A note on expectational stability under non-zero trend inflation*

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Abstract

This study examines the expectational stability of the rational expectations equilibria (REE) under alternative Taylor rules when trend inflation is non-zero. We find that when trend inflation is high, the REE is likely to be expectationally unstable. This result holds true regardless of the nature of the data (such as contemporaneous data, forecast, and lagged data) introduced in the Taylor rule. Our results suggest that a high macroeconomic volatility during the period of high trend inflation can be well explained by introducing the concept of expectational stability.

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1 Introduction

Many of the monetary policy analyses based on the New Keynesian framework have neglected the existence of non-zero trend inflation. However, several recent studies point out that the introduction of non-zero trend inflation has profound implications for monetary policy. Among them, Kiley (2007) and Ascari and Ropele (2009) introduce alternative versions of Taylor rules into New Keynesian models with non-zero trend inflation. They show that the so-called Taylor principle, which requires the central bank to adjust the nominal interest rate more than one-for-one with the variations of the inflation rate, does not necessarily guarantee the determinacy of the rational expectation equilibrium (REE) when the level of trend inflation is positive. Coibion and Gorodnichenko (2010) recently argue that high trend inflation in the 1970s made the U.S. economy indeterminate even though the Fed's policy likely satisfied the Taylor principle during that period. This implies that the central bank should carefully choose its policy rule coefficients by recognizing the relationship between the level of trend inflation and the determinacy of REE.

However, these studies assume that the economic agents have perfect knowledge of macroeconomic structures and always form rational expectations. If we instead assume that the agents only have imperfect knowledge and are learning about the structure of the economy over time, then the determinacy of REE is not the sole requirement for the central bank. Bullard and Mitra (2002) propose the expectational stability (E-stability) of REE, which ensures convergence of expectations to the REE under the standard learning algorithm, as another requirement for monetary policy rules. They show the parameter regions that satisfy the E-stability conditions as well as the determinacy conditions under alternative versions of Taylor rules. Their main finding is that the relationship between determinacy and E-stability depends on the version of policy rule being used. However, their study focuses on a relatively specific environment in which trend inflation is exactly equal to zero. It is still unclear how the introduction of non-zero trend inflation will affect their results.

In this study, we attempt to obtain the E-stability (as well as determinacy) conditions of REE, taking into account the presence of non-zero trend inflation. Several previous studies show that the introduction of non-zero trend inflation makes firms' pricing behavior more forward-looking compared to the case of zero trend inflation.[1] As a result, the rate of current inflation is inevitably affected by long-horizon inflation forecasts. This is the only, but important, departure from the study of Bullard and Mitra (2002).

Our analysis finds that when the level of trend inflation is high, the REE is likely to be E-unstable. This result holds true regardless of the nature of the data (such as contemporaneous data, forecast, and lagged data) used in the Taylor rule. We also show

[1] See, for example, Ascari (2004), Ascari and Ropele (2007), Sbordone (2007) and Cogley and Sbordone (2008).

that while the availability of current economic data in the conduct of monetary policy is a key to E-stability as well as REE determinacy in a low inflation environment, this is not necesarily the case in a high inflation environment.

Our results on E-stability conditions appear to be parallel with Ascari and Ropele's (2009) finding on determinacy conditions. However, there is an important difference. Ascari and Ropele (2009) show that in the case of the lagged-data rule, a rise in trend inflation does not necessarily narrow the determinacy region because the central bank can easily attain determinacy by responding strongly to the lagged output gap. In contrast, our study finds that higher trend inflation under the lagged rule makes the REE more likely to be E-unstable even if it is determinate. This means that high trend inflation is more robustly undesirable in terms of E-stability than determinacy. Therefore, the introduction of a learning mechanism can provide a better explanation for the well-established positive relationship between high trend inflation and macroeconomic instability.[2]

2 The model

2.1 A New Keynesian model under non-zero trend inflation

Some previous studies, such as Kiley (2007), Sbordone (2007), Cogley and Sbordone (2008) and Ascari and Ropele (2007, 2009), provide alternative expressions of New Keynesian models under non-zero trend inflation. Our model is based on that of Sbordone (2007) and of Cogley and Sbordone (2008), which is given as follows:

$$
\begin{aligned}
y_t &= y^e_{t+1} - \sigma(i_t - \pi^e_{t+1} - r^n_t), && (1)\\
\pi_t &= \kappa y_t + b_1 \pi^e_{t+1} + b_2 \sum_{j=2}^{\infty} \phi_1^{j-1} \pi^e_{t+j}, && (2)\\
r^n_t &= \rho_r r^n_{t-1} + \varepsilon_t. && (3)
\end{aligned}
$$

π_t is the percentage deviation of inflation from the (possibly non-zero) rate of trend inflation, which is assumed to be constant. y_t, i_t and r^n_t are the output gap, the nominal interest rate and the natural rate of real interest, respectively.[3] For an arbitrary variable

[2] Our study is closely related to Kurozumi (2011), who also investigates the E-stability of REE in a sticky price model with positive trend inflation. However, our analysis is distinct from his work mainly in two respects. First, we analyze the E-stability of REE under alternative versions (such as forward and lagged versions) of Taylor rules, although Kurozumi (2011) only examines the case under the contemporaneous Taylor rule. Second, since deflation has become a pressing concern in major developed countries, we investigate the case of negative (as well as positive) trend inflation, although Kurozumi (2011) focuses on the case of positive trend inflation.

[3] Preston (2005, 2006) argues that if adaptive learning is introduced for the process of agents' expectation formations, structural equations determining the output gap and inflation rate should involve long-horizon forecasts even when trend inflation does not exist. However, Honkapohja (2003) and Honkapohja, Mitra and Evans (2002) point out that bounded rationality itself does not call for long-horizon forecasts. They

x, x^e denotes the expectations of variable x. The technical reason for the deviation from the standard NKPC is that here log-linearization is done around a non-zero value of steady state inflation while steady state inflation is assumed to be zero in the standard model. Intuitively, if trend inflation is positive, firms try to internalize the influence of unavoidable relative price erosion in setting current prices. Thus, goods prices under non-zero trend inflation contain more information about the future compared to the case of zero trend inflation. As a result, long-horizon forecasts emerge in the third term of (2), and thus the parameters κ, b_1, b_2 and ϕ_1 are affected by the level of trend inflation.[4] [5]

To see how non-zero trend inflation influences firms' forward-lookingness, it would be useful to check how the values of b_1 and b_2 vary with the level of trend inflation. Following Sbordone (2007) and Cogley and Sbordone (2008), we set the benchmark parameter values as follows: $\alpha = .588$, $\theta = 9.8$, $\omega = .429$, $\tilde{\omega} = .63$, $\beta = .99$, and $\sigma = 6.25$.[6] With this parameterization, (b_1, b_2) takes the values of $(.968, -.009)$, $(.99, 0)$, $(1.033, .017)$, and $(1.073, .032)$ for the rate of (annualized) trend inflation -1%, 0%, 2%, 4%, respectively. Under zero trend inflation, the model reduces to the standard model ($b_1 = \beta$ and $b_2 = 0$). However, in other cases, the model departs from the standard one. Furthermore, at least within these numerical examples, the sum of coefficients on inflation expectations increases with the level of trend inflation. A rise in trend inflation also weakens the influence of the current output gap on current inflation: κ takes the value of .039, .035, .029 and .023 for the rate of (annualized) trend inflation -1%, 0%, 2%, 4%, respectively.

As for monetary policy rules, we introduce some versions of Taylor rules in which the central bank responds to (i) the contemporaneous data (y_t, π_t), (ii) the forecast (y^e_{t+1},

insist that the structural equations with one-period-ahead forecasts are still valid as long as agents have identical subjective expectations.

[4]Cogley and Sbordone (2008) derive the parameters as follows: $\phi_1 = \alpha\beta\overline{\Pi}^{(\theta-1)}$, $\phi_2 = \alpha\beta\overline{\Pi}^{\theta(1+\omega)}$, $\chi = \frac{1-\alpha\overline{\Pi}^{(\theta-1)}}{\alpha(1+\theta\omega)\overline{\Pi}^{(\theta-1)}}$, $b_1 = (1 + (1+\omega)\theta\chi)\,\phi_2 - (\theta-1)\chi\phi_1$, $b_2 = (\theta-1)\chi(\phi_2 - \phi_1)$, $\kappa = \tilde{\omega}\chi(1-\phi_2)$, where α is the probability of not changing prices, β is the discount factor, θ is the elasticity of substitution among different goods, ω is the elasticity of real marginal cost to its own output, $\tilde{\omega}$ is the elasticity of real marginal cost to the aggregate output and $\overline{\Pi}$ is the trend inflation in gross term.

[5]The original AS equation used by Cogley and Sbordone (2008, eq.8) also includes two additional terms: one is the term that depends on long-horizon forecasts of the output gap, and the other on past inflation. However, they report that the degree of price indexation is statistically not different from zero, and the estimated coefficient on the long-horizon forecasts of the output gap is at most 5×10^{-3} over the whole sample period (1960Q1:2003Q4). On that ground, we employ a simpler AS equation, and given the irrelevancy of those terms, our model is essentially the same as the one used by Ascari and Ropele (2007, 2009). Note that the model is exactly the same as the original one as long as log-utility is assumed. However, we also examined the case of a more general CRRA utility function, in which case long-horizon forecasts of the output gap need to be added. We nevertheless found that the main results of this paper do no change. The details are shown in the supplementary appendix, which is available from the authors upon request.

[6]Throughout this analysis, $\phi_1 > 0$ is assumed to be less than one. This implies that annual trend inflation must be less than 27.9% under our benchmark parameter values.

π^e_{t+1}), and (iii) the lagged data (y_{t-1}, π_{t-1}). The policy rule is generally given as

$$i_t = F_l X_{t-1} + F_c X_t + F_f X^e_{t+1} \tag{4}$$

where $X_t = [y_t \ \pi_t]'$, $F_i = [F_{iy} \ F_{i\pi}]$ for $i = c, f, l$. c, f and l represent the contemporaneous rule, the forecast-based rule, and the lagged-based rule, respectively.[7]

2.2 Rational expectations

Under rational expectations, the AS equation (2) can be made simpler by using an auxiliary variable h_t:

$$\begin{aligned} \pi_t &= \kappa y_t + (b_1 - b_2)\pi^e_{t+1} + h_t, \\ h_t &= b_2 \pi^e_{t+1} + \phi_1 h^e_{t+1}. \end{aligned}$$

This treatment is valid when the economic agents know the functional forms and parameters of the structural equations. However, if we assume that the agents do not have complete knowledge of the functional forms and parameters, then the agents cannot use the auxiliary variable to compute inflation expectations. Therefore, we need to employ the AS equation with long-horizon forecasts when examining expectational stability.

Under the contemporaneous rule or the forecast-based rule, the structure of the model under RE can be written as

$$G\bar{X}_t = H\bar{X}^e_{t+1} + Jr^n_t,$$

where $\bar{X}_t = [y_t, \pi_t, h_t]'$ and G, H and J are the matrices of coefficients. The minimal state variable (MSV) solution is given as

$$\bar{X}_t = D_{re} r^n_t.$$

The method of undetermined coefficients yields

$$D_{re} = [I - \rho_r G^{-1} H]^{-1} G^{-1} J.$$

If this is the only possible solution attained under RE, then the REE is said to be determinate. On the other hand, if sunspots or self-fulfilling expectations can determine the equilibrium, then the solution does not necessarily take the MSV form and the REE is indeterminate.

[7] As for the forecast-based rule, we assume that the central bank and private agents make identical forecasts. Honkapohja and Mitra (2005) and Muto (2011) introduce alternative setups in which the central bank and private agents make heterogeneous forecasts or one of their forecasts is influential to the other, although they assume that trend inflation is zero.

2.3 Adaptive learning

We now assume that agents are boundedly rational and that they are engaged in adaptive learning. Following the literature (e.g., Evans and Honkapohja, 2001), we assume that agents estimate their perceived law of motions (PLM), which takes the MSV form, by recursive least squares (RLS) with decreasing gain. The actual law of motions (ALM) are obtained by replacing the expectation terms in the structural model with the ones derived from the PLM. The mapping function, which maps PLM into ALM, is called "T-map" and the fixed point of the T-map constitutes the REE. The E-stability is defined as the local asymptotic stability of a REE under the learning mechanism. The REE is E-stable if all eigenvalues of the Jacobian of the T-map have real parts less than one. The "E-stability principle" advocated by Evans and Honkapohja (2001) states that if agents use RLS with decreasing gain and their information set remains bounded, then the E-stability guarantees that the economy eventually converges to the REE. As Bullard and Mitra (2002) argue, E-stability can be viewed as a minimum criterion for evaluating monetary policy rules.

2.3.1 The contemporaneous and the forecast-based rules

Under the contemporaneous and the forecast-based rules, the PLM is given as

$$X_t = A + Dr_t^n, \tag{5}$$

where A and D are 2 by 1 vectors of PLM coefficients. Given (5), the following expression is obtained.[8]

$$\sum_{j=2}^{\infty} \phi_1^{j-1} X_{t+j}^e = (1-\phi_1)^{-1}\phi_1 A + (1-\phi_1\rho_r)^{-1}\phi_1\rho_r^2 Dr_t^n. \tag{6}$$

The structural equations can be reformulated as

$$QX_t = WX_{t+1}^e + Ni_t + Ur_t^n + M\sum_{j=2}^{\infty} \phi_1^{j-1} X_{t+j}^e, \tag{7}$$

where

$$\begin{aligned} Q &= \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix}, W = \begin{bmatrix} 1 & \sigma \\ 0 & b_1 \end{bmatrix}, N = \begin{bmatrix} -\sigma \\ 0 \end{bmatrix}, \\ U &= \begin{bmatrix} \sigma \\ 0 \end{bmatrix}, M = \begin{bmatrix} 0 & 0 \\ 0 & b_2 \end{bmatrix}. \end{aligned}$$

[8]Following previous studies, it is assumed that agents know the value of ρ_r. This assumption is innocuous since agents can easily estimate the value by regressing (3) and this estimation process does not affect the E-stability of REE.

By inserting (4), (5) and (6) into (7), we can obtain the actual law of motion (ALM):

$$\begin{aligned} X_t &= (Q - NF_c)^{-1}\{[W + NF_f + (1-\phi_1)^{-1}\phi_1 M]A \\ &\quad +[(W + NF_f + (1-\phi_1\rho_r)^{-1}\phi_1\rho_r M)\rho_r D + U]r_t^n\}. \end{aligned} \tag{8}$$

The T-maps from the PLM to the ALM are then given as

$$T(A) = (Q - NF_c)^{-1}[W + NF_f + (1-\phi_1)^{-1}\phi_1 M]A \tag{9}$$

$$T(D) = (Q - NF_c)^{-1}[(W + NF_f + (1-\phi_1\rho_r)^{-1}\phi_1\rho_r M)\rho_r D + U]. \tag{10}$$

Here, let $DT_Z(\bar{Z})$ be the Jacobian matrix of the T-map evaluated at the corresponding RE value $\bar{Z}$:

$$DT_Z(\bar{Z}) = \left.\frac{\partial vec(T(Z))}{\partial vec(Z)'}\right|_{Z=\bar{Z}}.$$

The E-stability of the REE can be attained if and only if all of the eigenvalues of $DT_A(\bar{A})$ and $DT_D(\bar{D})$ have real parts less than one.[9]

2.3.2 The lagged-data rule

Under the rule based on lagged data, it is implicitly assumed that the central bank and private agents do not have current economic data. McCallum (1999) argues that policymakers are practically unable to obtain the data on contemporaneous macroeconomic variables, such as inflation and output, in making their policy decisions. In this situation it is natural to consider that private agents also do not have information about current economic data.[10] Under RE, the model can be expressed as

$$\tilde{G}\bar{X}_t = \tilde{H}\bar{X}_{t+1}^e + K\bar{X}_{t-1} + Jr_t^n.$$

The MSV solution is given as

$$\bar{X}_t = C_{re}\bar{X}_{t-1} + D_{re}r_{t-1}^n + E_{re}\varepsilon_t.$$

The method of undetermined coefficients yields the following three conditions:

$$\begin{aligned} C_{re} &= \tilde{G}^{-1}[\tilde{H}C_{re}^2 + K] \\ D_{re} &= [I - \tilde{G}^{-1}\tilde{H}(C_{re} + \rho_r I)]^{-1}\rho_r\tilde{G}^{-1}J \\ E_{re} &= G^{-1}J. \end{aligned}$$

[9] We find that the E-stability condition is generally stipulated by four inequalities, and which one is relevant depends on the level of trend inflation and other parameter values. Since the relevant inequality condition varies with the level of trend inflation in a non-proportional way, we report numerical results here. Analytical expression of the generalized E-stability conditions is shown in the supplementary appendix.

[10] Bullard and Mitra (2002) also assume this type of informational symmetry. If we instead assume that only private agents can use the contemporaneous data, the E-stability region coincides with the determinacy region. However, we consider this assumption practically implausible.

As is noted by Bullard and Mitra (2002), the coefficient matrix on the lagged vector, C_{re}, is a solution of a matrix quadratic.

We express the PLM under the lagged-data rule as follows:

$$X_t = A + CX_{t-1} + Dr^n_{t-1} + Q^{-1}U\varepsilon_t. \tag{11}$$

The coefficient on ε_t is assumed to be the same as that under ALM. This specification is innocuous because expectations are formed at $t-1$.[11] It follows that

$$\begin{aligned}
X^e_{t+1} &= (I+C)A + C^2X_{t-1} + (C+\rho_r I)Dr^n_{t-1} \\
X^e_{t+2} &= (I+C+C^2)A + C^3X_{t-1} + [C(C+\rho_r I) + \rho_r^2 I]Dr^n_{t-1} \\
&\vdots
\end{aligned}$$

The infinite summation term in eq. (2) leads to

$$\begin{aligned}
\sum_{j=2}^{\infty}\phi_1^{j-1}X^e_{t+j} &= (1-\phi_1)^{-1}\phi_1[I + C + (I-\phi_1 C)^{-1}C^2]A \\
&\quad +(I-\phi_1 C)^{-1}\phi_1 C^3 X_{t-1} \\
&\quad +(I-\phi_1 C)^{-1}\phi_1[C(C+\rho_r I) + (1-\phi_1\rho_r)^{-1}\rho^2 I]Dr^n_{t-1} \\
&= \tilde{A} + \tilde{C}X_{t-1} + \tilde{D}r^n_{t-1}.
\end{aligned}$$

The ALM can then be written as

$$\begin{aligned}
X_t &= Q^{-1}\{W(I+C)A + M\tilde{A} + (WC^2 + M\tilde{C} + NF_l)X_{t-1} \\
&\quad +[W(C+\rho_r I)D + \rho_r U + M\tilde{D}]r^n_{t-1} + U\varepsilon_t\}.
\end{aligned} \tag{12}$$

The T-maps from the PLM to the ALM are then given as

$$T(A) = Q^{-1}[W(I+C)A + M\tilde{A}], \tag{13}$$

$$T(C) = Q^{-1}(WC^2 + M\tilde{C} + NF_l), \tag{14}$$

$$T(D) = Q^{-1}[W(C+\rho_r I)D + \rho_r U + M\tilde{D}]. \tag{15}$$

The E-stability conditions are that all of the eigenvalues of $DT_A(\bar{A},\bar{C})$, $DT_C(\bar{C})$ and $DT_D(\bar{C},\bar{D})$ have real parts less than one.

3 The E-stability conditions under non-zero trend inflation

The combinations of the Taylor rule coefficients, $F_{i\pi}$ and F_{iy}, $i = c, f, l$, that ensure E-stability and determinacy of the REE are presented in Figures 1, 2 and 3. In all figures, the upper-right panel corresponds to the case of zero trend inflation, which is equivalent to

[11] Bullard and Mitra (2002) use the same specification of PLM.

the situation analyzed by Bullard and Mitra (2002). The other panels show the E-stable and determinate regions under non-zero trend inflation. Although the determinate regions (except for the case of negative trend inflation) are essentially the same as those presented by Ascari and Ropele (2009), the E-stable regions are novel contribution of our study.

Our main finding is that under all specifications of the rule, higher trend inflation makes the REE more likely to be E-unstable: the E-stable region always shrinks as the rate of trend inflation increases. This is in contrast to the case of determinacy since higher trend inflation does not necessarily make the REE more likely to be indeterminate. Under the contemporaneous rule the E-stable region corresponds exactly to the determinate region, while the E-stable region is broader than the determinate region under the forecast-based rule. With these two rules, both the determinate region and E-stable region shrink as the rate of trend inflation increases.[12]

Under the lagged-data rule, however, things are different. There exists a region in which the REE is determinate but E-unstable, as shown by Bullard and Mitra (2002). Figure 3 shows that this region is broader for higher trend inflation. When the level of trend inflation is high, the central bank can easily achieve determinacy by responding strongly to the output gap. However, our results show that this kind of policy action fails to make the REE E-stable. The REE under the lagged-data rule is more likely to be E-unstable when trend inflation is high even if it is determinate.

Under RE, the introduction of lagged variables in the policy rule causes two kinds of effects. First, the REE becomes more likely to be explosive since additional state variables require more stable roots to obtain a non-explosive solution. Second, indeterminacy becomes less likely to arise since the lagged variables make the equilibrium more history dependent. The upper-right panel of Figure 3 reveals that, when trend inflation is zero, the REE is likely to be explosive or determinate as long as the response to the output, F_{ly}, are not very small. As the level of trend inflation increases, the determinate and non-explosive region enlarges. This is because firms become more forward-looking and the lagged variables become less influential. Although the likelihood of indeterminacy also increases, as in the case of alternative rules, for a small value of F_{ly}, our numerical exercise suggests that this effect is relatively minor under the lagged rule. The central bank can easily guarantee determinacy by strongly responding to the lagged variables.

However, since higher trend inflation strengthens firms' forward-lookingness, it is likely that agents' learning makes the REE E-unstable. In our numerical exercises, the sum of coefficients on inflation expectations (b_1 and b_2 in (2)) exceeds unity when trend inflation is relatively high (such as 2% or 4%). Thus, there arises a negative relationship between

[12]Following Bullard and Mitra (2002), we also examined the policy rule based on contemporaneous expectations: $i_t = F_p X_t^e$. The analysis reconfirms their original result that the E-stability and determinacy regions are exactly the same as those under the contemporaneous rule. The details are shown in the supplementary appendix.

output and inflation in the long run. This implies that an upward deviation of inflation expectation from the RE value will push down the output gap. If the central bank lowers the nominal interest rate in response to the fall in the output gap, then inflation expectations rise further. This suggests that the equilibrium under adaptive learning may not converge to the REE even if it is determinate.

Next, let us focus on the case of negative trend inflation. Under contemporaneous and forecast-based rules, the determinate and E-stable region is broader when trend inflation is negative rather than positive. Therefore, the REE is less likely to be indeterminate or E-unstable in a deflationary environment. This result has an important policy implication for low inflation countries. When trend inflation is very low, the degree of freedom for the central bank to control the nominal interest rate is inevitably small due to the presence of the zero lower bound (ZLB). Fortunately, our result indicates that the REE is more likely to be E-stable and determinate for lower trend inflation. This implies that the necessity of cutting interest rates against downward shocks will to some extent be removed. This will mitigate the fear of ZLB that the central banks have in an era of very low inflation.

Our analysis also shows that the availability of current economic data for the central bank is especially important in a low inflation environment because, when trend inflation is very low, the E-stable region is much broader under the contemporaneous rule than under the lagged-data rule. However, in a high inflation environment, the E-stable regions are similarly narrow under all versions of Taylor rules. The central bank's usage of current economic data does not help much to ensure the E-stability of the REE. In this sense, higher trend inflation is very likely to be associated with higher macroeconomic volatility.

4 Concluding remarks

Our analysis has shown that higher trend inflation tends to make the REE E-unstable under various specifications of the Taylor rule. This result holds true regardless of the nature of the data employed in the Taylor rule. Although the availability of current economic data for the central bank helps to guarantee the expectational stability of REE in a low inflation environment, this is not necessarily the case in a high inflation environment.

Our results provide a plausible explanation about why macroeconomic variables tend to be quite volatile in a high inflation environment. Coibion and Gorodnichenko (2010) argue that the US economy was quite volatile in the 1970s because high trend inflation caused the indeterminacy of the REE. Although this is an intriguing explanation, their argument is sensitive to the data employed by the Fed. In the case of a lagged-data rule, the determinacy region is quite broad even in a high inflation environment. In contrast, higher trend inflation always narrows the E-stable region under all versions of the Taylor rule. Therefore, the positive relationship between the level of trend inflation and macroeconomic volatility is better explained by introducing the concept of E-stability as opposed to determinacy.

Finally, our main results also have an important implication for the recent dispute on whether the level of inflation targets should be set well above zero. Based on the recent experience of the global financial crisis, Blanchard et al. (2010) raised the issue of whether the central bank should aim for a higher inflation target, such as 4%, in normal times in order to avoid ZLB. Our results may provide a negative answer to this question. A rise in the level of trend inflation will change the price-setting behavior of firms in a way that a violation of the E-stability condition becomes more likely. To investigate this issue more formally, however, the influence of ZLB should be explicitly taken into account. This issue should be left for future research.

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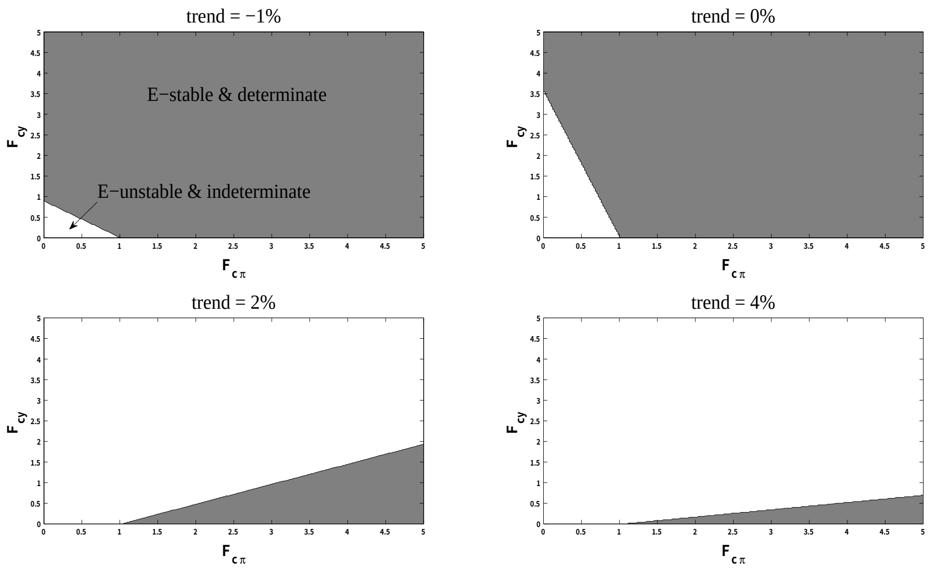

Figure 1: The E-stability and determinacy regions under the contemporaneous rule

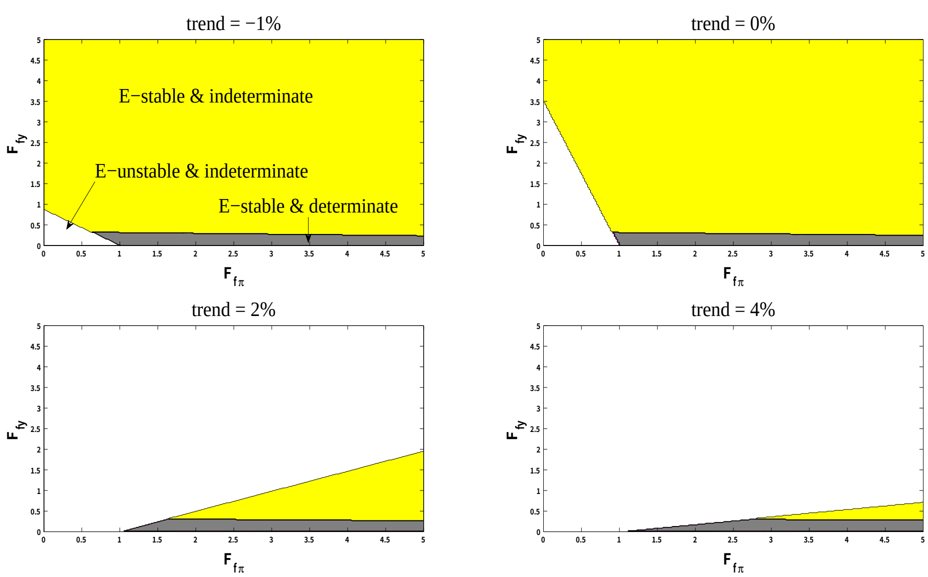

Figure 2: The E-stability and determinacy regions under the forecast-based rule

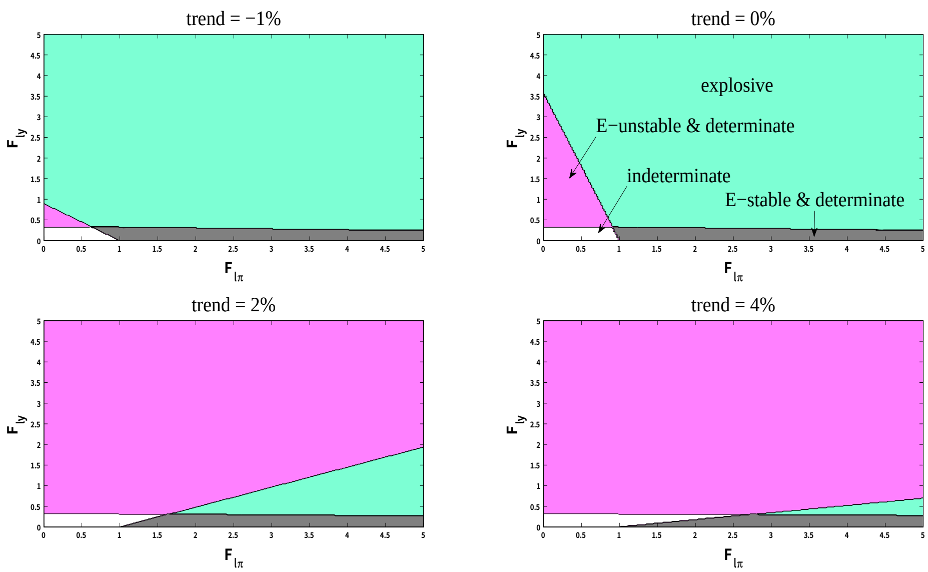

Figure 3: The E-stability and determinacy regions under the lagged-data rule

Supplementary appendix

A The full NKPC under non-zero trend inflation

In Cogley and Sbordone (2008, AER), the full version of the NKPC is given as

$$\pi_t = \zeta mc_t + \tilde{b}_1 E_t \pi_{t+1} + \tilde{b}_2 E_t \sum_{j=2} \phi_1^j \pi_{t+j} + b_3 E_t \sum_{j=0} \phi_1^j (q_{t+j,t+j+1} + \tilde{y}_{t+j+1} - \tilde{y}_{t+j}), \quad \text{(A.1)}$$

where $\tilde{y}_t$ and $q_{t+j,t+j+1}$ denote the (logged) level of output and the real stochastic discount factor, respectively.[1] The degree of price indexation is set at 0. Here, suppose that the utility function is given by the CRRA form: $U(C_t) = C_t^{1-\sigma^{-1}}/(1-\sigma^{-1})$. The last term of the RHS of (A.1) leads to

$$\begin{aligned} b_3 E_t \sum_{j=0} \phi_1^j (q_{t+j,t+j+1} + \tilde{y}_{t+j+1} - \tilde{y}_{t+j}) &= b_3(1-\sigma^{-1}) E_t[(\tilde{y}_{t+1} - \tilde{y}_t) + \phi_1(\tilde{y}_{t+2} - \tilde{y}_{t+1}) \\ &\quad + \phi_1^2(\tilde{y}_{t+3} - \tilde{y}_{t+2}) + \ldots]. \end{aligned} \quad \text{(A.2)}$$

Recall that in obtaining the IS equation (1), we defined the natural rate of interest as $r_t^n \equiv \sigma^{-1}(E_t \tilde{y}_{t+1}^n - \tilde{y}_t^n)$, where $\tilde{y}_t^n$ stands for the natural rate of output. Using this definition, (A.2) can be rewritten as

$$\begin{aligned} & b_3(1-\sigma^{-1}) E_t[(y_{t+1} - y_t + \tilde{y}_{t+1}^n - \tilde{y}_t^n) + \phi_1(y_{t+2} - y_{t+1} + \tilde{y}_{t+2}^n - \tilde{y}_{t+1}^n) + \ldots] \\ = \; & b_3(1-\sigma^{-1})[(y_{t+1}^e - y_t + \sigma r_t^n) + \phi_1(y_{t+2}^e - y_{t+1}^e + \sigma E_t r_{t+1}^n) + \ldots] \\ = \; & b_3(1-\sigma^{-1})[y_{t+1}^e - y_t + \sum_{j=2} \phi_1^{j-1}(y_{t+j}^e - y_{t+j-1}^e) + \sigma \frac{r_t^n}{1-\phi_1\rho_r}], \end{aligned}$$

where y_t denotes the output gap. Using the relation that $E_t \sum_{j=2} \phi_1^{j-1} y_{t+j-1}$ $= \phi_1 y_{t+1}^e + \phi_1 \sum_{j=2} \phi_1^{j-1} y_{t+j}^e$, the above equation leads to

$$b_3(1-\sigma^{-1})[(1-\phi_1)y_{t+1}^e - y_t + (1-\phi_1)\sum_{j=2} \phi_1^{j-1} y_{t+j}^e + \sigma \frac{r_t^n}{1-\phi_1\rho_r}].$$

It follows that

$$\begin{aligned} \pi_t &= [\zeta\tilde{\omega} - b_3(1-\sigma^{-1})]y_t + \tilde{b}_1 \pi_{t+1}^e + \tilde{b}_2 \phi_1 \sum_{j=2} \phi_1^{j-1} \pi_{t+j}^e + (1-\sigma^{-1}) b_3 (1-\phi_1) y_{t+1}^e \\ &+ (1-\sigma^{-1}) b_3 (1-\phi_1) \sum_{j=2} \phi_1^{j-1} y_{t+j}^e + (\sigma - 1) b_3 \frac{r_t^n}{1-\phi_1\rho_r}. \end{aligned} \quad \text{(A.3)}$$

where we used the relation $mc_t = \tilde{\omega} y_t$. This generalized NKPC and the IS equation can be summarized in a way consistent with eq.(7):

$$QX_t = WX_{t+1}^e + Ni_t + Ur_t^n + M \sum_{j=2}^{\infty} \phi_1^{j-1} X_{t+j}^e, \quad \text{(A.4)}$$

[1] $\tilde{b}_1 = \phi_2[1 + \theta\chi(1+\omega)] - \phi_1\chi(\theta - 1)$, $\tilde{b}_2 = \chi(\phi_2 - \phi_1)(\theta - 1)/\phi_1$ and $b_3 = \chi(\phi_2 - \phi_1)$.

where

$$Q = \begin{bmatrix} 1 & 0 \\ -\kappa + b_3(1-\sigma^{-1}) & 1 \end{bmatrix}, W = \begin{bmatrix} 1 & \sigma \\ b_3(1-\sigma^{-1})(1-\phi_1) & \tilde{b}_1 \end{bmatrix}, N = \begin{bmatrix} -\sigma \\ 0 \end{bmatrix},$$
$$U = \begin{bmatrix} \sigma \\ \frac{b_3(\sigma-1)}{1-\phi_1\rho_r} \end{bmatrix}, M = \begin{bmatrix} 0 & 0 \\ b_3(1-\sigma^{-1})(1-\phi_1) & \tilde{b}_2\phi_1 \end{bmatrix},$$

where $\kappa = \zeta\tilde{\omega}$.

Therefore, we can reexamine all the E-stability analyses done in the main text just by redefining the coefficient matrices. It should be noted that this generalized version is exactly the same as eq.(7) if we assume $b_3 = 0$ or $\sigma = 1$.

Under our benchmark parameter values, b_3 = -.001, .002, and .004 when annual trend inflation is -1%, 2%, and 4%, respectively. Figures 4 - 7, shown at the end of this appendix, suggest that introducing an additional term in the NKPC does not change the result because those figures are virtually identical to the corresponding ones shown in the main text.

B The contemporaneous expectations rule

Suppose that the policy rule is given by

$$i_t = F_{p\pi}\pi_t^e + F_{py}y_t^e, \tag{B.1}$$

where expectations are formed at $t-1$.

Following Bullard and Mitra (2002), the PLM is assumed to be given as

$$X_t = A_{t-1} + D_{t-1}r_{t-1}^n. \tag{B.2}$$

Notice that this is a simpler version of the PLM under the lagged-data rule ($C_{t-1} = 0$). It follows that

$$\sum_{j=2} \phi_1^{j-1} X_{t+j}^e = (1-\phi_1)^{-1}\phi_1 A_{t-1} + (1-\phi_1\rho_r)^{-1}\phi_1\rho_r^2 D_{t-1}r_{t-1}^n.$$

By defining $F_p = [F_{py}, F_{p\pi}]$, the ALM leads to

$$\begin{aligned} QX_t &= W(A_{t-1} + D_{t-1}\rho_r r_{t-1}^n) + NF_p(A_{t-1} + D_{t-1}r_{t-1}^n) \\ &+ Ur_t^n + M[(1-\phi_1)^{-1}\phi_1 A_{t-1} + (1-\phi_1\rho_r)^{-1}\phi_1\rho_r^2 D_{t-1}r_{t-1}^n]. \end{aligned} \tag{B.3}$$

It follows that

$$\begin{aligned} X_t &= Q^{-1}\{(W + NF_p + (1-\phi_1)^{-1}\phi_1 M)A_{t-1} \\ &+[W\rho_r D_{t-1} + NF_p D_{t-1} + \rho_r U + (1-\phi_1\rho_r)^{-1}\phi_1 M\rho_r^2 D_{t-1}]r_{t-1}^n + U\varepsilon_t\}. \end{aligned} \tag{B.4}$$

The T-maps are given by

$$T(A_{t-1}) = Q^{-1}(W + NF_p + (1-\phi_1)^{-1}\phi_1 M)A_{t-1} \tag{B.5}$$
$$T(D_{t-1}) = Q^{-1}\{[W\rho_r + NF_p + \phi_1(1-\phi_1\rho_r)^{-1}\rho_r^2 M]D_{t-1} + U\rho_r\}. \tag{B.6}$$

The E-stability condition is that all of the eigenvalues of $T(A)$ and $T(D)$ have real parts less than one. I show the analytical expression of the E-stability condition in the next section.

C Analytical expression of the E-stability conditions

C.1 The contemporaneous rule

From eqs. (9) and (10) of the main text, the rule based on contemporaneous data will be E-stable if and only if all of the eigenvalues of the following two matrices have real parts less than one:

$$DT_A = \Delta \begin{bmatrix} 1 & \sigma - \sigma F_{c\pi}[b_1 + \frac{\phi_1}{1-\phi_1}b_2] \\ \kappa & \kappa\sigma + (1+\sigma F_{cy})[b_1 + \frac{\phi_1}{1-\phi_1}b_2] \end{bmatrix}$$

and

$$DT_D = \rho_r\Delta \begin{bmatrix} 1 & \sigma - \sigma F_{c\pi}[b_1 + \frac{\phi_1\rho_r}{1-\phi_1\rho_r}b_2] \\ \kappa & \kappa\sigma + (1+\sigma F_{cy})[b_1 + \frac{\phi_1\rho_r}{1-\phi_1\rho_r}b_2] \end{bmatrix},$$

where $\Delta \equiv 1/(1+\sigma F_{cy} + \sigma\kappa F_{c\pi})$. Suppose that the characteristic equation of $DT_A - I$ is given as $\mu^2 + m_1\mu + m_2 = 0$, where $m_1 = -tr(DT_A - I)$ and $m_2 = det(DT_A - I)$. As is pointed out by Bullard and Mitra (2002), both of the eigenvalues of DT_A have real parts less than one if and only if $m_1 > 0$ and $m_2 > 0$. It follows that

$$1 + \sigma(2F_{cy} + 2\kappa F_{c\pi} - \kappa) - (1+\sigma F_{cy})(b_1 + \frac{\phi_1}{1-\phi_1}b_2) > 0, \tag{C.1}$$
$$\kappa(F_{c\pi} - 1) + F_{cy}(1 - b_1 - \frac{\phi_1}{1-\phi_1}b_2) > 0. \tag{C.2}$$

The corresponding conditions for DT_D are

$$1 + \sigma(2F_{cy} + 2\kappa F_{c\pi} - \kappa) - (1+\sigma F_{cy})(b_1 + \frac{\rho_r\phi_1}{1-\rho_r\phi_1}b_2) > 0, \tag{C.3}$$
$$\kappa(F_{c\pi} - 1) + F_{cy}(1 - b_1 - \frac{\rho_r\phi_1}{1-\rho_r\phi_1}b_2) > 0. \tag{C.4}$$

We obtain the following proposition:

Proposition C.1 *Assume that $\phi_1 = \alpha\beta\bar{\Pi}^{\theta-1} < 1$ and $\rho_r < 1$. The E-stability condition for the policy rule under the contemporaneous data is given by (C.1) and (C.2) if $b_2 \geq 0$, and (C.3) and (C.4) otherwise.*

Thus, the E-stability condition under nonzero trend inflation is generally stipulated by four inequalities.[2] Notice that in the special case of zero trend inflation, where $b_1 = \beta$ and $b_2 = 0$, the E-stability condition reduces to $\kappa(F_{c\pi} - 1) + F_{cy}(1 - \beta) > 0$. In the generalized version, the term $(1 - \beta)$ is replaced with $1 - b_1 - \phi_1 b_2/(1 - \phi_1)$, which describes the slope of the NKPC under nonzero trend inflation.

C.2 The forecast-based rule

DT_A and DT_D under the forecast-based rule are given as

$$DT_A = \begin{bmatrix} 1 - \sigma F_{fy} & \sigma(1 - F_{f\pi}) \\ \kappa(1 - \sigma F_{fy}) & \kappa\sigma(1 - F_{f\pi}) + b_1 + \frac{\phi_1}{1-\phi_1} b_2 \end{bmatrix}$$

and

$$DT_D = \rho_r \begin{bmatrix} 1 - \sigma F_{fy} & \sigma(1 - F_{f\pi}) \\ \kappa(1 - \sigma F_{fy}) & \kappa\sigma(1 - F_{f\pi}) + b_1 + \frac{\rho_r\phi_1}{1-\rho_r\phi_1} b_2 \end{bmatrix}.$$

Conditions $-tr(DT_A - I) > 0$ and $\det(DT_A - I) > 0$ lead to

$$\sigma F_{fy} + \kappa\sigma(F_{f\pi} - 1) + 1 - b_1 - \frac{\phi_1}{1 - \phi_1} b_2 > 0, \tag{C.5}$$

$$\kappa(F_{f\pi} - 1) + F_{fy}(1 - b_1 - \frac{\phi_1}{1 - \phi_1} b_2) > 0. \tag{C.6}$$

The corresponding conditions for DT_D are

$$\sigma F_{fy} + \kappa\sigma(F_{f\pi} - 1) + 1 - b_1 - \frac{\rho_r\phi_1}{1 - \rho_r\phi_1} b_2 > 0, \tag{C.7}$$

$$\kappa(F_{f\pi} - 1) + F_{fy}(1 - b_1 - \frac{\rho_r\phi_1}{1 - \rho_r\phi_1} b_2) > 0. \tag{C.8}$$

If b_2 is non-negative (negative), then the only relevant conditions are (C.5) and (C.6) ((C.7) and (C.8)). This suggests that the E-stability condition under the forecast-based rule is generally different from the one under the contemporaneous rule, whereas the forecast-based rule and the contemporaneous rule share the E-stability conditions in common under zero trend inflation.

Proposition C.2 *Assume that $\phi_1 = \alpha\beta\bar{\Pi}^{\theta-1} < 1$ and $\rho_r < 1$. The E-stability condition for the policy rule under the forecast-based data is given by (C.5) and (C.6) if $b_2 \geq 0$, and (C.7) and (C.8) otherwise.*

C.3 The contemporaneous expectations rule

Under the contemporaneous expectations rule, DT_A and DT_D are given as

$$DT_A = \begin{bmatrix} 1 - \sigma F_{py} & \sigma(1 - F_{p\pi}) \\ \kappa(1 - \sigma F_{py}) & \kappa\sigma(1 - F_{p\pi}) + b_1 + \frac{\phi_1}{1-\phi_1} b_2 \end{bmatrix}$$

[2] It can be shown that b_2 may take a negative value when $\bar{\Pi}$ is sufficiently large even if $\phi_1 < 1$.

and

$$DT_D = \rho_r \begin{bmatrix} 1-(\sigma/\rho_r)F_{py} & \sigma(1-F_{p\pi}/\rho_r) \\ \kappa(1-(\sigma/\rho_r)F_{py}) & \kappa\sigma(1-F_{p\pi}/\rho_r)+b_1+\frac{\rho_r\phi_1}{1-\rho_r\phi_1}b_2 \end{bmatrix}.$$

It is clear that the combination of policy coefficients that satisfy $-tr(DT_A - I) > 0$ and $\det(DT_A - I) > 0$ are exactly the same as those under the forecast-based rule:

$$\sigma F_{py} + \kappa\sigma(F_{p\pi}-1) + 1 - b_1 - \frac{\phi_1}{1-\phi_1}b_2 > 0, \tag{C.9}$$

$$\kappa(F_{p\pi}-1) + F_{py}(1 - b_1 - \frac{\phi_1}{1-\phi_1}b_2) > 0. \tag{C.10}$$

On the other hand, the conditions $-tr(DT_D - I) > 0$ and $\det(DT_D - I) > 0$ yield

$$\frac{\sigma}{\rho_r}F_{py} + \kappa\sigma\left(\frac{1}{\rho_r}F_{p\pi}-1\right) + 1 - b_1 - \frac{\rho_r\phi_1}{1-\rho_r\phi_1}b_2 > 0, \tag{C.11}$$

$$\kappa\left(\frac{1}{\rho_r}F_{p\pi}-1\right) + \frac{1}{\rho_r}F_{py}(1 - b_1 - \frac{\rho_r\phi_1}{1-\rho_r\phi_1}b_2) > 0. \tag{C.12}$$

If $b_2 \geq 0$, then condition (C.11) is redundant because the inequality always holds as long as (C.9) is satisfied. The difference between the LHSs of (C.10) and (C.12) ((C.10) - (C.12)) leads to

$$\left(1-\frac{1}{\rho_r}\right)(\kappa F_{p\pi} + (1-b_1)F_{py}) - \frac{\phi_1^2(1-\rho_r)}{(1-\phi_1)(1-\rho_r\phi_1)}b_2F_{py} < 0 \quad \text{if} \quad b_1 \leq 1, b_2 \geq 0, \kappa \geq 0.$$

It follows that the E-stability condition is given by (C.9) and (C.10) if $b_1 \leq 1$, $b_2 \geq 0$ and $\kappa \geq 0$. This suggests that the E-stability condition under the contemporaneous expectations rule becomes exactly the same as that under the forecast-based rule. More generally, however, any of the conditions (C.9) - (C.12) can be relevant.

Proposition C.3 *Assume that $\phi_1 = \alpha\beta\bar{\Pi}^{\theta-1} < 1$ and $\rho_r < 1$. The E-stability condition for the policy rule based on contemporaneous expectations is given by (C.9) and (C.10) if $b_1 \leq 1$, $b_2 \geq 0$ and $\kappa \geq 0$, and (C.9) - (C.12) otherwise.*

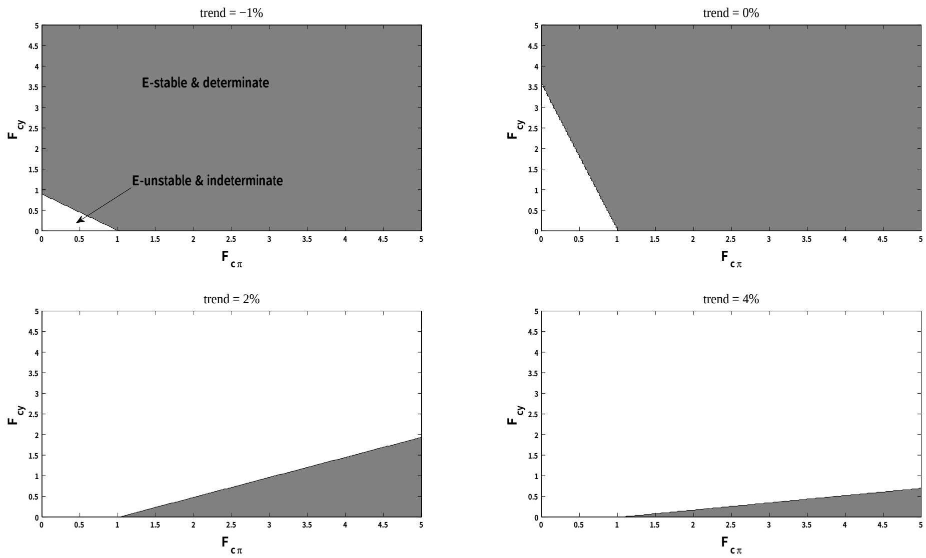

Figure 4: The E-stability and determinacy regions under the contemporaneous rule: $b_3 > 0$.

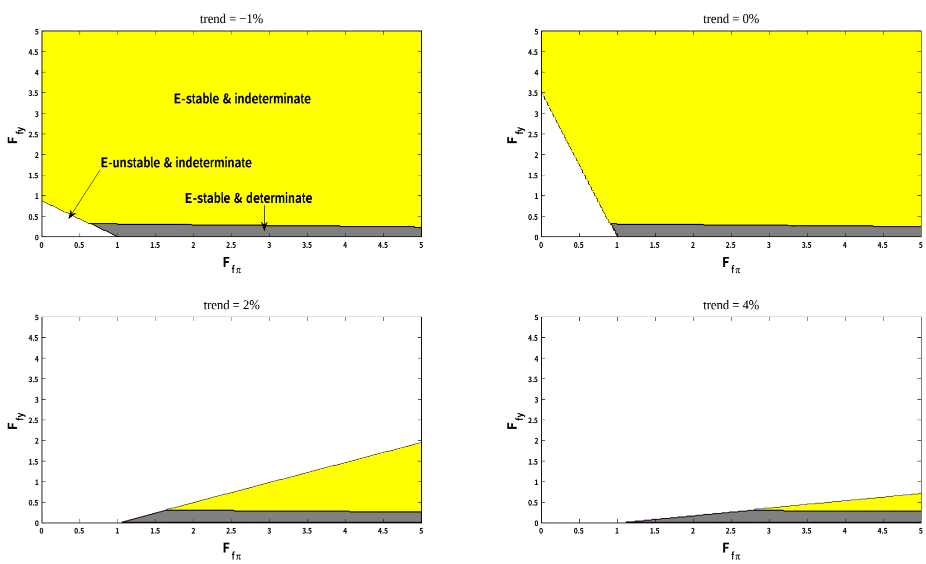

Figure 5: The E-stability and determinacy regions under the forecast-based rule: $b_3 > 0$.

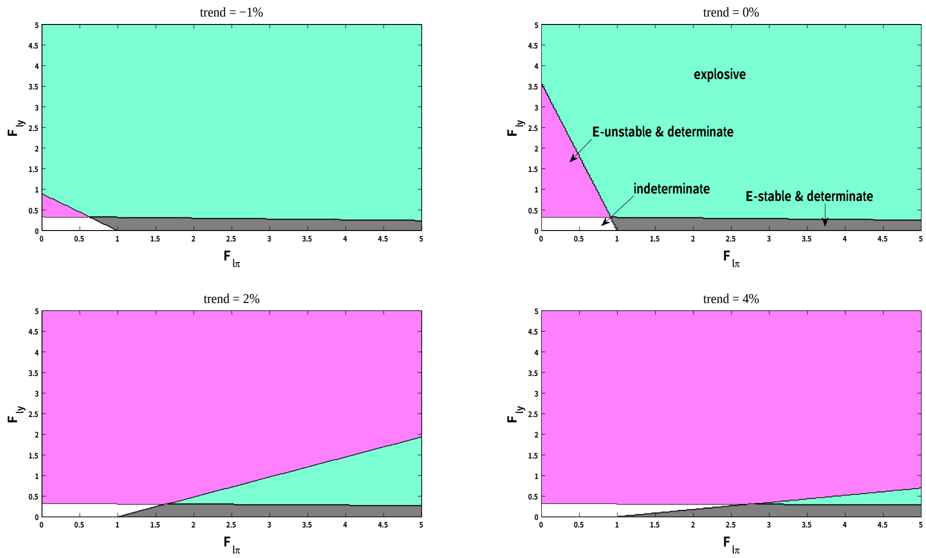

Figure 6: The E-stability and determinacy regions under the lagged-data rule: $b_3 > 0$.

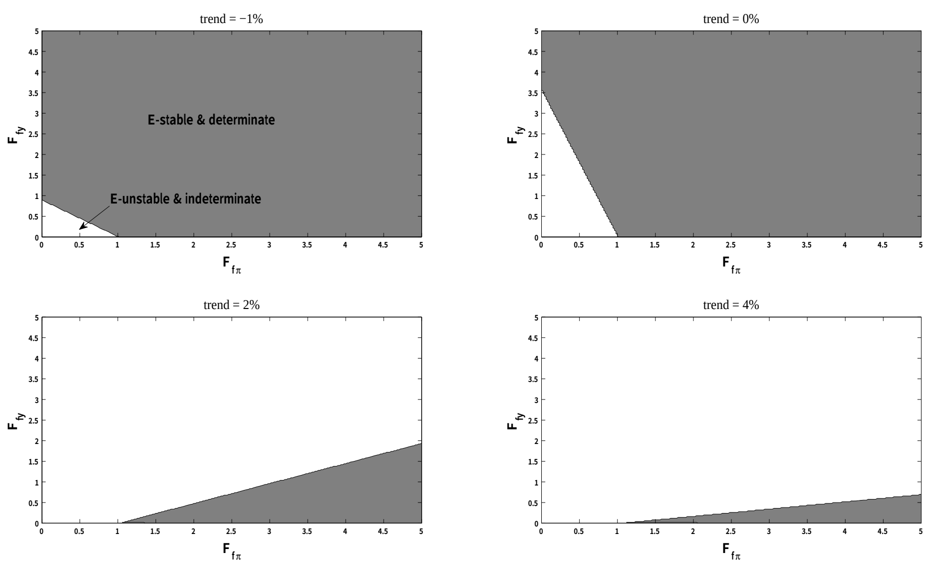

Figure 7: The E-stability and determinacy regions under the contemporaneous expectations rule: $b_3 > 0$.