\Phi_{T_f} \equiv -(A^*\Phi + B^*)^{-1} C^*, \quad \Gamma_{T_f} \equiv -(A^*\Phi + B^*)^{-1} D^* \tag{44}$$

Iterating this process, we obtain Φ_t and Γ_t for all $t \in [T_s, T_f]$. The path can then be simulated and verified. If the guessed solution is not verified, another guess can be tried by changing the value of T_s, T_f or both.

3.2 Benchmark impulse response

I set the deep parameters as the listed in Table 1. First, I simulate the model with the Taylor rule. I construct the vector of variables as follows.

$$\begin{aligned} X_t \equiv \big[\, & r_t,\ c_t,\ i_t,\ y_t,\ \pi_t,\ n_t,\ w_t,\ k_{t-1},\ r_t^k,\ s_t,\ q_t, \\ & E_t c_{t+1},\ E_t \pi_{t+1},\ E_t w_{t+1},\ E_t r_{t+1}^k,\ E_t i_{t+1},\ E_t q_{t+1},\ v_t^r \,\big]^\top \end{aligned} \tag{45}$$

Figure 1 illustrates the impulse response to the monetary easing shock when the policy rule is the Taylor type. I set the value of the policy shock so that the nominal interest rate responds to -1 at minimum. The solid blue line in the figure indicates the impulse responses without the ZLB constraint. Since the easing policy stimulate the economy, the decrease in the nominal interest rate raises the labor supply, output, inflation and investment. The responses of these variables, especially inflation and investment, are

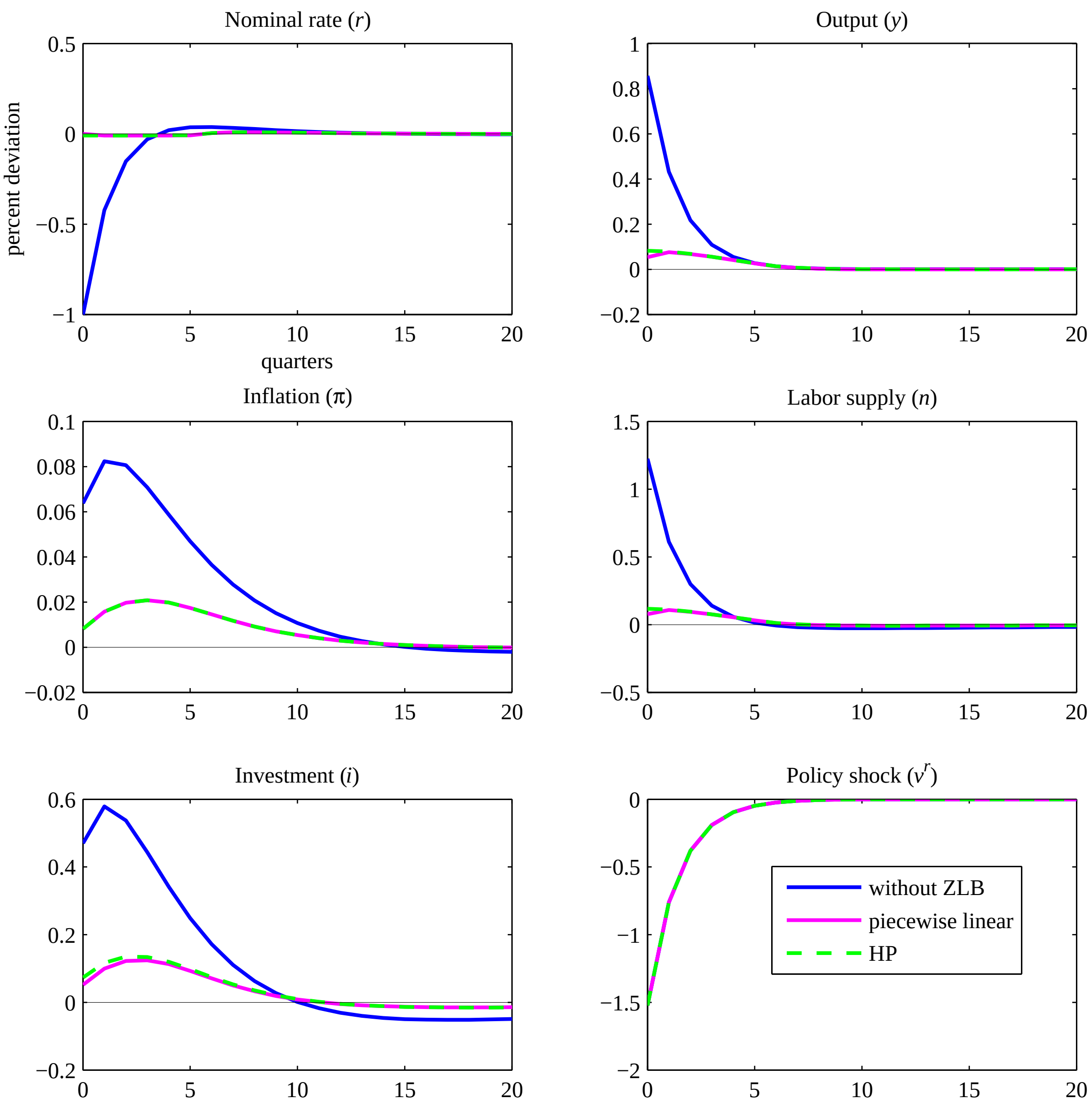

Figure 1: Impulse responses to monetary policy shocks under the Taylor rule. Solid blue line: the impulse response without the ZLB; Solid purple line: the impulse response with the ZLB by the piecewise linear solution; Dashed green line: the impulse response with the ZLB by the HP algorithm.

relatively small compared to the response of the nominal interest rate. Even though the responses of the labor supply and the output are larger than that of the nominal interest rate, they are not as large as twice the response of the nominal interest rate. Thus, the response of the nominal interest rate is not greatly different from those of the other variables.

In Figure 1, the dashed green and solid purple line show the impulse responses obtained by the HP algorithm and the piecewise linear solutions. The result from these two methods are extremely close. Figure 1 indicates that the nominal interest rate stays at the lower bound until the 4th quarter and then becomes small positive. This describes the zero lower bound on the nominal interest rate. The dynamics of the other variables change dramatically toward the result without the ZLB constraint. The responses of all of the variables decrease markedly. The interpretation maybe as follows. The central bank gives the monetary policy shock to stimulate the economy and then the nominal interest rate decreases. If there is the ZLB constraint, however, the nominal interest rate can no longer decrease under zero. This means that the nominal interest rate is not able to ease significantly because of the ZLB. The absence of significant easing affects the other macro variables delaying the positive response of the nominal interest rate further and further. The nominal interest rate's return to a positive status is delayed one quarter in this model toward the case without the ZLB constraint.

Next, I simulate the model with the money growth rule.

$$\begin{aligned} X_t \equiv \big[& r_t,\ m_t,\ c_t,\ i_t,\ y_t,\ \pi_t,\ n_t,\ w_t,\ k_{t-1},\ r_t^k,\ s_t,\ q_t, \\ & E_t c_{t+1},\ E_t \pi_{t+1},\ E_t w_{t+1},\ E_t r_{t+1}^k,\ E_t i_{t+1},\ E_t q_{t+1},\ \mu_t \big]^\top \end{aligned} \tag{46}$$

Figure 2 shows the impulse responses to the positive shock under the money growth rule. Similar to the result obtained with the Taylor rule, the easing policy in the money growth rule stimulates the economy and increase all of the variables indicated in Figure 2. The positive money growth shock lowers the nominal interest rate. This is called the '*liquidity effect*', which is defined as the negative relationship between money growth and the nominal interest rate. In the theoretical literature, the occurrence of the liquidity effect depends negatively on the persistence of money growth rate (Christiano et al. 1997). I set the money growth persistence, ρ_μ, at 0.5 to generate as strong a liquidity effect as possible. The response of the nominal interest rate is very small relative to those of the other macro variables. Both the dashed green line and the solid purple line in Figure 1 indicate the impulse responses dealing with the non-negative constraint. The impulse response of the nominal rate binds the zero lower bound for first 1 to

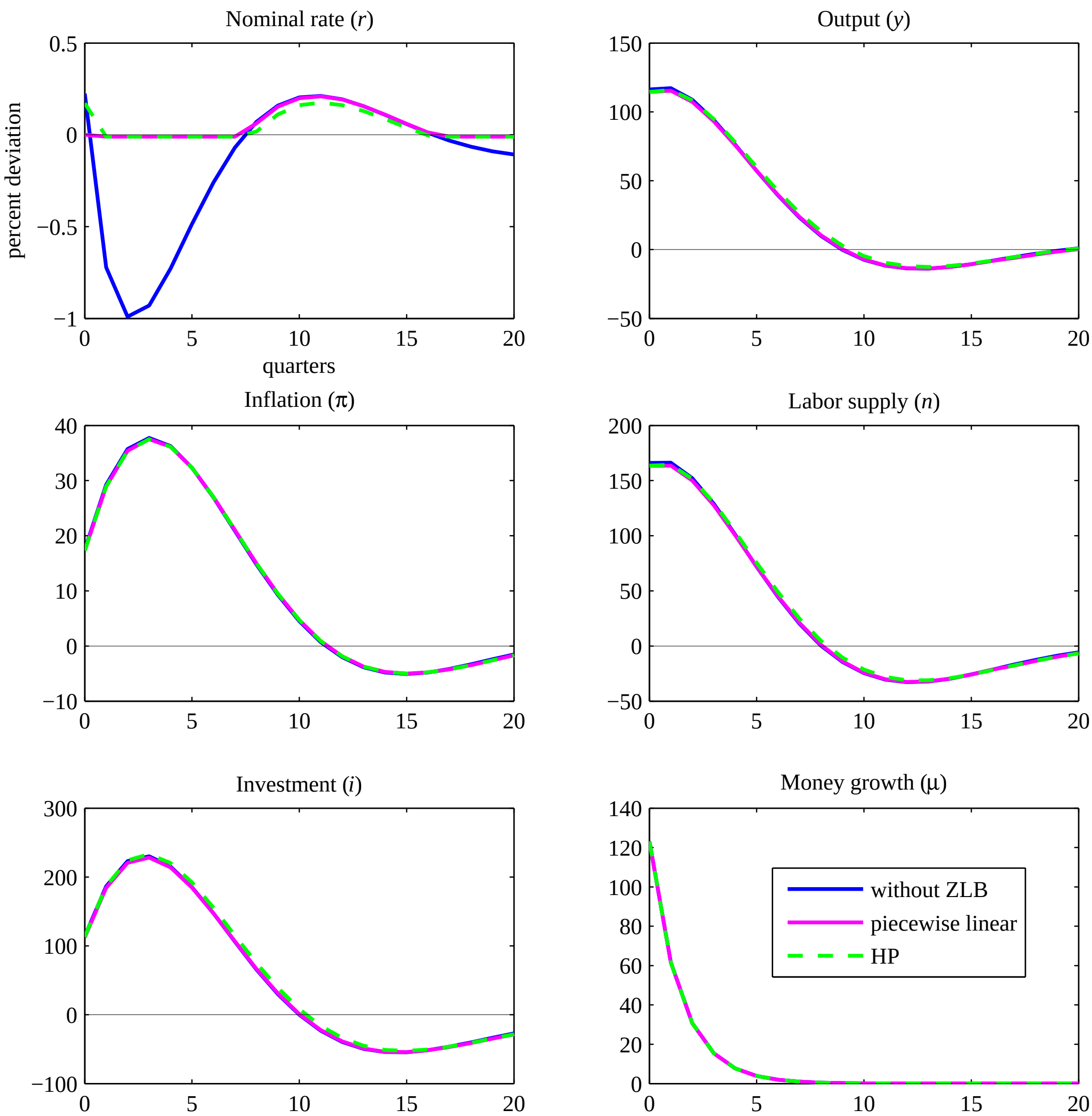

Figure 2: Impulse responses to monetary policy shocks under the money growth rule. Solid blue line: the impulse response without the ZLB; Solid purple line: the impulse response with the ZLB by the piecewise linear solution; Dashed green line: the impulse response with the ZLB by the HP algorithm.

Table 2:

Variables	Taylor rule		
	max(PW)	max(HP)	max(IRF)
y_t	0.075585	0.081866	0.85686
i_t	0.12439	0.13501	0.579
n_t	0.10742	0.11695	1.2241
π_t	0.020795	0.021413	0.082309

max(PW): maximum responses with the ZLB by the piecewise linear solution; max(HP): maximum responses with the ZLB by the HP algorithm; max(IRF): maximum responses without the ZLB to monetary policy shocks for each variables.

Table 3:

Variables	Money growth rule		
	max(PW)	max(HP)	max(IRF)
y_t	115.3241	115.8718	117.2244
i_t	228.1931	233.1472	230.1629
n_t	163.5437	164.3163	166.2422
π_t	37.4943	36.7371	37.7284

max(PW): maximum responses with the ZLB by the piecewise linear solution; max(HP): maximum responses with the ZLB by the HP algorithm; max(IRF): maximum responses without the ZLB to monetary policy shocks for each variables.

7 quarters and reaches the lower bound again for 17 quarters. The nominal interest rate could not ease significantly because of the ZLB. The result of this insignificant easing in the nominal interest rate spills over and then lowers the response of the other variables. However, the reductions in responses are extremely small. This is reflected by the extreme closeness of the solid blue line, the solid purple line and the dashed green line. Next I compare the effect of zero lower bound between the Taylor rule and the money growth rule. First, I compare the maximum responses of four variables. Tables 2 and 3 indicate the maximum responses of y_t, i_t, n_t and π_t to each monetary policy shock. The columns labeled max(PW) and max(HP) indicate the maximum value of responses to the monetary policy shock in the HP algorithm and the piecewise linear solution. The colunmns of max(IRF) indicate the maximum value of responses to the monetary policy shock without constraint. The result from max(PW) and max(HP) are close to each other. In Table 2, the results from max(PW) and max(HP) are less than those of max(IRF). On the other hand, the results from max(PW) and max(HP) are

Table 4: The Effects of ZLB indicate $\sum$|IRF−ZIRF|/$\sum$|IRF| for each variables.

variables	Taylor rule		Money growth rule	
Effects of ZLB	PW	HP	PW	HP
y_t	80.4885%	78.2123%	1.6348%	4.2977%
i_t	74.3258%	71.8679%	1.0829%	4.5272%
n_t	82.1438%	80.4424%	1.5695%	3.8598%
π_t	70.6225%	69.5775%	1.0385%	3.1222%

close to max(IRF) in Table 3. Thus, the maximum responses to monetary policy shocks are more greatly affected by the non-negative constraint on the nominal interest rate under the in Taylor rule than under the money growth rule.

Second, I compare the effect of the zero lower bound in impulse responses for 20 periods.

$$\sum_{t=0}^{20} |\mathrm{IRF}_t - \mathrm{PW}_t| / \sum_{t=0}^{20} |\mathrm{IRF}_t| \quad \text{and} \quad \sum_{t=0}^{20} |\mathrm{IRF}_t - \mathrm{HP}_t| / \sum_{t=0}^{20} |\mathrm{IRF}_t|$$

Table 4 indicates that the effects of constraint are much larger under the Taylor rule than the money growth rule. The responses with constraint were about 70%-80% of the size of the responses without constraint under the Taylor rule. The responses with constraint were about 1%-4% of the size of the responses without constraint under the money growth rule.

3.3 Flexible wages

Next, I derive the impulse responses without the wage stickiness. Since there is no stickiness in wage, the version of the Philips curve equation is substituted for the labor supply equation from which the household's first order conditions are derived. The log-linearized version is

$$n_t + \sigma c_t = w_t. \tag{47}$$

Figure 3 indicates the impulse responses to the negative monetary policy shock under the Taylor rule. Some standard responses without sticky wages become larger than the responses with sticky wages and others do not. The response of the nominal interest rate with a flexible wages rate is smaller than that of the nominal interest rate with sticky wages. Table 5 indicates the maximum responses to the monetary policy shock

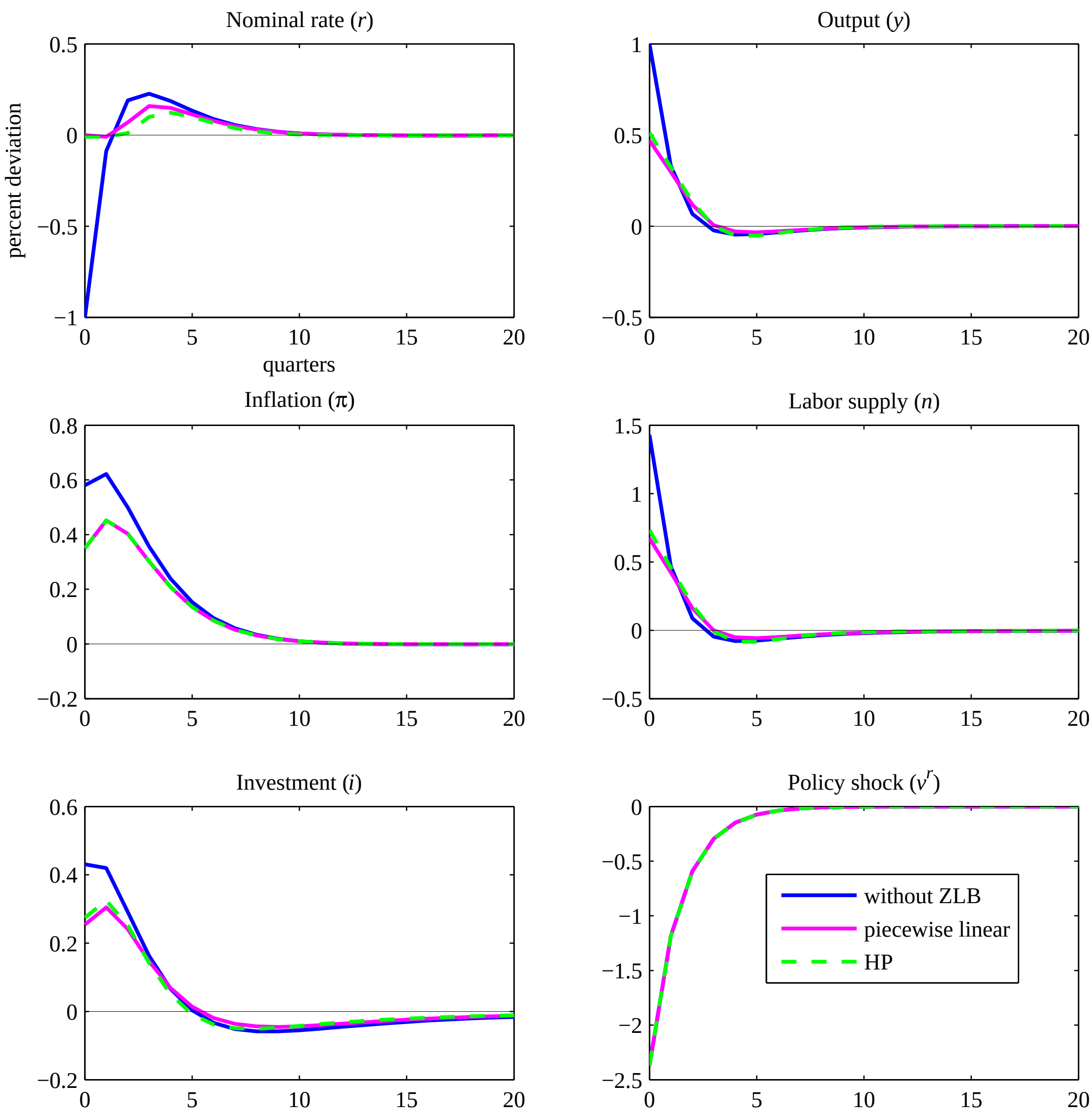

Figure 3: Impulse responses to monetary policy shocks under the Taylor rule and flexible wages. Solid blue line: the impulse response without the ZLB; Solid purple line: the impulse response with the ZLB by the piecewise linear solution; Dashed green line: the impulse response with the ZLB by the HP algorithm.

Table 5:

variables	Maximum responses			Effects of ZLB	
	max(PW)	max(HP)	max(IRF)	PW	HP
y_t	0.46998	0.5136	0.99972	42.2759%	38.4409%
i_t	0.3045	0.32513	0.43061	25.9485%	24.1256%
n_t	0.67139	0.73371	1.4282	41.3739%	37.1486%
π_t	0.45146	0.47694	0.62137	23.2359%	27.5527%

max(PW): maximum responses with the ZLB by the piecewise linear solution; max(HP): maximum responses with the ZLB by the HP algorithm; max(IRF): maximum responses without the ZLB to monetary policy shocks under flexible wages. The columns of Effects of ZLB indicate $\sum|\text{IRF}-\text{ZIRF}|/\sum|\text{IRF}|$ for each variable, where, the monetary policy rule is the Taylor type.

in some variables. The standard maximum response of inflation becomes much larger than that with sticky wages. This feature is consistent with the study by Christiano et al. (2005) in that the nominal rigidities, especially the wage rigidity, contributes to the initial dynamics of inflation.

The responses with the ZLB constraint become larger than those with sticky wage. The intuitive reason for the reduction in the effects of the ZLB is that the inflation dynamics becomes larger because of the absence of sticky wage. The inflation can abosorb the effects of the constraint more than before since the dynamics of inflation become larger. Table 5 provides the sticky wage versions of Tables 2, 3 and 4. First, the maximum responses of variables with the constraint are more close to max(IRF) than the case with sticky wages. Second, the effects of the constraint decrease in all variables. Thus, the effects of monetary policy shock under the Taylor rule with frexible wages are larger than those with the sticky wages.

3.4 The persistency in policy shock under the Taylor rule

The persistency in policy shock under the Taylor rule, ρ_r, contributes to the dynamics of the nominal interest rate. Figure 4 indicates the impulse responses when ρ_r is 0.9. The impulse response of the nominal intrerest rate with the ZLB stays zero for initial periods, but it departs from zero earlier than the case without the ZLB. The responses of other variables with the ZLB became close to those without the ZLB. An increase in the persistence of policy shock lowers the negative response of the nominal interest rate because high persistence of policy shock indicates that monetary easing continues longer and the nominal interest rate takes on a long term aspect in the model. These

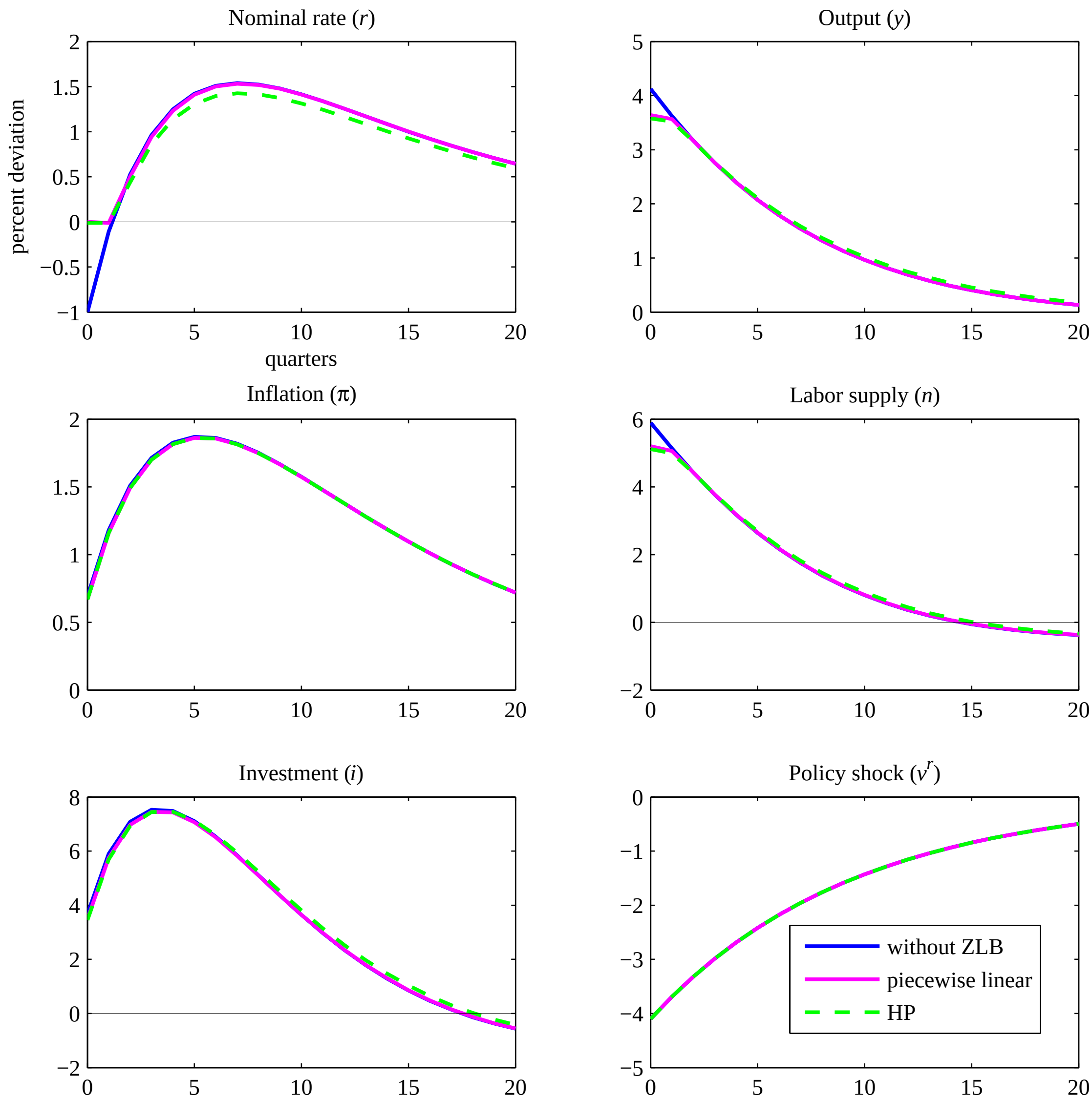

Figure 4: Impulse responses to monetary policy shocks under the Taylor rule when $\rho_r = .9$. Solid blue line: the impulse response without the ZLB; Solid purple line: the impulse response with the ZLB by the piecewise linear solution; Dashed green line: the impulse response with the ZLB by the HP algorithm.

results indicate that an increase in the persistence of policy shock under the Taylor rule dramatically mitigates the effects of ZLB through the reduction of the response of the nominal interest rate.

4 Conclusion and Future Task

The main finding of this paper is that the influence of the ZLB on the effects of monetary policy shock under the Taylor rule is larger than under the money growth rule in a typical DSGE model. The reduction in the nominal interest rate is small compared to the money growth shock. The ZLB constraint on the nominal interest rate has little effect on the other variables because of the insignificant negative responses of the nominal interest rate to money growth shocks. In other words, the ZLB does not affect the economy so much because of the weak liquidity effect. However, this result is unrealistic because there might be a strong liquidity effect in the actual economy. It is thus important to use models that can generate a strong liquidity effect. This implication is related to the third analysis described herein, which assessed the effects of ZLB in high persistence of shocks to Taylor rule. The ZLB might have a significant effect on the impact of monetary policy shocks under the Taylor rule in models that generate strong liquidity effects even though shock persistence is high.

Second, flexible wages reduce the effects of the ZLB and increase some variables' responses. Wage stickiness affects the dynamics of inflation through the marginal costs of the intermediate goods sector. A reduction in the stickiness of wages raises inflation and then reduces the response of the nominal interest rate. A decrease in the response of nominal rate of interest mitigates the amplification of the effects of anticipated bindings in the HP algorithm, but this result does not necessally indicate that reduce wage stickiness is good method of decreasing the effects of the ZLB. This issue should be explored further in the field of optimal monetary policy.

Third, the results obtained with the HP algorithm and those obtained with the piecewise linear solution are close to each other in all analyses in this model.

Finally, these results are not necessarily consistent with the traditional IS-LM literature since monetary policy shocks affect the economy under some cases. This result may indicate that the model cannot explain the real economy or that monetary policy is effective even though the nominal interest rate cannot decrease further than zero. It is, however, difficult to suggest the latter implication. As noted in above, it is important to search for a way to greatly reduce response of of the nominal interest rate to the money growth shocks. The effects of monetary policy employing the Taylor rule might become close to completely ineffective in such models. The liquidity effect might be more important, because of the ZLB.

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