

Adult Longevity and Growth Takeoff

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Abstract

This paper develops an overlapping generations model in which agents make educational and fertility decisions under life-cycle considerations, and retirement from work is distinguished from death. This model sheds light on a novel mechanism that links life expectancy, retirement, education, fertility, and growth. Gains in adult longevity induce agents to save more for retirement, reduce fertility, invest in education, and achieve sustained growth. Even if the length of working life is shortened by early retirement, this mechanism works as long as adult longevity increases sufficiently. Our model replicates the stylized facts of the transition from stagnation to growth in terms of longevity, time in retirement, fertility, education, and income, as well as reconciles the theory that gains in life expectancy trigger a growth takeoff by increasing education with the observation that the length of working life is not substantially prolonged because of retirement. This study provides a framework for considering the joint determination of education, fertility, and retirement.

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1 Introduction

It is sometimes argued that increased life expectancy triggers rises in investment in human capital, a demographic transition, and a takeoff from stagnation to growth.¹ This argument is based on the life-cycle model à la Ben-Porath (1967): increased life expectancy prolongs the working life over which investments in human capital are paid off, thereby positively affecting human capital investment. This intuitively appealing mechanism has attracted many growth theorists, and a number of growth theories have been constructed based on it. Examples include Ehrlich and Lui (1991), Meltzer (1992), de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Blackburn and Cipriani (2002), Boucekkine et al. (2002, 2003), Kalemli-Ozcan (2002), Chakraborty (2004, 2005), Cervellati and Sunde (2005, 2007), Soares (2005), Boucekkine et al. (2007), Soares and Falcão (2008), and de la Croix and Licandro (2011). However, this conventional theory is challenged by the fact that even if mortality declines, the period of working life, over which the investments in human capital are paid off, might not be prolonged, because of early retirement. The estimation of Lee (2001) shows that over the last 150 years, the expected length of retirement life at age 20 for U.S. male workers increased considerably, while their expected length of working life exhibited little change.² Hazan (2009) finds a similar trend.³ Gendell and Siegel (1992) estimate that age at final retirement has fallen by 4 to 5 years for both men and women since 1950.⁴ Whether increased life expectancy promotes education through

¹Life expectancy generally reflects both child mortality and adult longevity. The focus of this paper is on adult longevity. We use the term “life expectancy” to mean adult longevity unless otherwise noted.

²According to Lee (2001), the expected length of the retirement period at age 20 rose from 2.65 years for the cohort born in 1830 to 5.50 years for the cohort born in 1880 and 13.13 years for the cohort born in 1930, while the expected lengths of working life for these cohorts were 41.05, 41.79, and 41.89, respectively.

³According to Hazan (2009), in the United States, the expected length of retirement life increased more than that of working life in the 19th and 20th centuries: 20-year-old men who belong to the cohort born in 1840 were expected to live for another 43.2 years and work for 37.23 years, while their counterparts born in 1930 were expected to live for another 53.01 and work for 41.73 years. Note that Hazan (2009) explores not only the expected number of years in the labor market but also yearly hours worked and shows that the expected total working hours over a lifetime decreased because of declining hours worked per year. In our model, as will be apparent, decreases in child-rearing time result in increases in working time, stimulating educational investments. However, taking into account that female participation in paid work increased and female participation in housework decreased over the last century (see Greenwood et al. 2005, Ramey and Francis 2009, and Kimura and Yasui 2010) and that women have taken the central role in child rearing, on which this paper focuses, our mechanism would not contradict the historical evidence on households’ time allocation.

⁴Although the studies referred to above are based on U.S. data, there is little doubt that most developed countries have experienced both rises in life expectancy and declines in labor force participation by the

the Ben-Porath channel mentioned above is controversial, but there is little doubt that increased life expectancy has resulted in a considerable increase in the length of retirement life.⁵ This paper aims to construct a growth model focusing on the role of the length of retirement life, rather than that of working life, in individual life-cycle behavior.

The key mechanism in our model is as follows. Gains in life expectancy induce agents to save more for their retirement life and to reduce fertility to increase their labor supply and earn more income.⁶ With strong motives to save, agents allocate resources to their own education, thereby escaping stagnation and reaching a state of sustained growth. Even if the length of working life is shortened by early retirement, this mechanism works as long as adult longevity increases sufficiently.⁷ That is, our model reconciles the theory that gains in life expectancy trigger a growth takeoff by increasing educational investment with the observation that because of retirement, the length of working life has not been substantially prolonged.⁸ Incorporating fertility choice is crucial for deriving the result that increased adult longevity promotes education without the prolongation of working life: declines in fertility induced by gains in adult longevity imply increases in labor supply, and thus the return on education can rise despite falls in retirement age. Our results indicate that economies (dynasties) with low longevity are likely to stagnate and those with high

elderly. See the World Bank's World Development Indicators for life expectancy and Kalemli-Ozcan and Weil (2010) and the references therein for labor force participation of the elderly.

⁵Regarding the controversy on the Ben-Porath channel, see Hazan (2009), Sheshinski (2009), Cervellati and Sunde (2010), and Cai and Lau (2011) and the references therein. Sheshinski (2009), Cervellati and Sunde (2010), and Cai and Lau (2011) aim to overcome the challenge raised by Hazan (2009) by generalizing survival law. On the other hand, we focus on fertility choice, that is, household time allocation, while keeping the rectangularity on survival function as in Hazan (2009). Our model will complement rather than contradict the attempts of these studies.

⁶Empirical analyses in Bloom et al. (2003) and Zhang and Zhang (2005) indicate that life expectancy has positive effects on savings rate.

⁷Bar and Leukhina (2010) present a growth model in which reductions in adult mortality trigger a growth takeoff by improving knowledge transmission and encouraging innovation activity. Their mechanism also works without rises in the length of working life.

⁸Ferreira and Pessôa (2007) have a similar motivation. They attempt to reconcile the increases in education and rises of retirement by incorporating labor-leisure choice, not fertility choice, into a life-cycle model. In their model, to reconcile the two facts, agents must work more intensively at the expense of leisure when they are young. According to the estimation of Ramey and Francis (2009), however, leisure time did not decrease for all age groups in the twentieth-century United States. Furthermore, it should be noted that there is more to incorporating fertility choice, not labor-leisure choice, than consistency with the data. Under the conventional assumption that the utility from leisure is discounted at the same rate as the utility from consumption, as in the model of Ferreira and Pessôa (2007), increased longevity does not alter the relative marginal utility of consumption compared to leisure, and thus the effect proposed in this paper does not exist (see also the discussion below equation (1) in Section 2).

longevity are likely to grow, and within an intermediate range of longevity, multiple long-run equilibria arise; whether an economy (a dynasty) is stuck in a poverty trap or grows persistently depends on the initial amount of ancestral human capital.

Particularly important results in the growth literature are (I) significant increases in life expectancy are needed to promote education and generate a growth takeoff (i.e. a small increase might have no effect on schooling and growth); (II) under a given life expectancy, some economies (dynasties) succeed in growth while others remain poor, depending on the initial amount of ancestral human capital; and (III) rises in education, declines in fertility, and falls in retirement age can occur simultaneously. The first two results can account for two historical facts often viewed as evidence against the growth models that focus on life expectancy. Result (I) is consistent with the fact that in the Western world, life expectancy began to rise more than a century prior to the significant increases in schooling that took place in the second half of the nineteenth century.⁹ Result (II) can account for the fact that although the spread of health and medical knowledge to less-developed countries has led to dramatic cross-country convergence in life expectancy, convergence in per-capita income has not been observed.¹⁰ Result (II) also has important policy implications: policies promoting education as well as those improving health are needed to put an economy stuck in a poverty trap onto a growth path. Result (III) captures a salient feature of the process of economic growth broadly observed across industrialized economies in the last century. To the best of our knowledge, this is the first study to generate rises in education, declines in fertility, and falls in retirement age simultaneously in a single framework.

We share with Soares (2005) the motivation to explain the changes in fertility and educational attainment in terms of exogenous changes in life expectancy.¹¹ Soares (2005)

⁹We are aware that in Western Europe, there were gains in literacy prior to the establishment of formal schooling in the 19th century (see, for instance, Boucekine et al. 2003). On the other hand, secondary schooling did not rise in those days. Since this paper addresses the agents' decision on their own education, it is reasonable to focus on the data on secondary schooling. Galor (2005a) provides a comprehensive survey on demographic changes and related facts.

¹⁰See Deaton (2004), Becker et al. (2005), and Acemoglu and Johnson (2007) for details.

¹¹Although our theory, as well as that of Soares (2005), concentrates on exogenous reductions in mortality as the main driving force behind the demographic transition and growth takeoff, we do not deny the possibility that individuals can invest in their own health to reduce mortality. However, several studies support the view that a large fraction of mortality declines are unrelated to economic conditions and exogenous to individuals and countries (see, for instance, Preston 1975, 1980; Becker et al. 2005).

provides the cross-country data for 1960 and 1995, suggesting that for constant levels of income, life expectancy and educational attainment rose and fertility declined, and changes in fertility and educational attainment closely followed the changes in life expectancy. We are also motivated by this evidence. Our model departs from the model of Soares (2005) by explicitly considering individual life-cycle behavior. In particular, since our model distinguishes between retirement from the labor market and death, we can separately explore the effects of the prolongation of working life and those of retirement life. This distinction is important because the prolongation of working life and that of retirement life might provide very different incentives for individuals, for example, our model suggests that the prolongation of working life increases fertility while that of retirement life reduces fertility.

Many data, including those presented in Soares (2005), suggest a strong correlation between life expectancy and economic development and growth.¹² Nevertheless, the causal interpretation of this relationship is controversial. In recent studies, for example, Acemoglu and Johnson (2007) find that increases in life expectancy had either an insignificant or a small negative effect on income per capita while Lorentzen et al. (2008) find that life expectancy is a significantly positive predictor of income per capita.¹³ Cervellati and Sunde (2011) reconcile the seemingly contradictory empirical findings by allowing for a non-monotonic effect of life expectancy on income growth. Their estimation suggests that the effect of life expectancy on income per capita may be ambiguous before the demographic transition, but should be unambiguously positive after its onset. Our theory predicts that significant increases in life expectancy and/or parents' human capital are required to generate a growth takeoff, and that increased life expectancy promotes education after the takeoff but does not promote education before the takeoff, a prediction consistent with the non-monotonicity result of Cervellati and Sunde (2011).

¹²See, for instance, Shastry and Weil (2003) for the cross-country relationship between adult longevity and income per capita. Jayachandran and Lleras-Muney (2009) find that life expectancy positively affects educational attainments at the individual level using data for Sri Lanka.

¹³The difference between their results might partly come from the difference between the indicators used in their estimation. The focus of Acemoglu and Johnson (2007) is largely on child mortality while that of Lorentzen et al. (2008) is on adult mortality. Lorentzen et al. (2008) find that adult mortality strongly affects growth by affecting investment and fertility even when controlling for infant mortality. The prediction of our model is consistent with their empirical results.

This paper is not the first study to focus on the motives for savings as a channel through which increased life expectancy reduces fertility. This channel was already observed by Zhang and Zhang (2005) and Chen (2010). What differentiates our work from theirs is the detailed investigation into life-cycle behavior. We employ a continuous-time overlapping-generations model, and the periods of working life and retirement each have a positive length. In contrast, Zhang and Zhang (2005) and Chen (2010) use a discrete-time overlapping-generations model in which an individual's life consists of childhood, adulthood, and old age, and increased life expectancy is modeled as the increased probability of surviving to old age. Since increased life expectancy is tantamount to prolonged retirement life in their model, they could not separately explore the effects of the prolongation of working life and that of the retirement period. We can conduct such an exploration and consider the endogeneity of the length of working life (Subsection 3.1) and the effects of increased longevity in the earlier stages of development where there is little retired life (Subsection 3.3).¹⁴

This paper follows a large body of literature that addresses the interaction of education and fertility in economic development and growth (e.g. Becker et al. (1990), Tamura (1996), Galor and Weil (2000), Galor and Moav (2002), Greenwood and Seshadri (2002), Hazan and Berdugo (2002), Lucas (2002), de la Croix and Doepke (2003, 2004), Kalemli-Ozcan (2003), Lagerlöf (2003), Doepke (2004, 2005), Doepke and Zilibotti (2005), Soares (2005), Moav (2005), Soares and Falcão (2008), and de la Croix and Licandro (2011)). With respect to the transmission of poverty across generations and its relationship with fertility, this paper is similar to Hazan and Berdugo (2002) and Moav (2005). Their models and ours predict that dynasties within a country can converge to one of two long-run equilibria: poor dynasties with human capital below a threshold level converge to a low-education and high-fertility equilibrium, whereas wealthy dynasties with human capital above the threshold level converge to a high-education and low-fertility equilibrium.¹⁵ This

¹⁴The focus of Zhang and Zhang (2005) is rather empirical, while ours is theoretical. Despite the differences between their model and ours mentioned above, their empirical results that life expectancy has a significant positive effect on savings rate, secondary school enrollment, and growth, but a significant negative effect on fertility, support our theory.

¹⁵This prediction implies that if the initial average level of human capital is above the threshold, then a larger fraction of dynasties converge to a high-education and low-fertility equilibrium in a more equal

prediction implies that poverty can persist even in rich countries, while wealthy households can carry through even in poor countries.

We focus on adult longevity, whereas several studies (e.g. Kalemli-Ozcan (2003), Lagerlöf (2003), Doepke (2005), Soares (2005), Hazan and Zoabi (2006), and Soares and Falcão (2008)) explore the effects of declining child mortality on the quantity-quality tradeoff first introduced by Becker (1960).¹⁶ There is controversy, however, as to whether declining child mortality induces parents to substitute quality for quantity and reduce fertility.¹⁷ Nevertheless, this controversy does not detract from the contribution of our paper because we address the individual's own education and the effects of rising adult longevity.¹⁸ Our paper is related to Boucekkine et al. (2002) and de la Croix and Licandro (2011) in that the role of adult longevity is considered in a continuous-time overlapping-generations framework. Our paper differs from these studies as our model jointly considers the endogeneity of fertility and the presence of the retirement period.¹⁹ The joint consideration of fertility and retirement enables us to derive some interesting results: for example, under the assumption of exogenous retirement, the prolongation of working life and retirement life might provide very different incentives for individuals; under the assumption of endogenous retirement, fertility reduction induced by gains in adult longevity decreases the retirement age.

This paper also provides a new insight concerning the individual choice of retirement date. In previous decades, explaining the decline in labor force participation of the elderly

economy. In this respect, our paper, like Hazan and Berdugo (2002) and Moav (2005), is related to the literature on the effect of income inequality on economic development and growth (e.g. Banerjee and Newman (1993) and Galor and Zeira (1993)).

¹⁶Soares (2005) and Soares and Falcão (2008) explore the effects of both adult longevity and child mortality.

¹⁷Hazan and Zoabi (2006) argue that declining child mortality does not affect the quantity-quality tradeoff because it affects not only the returns to quality but also the returns to quantity. Kalemli-Ozcan (2003) shows that when individuals decide on fertility in the face of uncertainty concerning child survival and a precautionary demand for children exists, declining child mortality reduces fertility. Doepke (2005) argues that precautionary demand should not have a strong effect because deceased children can be replaced through higher subsequent fertility. Galor (2005b) points out the implausibility of precautionary demand from an evolutionary perspective.

¹⁸Dahan and Tsiddon (1998), Blackburn and Cipriani (2002), Soares (2005), Zhang and Zhang (2005), Kimura and Yasui (2007), Soares and Falcão (2008), Chen (2010), and de la Croix and Licandro (2011) also consider agents' investments in their own education in a framework with endogenous fertility.

¹⁹Boucekkine et al. (2002) distinguish retirement from death, but assume exogenous fertility; de la Croix and Licandro (2011) endogenize fertility, but do not distinguish retirement from death.

has been a major concern for economists. For example, a well-developed public pension system, income effects of rising wages, and technological progress that makes obsolete the skills of the elderly are well known as possible causes of the decline in retirement age. Our work is related to Boucekkine et al. (2002), Ferreira and Pessôa (2007), and Kalemli-Ozcan and Weil (2010), which explore the effects of increased longevity on the retirement decision.²⁰ We show that declining fertility reduces the retirement age, an effect not identified in the literature as a possible mechanism that induces early retirement. Declines in fertility mean increases in working time and increases in earnings for agents when they are young adults, thereby enabling them to save more for the future and retire early via the income effect. We refer to this as the ‘fertility effect’ on the individual retirement decision. This effect allows our model to simultaneously generate rises in education, declines in fertility, and falls in retirement age.

The remainder of this paper is organized as follows. Section 2 presents the basic model and illustrates the main results of our paper. In Section 3, we consider the individual retirement decision and explore the joint determination of retirement and fertility, introduce a quantity-quality tradeoff as an extension, and discuss the possibility that our model is applicable to an explanation of the long-run fertility transition. Section 4 concludes.

2 Basic Model

2.1 Economic Environment

Consider a small-open economy in which time is continuous and the population consists of a discrete number of overlapping generations.

An individual’s life consists of three periods, childhood, adulthood, and old age, each of which has a time interval with positive measure. In childhood, agents do not make any decisions and only consume a fixed quantity of time from their parents. In adulthood, they invest in their own education, raise children, supply labor to the market, and con-

²⁰See Kalemli-Ozcan and Weil (2010) and the references therein for details on such works. Kalemli-Ozcan and Weil (2010) present a novel explanation for the rise in retirement, which they call the ‘uncertainty effect’: as mortality falls, the risk of dying before consuming retirement savings falls, and it becomes optimal to plan and save for retirement.

sume goods. In old age, they only consume goods. We denote the lengths of childhood, adulthood, and old age by D , W , and R , respectively. That is, the length of the period over which agents consume goods is $T \equiv W + R$. We treat T as a proxy variable of adult longevity: ‘increases in life expectancy’ and ‘the prolongation of lifetime’ are defined as rises in T in the model.²¹ To keep our primary mechanism as simple as possible, we assume that there is no uncertainty about the length of each period. All decisions are made at the beginning of adulthood (adult age 0): agents decide the number of children, the amount of investment in their own education, and the consumption plan over their lifetime.²² For simplicity, the time required for education and childbirth is assumed to be zero: both events are instantaneously completed at adult age 0. The educational investment entails only pecuniary costs, and the expenditure on education must be repaid during the remainder of life.²³ The world rate of interest is equal to $r > 0$, which is constant over time. Individuals can lend and borrow any amount at this rate. Having children entails only time costs for child rearing. Each adult is endowed with a unit of time that can be devoted to market work and child rearing at each time. Rearing a child requires a fraction $z \in (0, 1)$ of the time endowment at every time from adult age 0 to D , during which period children are dependent family members. Agents retire from work at adult age W and die at adult age $T \equiv W + R$. We assume $D < W$, which seems realistic. Figure 1 illustrates the timing of the model.

Agents receive utility from the number of children they have and from the consumption

²¹In this section, we assume that D , W , and R are exogenously fixed. In Section 3, as an extension, we endogenize the length of working life, W , under the assumption that T is exogenously fixed.

²²The basic model excludes the educational investments made for children to concentrate on our main contribution. In Section 3, we incorporate the investments in a child’s education, showing that the so-called quantity-quality interaction does not change our main results.

²³As suggested by a large body of literature on human capital accumulation, for example, Bils and Klenow (2000), the opportunity cost constitutes a major part of education costs in the real world. The assumption that education is goods-consuming is for analytical convenience; we can introduce time-consuming education in place of goods-consuming education without affecting the qualitative results. A possible way is to divide childhood into two sub-periods, early childhood and later childhood, and assume that agents can choose how much time to spend in schooling and work in later childhood; human capital in later childhood is exogenously given and schooling in later childhood increases human capital used in adulthood. Under such assumptions, the results remain qualitatively the same as in the case of goods-consuming education.

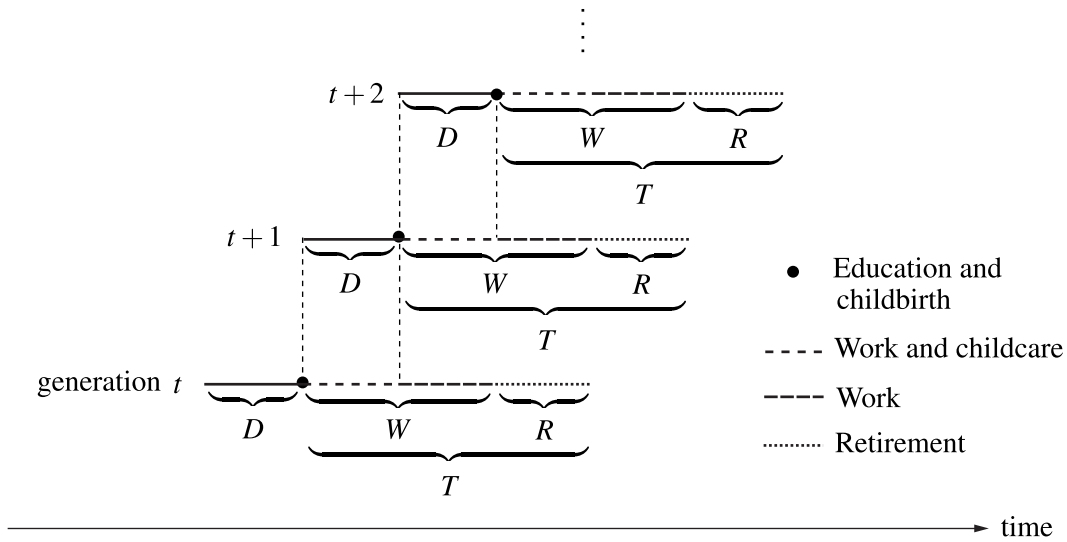


Figure 1: Timing of events

stream over their lifetime. The utility function of the agents of generation t is

$$\gamma \ln n_t + (1 - \gamma) \int_0^T e^{-\rho\tau} \ln c_t(\tau) d\tau, \quad (1)$$

where n_t and $c_t(\tau)$, respectively, represent the number of children and consumption at adult age τ , and $\gamma \in (0, 1)$ and $\rho > 0$ denote the relative weight given to children and the subjective discount rate, respectively. The first term of (1) denotes the utility that parents derive from children and the second term denotes the utility that they derive from consumption. The main result of this paper comes directly from this specification of preferences: the prolongation of lifetime implies that more weight is given to self consumption. Alternatively, if parents simply regard their children as consumer durables from which they receive utility at each time, the first term also must be integrated from 0 to T and discounted at rate ρ . Given that there are motives to demand children other than the consumption motive (e.g. the procreation motive), however, it seems reasonable to assume the utility function given by (1) to concentrate on our main mechanism.²⁴ For

²⁴A specification such as (1) is only one extreme, as is the specification that the term on children is integrated from 0 to T and discounted at rate ρ ; the reality might be somewhere between these two specifications. The effect proposed in this paper does not vanish as long as the latter extreme is not assumed.

example, Soares (2005), Zhang and Zhang (2005), Doepke et al. (2007), and Chen (2010) also assume a utility function in which the utility from children is not discounted in the same way as the utility from consumption. Note that our main result is robust as long as the prolongation of lifetime raises the relative weight given to self consumption (e.g. the log utility is just for convenience).

Denote the wage rate by $w > 0$. Agents with human capital h_t and n_t children earn $(1 - zn_t)wh_t$ at each time from adult age 0 to D and earn wh_t at each time from D to W . The production function of human capital is given by

$$h_t = \max \left\{ \eta x_t^\theta h_{t-1}^{1-\theta}, \underline{h} \right\}, \quad (2)$$

where x_t and h_{t-1} are educational investment and parents' human capital, respectively, and $\eta > 0$, $\theta \in (0, 1)$ and $\underline{h} > 0$ are the parameters. This formulation of human capital production technology implies that the minimum level of human capital, \underline{h} , is guaranteed irrespective of educational investment and parents' human capital. Alternatively, we can interpret that there exist two sectors, a modern market sector and a traditional agriculture sector (or home production), and agents can freely decide which sector to choose. Human capital matters, and earnings depend on the amount of human capital in the former sector but not in the latter.

The accumulation of assets is described by

$$\dot{a}_t(\tau) = ra_t(\tau) + (1 - zn_t)wh_t - c_t(\tau) \text{ for } \tau \in [0, D], \quad (3)$$

$$\dot{a}_t(\tau) = ra_t(\tau) + wh_t - c_t(\tau) \text{ for } \tau \in [D, W], \quad (4)$$

and

$$\dot{a}_t(\tau) = ra_t(\tau) - c_t(\tau) \text{ for } \tau \in [W, T], \quad (5)$$

where $a_t(\tau)$ is the quantity of assets at adult age τ and $\dot{a}_t(\tau)$ denotes its time derivative. The differences among (3), (4), and (5) indicate that agents can supply labor in adulthood but not in old age, and that they must spend time zn_t on child rearing until adult age D .

The initial and terminal conditions for assets are given by, respectively,

$$a_t(0) = -x_t \text{ and } a_t(T) = 0.$$

Agents start their working lives with education expenditure debts, and they cannot leave debts or bequests for their offspring.

The intertemporal budget constraint for the agents of generation t is

$$\int_0^T e^{-r\tau} c_t(\tau) d\tau + x_t = \int_0^D e^{-r\tau} (1 - zn_t) wh_t d\tau + \int_D^W e^{-r\tau} wh_t d\tau.$$

The present discounted value of expenditure on consumption and education must equal that of lifetime earnings.

2.2 Agent's Problem

Consider the agent's problem. The optimal path of consumption is given by the familiar Euler equation:

$$\frac{\dot{c}_t(\tau)}{c_t(\tau)} = r - \rho. \quad (6)$$

It follows that consumption at adult age τ can be written as

$$c_t(\tau) = C_t e^{(r-\rho)\tau} \text{ for } \tau \in [0, T],$$

where $C_t > 0$ is the initial consumption.

Therefore, the problem faced by the agent can be written as

$$\max_{n_t, C_t, x_t} \gamma \ln n_t + (1 - \gamma) \int_0^T e^{-\rho\tau} \ln C_t e^{(r-\rho)\tau} d\tau, \quad (7)$$

$$\text{s.t. } \int_0^T e^{-\rho\tau} C_t d\tau + x_t = \int_0^D e^{-r\tau} (1 - zn_t) wh_t d\tau + \int_D^W e^{-r\tau} wh_t d\tau, \quad (8)$$

$$\text{and } h_t = \max \left\{ \eta x_t^\theta h_{t-1}^{1-\theta}, \underline{h} \right\}.$$

We focus on the interior solutions for fertility choice throughout this paper. Depending

on the parameters, the time constraint at each time might be binding for $\tau \leq D$, that is, $zn_t = 1$. We concentrate on the parameter configurations such that the time constraint does not bind.²⁵

The solution to the above problem can either be interior or at a corner for educational choice, that is, $x_t > 0$ or $x_t = 0$.²⁶ First, consider the case of an interior solution. The first-order conditions imply

$$n_t^E = \frac{\gamma (1 - e^{-rW})}{z (1 - e^{-rD}) \left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} \right)}, \quad (9)$$

$$C_t^E = \frac{\rho \theta^{\frac{\theta}{1-\theta}} (1-\theta)}{1 - e^{-\rho T}} \left(\frac{1 - e^{-rW}}{r} \frac{\frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} w \eta}{\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho}} \right)^{\frac{1}{1-\theta}} h_{t-1}, \quad (10)$$

and

$$x_t^E = \left(\frac{1 - e^{-rW}}{r} \frac{\frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} w \eta \theta}{\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho}} \right)^{\frac{1}{1-\theta}} h_{t-1}, \quad (11)$$

where the superscript E represents ‘educated.’ Consumption and education increase with parents’ human capital, h_{t-1} , whereas fertility is constant with respect to the changes in h_{t-1} because the positive income and negative substitution effects on fertility cancel each other out.

The important comparative-static results are that fertility, n_t^E , is decreasing in the length of lifetime, T , increasing in the length of working life, W , and decreasing in the length of child-rearing period, D ; and the amount of educational investment, x_t^E , is increasing in T and W . Let us elaborate the mechanism behind this. The prolongation of lifetime stimulates the demand for consumption relative to the demand for children because agents must consume goods over a longer period. In response to such a change, agents shift their time from child rearing to work.²⁷ The resultant increases in working

²⁵As will be apparent, the number of children is exclusively determined by the parameters (see n_t^E given by (9) and n_t^U given by (12)). Since $n_t^E < n_t^U$ (see Proposition 2), we can exclude the corner case by concentrating on the parameter configurations under which $zn_t^U < 1$.

²⁶In contrast to fertility choice, we cannot exclude the corner solutions for educational choice by imposing restrictions on the parameters, except for limiting cases (e.g. the case where $\theta \rightarrow 0$). Whether the agent chooses an interior solution or a corner one depends on the amount of parental human capital, h_{t-1} , which is the state variable of this economy.

²⁷Since n_t^E is constant with respect to h_{t-1} and decreasing in T , the model predicts that the amount of resources allocated to children decreases as economy develops and adult mortality declines. One might

time raise the return on education, and thus the amount of educational investment increases. Conversely, the prolongation of working life raises fertility. Since agents have already been liberated from child care at the retirement date (adult age W), the prolongation of working life has only a positive income effect on fertility. An increase in W has two opposing effects on education: the prolongation of working life directly increases working time while the increase in fertility induced by the prolongation of working life implies a decline in working time. The former direct effect dominates the latter effect, and thus the amount of educational investment increases.

Next, consider the case of a corner solution. The first-order conditions imply

$$n_t^U = \frac{\gamma (1 - e^{-rW})}{z (1 - e^{-rD}) \left[\gamma + (1 - \gamma) \frac{1 - e^{-\rho T}}{\rho} \right]} \quad (12)$$

and

$$C_t^U = \frac{1 - e^{-rW}}{r} \frac{(1 - \gamma) w \underline{h}}{\gamma + (1 - \gamma) \frac{1 - e^{-\rho T}}{\rho}}, \quad (13)$$

where the superscript U represents ‘uneducated.’ As in the case of an interior solution, fertility, n_t^U , is decreasing in T , increasing in W , and decreasing in D . These comparative-static results are derived using a mechanism identical to that in the case of positive educational investment. It should be noted that in either case, interior or corner, increases in T and W yield opposite effects on fertility, suggesting the importance of distinguishing between retirement from the labor market and death.

Comparing the case of an interior solution with that of a corner solution, we obtain the following two propositions.

Proposition 1 *There is a threshold level of parents’ human capital, h^* , above which agents choose an interior solution of investing in education.*

Proof. Denote the indirect utility function of the educated by $V^E(h_{t-1})$ and that of the uneducated by V^U . Substituting (9) and (10) into (7), and (12) and (13) into (7), we

think that this prediction is unreasonable, but such a criticism is not crucial. If we incorporate educational investment for children, the model predicts that the amount of resources allocated to children is increasing in h_{t-1} and not always decreasing in T (see equations (26), (27), and (28) in Subsection 3.2).

obtain, respectively,

$$V^E(h_{t-1}) = A - \gamma \ln \left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} \right) \\ + (1-\gamma) \frac{1-e^{-\rho T}}{\rho} \ln \frac{\rho \theta^{\frac{\theta}{1-\theta}} (1-\theta)}{1-e^{-\rho T}} \left(\frac{1-e^{-rW}}{r} \frac{\frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} w \eta}{\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho}} \right)^{\frac{1}{1-\theta}} h_{t-1}$$

and

$$V^U = A - \gamma \ln \left[\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho} \right] \\ + (1-\gamma) \frac{1-e^{-\rho T}}{\rho} \ln \frac{1-e^{-rW}}{r} \frac{(1-\gamma) w \underline{h}}{\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho}},$$

where A is a constant term common to $V^E(h_{t-1})$ and V^U .

$V^E(h_{t-1})$ is increasing in h_{t-1} , $\lim_{h_{t-1} \rightarrow 0} V^E(h_{t-1}) = -\infty$, and $\lim_{h_{t-1} \rightarrow \infty} V^E(h_{t-1}) = \infty$, while V^U does not vary with h_{t-1} . Thus, there exists a unique h^* such that the agents with parents whose human capital level is h^* are indifferent to the choice between investing in education and remaining uneducated; that is, $V^E(h^*) = V^U$, $V^E(h_{t-1}) < V^U$ if $h_{t-1} < h^*$, and $V^E(h_{t-1}) > V^U$ if $h_{t-1} > h^*$. ■

The larger the amount of parents' human capital, the higher the return on educational investment. The agents whose parents' human capital level is high are more likely to be educated.

Proposition 2 (i) *The fertility of the educated is lower than that of the uneducated.* (ii) *The threshold, h^* , is decreasing in T and W .*

Proof. (i) It follows from (9) and (12) that

$$n_t^E - n_t^U = - \frac{\frac{\gamma}{z} \frac{\theta}{1-\theta} (1-\gamma) \frac{1-e^{-\rho T}}{\rho}}{\left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} \right) \left[\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho} \right]} < 0.$$

(ii) Totally differentiating $V^E(h^*) = V^U$, we obtain

$$\frac{dh^*}{dT} = \frac{\gamma}{1-\gamma} \frac{\rho^2 e^{-\rho T} h^*}{(1-e^{-\rho T})^2} \left\{ \ln \left[\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho} \right] - \ln \left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} \right) \right\} < 0$$

and

$$\frac{dh^*}{dW} = -\frac{\theta}{1-\theta} \frac{re^{-rW}h^*}{1-e^{-rW}} < 0.$$

■

Result (i) simply comes from the fact that the opportunity costs for child rearing are larger for the educated than for the uneducated. When choosing to be educated, agents face a tradeoff between increasing the efficiency units of labor at each time and saving on educational expenditure. The educated with larger efficiency units of labor have a comparative advantage in earning income, whereas the uneducated with smaller efficiency units have a comparative advantage in raising children. Therefore, the utility-maximizing behavior of the educated (resp. uneducated) is characterized by a lesser (resp. higher) number of children.

Result (ii) states that as lifetime and working life increase in length, agents are more likely to choose to be educated: as T rises, agents come to attach importance to consumption relative to children. Then, it is advantageous for agents to acquire education because it gives them a comparative advantage in earning income for consumption. As such, $dh^*/dT < 0$. The result $dh^*/dW < 0$ reflects the conventional Ben-Porath mechanism: prolongation of working life over which investments in human capital are paid off promotes human capital investment.

2.3 Dynamics of Human Capital

It follows from the results obtained above that the dynamic equation of human capital is given by

$$h_t = \underline{h} \text{ if } h_{t-1} \leq h^* \quad (14)$$

and

$$h_t = \eta \left(\frac{1 - e^{-rW}}{r} \frac{\frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} w\eta\theta}{\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho}} \right)^{\frac{\theta}{1-\theta}} h_{t-1} \equiv \Omega h_{t-1} \text{ if } h_{t-1} > h^*. \quad (15)$$

Using this dynamic equation, we obtain the following proposition.

Proposition 3 (i) If $\Omega \leq 1$, the economy is characterized by stagnation. If $\Omega > 1$ and $h^* \geq \underline{h}$, multiple long-run equilibria exist: whether the economy persistently grows depends on the initial amount of ancestral human capital. If $\Omega > 1$ and $h^* < \underline{h}$, the economy is characterized by sustained growth. (ii) Rises in T and W increase Ω and decrease h^* , thereby facilitating the growth takeoff.

Proof. (i) It is evident that depending on the parameters, Ω can be larger or smaller than 1 and h^* can be larger or smaller than \underline{h} . Thus, it suffices to show that $\Omega h^* > \underline{h}$. Suppose that $\Omega h^* \leq \underline{h}$. Then, the relationship $V^E(\underline{h}/\Omega) \geq V^E(h^*) = V^U$ must hold; that is, the following must hold:

$$\begin{aligned} & \gamma \ln n_t^E + (1 - \gamma) \frac{1 - e^{-\rho T}}{\rho} \ln \frac{1 - \gamma}{\gamma + \frac{1 - \gamma}{1 - \theta} \frac{1 - e^{-\rho T}}{\rho}} \\ & \geq \gamma \ln n_t^U + (1 - \gamma) \frac{1 - e^{-\rho T}}{\rho} \ln \frac{1 - \gamma}{\gamma + (1 - \gamma) \frac{1 - e^{-\rho T}}{\rho}}. \end{aligned}$$

This relationship cannot hold because $n_t^E < n_t^U$. It follows that $\Omega h^* > \underline{h}$.

(ii) It follows from differentiating (15) that $\partial\Omega/\partial T > 0$ and $\partial\Omega/\partial W > 0$. The results $\partial h^*/\partial T < 0$ and $\partial h^*/\partial W < 0$ have already been obtained in Proposition 2. ■

Figure 2 depicts the dynamic system of human capital. Depending on the parameter configuration, we can classify the dynamic system into three cases. Panel (a) describes the case in which the only long-run equilibrium is characterized by stagnation; panel (b) describes the case in which multiple long-run equilibria occur, characterized by stagnation and sustained growth; and panel (c) describes the case in which the only long-run equilibrium is characterized by sustained growth. The result that rises in T and W increase Ω and decrease h^* suggests that as T and W increase, the characteristics of the dynamic system evolve as depicted in panel (d).²⁸ The dynasties with low longevity are likely to stagnate and those with high longevity are likely to grow, and within an intermediate range of longevity, multiple long-run equilibria occur: whether a dynasty is stuck in a poverty trap or takes off depends on the initial amount of ancestral human capital. Multiplicity

²⁸Whether Ωh^* increases with T and W is ambiguous.

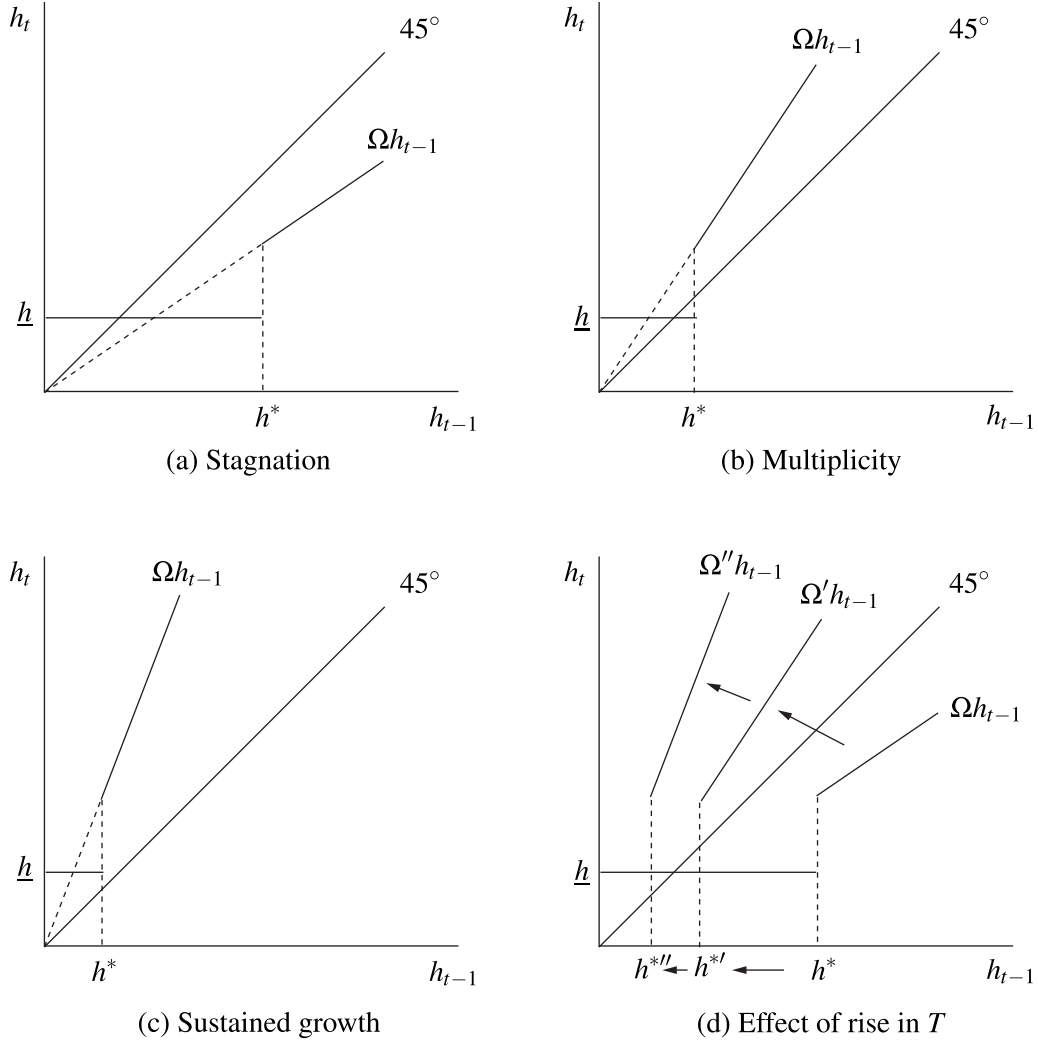


Figure 2: Dynamics of human capital

of long-run equilibria is based on the effect of parental human capital on the returns on education. The larger the parents' human capital, the larger is the offspring's educational investment, and thus the difference in the amount of ancestral human capital is inherited by the following generation. However, even if a dynasty is initially caught in a poverty trap, a sufficiently large increase in life expectancy can liberate the dynasty from poverty.

The dynamic system has an important implication for the links among life expectancy, human capital, and growth takeoff. Some have argued that increased life expectancy cannot result in growth taking off based on the observation that life expectancy in today's

developing countries is comparable to that during the onset of the takeoff of growth in today's developed countries, but they remain poor and their growth seems not to have taken off. Our model can provide a reasonable explanation for this observation without denying the role of life expectancy as the driving force of growth. According to our model, increased life expectancy alone is not sufficient for an economy to take off from stagnation if the dynamic system is characterized by multiple long-run equilibria; human capital is also necessary. Since the late nineteenth century, universal education systems have been established in many developed countries. Our theory suggests that such education reforms, combined with increased adult longevity, may have put these countries onto a growth path.²⁹

We conclude this subsection by investigating the effect of simultaneous changes in T and W . To see whether such changes promote education and growth, we need only examine the effect on x_t^E :

$$\begin{aligned} dx_t^E &= \frac{\partial x_t^E}{\partial W} dW + \frac{\partial x_t^E}{\partial T} dT \\ &= \frac{(1-\gamma)w\theta\Omega h_{t-1}}{(1-\theta)^2 \left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} \right)} \left(\frac{1-e^{-\rho T}}{\rho} e^{-rW} dW + \gamma \frac{1-e^{-rW}}{r} e^{-\rho T} dT \right) \end{aligned}$$

Early retirement implies $dW < 0$ and increased life expectancy implies $dT > 0$. Even if early retirement has a negative effect on education through the conventional Ben-Porath mechanism, the total effect on education can be positive if life expectancy sufficiently increases. According to Lee (2001), over the last 150 years, the expected length of post-retirement period at age 20 for U.S. male workers considerably increased, while their expected length of working life exhibited little change. Our model predicts that agents increase educational investment in response to such changes.

²⁹Galor and Moav (2006) provide a concise survey on the history of education reforms in the Western world.

3 Extensions and Discussion

3.1 Endogenous Retirement

Although life expectancy, which relies on knowledge and technological development in the medical sciences, is largely exogenous for individuals (see, for example, Soares 2005), retirement is usually an individual choice. Here, we extend the basic model by endogenizing the retirement date. We reformulate the utility function of agents so that they receive disutility from work:

$$\gamma \ln n_t + (1 - \gamma) \int_0^T e^{-\rho\tau} \ln C_t e^{(r-\rho)\tau} d\tau - \int_0^{W_t} e^{-\rho\tau} f(\tau) d\tau, \quad (16)$$

where $f(\tau)$ represents the disutility from work at adult age τ and W_t is the retirement date, which is determined by individual choice. Note that the consumption path given by (6) is already incorporated in this expression. We assume the following instantaneous disutility function:

$$f(\tau) = \begin{cases} 0 & \text{if } \tau \leq D, \\ e^{\lambda\tau} \sigma & \text{if } \tau > D, \end{cases} \quad (17)$$

where $\lambda > \rho$ and $\sigma > 0$. This function indicates that work becomes increasingly hard with age.³⁰ The assumption that $f(\tau) = 0$ for $\tau \leq D$ ensures that agents do not retire before completing child care, which is employed for simplification. All the other assumptions are the same as in the basic model.

The agent maximizes (16) with respect to n_t , C_t , x_t , and W_t , subject to equations (2), (8), and (17). We concentrate on the interior solutions for retirement choice, that is, we assume that λ and σ are sufficiently large that agents retire before death.

As in the basic model, the solution can either be interior, $x_t > 0$, or at a corner, $x_t = 0$. First, consider the case of an interior solution. The first-order conditions with respect to

³⁰One interpretation is that the disutility of work (and the relative utility of leisure) is higher when health is poorer. This is a convenient and occasionally implemented mechanism to model the retirement motive. See, for instance, Bloom et al. (2007) and Hazan (2009).

n_t and W_t are given by, respectively,

$$n_t^E = \frac{\gamma \left(1 - e^{-rW_t^E}\right)}{z(1 - e^{-rD}) \left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho}\right)} \quad (18)$$

and

$$\frac{\frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho} e^{-rW_t^E}}{\frac{1-e^{-rD}}{r} (1 - zn_t^E) + \frac{e^{-rD}-e^{-rW_t^E}}{r}} = e^{(\lambda-\rho)W_t^E} \sigma. \quad (19)$$

Equation (19) gives that agents choose their retirement date so that the disutility from work at retirement (RHS) is equal to the marginal cost of retiring, measured in terms of the loss in utility from foregone consumption (LHS). Solving these two equations, we obtain

$$e^{-(\lambda-\rho+r)W_t^E} \left(\gamma + \frac{1-\gamma}{1-\theta} \frac{1-e^{-\rho T}}{\rho}\right) = \frac{1-e^{-rW_t^E}}{r} \sigma, \quad (20)$$

which implicitly defines the retirement date, W_t^E . The other choice variables, C_t^E , x_t^E , and n_t^E , are given by (10), (11), and (18). We see that other than the result that the retirement date is endogenously determined by (20), the individual decision is the same as in the basic model.

Next, consider the case of a corner solution. The first-order conditions with respect to n_t and W_t are given by, respectively,

$$n_t^U = \frac{\gamma \left(1 - e^{-rW_t^U}\right)}{z(1 - e^{-rD}) \left[\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho}\right]} \quad (21)$$

and

$$\frac{(1-\gamma) \frac{1-e^{-\rho T}}{\rho} e^{-rW_t^U}}{\frac{1-e^{-rD}}{r} (1 - zn_t^U) + \frac{e^{-rD}-e^{-rW_t^U}}{r}} = e^{(\lambda-\rho)W_t^U} \sigma. \quad (22)$$

The interpretation of (22) is the same as that of (19). Solving (21) and (22), we obtain

$$e^{-(\lambda-\rho+r)W_t^U} \left[\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho}\right] = \frac{1-e^{-rW_t^U}}{r} \sigma, \quad (23)$$

which implicitly defines the retirement date, W_t^U . The other choice variables, C_t^U and n_t^U ,

are given by (13) and (21). We see that other than the result that the retirement date is endogenously determined by (23), the individual decision is the same as in the basic model.

Totally differentiating (20) and (23), we have $dW_t^E/dT > 0$ and $dW_t^U/dT > 0$; in either case, interior or corner, the length of working life increases with the length of lifetime. On the other hand, in contrast to the basic model, the effect of increases in T on fertility is ambiguous. Let us elaborate on the interaction between fertility and retirement decisions by exploiting the first-order conditions. We proceed by focusing on the case of an interior solution because similar reasoning is applicable to the corner case.

Equation (18) indicates the positive relationship between n_t^E and W_t^E , reflecting that, as mentioned in Section 2, the prolongation of working life has only positive income effects on fertility because agents have already been liberated from child care at the retirement date. Equation (19) indicates the positive association between n_t^E and W_t^E , reflecting that rises in fertility imply declines in working time and decreases in earnings, thereby increasing the loss of utility from foregone consumption because of the diminishing marginal utility and inducing agents to work longer. As depicted in Figure 3, the individual choice on fertility and retirement is characterized by the intersection of these two equations.³¹

Consider the effect of a rise in T . Equation (18) indicates that the rise in T decreases n_t for any given W_t . The prolongation of lifetime stimulates the demand for consumption relative to the demand for children, which is the main mechanism in this paper, inducing agents to have fewer children. Equation (19) indicates that the rise in T increases W_t for any given n_t . The rise in T implies the prolongation of lifetime over which agents must consume goods, and thus an increase in the loss in utility from foregone consumption, thereby extending the length of working life. This effect is what Kalemli-Ozcan and Weil (2010) call the ‘horizon effect,’ through which gains in adult longevity delay retirement. In the diagram, the total effects on fertility and retirement are ambiguous. As mentioned above, however, it follows from (20) that $dW_t^E/dT > 0$, whereas the effect on n_t is analytically ambiguous. Depending on the parameters, fertility might rise or fall as a result

³¹The uniqueness can be easily verified from (20). We also have $d^2n_t^E/dW_t^{E2} < 0$ from (18) and $d^2W_t^E/dn_t^{E2} < 0$ from (19).

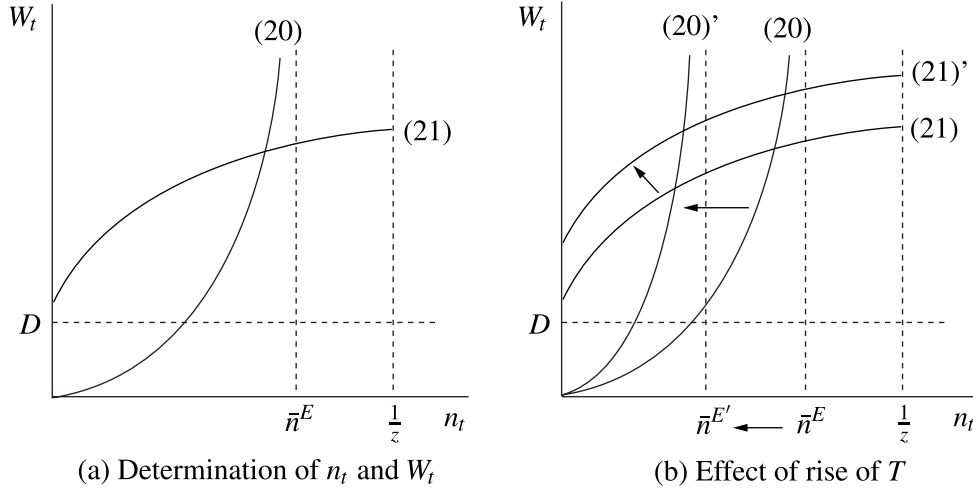


Figure 3: Individual choice of n_t and W_t . Notes: \bar{n}^E is defined by $\lim_{W_t^E \rightarrow \infty} n_t^E$ in (18).

of increased adult longevity.

There are many factors affecting the individual retirement decision. For example, the establishment of a public pension system, income effects of rising wages, and technological progress obsoleting the skills of the elderly are well known as possible causes of early retirement. It is natural that our model that excludes these factors cannot explain the declining trends in the labor force participation of the elderly. What is more important here is that endogenizing fertility mitigates the ‘horizon effect’ of increased adult longevity. Declining fertility, that is, increasing labor supply, enables agents to earn more and save more for the future when they are young adults and retire early through the income effect. This mechanism possibly induces early retirement, although this has not been identified in the literature. This ‘fertility effect,’ as will be shown below, allows us to generate a salient feature of the process of economic growth: simultaneous rises in education, declines in fertility, and declines in the labor force participation of the elderly.

As a thought experiment, consider a technological change prejudicial to the elderly.³² In our model, such a change can be expressed as a rise in λ , which represents the pace at which the disutility from work increases with age.³³ The rise in λ shifts (19) so that

³²Introducing a transfer promoting retirement also permits us to conduct a similar thought experiment.

³³Such an formulation of the effect of technological change can be interpreted as meaning that technological progress promotes skill obsolescence and calls for increased efforts to continue working. Alternatively,

W_t decreases for any given n_t , but does not shift (18). Hence, by increasing T and λ appropriately, we can decrease n_t and W_t simultaneously. Note that fertility and retirement affect the return on education only by affecting the present discounted value of the labor supply, $\int_0^D e^{-r\tau} (1 - zn_t) d\tau + \int_D^{W_t} e^{-r\tau} d\tau$, and thus simultaneous changes in n_t and W_t that do not change this value do not change the expenditure on education.³⁴ Suppose that such a change is caused by simultaneous increases in T and λ . This change then leads to decreases in n_t and W_t but does not change x_t . Then, suppose an infinitesimal increase in T . As a result, x_t increases while n_t and W_t are still lower than their levels before the initial changes in T and λ . It follows that we obtain the following proposition.

Proposition 4 *There are changes in T and λ such that rises in schooling, declines in fertility, and declines in retirement age are simultaneously generated*

Finally, we explore the implication of endogenizing the retirement date for the dynamics of human capital. As in the basic model, there is a threshold level of parents' human capital, above which agents choose an interior solution of investing in education. The dynamic system is still given by (14) and (15), except that W_t in (15) is endogenously determined by (20). It is obvious that result (i) of Proposition 3 holds entirely even in the case of endogenous retirement. Thus, the dynamic system is still classified into three cases: stagnation, multiple long-run equilibria, and sustained growth. It follows from (11) that x_t^E positively depends on T and W_t^E , and thus, rises in T have the effects of increasing x_t^E directly and by raising W_t^E . This implies that result (ii) of Proposition 3 (rises in T facilitate the growth takeoff) also holds.

3.2 Quantity-Quality Model

It is rather standard in the literature to include in the utility of parents not only the number of children, but also some measure of the quality of children, the so-called quantity-quality model à la Becker (1960). In this subsection, we extend the model so that parents care

we can assume that human capital depreciates with age and technological change increases the depreciation rate; then we obtain a qualitatively similar result, but the model becomes analytically complicated.

³⁴There exists an upward isoquant curve of the present discounted value of labor supply in the (n_t, W_t) space.

about the amount of education for their children, and we explore the robustness of our model to such an extension. The utility function and the production function of human capital are modified as, respectively,

$$\gamma \ln n_t + \gamma \phi \ln b_t + (1 - \gamma) \int_0^T e^{-\rho\tau} \ln c_t(\tau) d\tau$$

and

$$h_t = \max \left\{ \eta x_t^\theta b_t^\xi h_{t-1}^{1-\theta-\xi}, \underline{h} \right\}, \quad (24)$$

where b_t is the amount of education for each child and $\phi \in (0, 1)$ and $\xi \in (0, 1)$ are parameters.³⁵³⁶

The problem faced by the agent can be rewritten as

$$\begin{aligned} \max_{n_t, C_t, x_t, b_t} \quad & \gamma \ln n_t + \gamma \phi \ln b_t + (1 - \gamma) \int_0^T e^{-\rho\tau} \ln C_t e^{(r-\rho)\tau} d\tau, \\ \text{s.t.} \quad & \int_0^T e^{-\rho\tau} C_t d\tau + x_t + b_t n_t = \int_0^D e^{-r\tau} (1 - z n_t) w h_t d\tau + \int_D^W e^{-r\tau} w h_t d\tau, \\ & \text{and } h_t = \max \left\{ \eta x_t^\theta b_t^\xi h_{t-1}^{1-\theta-\xi}, \underline{h} \right\}. \end{aligned} \quad (25)$$

The product of the amount of a child's education and the number of children, $b_t n_t$ in the LHS of (25), represents the quantity-quality tradeoff. The larger the amount of education, the higher the cost of children; the larger the number of children, the higher the cost of education.

As in the basic model, the solution can either be interior, $x_t > 0$, or at a corner, $x_t = 0$. First, consider the case of an interior solution. The first-order conditions imply

$$n_t^E = \frac{\gamma (1 - e^{-rW})}{z (1 - e^{-rD}) \left[\gamma + \frac{\gamma \phi}{(1-\phi)(1-\theta)} + \frac{1-\gamma}{(1-\phi)(1-\theta)} \frac{1-e^{-\rho T}}{\rho} \right]}, \quad (26)$$

³⁵We assume that parents value the wealth, in the form of education, they pass on to their descendants. This motive is much more tractable than other bequest motives and is often used in the development and growth literature, for example, Banerjee and Newman (1993) and Galor and Zeira (1993).

³⁶The assumption $\phi \in (0, 1)$ is conventional in literature. This assumption guarantees that the second-order condition for maximization is satisfied in the household's maximization problem: if $\phi \geq 1$, households obtain infinite utility by reducing the number of children close to zero and thereby increasing their education level close to infinity.

$$x_t^E = \left[\frac{1 - e^{-rW}}{r} \frac{\left(\frac{\gamma\phi}{(1-\phi)(1-\theta)} + \frac{1-\gamma}{(1-\phi)(1-\theta)} \frac{1-e^{-\rho T}}{\rho} \right) w\eta\theta}{\gamma + \frac{\gamma\phi}{(1-\phi)(1-\theta)} + \frac{1-\gamma}{(1-\phi)(1-\theta)} \frac{1-e^{-\rho T}}{\rho}} b_{t-1}^\xi h_{t-1}^{1-\theta-\xi} \right]^{\frac{1}{1-\theta}}, \quad (27)$$

and

$$b_t^E = \frac{1 - e^{-rD}}{r} \frac{\phi zw h_t}{1 - \phi} = \frac{1 - e^{-rD}}{r} \frac{\phi zw}{1 - \phi} \eta (x_t^E)^\theta b_{t-1}^\xi h_{t-1}^{1-\theta-\xi}. \quad (28)$$

Next, consider the case of a corner solution. The first-order conditions imply

$$n_t^U = \frac{\gamma(1-\phi)(1-e^{-rW})}{z(1-e^{-rD}) \left[\gamma + (1-\gamma) \frac{1-e^{-\rho T}}{\rho} \right]}$$

and

$$b_t^U = \frac{1 - e^{-rD}}{r} \frac{\phi zw h}{1 - \phi}. \quad (29)$$

Although the calculation is a bit more complex than in the basic model, the main results concerning the household's decision problem remain qualitatively the same. There is a threshold level of the return on own education, which is determined by h_{t-1} and b_{t-1} , and agents faced with returns higher than the threshold level choose an interior solution of investing in own education. The comparative-static results are $\partial n_t^E / \partial T < 0$, $\partial n_t^E / \partial W > 0$, $\partial x_t^E / \partial T > 0$, $\partial x_t^E / \partial W > 0$, $\partial n_t^U / \partial T < 0$, $\partial n_t^U / \partial W > 0$, and $n_t^E < n_t^U$, all of which have the same sign of inequality as in the basic model. The quantity of education for each child, b_t , which is newly introduced here, is increasing in T in the case of positive own education (through the increase in x_t^E) and constant with respect to T otherwise. When adult longevity, T , increases, in spite of the decreased weight given to children, b_t does not decrease. This is because there exists a quantity-quality interaction: since increased adult longevity decreases the number of children, as in the basic model, the price of education for children falls and parents are induced to educate children, which offsets the effect of decreased weight given to children on b_t .

Consider the dynamics of human capital. If initial values of h_t and b_t are given, then in one period, the relationship between h_t and b_t becomes linear by the household's choice, (28) and (29). Thereafter, the economy (the dynasty) evolves according to the dynamic system similar to that of the basic model: substituting (27) and (28) into (24), we find

that h_t linearly depends on h_{t-1} in the case of $x_t > 0$, and $h_t = \underline{h}$ in the case of $x_t = 0$.

3.3 Hump-Shaped Fertility Transition

The model presented above predicts a negative, monotonic relationship between adult longevity and fertility: it follows from (9) and (12) that $\partial n_t^E / \partial T < 0$, $\partial n_t^U / \partial T < 0$ and $n_t^E < n_t^U$. While such a model may be a good description of the dynamics of fertility in countries that have already experienced the onset of the demographic transition, for example, in the United Kingdom or the United States over the last 150 years, it is not universal. It is widely known that many Western countries experienced upward fertility trends before the onset of the demographic transition. In this subsection, we discuss the possibility that our model provides a novel explanation of the reversal in the fertility transition.

This paper is motivated by the view that increased life expectancy has led to the prolongation of retirement life, rather than the prolongation of working life. Lee (2001) and Hazan (2009) support this view for the United States in the 19th and 20th centuries, but this might not be the case for earlier stages of development. According to Lee (2001), the expected length of the retirement period for the U.S. male workers born in 1830 was 2.65 years. Since there is a lower bound on the length of the retirement period, 0, it is likely that the length of working life was closely linked with adult longevity in earlier times. As the simplest case of such environments, now consider the case that $W = T$, that is, rises in adult longevity lengthen working life by exactly the same length. Furthermore, suppose for simplicity that the subjective discount rate is equal to the interest rate, $\rho = r$. Then, it follows from (9) and (12) that $\partial n_t^E / \partial T > 0$ and $\partial n_t^U / \partial T > 0$. As already mentioned in the paragraph below equation (9), rises in T and W have opposing effects on fertility. In the case we are now considering, the positive income effect of the prolongation of working life dominates, and thus increased adult longevity raises fertility.

Our model suggests that the association between fertility and adult longevity is positive when rises in T are accompanied by rises in W , and negative otherwise. The result of Lee (2001) indicates that in the 19th century in the United States, the expected length

of working life began to cease being tied to adult longevity because of retirement. That the U.S. fertility rate started to decline in the early 19th century may or may not be a coincidence.

4 Concluding Remarks

There is ample evidence that life expectancy is positively correlated with economic development and growth. Motivated by this evidence, a number of researchers have developed growth models in which gains in life expectancy promote education and growth via the Ben-Porath mechanism, which offers a clear explanation for the positive association between life expectancy and education. However, since the mid-nineteenth century, the length of working life has not increased substantially because of falls in retirement age, which casts doubt on the role of the Ben-Porath mechanism in the growth process. This paper presents an overlapping generations model in which agents make educational and fertility decisions under a life-cycle consideration, and retirement from the labor market is explicitly distinguished from death. We reconcile the theory that gains in life expectancy trigger a growth takeoff by increasing education with the observation that increased life expectancy has predominantly resulted in the prolongation of retirement life rather than the prolongation of working life. In spite of its simple form, the model provides fairly rich predictions about the links among life expectancy, retirement, education, fertility, and growth, for example, the prolongation of working life and that of retirement life have opposing effects on fertility choice; gains in adult longevity could shift the dynamic system of economies (dynasties) from stagnation to growth; and rises in education, declines in fertility, and falls in retirement age can be simultaneously generated.

The model does not consider several factors relevant to economic growth. A prominent example is the effect of technological progress. As initially pointed out by Galor and Weil (1999, 2000), technological progress must have played an important role in increasing the demand for human capital and raising education during the transition from stagnation to growth. If such a demand shift occurs, the amount of educational investment could increase even without increases in working time, which in our model is caused by declines

in child-rearing time. This argument might render moot our endeavor to establish the mechanism through which higher longevity promotes education without the prolongation of working life. Historical evidence, however, suggests that the return on human capital did not increase during the period when European countries transitioned into modern growth regimes, indicating that the rise in education over this period cannot be explained by the demand shift alone (see Clark 2005 and Galor 2005b for this discussion). This paper provides an explanation for the supply shift in terms of changes in individual life-cycle behavior without contradicting the observation that the length of working life has not been substantially prolonged because of retirement. Our model is more likely to complement rather than contradict the models focusing on demand shifts, such as that of Galor and Weil (2000). Models explicitly incorporating the life-cycle consideration, like ours, add new insight into the literature.

This model is simple and can be extended in several ways to explain more observations. We conclude by suggesting some potential directions for future research. In our model, acquiring education entails only pecuniary costs, and the timing of birth is fixed and does not vary across generations. Incorporating time-consuming education and endogenizing the timing of birth would allow us to study changes in individual life-cycle behavior and the demographic structure of the population. Introducing endogenous physical capital accumulation in a closed-economy framework would generate some interesting predictions. For example, such an extension would allow the model to capture the diluting effect of fertility and longevity on per-capita capital, and possibly improve our understanding of the results of the recent empirical literature on the relationship between life expectancy and growth (e.g. Acemoglu and Johnson (2007), Lorentzen et al. (2008), and Cervellati and Sunde (2011)). Furthermore, our model can be applied to the analysis of public policy, for example, the effects of improvement in a public pension system and public education.

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