Competing process and quality innovation in a model of occupational choice

Colin Davis[*] Yasunobu Tomoda[†]

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Abstract

We develop a simple model of endogenous growth and occupational choice in which skill differentiated workers choose between three types of employment activity: production, process innovation, and quality innovation. Incumbent firms invest in process innovation to reduce production costs and market entrants invest in quality improvements in order to capture the market from vintage product lines. We use this framework to examine innovation incentives for incumbent firms in an environment of creative destruction and find that there are two plausible and stable patterns of product evolution: a corner equilibrium with quality growth alone, and an interior equilibrium with both productivity growth and quality growth. We also show that the process innovation of an interior equilibrium has important policy implications for economic growth.

Keywords: Process innovation; Quality innovation; Endogenous growth; Occupational choice
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1. Introduction

In a modern industrial society, research and development (R&D) is critical for the survival of firms in a competitive market place, and for the growth of the aggregate economy. This competition induced R&D activity takes several forms including the creation of new markets through the development of new products, quality improvements on existing products, and improvements in existing production processes. While the growth literature has examined these types of research and development activities extensively, an implicit assumption has been that endogenous growth which stems from quality or process innovation essentially reflects the same mechanism. This paper develops a framework that examines improvements in product quality and advancements in production technology as distinct processes.

There are many studies that explore the three types of innovation activity described above. Romer (1990) and Grossman and Helpman (1990) introduce frameworks of endogenous growth that involve the introduction of new products with separate markets through a process of product innovation similar to the first type of R&D described above. Quality innovation corresponding with the second type of R&D has been investigated in many studies that build on the quality ladders framework of Grossman and Helpman (1991). Finally, the process innovation associated with the third type of R&D discussed above has been examined by Smulders and van de Klundert (1995) and Peretto (1996). There is also a number of studies that examine the relationship between two different types of innovation-based growth. For example, Young (1998) and Thompson (2001) explore models that include both product and quality innovation and Peretto and Smulders (2002) investigate a model with both product and

[*]Kobe University, Graduate School of Economics, 2-1 Rokko-dai, Nada, Kobe, Hyogo, Japan, 657-8501. *davis@econ.kobe-u.ac.jp*.

[†]Kyoto University, Institute of Economic Research, Yoshida-Honmachi, Sakyo-ku, Kyoto, Japan, 606-8501. *tomoda@kier.kyoto-u.ac.jp*

process innovation. To our knowledge, however, no study has been made of the relationship between process and quality innovation.

The distinction between these two types of R&D is important. While process innovation is usually undertaken by incumbent firms for the purpose of reducing production costs, quality innovation is typically associated with a process of creative destruction whereby the market entry of new product designs supplants vintage product lines. Studies that model process and product innovation, for example Peretto and Smulders (2002), describe the incentive for process innovation by incumbent firms that face decreasing market shares as a result of market entry by new firms. To counter falling market shares, incumbents increase firm value by developing cost reducing process innovations. Once a firm has entered the market, however, it faces no risk of exit, and therefore has a strong incentive to invest in process innovation as it is ensured the full return on investment. In contrast, we examine the incentive for in-house process innovation in an environment of creative destruction, and accordingly incumbent firms face a risk of losing the market to a higher quality product line, and thus may not be able to capture the full value of investment in process innovations.

In this paper, we develop a framework of endogenous growth and occupational choice in which workers sort into three activities: quality innovation, process innovation, and production. Quality innovation is undertaken by specialists employed at research centers. Process innovation, on the other hand, is the result of improvements made to production processes by plant managers. Production is carried out by factory employees. This division of labour into specialized occupations is based on a pattern of comparative advantage that arises with heterogeneous skill endowments. Specifically, low-skilled workers choose employment in production, mid-skill workers in process innovation, and high-skilled workers in quality innovation. The inclusion of this occupational choice framework introduces a mechanism that determines the allocation of investment across process and quality R&D on the basis of the effective labour productivity of each type of innovation activity.

We use this framework to investigate patterns of product evolution and find that long-run equilibrium may be characterized by either a corner solution in which only quality growth occurs, or an interior equilibrium with both productivity and quality growth. In particular, an interior equilibrium is more likely when the effective productivity of labour in process innovation is high, the effective productivity of labour in quality innovation is low, and the size of quality improvements is small. These results are strongly dependent, however, on the ranking of skill-based productivity for each type of innovation, and as such an occupational reversal whereby mid-skilled workers are employed in quality innovation and high-skilled workers are employed in process innovation would rule out a corner solution with quality innovation alone, and might lead to a corner solution in which only productivity growth occurs.

Several secondary results of the model are as follows. First, steady-state comparative statics for the interior equilibrium indicate that an increase in the effective labour productivity of either type of innovation increases the rate of economic growth. In contrast, an increase in the size of quality improvements has a negative impact on the growth rate. While the first result is consistent with Grossman and Helpman (1991), the second is not. Just as in the standard quality ladders model, an increase in the quality increment spurs the rate of quality innovation, but it also has a negative effect on the rate of process innovation. The second effect dominates the first and the growth rate falls. Second, in contrast to the market based system, regardless of the ranking of skill-based productivities for innovation, all three types of long-run equilibria are possible in the socially optimal equilibrium. Third, a simple comparison of the market equilibrium with the social optimum indicates that the market incentives for innovation are always insufficient and that the market based growth rate is too low.

The remainder of this paper is organized as follows. In Section 2 we introduce our basic model of endogenous growth. Section 3 investigates the characteristics of long-run equilibria, and Section 4 presents a steady-state comparative analysis of key model parameters. Section 5 derives the social optimum and compares it with the market based system. Section 6 provides some brief concluding remarks.

2. The model

Consider an economy with three economic activities: production (L), process innovation (M), and quality innovation (H). The production sector consists of a unitary mass of industries, indexed by ω, within each of which firms produce goods for consumption and compete according to Bertrand competition. Process innovation refers to research and development (R&D) undertaken by incumbent firms with the objective of reducing production costs. Quality innovation, on the other hand, refers to market entry through the development of new product designs that improve the quality of existing product lines. The sole factor of production is a labour force of workers who choose employment based on heterogeneous skill endowments.

An illustration of the product space for a representative industry is provided in Figure 1. The vertical axis measures the level of the product quality, and the horizontal axis measures the productivity of production technology. We define the state-of-the-art of a given industry as the product that has the highest available quality and is produced using the most efficient technology, and refer to the firm producing the state-of-the-art as the industry leader. Focusing on discrete quality and continuous process improvements, a successful research effort in the quality innovation sector leads to the introduction of a new product line with a quality of λ times the quality level of the current state-of-the-art. The dashed lines in Figure 1 denote the product lines associated with different quality levels, and the parameter $\lambda > 1$ is the vertical distance between them. In contrast, the in-house process innovation of incumbent firms results in an improvement in the effective productivity of workers producing the current state-of-the-art. This productivity growth is illustrated by the horizontal arrows in Figure 1.

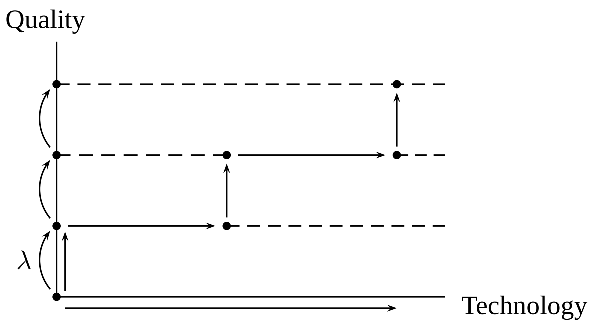


Figure 1: The process and quality dimensions of product evolution

The product space described in Figure 1 suggests three possible paths for product evolution. In the first, consecutive quality innovations improve the quality of the state-of-the-art, but no process innovation occurs, as described by the arced arrows running up the the vertical axis in Figure 1. In the second, there are consecutive process innovations, but quality innovation never occurs. This path is described by the arrow running along the horizontal axis. The third potential path of product evolution consists of continuous process innovation with the intermittent introduction of quality improvements. In what follows, we will argue that, depending on the skill-based productivities of labour in process and quality innovation, only the first and third types of product evolution are feasible in long-run equilibrium.

2.1. Households

The demand side of the economy consists of a representative dynastic household that chooses optimal saving and expenditure paths with the objective of maximizing lifetime utility. Intertemporal preferences are described by

$$U = \int_0^\infty e^{-\rho t} \log u(t) dt, \tag{1}$$

where ρ is the subjective discount rate, and instantaneous utility $u(t)$ takes the form of a quality-augmented Dixit-Stiglitz consumption index with a unitary elasticity of substitution between industries:

$$\log u(t) = \int_0^1 \log \left[\sum_j \lambda^j x(m, \omega, t) d\omega \right]. \tag{2}$$

Product quality and quantity are respectively denoted by λ^j and $x(j, \omega, t)$. Quality is increasing in j, the number of quality innovations that have been introduced by time t in industry ω, and hence consumers prefer higher quality products.

Intertemporal optimization requires that the representative household select an expenditure path that maximizes lifetime utility (1) subject to an intertemporal budget constraint $B(0) \geq \int_0^\infty e^{-R(t)} E(t) dt$, where $R(t) = \int_0^t r(s) ds$ is the cumulative interest factor, $r(t)$ is the risk free rate of return, $E(t)$ is expenditure, and $B(0)$ is the present value of the future flow of household income plus the initial value of assets. It is well known that the solution to this optimization problem is the expenditure path described by the following Euler equation:

$$\frac{\dot{E(t)}}{E(t)} = r(t) - \rho, \tag{3}$$

where a dot over a variable denotes time differentiation. Henceforth, we set expenditure as the model numeraire, and $r(t) = \rho$ at all moments in time.

With a unitary elasticity of substitution across industries, households allocate expenditures evenly across all product lines, and the demand function for the state-of-the-art in industry ω becomes

$$x(j, \omega, t) = \frac{1}{p(j, \omega, t)}, \tag{4}$$

where $p(j, \omega, t)$ is price. In each industry, households consume only the good with the lowest quality-adjusted price and demand is therefore zero for all products that are not state-of-the-art. For the remainder of the paper we suppress time notation where possible.

2.2. Occupational choice

The workforce has a mass of one, and workers are indexed by heterogeneous skill levels, z, that are distributed according to a time-invariant finite distribution $F(z)$, with density $f(z)$, and support $[0, 1]$. Employment is modeled after the occupational choice framework of Roy (1951). In a perfectly competitive labour market workers are free to select employment from the economic activities available: production (L), process innovation (M), and quality innovation (H). We follow Saint-Paul (2004) and assume that a worker's skill-based productivity in each activity is determined by $\phi_i(z)$, $i \in \{L, M, H\}$. These productivities are time-invariant, strictly increasing in skill level, and satisfy the following assumptions: $\phi_L(0) = \phi_M(0) = \phi_H(0) = 1$, and

$$\frac{\phi'_H(z)}{\phi_H(z)} > \frac{\phi'_M(z)}{\phi_M(z)} > \frac{\phi'_L(z)}{\phi_L(z)} \tag{5}$$

for all z, where a prime indicates partial differentiation with respect to the variable shown in parenthesis. These assumptions ensure that each worker has a comparative advantage in one type of activity, that is, production for low-skilled workers, process innovation for mid-skilled workers, and quality innovation for high-skilled workers.[1]

Given the competitive nature of the labour market, all firms operating in the same sector must pay employees the same effective, or per-unit, wage. Workers choose employment in the activity that offers

[1] Yeaple (2005) uses similar assumptions for the labour market in a static model of trade.

the highest wage income: $w(z) = \max\{w_L\phi_L(z), w_M\phi_M(z), w_H\phi_H(z)\}$, where w_L, w_M, and w_H are the effective wages earned respectively for each effective unit of low-, mid-, and high-skilled labour. The wage distribution is therefore

$$w(z) = \begin{cases} w_L\phi_L(z) & 0 \leq z \leq z_1 \\ w_M\phi_M(z) & z_1 \leq z \leq z_2 \\ w_H\phi_H(z) & z_2 \leq z \leq 1 \end{cases}, \tag{6}$$

where z_1 and z_2 are the threshold skill levels for workers that can earn the same wage income in either production and process innovation, or process innovation and quality innovation, respectively, and are therefore indifferent between employment in either of these activities.

The threshold skill levels determine the effective labour supplies and the effective relative wages for each activity:

$$L(z_1) = \int_0^{z_1} \phi_L(z)dF(z), \quad M(z_1, z_2) = \int_{z_1}^{z_2} \phi_M(z)dF(z), \quad H(z_2) = \int_{z_2}^{\bar{z}} \phi_H(z)dF(z), \tag{7}$$

where $L(z_1)$, $M(z_1, z_2)$, and $H(z_2)$ are the effective labour supplies for production, process innovation, and quality innovation, respectively. Note that $L'(z_1) > 0$, $M'(z_1) < 0$, $M'(z_2) > 0$, and $H'(z_2) < 0$, where a prime indicates a partial derivative with respect to the variable shown in parentheses. To simplify the mechanics of the model we introduce the variables $\mu(z_1) = \phi_L(z_1)/\phi_M(z_1)$ and $\theta(z_2) = \phi_M(z_2)/\phi_H(z_2)$ to describe the relative productivities of the marginal workers that correspond with each skill threshold.

2.3. Production

Firms in the production sector use an industry-specific technology that requires the employment of low-skilled workers and depends on the current level of productivity in the industry. In particular, the production function is

$$x(m, \omega) = m(\omega)L(\omega), \tag{8}$$

where $m(\omega)$ is current productivity and $L(\omega)$ is firm-level employment of effective low-skilled labour.

Firms operating in the same industry compete according to Bertrand competition, and as a result the industry leader producing the state-of-the-art captures the entire market. The profit maximizing price of the industry leader is a limit price that is set just low enough to force the closest rival firm out of the market. We denote the productivity of the closest rival firm as $\overline{m}(\omega)$. Then the industry leader sets a quality-adjusted price that is just equal to the marginal cost of the closest rival, $p(\omega)/\lambda = w_L/\overline{m}(\omega)$. This limit price allows the industry leader to earn operating profit on sales equal to

$$\pi(\omega) = 1 - \frac{\overline{m}(\omega)}{\lambda m(\omega)}, \tag{9}$$

where we have used the demand function (4), the limit pricing rule, and the marginal cost of the industry leader, w_L/m.

2.4. Process innovation

As discussed above, in each industry productivity growth may arise from in-house process innovation undertaken by the incumbent industry leader. The development and adoption of new technologies for the production process follows

$$\dot{m}(\omega) = \alpha m(\omega)M(\omega), \tag{10}$$

where α is a positive parameter, $m(\omega)$ is a firm-specific technology spillover, and $M(\omega)$ is firm-level employment of effective mid-skilled labour in process innovation for industry ω. The technology constraint (10) includes both the development and adoption of innovations that improve the production process.

The effective labour cost associated with process innovation is $w_M M(\omega)$, and in view of operating profit on sales (9), an incumbent firm with positive productivity growth earns the following instantaneous profits:

$$\Pi(\omega) = 1 - \frac{\overline{m}(\omega)}{\lambda m(\omega)} - w_M M(\omega). \tag{11}$$

The incentive to invest in process innovation is clear. An improvement in the productivity m of the incumbent firm decreases the limit price to marginal cost ratio, $\overline{m}/\lambda m$, and increases operating profit on sales (9). It is important to note that while advances in production technology are known to all firms in the industry, rival firms have no incentive to invest in similar process innovations for the production of vintage product lines as they would still have to set a higher-quality adjusted price than that of the state-of-the-art, and would therefore not be able to capture a positive market share.

At any given time the value of the industry leader is equal to the present value of expected profit flows: $v(\omega) = \int_0^\infty e^{-\int_0^t \rho + \iota(\omega,s)ds}\Pi(\omega,t)dt$, where $\iota(\omega)$ is the risk associated with loss of the market from the introduction of a new state-of-the-art product design. The industry leader chooses an optimal path for investment in process innovation $M(\omega)$ with the objective of maximizing the value of the firm $v(\omega)$ subject to the technology constraint (10). The current value Hamiltonian function for this optimization problem is $\mathcal{H}(\omega) = 1 - \overline{m}(\omega)/\lambda m(\omega) - w_M M(\omega) + \zeta(\omega)\alpha m(\omega)M(\omega)$, where $\zeta(\omega)$ is the shadow value of productivity m. Noting that the industry leader perceives a constant value for the productivity of the closest rival firm, $\overline{m}(\omega)$, the optimal paths for $M(\omega)$, $m(\omega)$, and $\zeta(\omega)$ satisfy the first order conditions $\zeta(\omega) = w_M/\alpha m(\omega)$ and

$$\rho + \iota(\omega) \geq \frac{\alpha\overline{m}(\omega)}{\lambda m(\omega) w_M} + \frac{\dot{w_M}}{w_M}, \tag{12}$$

and a standard transversality condition for an infinite horizon, $\lim_{t\to\infty} e^{-\int_0^t \rho+\iota(\omega,s)ds}\zeta(\omega,t)m(\omega,t) = 0$. The asset condition (12) states that when the industry leader exhibits positive productivity growth, the return on investment in process innovation must equal the risk free interest rate (ρ) plus the risk-adjustment associated with market entry $\iota(\omega)$. Otherwise, $M(\omega)$ is set equal to zero and $\dot{m}(\omega) = 0$.

2.5. Quality innovation

We now turn to the quality innovation sector where perfectly competitive firms invest in R&D in order to enter the market with new product designs that improve upon the qualities of current state-of-the-art products. Each new product design includes a quality improvement and a production process that adopts all of the quality improvements and process innovations that have been introduced to date in the respective industry. Therefore, a new product design has a quality that is one increment greater than the current state-of-the-art and reproduces the productivity level m of the current industry leader.

A new quality innovation is successfully developed in industry ω with probability $\iota(\omega)dt$ if research is undertaken for a time interval of dt at an intensity of $\iota(\omega)$. This research intensity requires the employment of $\beta H(\omega)$ units of effective high-skilled labour, where β is a positive parameter. With free entry and exit, there is active quality innovation when the expected cost of successfully developing a new quality innovation is equal to the present value of the potential profit stream that is earned with successful market entry. The value of a new quality innovation is thus $v(\omega)$ and the free entry condition for quality innovation is[2]

$$v(\omega) \leq \frac{w_H}{\beta}. \tag{13}$$

This entry condition binds when there is active quality innovation, and taking its time derivative yields the following asset equation:

$$\rho + \iota(\omega) \geq \frac{\Pi(\omega)}{v(\omega)} + \frac{\dot{v}(\omega)}{v(\omega)}, \tag{14}$$

where $\iota(\omega)$ is once again the risk that a subsequently developed quality improvement will allow a later entrant to capture the market. This asset equation states that the rate of return on a quality innovation must equal the rate of return on a risk-free asset of size $v(\omega)$ plus the risk premium $\iota(\omega)$.

2.6. Short-run equilibrium

We now close the model and characterize the short-run equilibrium by combining the equilibrium conditions for each activity with the effective supplies of low-, mid-, and high-skilled labour. As the effective mid- and high-skilled wages are based on the effective low-skilled wage, it is convenient to begin by deriving a condition that ties the product market to the effective supply of low-skilled labour. Substituting the limit pricing rule $p(\omega) = \lambda w_L/\overline{m}(\omega)$ into the demand function (4) and setting the result equal to the production function (8) for the average industry, we obtain the following condition for the effective low-skilled wage rate:

$$w_L = \frac{\overline{m}}{\lambda m L(z_1)}, \tag{15}$$

where the index has been dropped to indicate average values, that is, $\overline{m} = \int_0^1 \overline{m}(\omega) d\omega$, $m = \int_0^1 m(\omega) d\omega$, and $L(z_1) = \int_0^1 L(\omega) d\omega$ are respectively average rival firm productivity, average industry leader productivity, and average firm-level employment in production. The effective low-skilled wage rate is, therefore, a negative function of the threshold skill level z_1, and a positive function of the limit price to marginal cost ratio $\overline{m}/\lambda m$ for the average industry leader.

Next, we examine the market clearing conditions for effective mid-skilled and high-skilled labour. These segments of the labour force are fully employed in process and quality innovation, respectively, and therefore the average research effort in each type of innovation equals the effective labour supply:

$$\frac{\dot{m}}{m} = \alpha M(z_1, z_2), \qquad \iota = \beta H(z_2), \tag{16}$$

where $\dot{m}/m$ is average productivity growth for the economy, and ι is the expected probability that a new quality innovation will arrive in a representative industry at each moment in time. While the effort in process innovation is a function of both skill thresholds, z_1 and z_2, the effort in quality innovation is a function of z_2 alone.

3. Long-run product evolution

This section characterizes long-run equilibria and their associated patterns of product evolution. In particular, we show that under the skill-based labour productivities assumed in (5), there are two possible patterns of long-run product evolution: a pattern with quality innovation alone, and a pattern with both process and quality innovation. We refer to the former as a corner solution and the latter as an interior solution.

[2]This specification for quality innovation closely follows that of Grossman and Helpman (1991), and therefore includes a scale effect. However, as our objective is a comparison of the incentives for process and quality innovation, to keep the model tractable, we normalize the population to unity and do not consider issues relating to this scale effect.

Before deriving dynamic equations for the skill thresholds, we introduce a variable to describe the average length of a product cycle. Figure 2 illustrates the evolution of production technology for both a representative industry and the average industry. The horizontal and vertical axes respectively measure the productivity of the industry leader and the productivity of the closest rival firm. In the representative industry, shown in Figure 2a, when an entering firm captures the market with a new quality innovation, its initial production technology is the same as that of its nearest rival. Thus, each new entrant begins with a productivity level on the $\overline{m} = m$ locus. As an incumbent, the firm then invests in process innovation, and its productivity moves to the right along a path described by a dashed arrow. The level of productivity growth depends on the length of time before the next market entry, the timing of which is determined stochastically. During this time interval there may be no productivity growth if subsequent entry is immediate, and infinite productivity growth if subsequent entry never occurs. When entry does occur, the industry jumps vertically to the $\overline{m}(\omega) = m(\omega)$ locus. In this way, the pace of quality innovation matches the speed of growth in $\overline{m}(\omega)$, which in turn depends on the length of the interval with process innovation.

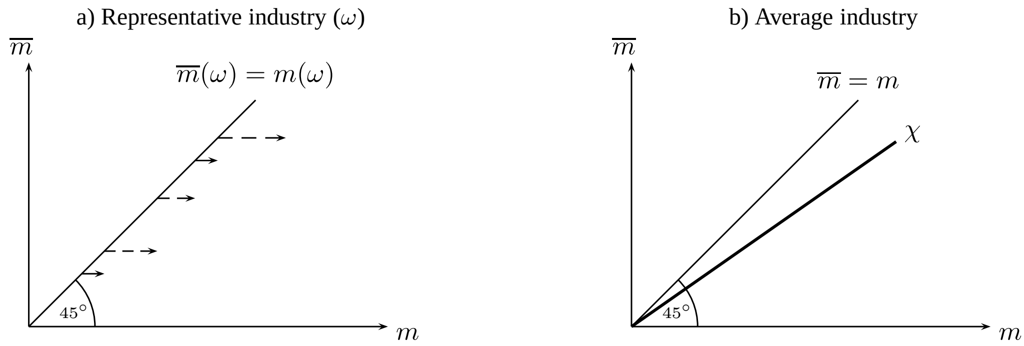


Figure 2: Average product cycle duration (χ)

The average industry, depicted in Figure 2b, captures the average time interval between quality innovations for all industries. We define a new variable $\chi \equiv \overline{m}/m$ to describe the ratio of average industry leader productivity $m = \int_0^1 m(\omega)d\omega$ to average rival firm productivity $\overline{m} = \int_0^1 \overline{m}(\omega)d\omega$. This ratio must, by definition, take values between zero and one. Noting that the time derivative of average rival firm productivity is $\dot{\overline{m}} = (m - \overline{m})\beta H(z_2)$, which simply states that the current industry leader becomes the closest rival when a firm enters the market with a new quality innovation, we can derive the motion for the average productivity ratio by substituting (16) into the time derivative of $\chi \equiv \overline{m}/m$:

$$\dot{\chi} = (1 - \chi)\beta H(z_2) - \chi\alpha M(z_1, z_2). \tag{17}$$

The average productivity ratio X converges to one when there is quality innovation, and market entry, but no process innovation. On the other hand, when there is process innovation, but no quality innovation, the average productivity ratio converges to zero. These cases are respectively described by the $45°$ line and the horizontal axis in Figure 2b.

Turning next to the skill thresholds, the motion for z_1 is obtained from the asset condition for process innovation (12) using (15) and (16), with $w_M = \mu(z_1)w_L$ and $\chi \equiv \overline{m}/m$:

$$\dot{z}_1 = \left[\frac{\alpha L(z_1)}{\mu(z_1)} + \frac{\dot{\chi}}{\chi} - \rho - \beta H(z_2)\right]\left[\frac{L'(z_1)}{L(z_1)} - \frac{\mu'(z_1)}{\mu(z_1)}\right]^{-1}. \tag{18}$$

In a similar manner, the motion for z_2 is derived from the asset condition for quality innovation (14) using (11), (13), (15), (16), and $w_H = \theta(z_2)w_M$ with (18):

$$\dot{z}_2 = \left[\frac{\alpha L(z_1)}{\mu(z_1)} - \frac{\beta(\lambda - \chi)L(z_1) - \beta\mu(z_1)\chi M(z_1, z_2)}{\mu(z_1)\theta(z_2)\chi}\right]\left[\frac{\theta'(z_2)}{\theta(z_2)}\right]^{-1}. \tag{19}$$

Together (18) and (19) describe the dynamics of labour allocation. The model is thus reduced to a system of three differential equations in three variables.

We are interested in long-run equilibria with a constant allocation of labour across sectors. Accordingly, we examine steady-state equilibria with constant effective labour supplies. Setting $\dot{z_1}$, $\dot{z_2}$, and $\dot{\chi}$ equal to zero, the steady-state system is described by the following conditions:

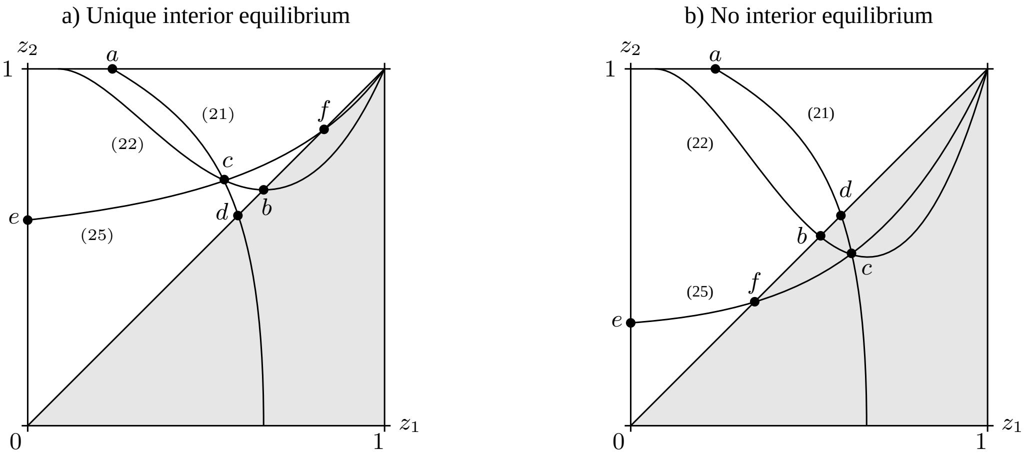
$$\chi = \frac{\beta H(z_2)}{\beta H(z_2) + \alpha M(z_1, z_2)}, \tag{20}$$

$$\rho + \beta H(z_2) = \frac{\alpha L(z_1)}{\mu(z_1)}, \tag{21}$$

$$\rho + \beta H(z_2) = \frac{\beta(\lambda - 1)L(z_1)}{\mu(z_1)\theta(z_2)} + \left[\frac{\alpha\lambda L(z_1)}{\mu(z_1)H(z_2)} - \beta\right]\frac{M(z_1, z_2)}{\theta(z_2)}, \tag{22}$$

where (22) has been obtained by substituting (20) and (21) into (19) set equal to zero.

Figure 3 provides numerical illustrations of the steady-state asset conditions for process innovation (21) and quality innovation (22). By definition the skill thresholds take values between zero and one, and $z_1 \leq z_2$. Thus, the shaded area below the 45° line indicates unfeasible combinations of the skill thresholds. There are three possible equilibria: an interior equilibrium that occurs at point c where conditions (21) and (22) intersect, a corner solution with process innovation that occurs at point a, and a corner solution with quality innovation that occurs at point b.



These figures are drawn using a uniform distribution for $F(z)$ and the following assumptions: $\phi_L = 1$, $\phi_M = z$, $\phi_H = z^2$, $\alpha = 1$, $\beta = 1$, and $\rho = 0.05$. In panel a) $\lambda = 2$ and in panel b) $\lambda = 3.5$.

Figure 3: The process and quality dimensions of product evolution

We begin with an examination of the corner solution with process innovation (point a). Substituting $z_2 = 1$ and $H = 0$ into (18) and setting the result equal to zero, this long-run equilibrium is described by

$$\rho = \frac{\alpha L(z_1)}{\mu(z_1)}. \tag{23}$$

Note that χ drops out of the system. Given that this condition binds, there is active process innovation in all industries and the average rate of productivity growth is determined by $\dot{m}/m = \alpha M(z_1)$. As such, z_1 will take a value between zero and one, similar to point a in Figure 3. Setting $H = 0$ and $\dot{\chi} = 0$ in (18), and taking the derivative with respect to z_1, we find that this corner solution is clearly a saddle point.

Next, we investigate the characteristics of the corner solution with quality innovation (point b). Setting $z_1 = z_2$, $M = 0$, and $\chi = 1$ in (22), this long-run equilibrium is described by

$$\rho + \beta H(z_2) = \frac{\beta(\lambda - 1)L(z_2)}{\mu(z_2)\theta(z_2)}. \tag{24}$$

This condition implicitly pins down the threshold skill level z_2 that allocates labour between production and quality innovation. The basic features of this long-run equilibrium are similar to those of the standard quality ladders model. Accordingly, setting $z_1 = z_2$, $M = 0$ and $\chi = 1$ in (19), and taking the derivative with respect to z_2, we can show that this corner solution is also a saddle point.

The characterization of the interior equilibrium with both process and quality innovation (point c) is a little more involved. Taking the total derivatives of (21) and (22), we find that while the slope of the asset condition for process innovation is positive, the slope of the asset condition for quality innovation is non-monotonic. In order to show that the steady-state asset conditions only intersect once, we substitute (21) into (22) to derive the following investment locus:

$$[\rho + \beta H(z_2)]\, H(z_2) \left[1 - \frac{\beta(\lambda - 1)}{\alpha\theta(z_2)}\right] = [\beta(\lambda - 1)H(z_2) + \rho\lambda]\, \frac{M(z_1, z_2)}{\theta(z_2)}, \tag{25}$$

where the positive value for the right-hand side indicates that $\alpha\theta(z_2) > \beta(\lambda-1)$ is a necessary condition for the existence of an interior solution. This locus describes all threshold skill combinations for which the returns to process and quality innovation are equal, including the intersection of the steady-state asset conditions (21) and (22). Moreover, the total derivative for (25) indicates that the slope of the investment locus is positive. This fact, combined with the negative slope of (21) ensures that an interior equilibrium is always unique when it exists.

The existence of an interior equilibrium requires that the asset conditions and the investment locus intersect above the 45° line shown in Figure 3. This will be the case when z_2 satisfies $\alpha\theta(z_2) > \beta(\lambda - 1)$. To show this, we first examine (21). There are two points of interest, the corner solution at point a and the intersection of (21) and the 45° line (point d), which satisfies $\rho + \beta H(z_2) = \alpha L(z_2)/\mu(z_2)$. Both points occur for values of z_1 that lie between zero and one, and if the investment locus crosses between these two points, a feasible interior equilibrium exists. Setting $z_1 = 0$ in (25), we can derive a condition that allocates labour between process and quality innovation (L=0). By definition the value of z_2 that determines this labour allocation must lie between zero and one (point e). Similarly, setting $z_1 = 1$ in (25), we find that $z_2 = 1$ (M=H=0). Finally, setting $z_1 = z_2$ in (25), we find that if the investment locus crosses the 45° line, the intersection will occur where $\alpha\theta(z_2) = \beta(\lambda-1)$, for example, point f. The investment locus lies above the 45° line for values of z_1 to the left of this intersection, and below the 45° line for values of z_1 to the right of this intersection. The points outlined above imply that a feasible interior equilibrium exists if $\alpha\theta(z_2) > \beta(\lambda - 1)$. We summarize this result in the following proposition.

Proposition 1 *A unique interior equilibrium exists for* $\alpha\theta(z_2) > \beta(\lambda - 1)$.

The existence of an interior equilibrium is likely when the skill-based labour productivity of workers in process innovation is large relative to the skill-based labour productivity of workers in quality innovation, that is, when $\phi_M(z_2)/\phi_H(z_2)$ is large. Similarly, a large effective labour productivity in process innovation (α), a small effective labour productivity in quality innovation (β), and a small quality increment (λ) are all conducive to the existence of an interior equilibrium.

In the Appendix, we example the local dynamics around the interior equilibrium and obtain the following proposition for the local stability of the interior equilibrium.

Proposition 2 *All feasible equilibria are saddlepath stable.*

Proof: See Appendix A.1.

We have shown that there are three possible stable steady-states, but which of these equilibria actually occurs in the long-run? Under the dynamic framework we have adopted all agents have perfect foresight, and investors with rational expectations adjust their investments in process and quality innovation until all arbitrage opportunities have been exploited. Holding the skill threshold z_2 constant and taking the partial derivatives of (21) and (22) with respect to z_1, we find that the rates of return to process innovation and quality innovation are less than the risk-adjusted nominal interest rate for skill threshold combinations that lie below (21) and (22), respectively. Additionally, comparing the intersection of (21) and the 45° line with the intersection of (22) and the 45° line, we find that (21) lies above (22) for all values of z_1 when $\alpha\theta(z_2) < \beta(\lambda - 1)$. Therefore, only the interior equilibrium erases all arbitrage opportunities when it lies above the 45° line. On the other hand, when the interior equilibrium lies below the 45° line, the only equilibrium with no arbitrage opportunities is the corner solution with quality innovation.

Proposition 3 *There are two possible long-run equilibria: (i) for $\alpha\theta(z_2) > \beta(\lambda - 1)$ the interior equilibrium with process and quality innovation describes the long-run pattern of product evolution, and (ii) for $\alpha\theta(z_2) < \beta(\lambda - 1)$ the corner solution with quality innovation describes the long-run pattern of product evolution. The corner solution with process innovation alone is never a long-run equilibrium.*

The results summarized in Proposition 3 correspond with Figure 3. The long-run interior equilibrium that occurs for $\alpha\theta(z_2) > \beta(\lambda - 1)$ corresponds with point c in Figure 3a. On the other hand, the long-run equilibrium that arises for $\alpha\theta(z_2) < \beta(\lambda - 1)$ corresponds with point b in Figure 3b. The second result given in Proposition 3 is strongly dependent on the ranking of skill-based labour productivities (5) for process and quality innovation. Specifically, the higher productivity of workers in quality innovation allows for a greater allocation of labour to production. Since aggregate demand is an increasing function of the wage income of low-skilled workers, the larger allocation of labour to production leads to greater demand and increases the return to investment in quality innovation. This allows for arbitrage opportunities through the redirection of investment from process innovation to quality innovation and ensures that the corner solution with quality innovation becomes the long-run equilibrium when an interior equilibrium is not feasible.

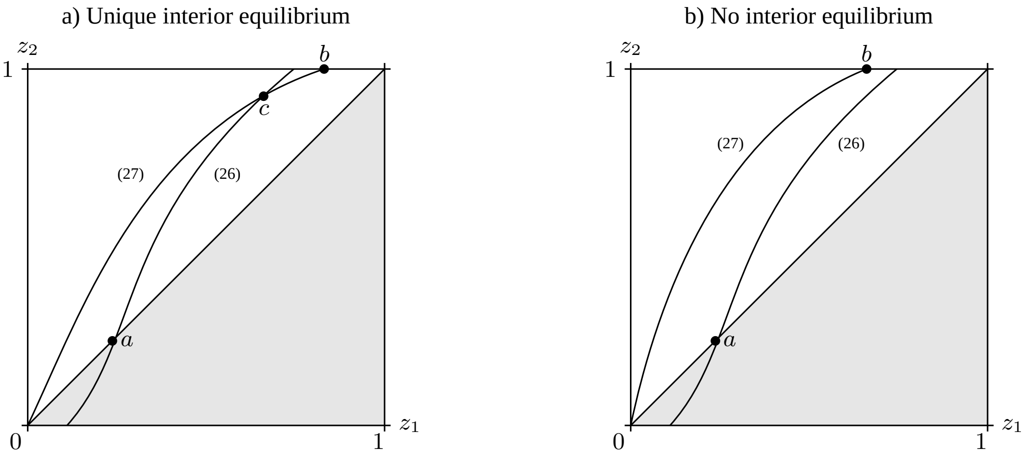
In order to emphasize the dependence of Proposition 3 on assumption (5) for skill-based labour productivities, we consider an occupation reversal whereby the skill-based productivities of workers in process and quality innovation respectively become $\phi_H(z)$ and $\phi_M(z)$. Under this occupation reversal high-skilled workers are employed in process innovation and mid-skilled workers are employed in quality innovation. Reconfiguring the model, the steady-state asset conditions for process and quality innovation respectively become

$$\rho + \beta M(z_1, z_2) = \frac{\alpha L(z_1)}{\mu(z_1)\theta(z_2)}, \tag{26}$$

$$\rho + \beta M(z_1, z_2) = \left[\beta(\lambda - 1) + \frac{\alpha\lambda H(z_2)}{M(z_1, z_2)}\right] \frac{L(z_1)}{\mu(z_1)}. \tag{27}$$

The basic features of the reconfigured system can be examined using the procedure employed above to characterize the original system. Here our objective is to emphasize the dependence of Proposition 3 on the skill-based productivities for workers employed in each type of innovation. To this end, we simply describe the reconfigured system using the numerical illustrations provided in Figure 4.

Once again two cases for the long-run equilibrium emerge. The first is shown in Figure 4a where a long-run equilibrium with both process and quality innovation occurs at point c. The second is shown in Figure 4b where the corner solution with process innovation at point a dominates the corner solution with quality innovation at point b. An occupation reversal for mid- and high-skilled workers therefore reverses the second result of Proposition 3.



These figures are drawn using a uniform distribution for $F(z)$ and the following assumptions: $\phi_L = 1$, $\phi_M = z$, $\phi_H = z^2$, $\alpha = 1$, $\beta = 1$, and $\rho = 0.05$. In panel a) $\lambda = 1.5$ and in panel b) $\lambda = 3$.

Figure 4: Occupation reversal for mid-skilled and high-skilled workers

4. Comparative steady-state analysis and long-run growth

In this section we investigate the effects of changes in model parameters on the long-run allocation of labour and the rate of economic growth. In the Appendix, we use a steady-state comparative static analysis of (21) and (25) to obtain the following proposition.

Proposition 4 *The long-run skill threshold z_1 depends positively on the discount rate (ρ), effective labour productivity in quality innovation (β), and the size of quality innovations (λ), but depends negatively on effective labour productivity in process innovation (α). The long-run skill threshold z_2 depends negatively on λ, but the relationships between z_2 and ρ, α, and β are ambiguous.*

Proof: See Appendix A.2.

A rise in the discount rate (ρ) increases the opportunity cost of investment in innovation, and leads to an increase in the employment share for production. An increase in the quality increment (λ) also leads to a decrease in the overall employment share of innovation, but increases the share of quality innovation. The lack of a specific functional form for the skill distribution $F(z)$, however, leads to ambiguous results for α and β. While z_1 is negatively and positively related to α and β, respectively, the relationship between these parameters and z_2 is unclear.

In order to pin down the relationships between z_2, and ρ, α, and β, we assume that the distribution of skills is described by $F(z) = z^k$, where k is a shape parameter. Table 1 gives the signs of comparative statics for several possible shapes of this skill distribution.

Table 1: Comparative statics: numerical examples

k	$\frac{dz_2}{d\rho}$	$\frac{dz_2}{d\alpha}$	$\frac{dz_2}{d\beta}$
k=0.1	$-$	$+$	$+$
k=1.0	$+$	$+$	$+$
k=2.0	$+$	$+$	$+$

These comparative statics are signed using $L = z_1^k$, $M = (z_2^{1+k} - z_1^{1+k})/(1+k)$, and $H = -kz_2^{2+k}/(2+k)$, and the following assumptions: ρ=0.05, α=1, β=1, and λ=2.

Based on the comparative statics summarized in Proposition 4 and the numerical examples presented in Table 1, we make the following conclusions. An increase in α unambiguously decreases employment in production (a fall in z_1), and decreases employment in quality innovation (a rise in z_2).

Employment in process innovation increases and thus while the pace of quality innovation slows, the rate of productivity growth rises. An increase in β, on the other hand, unambiguously increases employment in production (a rise in z_1) and decreases employment in quality innovation (a rise in z_2). In this case the direction of change in employment in process innovation is determined by the relative size of changes in z_1 and z_2 and therefore depends on the shape of the skill distribution. As such, the effects of changes in β on the pace of quality innovation and the rate of productivity growth are ambiguous.

The comparative statics discussed above have implications for policies that target the long-run rate of economic growth through changes in α and β. In this model, the long-run economic growth rate is determined by the rate of growth in instantaneous utility (2). Focusing once again on the average industry and noting that households only consume the state-of-the-art in each industry, steady-state instantaneous utility can be rewritten as $\log u(t) = (\log \lambda)I(t) + \log x$, where $I(t) = \int_0^t \iota(s)ds$ is the expected number of quality improvements before time t. Noting that $I(t) = \beta H(z_2)t$ in long-run equilibrium and using the production function (8) gives

$$\log u(t) = \beta H(z_2)t \log \lambda + \log m + \log L(z_1), \tag{28}$$

where the first term on the right-hand side captures the utility derived from the level of product quality, and the second and third terms capture the utility derived from the quantity of goods consumed. Taking the time derivative of (28) yeilds

$$g = \frac{\dot{u}(t)}{u(t)} = \beta H(z_2) \log \lambda + \alpha M(z_1, z_2), \tag{29}$$

where we have used (16), and the fact that $\dot{z}_1 = \dot{z}_2 = 0$ in long-run equilbrium.

Given the ambiguity associated with comparative statics for the general case, we cannot resolve the relationship between model parameters and the long-run growth rate without once again resorting to a specific form for the skill distribution. Table 2 presents numerical examples for different shapes of the specific skill distribution assumed above.

Table 2: Determinants of the growth rate: numerical examples

k	$\frac{dg}{d\rho}$	$\frac{dg}{d\alpha}$	$\frac{dg}{d\beta}$	$\frac{dg}{d\lambda}$
k=0.1	$-$	$+$	$+$	$-$
k=1.0	$-$	$+$	$+$	$-$
k=2.0	$-$	$+$	$+$	$-$

These comparative statics are signed using $L = z_1^k$, $M = (z_2^{1+k} - z_1^{1+k})/(1+k)$, and $H = -kz_2^{2+k}/(2+k)$, and the following assumptions: ρ=0.05, α=1, β=1, and λ=2.

First, the long-run growth rate is negatively related to the discount rate (ρ). This result replicates the results of quality-based innovation models where more patient households are associated with a higher rate of growth. Second, an increase in the quality increment (λ) also leads to a decrease in long-run growth. This result differs from existing literature where an increase in λ raises the growth rate through an increase in the pace of quality growth (Grossman and Helpman (1991)). The rate of quality innovation also rises in our model, but this positive effect is dominated by the decrease in productivity growth that occurs as workers are shifted out of process innovation and into production and quality innovation. Third, an increase in the productivity of effective labour in either type of innovation has a positive effect on the growth rate. Last, we note that an improvement in the shape of the skill distribution, that is, an increase in k, unambiguously leads to an increase in the long-run growth rate (not shown in Table 2). This result supports empirical results that link innovation output with labour force quality, for example, the share of college graduates in the labour force (Bottazi and Peri (2003); Hanushek and Kimko (2000)).

5. Social optimum

In this section we investigate the socially optimal allocation of labour. Focusing on the average industry the planner's objective is to maximize $U = \int_t^\infty e^{-\rho t}\left[(\log\lambda)I(t) + \log m(t) + \log L(z_1(t))\right] dt$ subject to the technology constraints $\dot{m}(t) = m(t)\alpha M(z_1(t), z_2(t))$ and $\dot{I}(t) = \beta H(z_2(t))$, where $I(t) = \int_0^t \beta H(z_2(s))ds$ is once again the expected number of quality improvements introduced before time t. This utility maximization problem can be solved using the following current value Hamiltonian function: $\mathcal{H} = (\log\lambda)I + \log m + \log L(z_1) + \zeta_1 \alpha m M(z_1, z_2) + \zeta_2 \beta H(z_2)$, where ζ_1 and ζ_2 are respectively the shadow values of average productivity and a quality improvement.

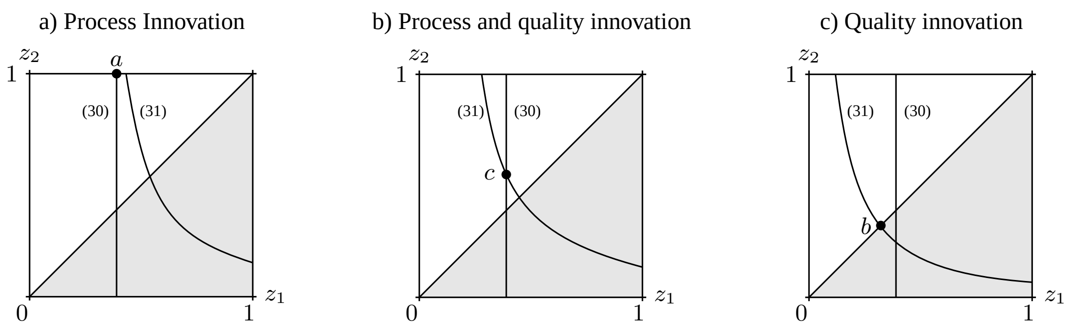
A long-run equilibrium with a constant allocation of labour across activities is characterized by $\dot{\zeta}_1/\zeta_1 = -\dot{m}/m$ and constant values for z_1, z_2, and ζ_2. Solving for the necessary conditions, the steady-state system can be reduced to two conditions in the skill thresholds:

$$\rho \geq \frac{\alpha L(z_1)}{\mu(z_1)}, \tag{30}$$

$$\rho \geq \frac{(\log\lambda)\beta L(z_1)}{\mu(z_1)\theta(z_2)}, \tag{31}$$

where (30) must bind for positive productivity growth, and (31) must bind for positive quality growth. These conditions assume the role of the asset conditions (21) and (22) in the market based system of Section 2.

Figure 5 illustrates three possible cases for the social optimum using numerical examples. In the first panel the optimal labour allocation is at point a where only process innovation occurs. In the second panel the social optimum is at point c where both types of innovation occur. In the third panel the social optimum is at point b where only quality innovation occurs.



These figures are drawn using a uniform distribution for $F(z)$ and the following assumptions: $\phi_L = 1$, $\phi_M = z$, $\phi_H = z^2$, $\alpha = 0.3$, $\beta = 2$, and $\rho = 0.05$. In panel a) $\lambda = 1.1$, in panel b) $\lambda = 1.45$ and in panel c) $\lambda = 2.2$.

Figure 5: Social optimum

The socially optimal equilibrium can be characterized as follows. While (30) is a vertical line in z_1-z_2 space, (31) has a strictly negative slope. These conditions will cross above the $z_1 = z_2$ diagonal if z_2 takes a higher value than the z_1 that satisfies (30). This will be the case when $\theta(z_1) > (\log\lambda)\beta/\alpha$. Next, the conditions will cross below $z_2 = 1$ if the value of z_1 that satisfies (31) at $z_2 = 1$ is lower than the value of z_1 that satisfies (30). This will be the case when $(\log\lambda)\beta/\alpha > 1$. Finally, holding z_2 constant, the partial derivatives of (30) and (31) with respect to z_1 indicate that the return to process innovation is greater than the discount rate for skill threshold combinations to the right of (30), and that the return to quality innovation is greater than the discount rate for skill threshold combinations to the right of (31). We summarize these points in the following proposition.

Proposition 5 *The socially optimal steady-state labour allocation entails (i) process innovation only for* $\theta(z_1) > 1 > (\log \lambda)\beta/\alpha$, *(ii) process and quality innovation for* $\theta(z_1) \geq (\log \lambda)\beta/\alpha \geq 1$, *and (iii) quality innovation only for* $(\log \lambda)\beta/\alpha > \theta(z_1) > 1$.

In contrast to the market based system, where a long-run pattern of product evolution with process innovation alone was not feasible, the socially optimal pattern of product evolution may entail process or quality innovation alone, or both types of innovation occurring simultaneously depending on parameter values and the shape of the skill distribution.

Before concluding, we note that the socially optimal allocation of labour to innovation activity is greater than that of the market based system. This can be seen from a comparison of (21) and (30) which indicates that the skill threshold z_1 is always greater for the market based system.

Proposition 6 *Market incentives for innovation activity are always insufficient.*

This result differs from that of the quality ladders model of Grossman and Helpman (1991) where the market incentives for R&D may be excessive or insufficient depending on the size of the quality increment λ. In our model while a general comparison of the investment allocation between process and quality innovation is not possible, overall market incentives for labour allocation to innovation activity are insufficient and the socially optimal growth rate is always higher than that of the market based system.

6. Conclusion

The innovation activity of firms is critical for both their survival in the competitive market place and the growth of the aggregate economy. This innovation activity takes several forms including the creation of new markets through the development of new products, quality improvements on existing products, and improvements in existing production processes. While the growth literature has examined these types of R&D activities extensively, an implicit assumption of this literature has been that endogenous growth which stems from quality or process innovation essentially reflects the same mechanism. This paper develops a model of endogenous growth and occupational choice that examines improvements in product quality and advancement in production technology as distinct processes.

In particular, workers sort into three activities on the basis of heterogenous skill levels and different skill-based productivities for each activity; low-skilled workers are employed in production, mid-skilled workers in process innovation, and high-skilled workers in quality innovation. While process innovation is undertaken by incumbent firms with the objective of reducing the costs of production, quality innovations are developed by new firms entering the market. The model therefore allows for a characterization of goods according to both the quality perceived by consumers and the technology employed in production. We use the model to investigate patterns of product evolution and find that long-run equilibria may be characterized by either a corner solution with only quality growth, or an interior equilibrium with both productivity and quality growth. This conclusion is strongly dependent, however, on the ranking of skill-based productivity for workers in each type of innovation, and an occupational reversal whereby mid-skilled and high-skilled workers respectively sort into quality and process innovation rules out a corner solution with quality growth alone, and may lead to a corner solution in which only process innovation occurs.

There are two possible extensions. First, the endogenous growth model includes a scale effect whereby an increase in population size leads to an increase in employment in innovation and raises the long-run rate of economic growth. A large body of research concludes, however, that the scale effect is empirically implausible (see, for example, Jones (1995)). Second, in our model the elasticity of substitution between industries is one. This assumption preempts an analysis of the incentives for R&D when market entry is characterized by drastic quality innovations (see, for example, Li (2003)). We leave these issues for future research.

Appendix

This appendix provides details of the stability analysis and the steady-state comparative static analysis for an interior equilibrium.

A.1 Stability analysis for an interior equilibrium

The dynamic system around a long-run interior equilibrium consists of one state variable (χ) and two control (z_1 and z_2) variables, and saddlepath stability therefore requires one negative and two positive eigenvalues. Using a Taylor expansion of (17), (18), and (19), we obtain the Jacobian matrix for a linearized system as follows:

$$J = \begin{bmatrix} -\frac{\beta H}{\chi} & -\chi\alpha M'(z_1) & [\chi\alpha\theta H + (1-\chi)\beta H]\frac{H'(z_2)}{H} \\ -\frac{\beta H}{\chi^2}\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu}\right]^{-1} & j_{22} & j_{23} \\ \frac{\beta\lambda L}{\mu\theta\chi^2}\left[\frac{\theta'(z_2)}{\theta}\right]^{-1} & j_{32} & \frac{\alpha L}{\mu} - \beta H\frac{H'(z_2)}{H}\left[\frac{\theta'(z_2)}{\theta}\right]^{-1} \end{bmatrix},$$

where

$$\begin{aligned} j_{22} &= \frac{\alpha L}{\mu} - \alpha M'(z_1)\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu}\right]^{-1}, \\ j_{23} &= \left[\frac{(1-2\chi)\beta H}{\chi} + \alpha\theta H\right]\frac{H'(z_2)}{H}\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu(z_1)}\right]^{-1}, \\ j_{32} &= \frac{\beta M}{\theta}\left[\frac{M'(z_1)}{M} + \frac{\mu'(z_1)}{\mu} - \frac{L'(z_1)}{L}\right]\left[\frac{\theta'(z_2)}{\theta}\right]^{-1}, \end{aligned}$$

and we have used (22), (23), (24), and the fact that $M'(z_2) = -\theta(z_2)H'(z_2)$. The leading principal minors of J are $-\beta H/\chi < 0$, $-\beta H\alpha L/\mu\chi < 0$, and

$$\begin{aligned} |J| = &-\frac{\beta H}{\chi}\left[\frac{\alpha L}{\mu}\right]^2 - \frac{\beta H}{\chi}\left[\rho\left[\frac{\alpha L}{\mu} + \frac{\beta M}{\theta}\right] + \frac{M}{\theta}\left[\beta(\lambda-1) + \frac{\rho\lambda}{H}\right]\right]\frac{H'(z_2)}{H}\left[\frac{\theta'(z_2)}{\theta}\right]^{-1} \\ &- \frac{\beta M}{\theta}\left[\frac{\alpha(\lambda-1)L}{\mu} + \rho\right]\frac{M'(z_1)}{M}\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu(z_1)}\right]^{-1}\frac{H'(z_2)}{H}\left[\frac{\theta'(z_2)}{\theta}\right]^{-1} < 0. \end{aligned}$$

Following from $|J| < 0$ there are either three negative eigenvalues or one negative and two positive eigenvalues. All of the leading principal minors are negative, however, and J is not negative definite. The former case can therefore be ruled out, and the interior long-run equilibrium with process and quality innovation is saddlepath stable. This proves Proposition 2.

A.2 Steady-state comparative static analysis of the interior equilibrium

The steady-state comparative statics summarized in Proposition 4 are derived using the total derivatives of (21) and (25):

$$\begin{bmatrix} \frac{\alpha L}{\mu}\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu}\right] & -\beta H'(z_2) \\ -\left[\beta(\lambda-1) + \frac{\rho\lambda}{H}\right]\frac{M'(z_1)}{\theta} & a_{22} \end{bmatrix}\begin{bmatrix} dz_1 \\ dz_2 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{L}{\mu} & H & 0 \\ \frac{\beta H}{\alpha L}\frac{\mu M}{\theta} & -\frac{\beta(\lambda-1)L}{\alpha\mu\theta} & b_{23} & b_{24} \end{bmatrix}\begin{bmatrix} d\rho \\ d\alpha \\ d\beta \\ d\lambda \end{bmatrix},$$

where

$$a_{22} = H'(z_2)\left[\beta - \frac{\beta(\lambda-1)}{\alpha\theta} + \frac{\rho\lambda M}{\theta H^2}\right] - \frac{M'(z_2)}{\theta}\left[\beta(\lambda-1) + \frac{\rho\lambda}{H}\right] + \frac{\theta'(z_2)}{\theta}\left[\frac{\beta(\lambda-1)}{\theta}\left[\frac{L}{\mu} + M\right] + \frac{\rho\lambda M}{\theta H}\right]$$
$$b_{23} = -H + \frac{(\lambda-1)}{\alpha\theta}\left[\beta H + \alpha M + \frac{\alpha L}{\mu}\right],$$
$$b_{24} = \frac{\beta}{\theta}\left[\frac{L}{\mu} + M\right].$$

Noting that $\triangle = a_{11}a_{22} - a_{12}a_{21} < 0$, we can use Cramer's rule to obtain the following:

$$\frac{dz_1}{d\rho} = \frac{a_{22} - a_{12}b_{21}}{\triangle} > 0,$$
$$\frac{dz_2}{d\rho} = \frac{1}{\triangle}\left[\beta H\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu}\right] + \left[\beta(\lambda-1) + \frac{\rho\lambda}{H}\right]\frac{M'(z_1)}{M}\right]\frac{M}{\theta},$$
$$\frac{dz_1}{d\alpha} = \frac{1}{\triangle}\left[-a_{22}\frac{L}{\mu} - b_{22}a_{12}\right] < 0,$$
$$\frac{dz_2}{d\alpha} = \frac{1}{\triangle}\left[\beta H'(z_2) + \frac{\beta(\lambda-1)L}{\mu\theta}\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu}\right]\right],$$
$$\frac{dz_1}{d\beta} = \frac{1}{\triangle}\left[\frac{L}{\mu}\frac{\beta(\lambda-1)H}{\theta}\frac{\theta'}{\theta} + \frac{\rho\lambda M}{\theta}\frac{H'}{H} + \frac{\beta(\lambda-1)H'}{\theta}\left[\frac{L}{\mu} + M\right]\right] > 0,$$
$$\frac{dz_2}{d\beta} = \frac{1}{\triangle}\left[\frac{\alpha L}{\mu}\left[\frac{L'(z_1)}{L} - \frac{\mu'(z_1)}{\mu}\right]\left[\frac{(\lambda-1)}{\theta}\left[\frac{L}{\mu} + M\right] - H\left[1 - \frac{\beta(\lambda-1)}{\alpha\theta}\right]\right] + \frac{[(\lambda-1) + \rho\lambda]M'(z_1)}{\theta}\right],$$
$$\frac{dz_1}{d\lambda} = -\frac{a_{12}b_{24}}{\triangle} > 0,$$
$$\frac{dz_2}{d\lambda} = \frac{a_{11}b_{24}}{\triangle} < 0.$$

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