The Cleansing Effect of R&D Subsidies

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Abstract

The paper develops a patent race model of firms which differ in R&D productivity. It is demonstrated that R&D subsidies generate the *cleansing* effect where relatively lower productivity firms drop out of the race and innovation accelerates due to expanded R&D investment by the remaining firms and new entrants with higher productivity than those that exit.

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1 Introduction

Governments affect private R&D directly (e.g. grants) and indirectly (e.g. tax credits) in order to alleviate inefficiency caused by, e.g. the "public good" characteristics of its output. A key question in the theoretical literature on R&D is to understand how firms respond to policy changes in making R&D decisions and to inform policy makers of policy options to tackle inefficiency.

The present paper complements the literature by showing that R&D subsidies generate the *cleansing effect* where relatively lower productivity firms drop out of R&D competition and innovation accelerates due to expanded R&D investment by the remaining firms and new entrants with higher productivity than those that exit. This new result is established in an extended model of a patent race where firms differ in R&D productivity. Intuitively, greater R&D subsidies increase the value of all R&D firms, inducing even relatively lower productivity firms to start R&D. This is called the *value-boosting effect*. On the other hand, greater R&D subsidies create incentives to expand R&D investment and induce more firms to enter a patent race. As a result, competition intensifies and the value of all firms falls, which in turn causes relatively lower productivity firms to exit an R&D race. This is called the *competition effect*. The latter effect dominates the former, giving rise to the cleansing effect of R&D subsidies.

The present paper is structured as follows. In Section 2, the basic model is developed, and equilibrium is characterized. Section 3 explores the effects of R&D subsidies on variables related to the distribution of research firms and their flow R&D expenditure.

2 A Patent Race Model

2.1 Assumptions

We consider a one-shot patent race in continuous time. Firms have to incur two types of sunk costs before investing in flow R&D. First, firms do not know their own R&D productivity before entry, and their true value, labelled a, is revealed once the first type of sunk costs f_N are incurred. R&D productivity is randomly drawn according to the known probability density function $z(a)$, $a \in [0, a_H]$, $a_H < \infty$.[1] The probability distribution function is denoted by $Z(a)$. Second, after R&D productivity a is revealed, firms have to incur another type of fixed costs f_A in order to stay in the race. Those which cannot afford to pay f_A simply exit the race at this stage.

Next consider flow R&D expenditure. For a firm with productivity a, innovation occurs with the Poisson rate $ah(R(a))$, $h' > 0 > h''$ where $R(a)$ is the number of workers employed. The race ends when one of firms succeeds in R&D and gains a prize V. The model is solved "backward".

2.2 R&D Investment

Let us consider a firm investing in flow R&D. The value of the firm is given by[2]

$$v(a) = \frac{Vah(R(a)) - (1 - s_R)R(a)}{r + ah(R(a)) + I_{-1}} \tag{1}$$

where r is an interest rate and s_R is the rate of R&D subsidy to flow expenditure. The first term in the numerator is the expected gross benefit of R&D, and the second term is flow R&D costs. In the denominator, the second term is the instantaneous probability that the firm loses its value $v(a)$ due

[1] In this paper, information on R&D productivity is updated after sunk costs are paid. In Malueg and Tsutsui (1997) and Choi (1991), updating occurs as flow costs are incurred.

[2] (1) can be derived from a recursive equation $rv(a) = [V - v(a)]\, ah(R(a)) - (1 - s_R)R(a) - v(a)I_{-1}$. The first-order condition for maximizing its right-hand side is equivalent to (2).

to its own innovation. The last term I_{-1} is the expected instantaneous probability that one of rival firms succeeds first. It is independent of a because rival firms' productivity levels are assumed to be unobservable, although the number of firms investing in flow R&D is known.

A firm chooses $R(a)$ to maximize the right-hand side of (1), taking I_{-1} as given. The FOC is

$$[V - v(a)]\, ah'(R(a)) = 1 - s_R \tag{2}$$

after rearrangement. The left-hand side is the expected marginal net benefit of employing a research worker, which is equated to the marginal cost on the right-hand side. Substituting (1) into (2) gives

$$\left(V - \frac{Vah(R(a)) - (1-s_R)R(a)}{r + ah(R(a)) + I_{-1}}\right) ah'(R(a)) = 1 - s_R. \tag{3}$$

It implicitly defines the optimal number of R&D workers employed, which is rewritten in an implicit form:

$$R(a) = X(a, I_{-1}; s_R), \qquad \frac{\partial X(a)}{\partial j} > 0, \qquad j = a, I_{-1}, s_R \qquad \text{for } a > A \tag{4}$$

where A is a certain threshold value, which will be defined below. It shows that employment is higher in firms with higher productivity.

2.3 Stay or Exit Decisions

Before the start of making flow R&D investment, firms have to pay sunk costs f_A. To identify types of firms which compete in flow R&D, substitute (4) into (1) to derive

$$v(a) = \frac{Vah(X(a, I_{-1}; s_R)) - (1-s_R)X(a, I_{-1}; s_R)}{r + ah(X(a, I_{-1}; s_R)) + I_{-1}}. \tag{5}$$

It is the gross value of staying in the patent race. Let us consider how it changes as productivity a changes. Using the fact that the indirect effect through $X(a, I_{-1}; s_R)$ can be ignored due to the envelope theorem, we can easily confirm $\left.\frac{\partial v(a)}{\partial a}\right|_{\text{given } I_{-1}} > 0$. Therefore, firms with a non-negative net value $v(a) - f_A$ stay in the race, and others exit the race at this stage. Hence, for a sufficiently high a_H, there is the threshold productivity $a_H > A > 0$ such that

$$v(A) = (1 - s_A) f_A \tag{6}$$

where s_A is the rate of subsidy to sunk costs f_A.

Making use of (6), the first-order condition (2) for firms with the threshold productivity becomes $[V - (1-s_A)f_A]\, Ah'(R(A)) = 1 - s_R$ or

$$R(A) = X(A, I_{-1}; s_R, s_A), \quad \frac{\partial X(A)}{\partial j} > 0, \quad j = A, I_{-1}, s_R, s_A. \tag{7}$$

2.4 Poisson Rates

Before we proceed further, we explore various Poisson rates which are useful in analysis. First, the average Poisson rate of innovation across firms is given by

$$\iota = \int_A^{a_H} ah(X(a, I_{-1}; s_R)) \frac{dZ(a)}{1 - Z(A)} \equiv \iota(A, I_{-1}; s_R), \qquad \frac{\partial \iota}{\partial j} > 0, \quad j = A, I_{-1}, s_R. \tag{8}$$

Therefore, from the viewpoint of a given firm which cannot observe rival firms' productivity, the expected Poisson rate of rival firms is

$$I_{-1} = [N(1 - Z(A)) - 1]\iota = \left(1 - \frac{1}{N[1 - Z(A)]}\right) \int_A^{a_H} ah(X(a, I_{-1}; s_R))dZ(a) \tag{9}$$

where $\frac{\partial I_{-1}}{\partial A} < 0$, $\frac{\partial I_{-1}}{\partial j} > 0$, $j = N, s_R$ and $N(1-Z(A))-1$ is the number of rival firms. The signs of those derivatives are based on an assumption regarding the "stability" of equilibrium.[3] Using (8) and (9), the expected industry-wide Poisson rate of innovation, denoted by I, can be written as

$$I = I_{-1} + \iota(A, I_{-1}; s_R). \tag{10}$$

2.5 Entry into a Patent Race

Firms have to pay sunk costs f_N in order to enter the race. Given that firms do not know their own productivity, the expected benefit of entry is

$$\Pi = \int_A^{a_H} [v(a) - (1 - s_A)f_A]\, dZ(a). \tag{11}$$

Note that $v(a)$, hence Π, falls as the number of entrant firms N rises. To show it, note that in (9) the Poisson rate of rival firms I_{-1} increases with N, i.e. a greater number of entrant firms means a higher possibility that one of rival firms innovates first. In addition, the value of an R&D firm $v(a)$ in (5) decreases in I_{-1} because intensified competition increases the risk of losing the race. Thus, using s_N to denote the rate of subsidy to sunk costs f_N, the following holds because of free entry:[4]

$$\Pi = (1 - s_N)f_N \tag{12}$$

3 Equilibrium and Policy Effects

3.1 Equilibrium Conditions

We characterize equilibrium in two variables (I_{-1}, A). First, let us consider the determination of the threshold productivity A, given I_{-1}. Using (1), (6) and (7), we obtain

$$(1 - s_A)\, f_A = \frac{VAh(X(A, I_{-1}; s_R, s_A)) - (1 - s_R)X(A, I_{-1}; s_R, s_A)}{r + Ah(X(A, I_{-1}; s_R, s_A)) + I_{-1}}. \tag{13}$$

This is called the *value cut-off* condition and shown as an increasing curve in Figure 1. An intuition is simple. An increase in the Poisson rate I_{-1} is tantamount to an increased risk of losing in the race, and the value of each of all firms falls, ceteris paribus. Hence, the net value of staying in the race $v(a) - f_A$ becomes negative for relatively lower productivity firms, and they exit the race before reaching the stage of investing in flow R&D. This results in a higher threshold productivity A.

Next, making use of (4), (5), (11) and (12), we can derive the following condition:

$$(1 - s_N)f_N = \int_A^{a_H} \left(\frac{Vah(X(a, I_{-1}; s_R)) - (1 - s_R)X(a, I_{-1}; s_R)}{r + ah(X(a, I_{-1}; s_R)) + I_{-1}} - (1 - s_A)f_A\right) dZ(a) \tag{14}$$

[3] Formally, $1 - \left(1 - \frac{1}{N[1-Z(A)]}\right) \int_A^{a_H} ah'(X(a, I_{-1}))\frac{\partial X(a,I_{-1})}{\partial I_{-1}} dZ(a) > 0$ is assumed. This is essentially equivalent to an assumption in Theorem 1 of Lee and Wilde (1980). The assumption is made in order to avoid taxonomic analysis, so that we focus on "normal" equilibrium.

[4] We assume parameter values such that $\Pi > (1 - s_N)f_N$ for $N = 1$.

It is the *free entry* condition which determines the Poisson rate I_{-1}, given A. It is depicted as a vertical line in Figure 1 because the condition is independent of the threshold A at the margin.

3.2 The Cleansing Effect of R&D Subsidies

This section conducts a comparative static analysis with respect to R&D subsidy rates without examining how the industry moves from one equilibrium to the other. The analysis should be interpreted as comparing two one-shot patent races with different subsidy rates rather than considering the effect of policy shifts on a given patent race.

First, consider subsidy to flow R&D expenditure. The result is summarized below:

Proposition 1. *A patent race with a higher s_R has a greater value of each of A, I_{-1}, I, N, $R(a)$ for $a \geq 0$.*

Proof. See Appendix. □

To develop an intuition behind a higher value of the threshold productivity A, consider the value cut-off condition (13). A greater s_R reduces R&D costs, and hence, boosts the net value of firms investing in flow R&D, $v(a) - f_A$. It induces low productivity firms (which are willing to pay f_N, but not f_A) to incur fixed costs f_A to compete in flow R&D. This shifts downward the value cut-off condition (13) in Figure 1, tending to lower the threshold productivity A. This is called the *value boosting effect.*

An opposing effect comes from the other condition. A higher s_R reduces the marginal cost of flow R&D, encouraging firms to expand flow R&D employment. In addition, more firms enter a patent race paying fixed costs f_N (a higher N). This increases the number of firms conducting flow R&D, $Nz(a)$, at productivity levels $a \geq A$.[5] These two facts shift rightward the free entry condition (14) in Figure 1, tending to increase the the threshold productivity A. An intuition for an upward pressure for A is that, as competition intensifies (a higher I_{-1}), the net value of staying in R&D $v(a) - f_A$ falls, causing relatively lower productivity firms to exit the race. This is called the *competition effect.*

The proposition shows that the competition effect dominates the the value boosting effect with the result of a higher threshold productivity A. That is, a greater rate of subsidy to flow R&D generates the *cleansing* effect where relatively lower productivity firms are driven out of a patent race, and an industry-wide Poisson rate I rises due to expanded flow R&D investment by the remaining firms and new entrant firms which have higher productivity than the exiting firms.

The cleansing effect also exists in the case of subsidies to two types of fixed costs:

Proposition 2. *A patent race with a higher s_A or s_N has a greater value of each of A, I_{-1}, I, N, $R(a)$ for $a \geq 0$.*

Proof. The proof is omitted since it is similar to that of Proposition 1. □

4 Conclusion

This paper developed a patent race model of firms which differ in R&D productivity. R&D subsidies are shown to generate the cleansing effect where lower productivity firms exit a patent race and the total R&D investment in the industry expands.

Appendix: Proof of Proposition 1

[5] The total number of firms investing in flow R&D is $N(1 - Z(A))$. The direction of its change after the policy shift is ambiguous.

Totally differentiating (13) and (14) gives

$$\begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} dI_{-1} \\ \\ dA \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} ds_R \tag{A1}$$

where

$$P_{11} = -\int_A^{a_H} \frac{v(a)}{r + ah(X(a)) + I_{-1}} dZ(a) < 0, \tag{A2}$$

$$P_{21} = -\frac{(1 - s_A) f_A}{r + Ah(X(A)) + I_{-1}} < 0, \quad P_{22} = \frac{[V - (1 - s_A) f_A] h(X(A))}{r + Ah(X(A)) + I_{-1}} > 0, \tag{A3}$$

$$S_1 = -\int_A^{a_H} \frac{X(a)}{r + ah(X(a)) + I_{-1}} dZ(a) < 0, \quad S_2 = -\frac{X(A)}{r + Ah(X(A)) + I_{-1}} < 0. \tag{A4}$$

after rearrangement and invoking the envelope theorem which allows us to ignore the impact of changes in I_{-1} and s_R through $X(a, I_{-1}; a_R)$ for $a \in [A, a_H]$. Note that

$$D = P_{11} P_{22} < 0, \qquad D_{I_{-1}} = S_1 P_{22} < 0, \tag{A5}$$

$$D_A = P_{11} S_2 - P_{21} S_1 = \frac{D(A)}{r + Ah(X(A)) + I_{-1}} < 0 \tag{A6}$$

where

$$D(A) = \int_A^{a_H} \frac{v(a) X(A) - (1 - s_A) f_A X(a)}{r + ah(X(a)) + I_{-1}} dZ(a) < 0 \tag{A7}$$

whose sign can be confirmed by noting that $\frac{dD(A)}{dA} > 0$ and $D(a_H) = 0$. Using Cramer's rule,

$$\frac{dI_{-1}}{ds_R} = \frac{D_{I_{-1}}}{D} > 0, \qquad \frac{dA}{ds_R} = \frac{D_A}{D} > 0. \tag{A8}$$

$\frac{dI}{ds_R} > 0$, $\frac{dN}{ds_R} > 0$ and $\frac{dR(a)}{ds_R} > 0$ can be confirmed from (4), (7), (9), (10) and (A8).

References

Choi, J. P. (1991): "Dynamic R&D Competition under "Hazard Rate" Uncertainty," *RAND Journal of Economics*, 22(4), 596–610.

Lee, T., and L. L. Wilde (1980): "Market Structure and Innovation: A Reformulation," *Quarterly Journal of Economics*, 94(2), 429–436.

Malueg, D. A., and S. O. Tsutsui (1997): "Dynamic R&D Competition with Learning," *RAND Journal of Economics*, 28(4), 751–772.

Figure 1: A patent race at E_1 has a higher rate of subsidy to flow R&D, s_R, than the one at E_0.